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Dynamic analysis of a magneto-electro-elastic material with a semi-infinite mode-III crack under point impact loads

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Abstract. The problem of a semi-infinite magneto-electro-elastically impermeable mode-III crack in a magneto-electro-elastic material is considered under the action of impact loads. For the case when a pair of concentrated anti-plane shear impacts, electric displacement and magnetic induction impacts are exerted symmetrically on the upper and lower surfaces of the crack, the magneto-electro-elastic field ahead of the crack tip is determined in explicit form. The dynamic intensity factors and dynamic energy density factor are obtained. The method adopted is to reduce the mixed initial-boundary value problem, by using the Laplace and Fourier transforms, into three simultaneous dual integral equations, one of which is converted into an Abel's integral equation and the others into a singular integral equation with Cauchy kernel. Based on the obtained fundamental solutions of point impact loads, the solutions of two kinds of different loading cases are evaluated by integration. For some particular cases, the present results reduce to the previous results.

Keywords: dynamic response; crack; magneto-electro-elastic field; intensity factor; dynamic energy density factor; magneto-electrically impermeable.

1. Introduction

Composite material consisting of a piezoelectric phase and a piezomagnetic phase has drawn significant interest in recent years, due to the rapid development and application of this material in adaptive control systems. The magnetoelectric coupling is a new product property of the composite, since it is absent in each component. In some cases, the coupling effect of piezoelectric/piezomagnetic composites can be even a hundred times larger than that in a single-phase magnetoelectric material. Consequently, they are extensively used as magnetic field probes, electric packaging, acoustic, hydrophones, medical ultrasonic imaging, sensors, and actuators with the functionality of magneto-electro-mechanical energy conversion (Wu and Huang 2000). When subjected to mechanical, magnetical and electrical loads in service, these magneto-electro-elastic composites can fail prematurely due to some defects, such as cracks, holes and inclusions arising during their manufacturing process. Therefore, it is of great importance to study the fracture

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behaviors of piezoelectric/piezomagnetic composites under magneto-electro-elastic interactions (Song and Sih 2003, Sih and Song 2003).

The development of piezoelectric-piezomagnetic composites has its root from the early work of Van Suchtelen (1972) who proposed that the combination of piezoelectric-piezomagnetic phases might exhibit a new material property - the magnetoelectric coupling effect. Since then, the magnetoelectric coupling effect of $BaTiO_3$-$CoFe_2O_4$ composites has been measured and much of the theoretical work for the investigation of magneto-electro-elastic coupling effect has been studied (Wu and Huang 2000, Song and Sih 2003, Sih and Song 2003, Harshe *et al.* 1993, Avellaneda and Harshe 1994, Nan 1994, Benveniste 1995, Wang and Shen 1996, Huang and Kuo 1997, Li and Dunn 1998, Li 2000, Pan 2001, Gao *et al.* 2003a, 2003b, Lage *et al.* 2004).

On the other hand, although the dynamic fracture problems of piezoelectric material with cracks are widely investigated (Dascalu and Maugin 1995, Khutoryansky and Sosa 1995, Li and Mataga 1996a, Li and Mataga 1996b, Shindo *et al.* 1996, Ding *et al.* 1996, Chen and Yu 1997, Chen and Yu 1998, Shindo *et al.* 1999, Wang and Yu 2000, Kwon and Lee 2000, Li 2001, Gu *et al.* 2002), to the best of our knowledge, the analysis of dynamic crack problems magneto-electro-elastic material is very limited. Du *et al.* (2004) obtained the scattered fields of SH waves by a partially debonded magneto-electro-elastic cylindrical inhomogeneity, and determined the numerical results of crack opening displacement. Feng *et al*. (2006) investigated both the near- and far- field properties of arc-shaped interfacial cracks. Chen *et al*. (2007) studied the propagation of harmonic wave in magneto-electro-elastic multilayered plates. Pan and Heyliger (2002) considered the free vibration problem of simply supported and multilayered magneto-electro-elastic plates. Buchanan (2003) considered the free vibration problem of an infinite magneto-electro-elastic cylinder. Hou and Leung (2004) analyzed the plane strain dynamic problem of a magneto-electro-elastic hollow cylinder by virtue of the separation of variables, orthogonal expansion technique and the interpolation method. Recently, Ramirez *et al.* (2006) further investigated free vibration response of two-dimensional magneto-electro-elastic laminated plates. For dynamic fracture problem of magneto-electro-elastic materials with cracks, Feng *et al.* (2005) studied the dynamic fracture behaviors of magneto-electrically impermeable interfacial crack between two dissimilar magneto-electro-elastic materials using the energy density criterion. Zhou *et al*. (2005) analyzed the dynamic behavior of two collinear interface cracks. Hu *et al*. (2006) studied the moving crack at the interface between two dissimilar magneto-electro-elastic materials. Feng and Su (2006, 2007) further studied the dynamic fracture behaviors of cracks in a functionally graded magneto-electro-elastic strip and plate, respectively.

In this paper, the dynamic problem of a magneto-electro-elastic material with a semi-infinite magneto-electrically impermeable crack under concentrated anti-plane shear, in-plane electric displacement and magnetic induction impact loads is studied. Using the Laplace and Fourier transforms, the mixed initial-boundary value problem is first reduced to three pairs of dual integral equations. Then, one pair is transformed into an Abel's integral equation and the others into a singular integral equation with Cauchy kernel. Both the Abel's integral equation and Cauchy integral equation can be solved analytically. Furthermore, the dynamic intensity factors and dynamic energy density factor (EDF) are obtained in explicit analytic form, rather than approximate expressions and numerical results. Finally, the solutions corresponding to two kinds of different loading cases are presented via simple integration.

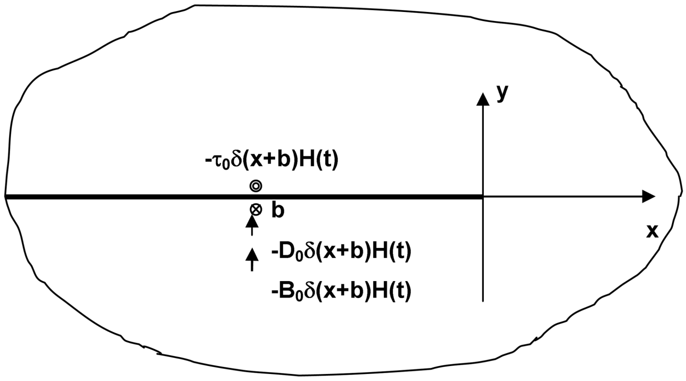

Fig. 1 A semi-infinite crack in a magneto-electro-elastic material subjected to anti-plane shear and in-plane electric displacement and magnetic induction point impact loads

2. Statement of problem

Consider an infinite magneto-electro-elastic material that contains a semi-infinite Griffith crack with reference to the rectangular coordinate system x, y, z, as shown in Fig. 1. The magneto-electro-elastic medium exhibits transversely isotropic behavior and is poled in z-direction. The anti-plane shear point impact and in-plane electric displacement and magnetic induction point impacts are suddenly applied on the crack surfaces at $t = 0$, and then maintain constants as imposed loads. In Fig. 1, $H(\bullet)$ denotes the Heaviside unit step function, $\delta(\bullet)$ is the Daric delta function.

The constitutive equations for anti-plane magneto-electro-elastic problem can be written as

$$\sigma_{zx} = c_{44}\frac{\partial w}{\partial x} + e_{15}\frac{\partial \phi}{\partial x} + f_{15}\frac{\partial \psi}{\partial x}, \quad \sigma_{zy} = c_{44}\frac{\partial w}{\partial y} + e_{15}\frac{\partial \phi}{\partial y} + f_{15}\frac{\partial \psi}{\partial y} \tag{1}$$

$$D_x = e_{15}\frac{\partial w}{\partial x} - \varepsilon_{11}\frac{\partial \phi}{\partial x} - g_{11}\frac{\partial \psi}{\partial x}, \quad D_y = e_{15}\frac{\partial w}{\partial y} - \varepsilon_{11}\frac{\partial \phi}{\partial y} - g_{11}\frac{\partial \psi}{\partial y} \tag{2}$$

$$B_x = f_{15}\frac{\partial w}{\partial x} - g_{11}\frac{\partial \phi}{\partial x} - \mu_{11}\frac{\partial \psi}{\partial x}, \quad B_y = f_{15}\frac{\partial w}{\partial y} - g_{11}\frac{\partial \phi}{\partial y} - \mu_{11}\frac{\partial \psi}{\partial y} \tag{3}$$

where $\sigma_{zk}, D_k, B_k (k = x, y)$ are the anti-plane shear stress, in-plane electric displacement and magnetic induction, respectively; c_{44}, ε_{11}, e_{15}, f_{15}, g_{11}, μ_{11} are the material constants; w, ϕ and ψ are the mechanical displacement, electric potential and magnetic potential, respectively.

The governing equations of the anti-plane magneto-electro-elastic boundary value problem are as follows

$$c_{44}\nabla^2 w + e_{15}\nabla^2 \phi + f_{15}\nabla^2 \psi = \rho\frac{\partial^2 w}{\partial t^2} \tag{4}$$

$$e_{15}\nabla^2 w - \varepsilon_{11}\nabla^2 \phi - g_{11}\nabla^2 \psi = 0 \tag{5}$$

$$f_{15}\nabla^2 w - g_{11}\nabla^2 \phi - \mu_{11}\nabla^2 \psi = 0 \tag{6}$$

where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the two-dimensional Laplace operator.

By introducing

$$\chi = e_{15}w - \varepsilon_{11}\phi - g_{11}\psi, \quad \zeta = f_{15}w - g_{11}\phi - \mu_{11}\psi \tag{7}$$

the governing Eqs. (4)-(6) can be expressed as

$$\nabla^2 w = c_2^{-2}\frac{\partial^2 w}{\partial t^2} \tag{8}$$

$$\nabla^2 \chi = 0 \tag{9}$$

$$\nabla^2 \zeta = 0 \tag{10}$$

where $c_2 = \sqrt{\mu/\rho}$ is the shear wave speed and

$$\mu = c_{44} + \frac{e_{15}^2\mu_{11} - 2e_{15}f_{15}g_{11} + f_{15}^2\varepsilon_{11}}{\mu_{11}\varepsilon_{11} - g_{15}^2} \tag{11}$$

The constitutive relations (1)-(3) can be rewritten as

$$\sigma_{zx} = \mu w_{,x} + \frac{f_{15}g_{11} - e_{15}\mu_{11}}{\mu_{11}\varepsilon_{11} - g_{11}^2}\chi_{,x} + \frac{e_{15}g_{11} - f_{15}\varepsilon_{11}}{\mu_{11}\varepsilon_{11} - g_{11}^2}\zeta_{,x} \tag{12a}$$

$$\sigma_{zy} = \mu w_{,y} + \frac{f_{15}g_{11} - e_{15}\mu_{11}}{\mu_{11}\varepsilon_{11} - g_{11}^2}\chi_{,y} + \frac{e_{15}g_{11} - f_{15}\varepsilon_{11}}{\mu_{11}\varepsilon_{11} - g_{11}^2}\zeta_{,y} \tag{12b}$$

$$D_x = \chi_{,x}, \quad D_y = \chi_{,y} \tag{13}$$

$$B_x = \zeta_{,x}, \quad B_y = \zeta_{,y} \tag{14}$$

Due to symmetry of the problem, it is sufficient to analyze the magneto-electro-elastic field in the upper half-plane, i.e., $y \geq 0$. For magneto-electrically impermeable crack considered here, the boundary conditions can be stated as follows

$$\left.\begin{aligned} \sigma_{zy}(x, 0, t) &= -\tau_0\delta(x + b)H(t), \quad D_y(x, 0, t) = -D_0\delta(x + b)H(t) \\ B_y(x, 0, t) &= -B_0\delta(x + b)H(t) \end{aligned}\right\} \quad x < 0 \tag{15}$$

$$w(x, 0, t) = \chi(x, 0, t) = \zeta(x, 0, t) = 0, \quad x > 0 \tag{16}$$

It is pointed out that Eq. (16) implies $w(x, 0, t) = \phi(x, 0, t) = \psi(x, 0, t) = 0$ for $x > 0$.

3. Derivation and solution of dual integral equations

In order to solve Eqs. (8)-(10), together with the boundary conditions (15)-(16), it is necessary to impose that the magneto-electro-elastic material is static at the initial time. Namely, we impose that the magneto-electro-elastic medium is subjected to the vanishing initial conditions

$$w(x,y,0) = \frac{\partial w(x,y,0)}{\partial t} = 0, \quad \chi(x,y,0) = \frac{\partial \chi(x,y,0)}{\partial t} = 0, \quad \zeta(x,y,0) = \frac{\partial \zeta(x,y,0)}{\partial t} = 0$$

$$-\infty < x < \infty, \quad 0 < y < \infty \tag{17}$$

By taking the Laplace transform with respect to t and the Fourier transform with respect to x to both sides of Eqs. (8)-(10), together with (17), we obtain

$$w^*(x,y,p) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} A(s,p)e^{-\alpha y}e^{-isx}ds \tag{18}$$

$$\chi^*(x,y,p) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} B(s,p)e^{-|s|y}e^{-isx}ds \tag{19}$$

$$\zeta^*(x,y,p) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} C(s,p)e^{-|s|y}e^{-isx}ds \tag{20}$$

respectively, where p and s are the Laplace and Fourier parameters, respectively. $A(s,p)$, $B(s,p)$, and $C(s,p)$ are the unknowns to be determined from given boundary conditions (15) and (16), and $\alpha(s,p) = \sqrt{s^2 + p^2/c_2^2}$. In the deriving of Eqs. (18)-(20), the unbounded terms for the upper half-plane have been discarded.

The stress $\sigma_{zy}^*(x,y,p)$, electric displacement $D_y^*(x,y,p)$ and magnetic induction $B_y^*(x,y,p)$ in the Laplace transform domain can be expressed in terms of $A(s,p)$, $B(s,p)$ and $C(s,p)$ as follows

$$\sigma_{zy}^*(x,y,p) = -\frac{1}{2\pi}\left[\mu\int_{-\infty}^{+\infty}\alpha A(s,p)e^{-\alpha y}e^{-isx}ds + \frac{f_{15}g_{11} - e_{15}\mu_{11}}{\mu_{11}\varepsilon_{11} - g_{11}^2}\right.$$
$$\left.\int_{-\infty}^{+\infty}|s|B(s,p)e^{-|s|y}e^{-isx}ds + \frac{e_{15}g_{11} - f_{15}\varepsilon_{11}}{\mu_{11}\varepsilon_{11} - g_{11}^2}\int_{-\infty}^{+\infty}|s|C(s,p)e^{-|s|y}e^{-isx}ds\right] \tag{21}$$

$$D_y^*(x,y,p) = -\frac{1}{2\pi}\int_{-\infty}^{+\infty}|s|B(s,p)e^{-|s|y}e^{-isx}ds \tag{22}$$

$$B_y^*(x,y,p) = -\frac{1}{2\pi}\int_{-\infty}^{+\infty}|s|C(s,p)e^{-|s|y}e^{-isx}ds \tag{23}$$

With the aid of the vanishing initial conditions (17), the boundary conditions in the Laplace domain can be written as

$$\sigma_{zy}^*(x,0,p) = -\frac{\tau_0\delta(x+b)}{p}, \quad D_y^*(x,0,t) = -\frac{D_0\delta(x+b)}{p}$$
$$B_y(x,0,t) = -\frac{B_0\delta(x+b)}{p} \qquad x < 0 \tag{24}$$

$$w^*(x,0,p) = \chi^*(x,0,p) = \zeta^*(x,0,p) = 0, \quad x > 0 \tag{25}$$

It is readily seen that substituting Eqs. (18)-(20) into Eq. (25) leads to

$$\int_{-\infty}^{+\infty} A(s,p)e^{-isx}ds = 0, \quad x>0 \tag{26}$$

$$\int_{-\infty}^{+\infty} B(s,p)e^{-isx}ds = 0, \quad x>0 \tag{27}$$

$$\int_{-\infty}^{+\infty} C(s,p)e^{-isx}ds = 0, \quad (x>0) \tag{28}$$

On the other hand, a comparison of the boundary conditions (24) with Eqs. (21)-(23), after a simple algebraic operation, yields

$$\int_{-\infty}^{+\infty} \alpha A(s,p)e^{-isx}ds = \frac{2\pi T_0 \delta(x+b)}{p}, \quad x<0 \tag{29}$$

$$\int_{-\infty}^{+\infty} |s| B(s,p)e^{-isx}ds = \frac{2\pi D_0 \delta(x+b)}{p}, \quad x<0 \tag{30}$$

$$\int_{-\infty}^{+\infty} |s| C(s,p)e^{-isx}ds = \frac{2\pi B_0 \delta(x+b)}{p}, \quad x<0 \tag{31}$$

where

$$T_0 = \frac{1}{\mu}\left(\tau_0 - \frac{f_{15}g_{11} - e_{15}\mu_{11}}{\mu_{11}\varepsilon_{11} - g_{11}^2}D_0 - \frac{e_{15}g_{11} - f_{15}\varepsilon_{11}}{\mu_{11}\varepsilon_{11} - g_{11}^2}B_0\right) \tag{32}$$

So far, three simultaneous systems of dual integral equations, Eqs. (26) and (29) for $A(s,p)$, Eqs.(27) and (30) for $B(s,p)$, Eqs. (28) and (31) for $C(s,p)$, are obtained, respectively.

4. Derivation and solution of singular integral equations

In the following, we will eliminate $A(s,p)$, $B(s,p)$ and $C(s,p)$ from dual integral equations and get three singular integral equations, respectively. One is two-dimensional weakly singular integral equation, which can further be transformed into an Abel's integral equation, and the others are singular integral equation with Cauchy kernel. For the three equations, the solutions can be obtained in closed-form.

To solve the resulting dual integral equations for $A(s,p)$, we introduce an auxiliary function $\sigma(x,y,t)$ such that $\sigma(x,y,0) = \partial\sigma(x,y,0)/\partial t = 0$ and

$$\sigma^*(x,y,p) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \alpha A(s,p)e^{-\alpha y}e^{-isx}ds \tag{33}$$

From Eq. (29), we get

$$\sigma^*(x,0,p) = \frac{T_0\delta(x+b)}{p}, \quad x<0 \tag{34}$$

$\sigma^*(x,0,p)$ is unknown to be determined for $x>0$.

Taking the inverse Fourier transform of (33), we find

$$A(s,p) = \frac{1}{\alpha}\int_{-\infty}^{+\infty}\sigma^*(x,0,p)e^{isx}dx \tag{35}$$

Substituting Eq. (35) into Eq. (26), we have

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{1}{\alpha}\sigma^*(x,0,p)e^{-is(x_0-x)}dsdx = 0, \quad x_0 > 0 \tag{36}$$

where x in Eq. (26) has been replaced with x_0 for convenience. Application of the inverse Laplace transform to Eq. (36) yields

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_0^{t_0}\sigma(x,0,t)J_0(sc_2(t_0-t))e^{-is(x_0-x)}dtdsdx = 0, \quad x_0 > 0,\ t_0 > 0 \tag{37}$$

where the following relation

$$\int_0^{\infty}J_0(t\xi)e^{-pt}dt = \frac{1}{\sqrt{\xi^2+p^2}} \tag{38}$$

has been used. Interchanging the order of integration and utilizing the following result

$$\int_{-\infty}^{\infty}J_0(a\xi)e^{-i\xi u}dt = \begin{cases}\dfrac{2}{\sqrt{a^2-u^2}}, & |u| < a\\ 0, & |u| > a\end{cases} \tag{39}$$

Eq. (37) is further transformed into the following form

$$\int_0^{t_0}\int_{x_0-c_2(t_0-t)}^{x_0+c_2(t_0-t)}\frac{\sigma(x,0,t)}{\sqrt{c_2^2(t_0-t)^2-(x_0-x)^2}}dxdt = 0, \quad x_0 > 0,\ t_0 > 0 \tag{40}$$

By making a transformation of variables: $u = (c_2t+x)/2$, $v = (c_2t-x)/2$, and applying the method described in Freund (1990), Eq. (40) becomes an Abel's integral equation

$$\int_0^{v_0}\frac{1}{\sqrt{v_0-v}}\int_{-v}^{u_0}\frac{\hat{\sigma}(u,0,v)}{\sqrt{u_0-u}}dudv = 0, \quad u_0 > v_0 > 0 \tag{41}$$

where $\hat{\sigma}(u,0,v) = \sigma(x,0,t)$. According to the Abel's integral equation theory, $\hat{\sigma}(u,0,v)$ must satisfy the following equation

$$\int_{-v}^{u_0}\frac{\hat{\sigma}(u,0,v)}{\sqrt{u_0-u}}du = 0, \quad u_0 > v > 0 \tag{42}$$

Eq. (42) can be further split up into two parts

$$\int_v^{u_0}\frac{\hat{\sigma}(u,0,v)}{\sqrt{u_0-u}}du = -\int_{-v}^{v}\frac{\hat{\sigma}(u,0,v)}{\sqrt{u_0-u}}du, \quad u_0 > v > 0 \tag{43}$$

Taking into consideration Eq. (34) and making a change of variables, $\xi = u-v$, $\xi_0 = u_0-v$, Eq. (43) finally simplifies a standard Abel's integral equation, i.e.,

$$\int_0^{\xi_0}\frac{\hat{\sigma}(\xi+v,0,v)}{\sqrt{\xi_0-\xi}}d\xi = -\frac{T_0}{\sqrt{\xi_0+b}}H\left(\frac{2v-b}{c_2}\right), \quad \xi_0 > 0, v > 0 \tag{44}$$

the solution of which is easily found to be

$$\hat{\sigma}(\xi + v, 0, v) = -\frac{T_0}{\pi}\frac{1}{\xi + b}\sqrt{\frac{b}{\xi}}H\left(\frac{2v - b}{c_2}\right), \quad \xi > 0, v > 0 \tag{45}$$

namely,

$$\sigma(x, 0, t) = -\frac{T_0}{\pi}\frac{1}{x + b}\sqrt{\frac{b}{x}}, \quad 0 < x < c_2 t - b \tag{46}$$

Now, the auxiliary function $\sigma(x, 0, t)$ is determined. The following is to find the solution of the dual integral equations for $B(s, p)$.

Performing the inverse Fourier transform of Eq. (22) reduces to

$$B(s, p) = -\frac{1}{|s|}\int_{-\infty}^{\infty} D_y^*(x, 0, p)e^{isx}dx \tag{47}$$

Replacing x in Eq. (27) with x_0 and inserting Eq. (47) into Eq. (27) results in

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{\infty}\frac{1}{|s|}D_y^*(x, 0, p)e^{-is(x_0 - x)}dsdx = 0, \quad x_0 > 0 \tag{48}$$

Differentiating (48) with respect to x_0 and using the known result

$$\int_0^{\infty}\sin(as)ds = \frac{1}{a}, \quad a > 0 \tag{49}$$

reduces Eq. (48) to

$$\int_{-\infty}^{\infty}\frac{D_y^*(x, 0, p)}{x - x_0}dx = 0, \quad x_0 > 0 \tag{50}$$

With the aid of Eq. (24), the inversion of Laplace transform of Eq. (50) yields a singular integral equation with Cauchy kernel defined on the positive x-axis

$$\int_0^{\infty}\frac{D_y(x, 0, t)}{x - x_0}dx = -\frac{D_0 H(t)}{x_0 + b}, \quad x_0 > 0 \tag{51}$$

Making a change of variables

$$x = \eta^2, \quad x_0 = \eta_0^2, \quad b = b_0^2 \tag{52}$$

and using the following relation

$$\frac{1}{\eta^2 - \eta_0^2} = \frac{1}{2\eta_0}\left(\frac{1}{\eta - \eta_0} - \frac{1}{\eta + \eta_0}\right) \tag{53}$$

Eq. (51) is transformed into a singular integral equation of the following form (Muskhelishvili 1953)

$$\int_{-\infty}^{\infty}\frac{|\eta|D_y^*(\eta^2, 0, t)}{\eta - \eta_0}d\eta = -\frac{\eta_0 D_0 H(t)}{\eta_0^2 + b_0^2}, \quad -\infty < \eta_0 < \infty \tag{54}$$

Based on the theory of a singular integral equation, the solution of Eq. (54) (Muskhelishvili 1953) is found to be

$$|\eta| D_y^*(\eta^2, 0, t) = \frac{D_0 H(t)}{\pi^2} \int_{-\infty}^{\infty} \frac{\eta_0}{(\eta_0 - \eta)(\eta_0^2 + b_0^2)} d\eta_0 \tag{55}$$

A straightforward evaluation of the singular integral at the right-hand side of Eq. (55) gives

$$|\eta| D_y^*(\eta^2, 0, t) = \frac{D_0 H(t)}{\pi} \frac{b_0}{\eta^2 + b_0^2} \tag{56}$$

Substituting Eq. (52) into Eq. (56) yields the solution of Eq. (51) as

$$D_y(x, 0, t) = \frac{D_0 H(t)}{\pi} \frac{1}{x + b} \sqrt{\frac{b}{x}}, \quad x > 0 \tag{57}$$

Similarly, the unknown function $C(s, p)$ and magnetic induction $B_y(x, 0, t)$ can be finally determined to solve dual integral Eqs. (28) and (31). They are respectively

$$C(s, p) = -\frac{1}{|s|} \int_{-\infty}^{\infty} B_y^*(x, 0, p) e^{isx} dx \tag{58}$$

$$B_y(x, 0, t) = \frac{B_0 H(t)}{\pi} \frac{1}{x + b} \sqrt{\frac{b}{x}}, \quad x > 0 \tag{59}$$

5. Analysis of crack tip field, field intensity factors and energy density factor

For the problem under consideration, using the obtained results Eqs. (46), (57) and (59), together with Eqs. (33), (47) and (58), the inversion of Laplace transform of Eqs. (21)-(23) results in the transient response of anti-plane shear stress σ_{zy}, in-plane electric displacement D_y and magnetic induction B_y along the crack line. They are respectively

$$\sigma_{zy}(x, 0, t) = \begin{cases} \dfrac{\tau_0}{\pi(x + b)} \sqrt{\dfrac{b}{x}}, & 0 < x < c_2 t - b \\ \dfrac{(f_{15} g_{11} - e_{15} \mu_{11}) D_0 + (e_{15} g_{11} - f_{15} \varepsilon_{11}) B_0}{\mu_{11} \varepsilon_{11} - g_{11}^2} \dfrac{1}{\pi(x + b)} \sqrt{\dfrac{b}{x}}, & x > c_2 t - b \end{cases} \tag{60}$$

$$D_y(x, 0, t) = \frac{D_0}{\pi(x + b)} \sqrt{\frac{b}{x}}, \quad x > 0 \tag{61}$$

$$B_y(x, 0, t) = \frac{B_0}{\pi(x + b)} \sqrt{\frac{b}{x}}, \quad x > 0 \tag{62}$$

It is concluded that stress field has apparent transient response due to impact loads, while both electric displacement and magnetic induction do not have transient response. For stress field, shear waves excited by a pair of concentrated impact loads applied at a point $(-b, 0)$ on the crack surfaces reach a position $(x, 0)$ when traveling time $t > (x + b)/c_2$, while for a position $(x, 0)$, $(x > x_0 = c_2 t - b)$, stress field is disturbed only by electric displacement and magnetic induction since shear waves do not reach this position within time $t \in [0, (x_0 + b)/c_2]$. Moreover, the electric displacement and magnetic induction are only related to corresponding electrical point impact loads and magnetical point impact loads, respectively. In addition, it should be noted that the stress field when $0 < x < c_2 t - b$, electric displacement and magnetic induction are all independent of material constants.

From Eqs. (1)-(3), we further obtain that for $t > b/c_2$

$$\begin{pmatrix} \gamma_{zy}(x,0,t) \\ E_y(x,0,t) \\ H_y(x,0,t) \end{pmatrix} = \begin{pmatrix} w_{,y}(x,0,t) \\ -\phi_{,y}(x,0,t) \\ -\psi_{,y}(x,0,t) \end{pmatrix} = \frac{1}{\pi(x+b)}\sqrt{\frac{b}{x}}\,\Pi^{-1}\begin{pmatrix} \tau_0 \\ D_0 \\ B_0 \end{pmatrix}, \quad 0 < x < c_2 t - b \tag{63}$$

$$\begin{pmatrix} \gamma_{zy}(x,0,t) \\ E_y(x,0,t) \\ H_y(x,0,t) \end{pmatrix} = \frac{1}{\pi(x+b)}\sqrt{\frac{b}{x}}\begin{pmatrix} 0 \\ (\mu_{11}D_0 - g_{11}B_0)/(\mu_{11}\varepsilon_{11} - g_{11}^2) \\ \varepsilon_{11}B_0 - g_{11}D_0/(\mu_{11}\varepsilon_{11} - g_{11}^2) \end{pmatrix}, \quad x > c_2 t - b \tag{64}$$

where

$$\Pi = \begin{pmatrix} c_{440} & -e_{150} & -f_{150} \\ e_{150} & \varepsilon_{110} & g_{110} \\ f_{150} & g_{110} & \mu_{110} \end{pmatrix} \tag{65}$$

As illustrated by Eqs. (63) and (64), when $x > c_2 t - b$, since shear waves do not arrive at $(x, 0)$, the strain component $\gamma_{zy}(x,0,t)$ vanishes, and both the electric field component $E_y(x,0,t)$ and magnetic field $H_y(x,0,t)$ are independent to shear loads, however, they both depend on electric displacement and magnetic induction impact loads simultaneously. When $0 < x < c_2 t - b$, $\gamma_{zy}(x,0,t)$, $E_y(x,0,t)$ and $H_y(x,0,t)$ are all disturbed by combined impact loads including mechanical, magnetical and electrical loads and exhibit transient response.

In general, of particular interest is transient feature due to impact loads, so now we focus our attention to this case, i.e., $0 < x < c_2 t - b$.

At the crack tip, we define the intensity factors of stress, strain, electric displacement, electric field, magnetic induction, and magnetic field, respectively, as

$$\begin{pmatrix} K_\sigma(t) \\ K_D(t) \\ K_B(t) \end{pmatrix} = \lim_{x\to 0^+}\sqrt{2\pi x}\begin{pmatrix} \sigma_{zy}(x,0,t) \\ D_y(x,0,t) \\ B_y(x,0,t) \end{pmatrix}, \quad \begin{pmatrix} K_\gamma(t) \\ K_E(t) \\ K_H(t) \end{pmatrix} = \lim_{x\to 0^+}\sqrt{2\pi x}\begin{pmatrix} \gamma_{zy}(x,0,t) \\ E_y(x,0,t) \\ H_y(x,0,t) \end{pmatrix} \tag{66}$$

According to these definitions, a straightforward evaluation gives their values as

$$\begin{pmatrix} K_\sigma(t) \\ K_D(t) \\ K_B(t) \end{pmatrix} = \sqrt{\frac{2}{\pi b}}\begin{pmatrix} \tau_0 \\ D_0 \\ B_0 \end{pmatrix}, \quad \begin{pmatrix} K_\gamma(t) \\ K_E(t) \\ K_H(t) \end{pmatrix} = \sqrt{\frac{2}{\pi b}}\,\Pi^{-1}\begin{pmatrix} \tau_0 \\ D_0 \\ B_0 \end{pmatrix} \tag{67}$$

for $t > b/c_2$.

Obviously, $K_\sigma(t), K_D(t)$ and $K_B(t)$ are all independent of material constants and only related to mechanical, electrical and magnetical loads, respectively, while $K_\gamma(t), K_E(t)$ and $K_H(t)$ all depend on both material constants and applied loads including mechanical loads, electrical loads and magnetical loads. It should be also noted that if all the magnetic quantities are made to vanish, the magneto-electro-elastic solution reduces to the dynamic anti-plane crack problem of piezoelectric material (Li 2001). This means our results are universal and correct.

Recently, some investigators found that the EDF is an essential quantity for analyzing the magneto-electro-elastic crack growth behavior (Sih and Song 2003, Song and Sih 2003). For the anti-plane problem, the EDF is defined as

$$S = \lim_{r \to 0} \frac{r}{2}(\sigma_{zx}\varepsilon_{zx} + \sigma_{zy}\varepsilon_{zy} + D_x E_x + D_y E_y + B_x H_x + B_y H_y) \tag{68}$$

in which r has been referred to as the core region within which microstructure effects become important. $\varepsilon_{z\Xi} = \frac{1}{2}w_{,\Xi}$, $\Xi = x, y$. After some algebra operation, the EDF can be directly expressed as

$$S(t) = \frac{\Omega_T \tau_0^2 + \Omega_D D_0^2 + \Omega_B B_0^2 + \Omega_{TD}\tau_0 D_0 + \Omega_{TB}\tau_0 B_0 + \Omega_{DB} D_0 B_0}{4\pi^2 b(f_{15}^2\varepsilon_{11} + c_{44}\mu_{11}\varepsilon_{11} - 2e_{15}f_{15}g_{11} - c_{44}g_{11}^2 + e_{15}^2\mu_{11})}, \quad t > b/c_2 \tag{69}$$

where

$$\begin{aligned} \Omega_T &= \mu_{110}\varepsilon_{110} - g_{110}^2, \quad \Omega_D = 2(c_{440}\mu_{110} + f_{150}^2), \quad \Omega_B = 2(c_{440}\varepsilon_{110} + e_{150}^2) \\ \Omega_{TD} &= f_{150}g_{110} - e_{150}\mu_{110}, \quad \Omega_{TB} = e_{150}g_{110} - f_{150}\varepsilon_{110}, \quad \Omega_{DB} = -4(c_{440}g_{110} + e_{150}f_{150}) \end{aligned} \tag{70}$$

It can be easily found that the EDF against time have no peak for concentrated combined impact loads, this coincides with field intensity factors. Eq. (69) also implies that the nearer the impact loads approaching to the crack tip, the easier the crack growth and propagation according to energy density criterion. In addition, for the pure mechanical case, the EDF is equivalent to the traditional definition of energy release rate.

6. Examples of application

Since the present results are derived from concentrated combined loads, Eqs. (60)-(63) and Eq. (67) can be taken as fundamental solutions, from which the corresponding general solutions may be obtained via simple integration. For example, if assuming that all the uniform impact loads

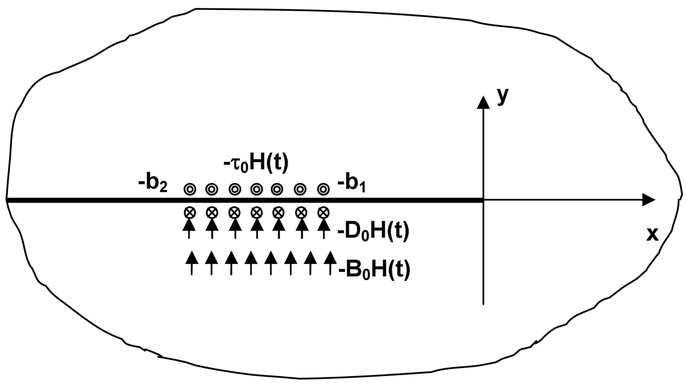

Fig. 2 A semi-infinite crack in a magneto-electro-elastic material subjected to anti-plane shear and in-plane electric displacement and magnetic induction impact loads on the crack surfaces of a finite length

are applied on a part of the crack faces (Fig. 2), i.e.,

$$\sigma_{zy}(x,0,t) = -\tau_0(x)H(t), \quad D_y(x,0,t) = -D_0(x)H(t), \quad B_y(x,0,t) = -B_0(x)H(t) \tag{71}$$

where

$$\Delta_0(x) = \begin{cases} \Delta_0, & x \in (-b_2, -b_1), \\ 0, & x \in (-\infty, -b_2) \cup (-b_1, 0), \end{cases}, \quad \Delta = \tau, D, B \tag{72}$$

the asymptotic expressions for the magneto-electro-elastic field ahead of the crack tip along the crack line and the field intensity factors can be directly evaluated by integrations of the corresponding expressions with respect to b from b_1 to b_2. The final results of them are respectively

$$\begin{pmatrix} \sigma_{zy}(x,0,t) \\ D_y(x,0,t) \\ B_y(x,0,t) \end{pmatrix} = \frac{2}{\pi}\left[\left(\sqrt{\frac{b_2}{x}} - \sqrt{\frac{b_1}{x}}\right) - \left(\tan^{-1}\sqrt{\frac{b_2}{x}} - \tan^{-1}\sqrt{\frac{b_1}{x}}\right)\right]\begin{pmatrix} \tau_0 \\ D_0 \\ B_0 \end{pmatrix}, \quad 0 < x < c_2 t - b_2 \tag{73}$$

$$\begin{pmatrix} \gamma_{zy}(x,0,t) \\ E_y(x,0,t) \\ H_y(x,0,t) \end{pmatrix} = \frac{2}{\pi}\left[\left(\sqrt{\frac{b_2}{x}} - \sqrt{\frac{b_1}{x}}\right) - \left(\tan^{-1}\sqrt{\frac{b_2}{x}} - \tan^{-1}\sqrt{\frac{b_1}{x}}\right)\right]\Pi^{-1}\begin{pmatrix} \tau_0 \\ D_0 \\ B_0 \end{pmatrix}, \quad 0 < x < c_2 t - b_2 \tag{74}$$

$$\begin{pmatrix} K_\sigma(t) \\ K_D(t) \\ K_B(t) \end{pmatrix} = 2\left(\sqrt{\frac{2b_2}{\pi}} - \sqrt{\frac{2b_1}{\pi}}\right)\begin{pmatrix} \tau_0 \\ D_0 \\ B_0 \end{pmatrix} \tag{75}$$

$$\begin{pmatrix} K_\gamma(t) \\ K_E(t) \\ K_H(t) \end{pmatrix} = 2\left(\sqrt{\frac{2b_2}{\pi}} - \sqrt{\frac{2b_1}{\pi}}\right)\Pi^{-1}\begin{pmatrix} \tau_0 \\ D_0 \\ B_0 \end{pmatrix} \tag{76}$$

With the aid of Eqs. (75) and (76), after an analogous procedure to derive Eq. (69), the EDF can be expressed as

$$S(t) = \frac{\Omega_T\tau_0^2 + \Omega_D D_0^2 + \Omega_B B_0^2 + \Omega_{TD}\tau_0 D_0 + \Omega_{TB}\tau_0 B_0 + \Omega_{DB} D_0 B_0}{2\pi(f_{15}^2\varepsilon_{11} + c_{44}\mu_{11}\varepsilon_{11} - 2e_{15}f_{15}g_{11} - c_{44}g_{11}^2 + e_{15}^2\mu_{11})}\left(\sqrt{\frac{2b_2}{\pi}} - \sqrt{\frac{2b_1}{\pi}}\right)^2 \tag{77}$$

As another example, when sudden anti-plane shear loads $\sigma_{zy}(x,0,t) = -\tau_0 H(t)$, $x < 0$, uniform electric displacement $D_y(x,0,t) = -D_0 H(t)$, $x < 0$ and uniform magnetic induction $H_y(x,0,t) = -H_0 H(t)$, $x < 0$ are applied on the entire semi-infinite crack surfaces, based on the basic solutions, the magneto-electro-elastic field ahead of the crack tip and the field intensity factors can be obtained via integrations of Eqs. (60)-(63) with respect to b from 0 to $c_2 t - x$ and integrations of Eq. (67) with respect to b from 0 to $c_2 t$, respectively. They are respectively

$$\begin{pmatrix} \sigma_{zy}(x,0,t) \\ D_y(x,0,t) \\ B_y(x,0,t) \end{pmatrix} = \frac{2}{\pi}\left(\sqrt{\frac{c_2 t - x}{x}} - \tan^{-1}\sqrt{\frac{c_2 t - x}{x}}\right)\begin{pmatrix} \tau_0 \\ D_0 \\ B_0 \end{pmatrix}, \quad 0 < x < c_2 t \tag{78}$$

$$\begin{pmatrix} \gamma_{zy}(x,0,t) \\ E_y(x,0,t) \\ H_y(x,0,t) \end{pmatrix} = \frac{2}{\pi}\left(\sqrt{\frac{c_2 t - x}{x}} - \tan^{-1}\sqrt{\frac{c_2 t - x}{x}}\right)\Pi^{-1}\begin{pmatrix} \tau_0 \\ D_0 \\ B_0 \end{pmatrix}, \quad 0 < x < c_2 t \tag{79}$$

$$\begin{pmatrix} K_\sigma(t) \\ K_D(t) \\ K_B(t) \end{pmatrix} = 2\sqrt{\frac{2c_2 t}{\pi}}\begin{pmatrix} \tau_0 \\ D_0 \\ B_0 \end{pmatrix} \tag{80}$$

$$\begin{pmatrix} K_\gamma(t) \\ K_E(t) \\ K_H(t) \end{pmatrix} = 2\sqrt{\frac{2c_2 t}{\pi}}\Pi^{-1}\begin{pmatrix} \tau_0 \\ D_0 \\ B_0 \end{pmatrix} \tag{81}$$

The EDF can then be obtained as

$$S(t) = \frac{\Omega_T \tau_0^2 + \Omega_D D_0^2 + \Omega_B B_0^2 + \Omega_{TD}\tau_0 D_0 + \Omega_{TB}\tau_0 B_0 + \Omega_{DB} D_0 B_0}{\pi^2 (f_{15}^2 \varepsilon_{11} + c_{44}\mu_{11}\varepsilon_{11} - 2e_{15} f_{15} g_{11} - c_{44} g_{11}^2 + e_{15}^2 \mu_{11})} c_2 t \tag{82}$$

Obviously, the fracture behaviors for the two cases considered above are the same as those of point impact. In addition, it should be noted that, if taking $b_1 \to 0, b_2 \to c_2 t \to x$ in Eqs. (73) and (74), and taking $b_1 \to 0, b_2 \to c_2 t$ in Eqs. (75)-(77), Eqs. (78)-(82) can be also recovered from Eqs. (73)-(77), respectively, and that the results of Chen and Yu (1998) for semi-infinite crack in piezoelectric material are, in fact, the corresponding degenerate expressions of the present work, i.e., Eqs. (78)-(82).

7. Conclusions

The dynamic response problem of a magneto-electro-elastic material with a semi-infinite magneto-electrically impermeable mode-III crack under anti-plane mechanical and in-plane magneto-electrical impact loads is analyzed. The magneto-electro-elastic field ahead of the crack tip under combined point impact loads is obtained in explicit form. The dynamic intensity factors and dynamic EDF of crack tips are derived and discussed in detail. From this work, the following conclusions can be drawn:

(1) Different to electric displacement and/or magnetic induction, the stress field has apparent transient response. For a definite position along the crack line, after the shear wave arriving, the stress, electric displacement and magnetic induction, respectively, depend only on the corresponding mechanical, electrical and magnetic loadings, and they all are independent of material properties. However, at this time, the strain component, electric field and magnetic field all depend on not only mechanical, magnetic and electrical impact loadings but also material properties.

(2) For a definite position alone the crack line, when shear waves do not arrive, the strain component vanishes. However, the electric field, magnetic field and stress component depend on electric displacement and magnetic induction impact loads simultaneously.

(3) After shear wave arrive crack tip, the field intensity factors of stress, electric displacement and

magnetic induction are all independent of material constants and only related to the correspondingly mechanical, electrical and magnetic loads, respectively. However, the intensity factors of strain, electric field and magnetic field all depend on both material constants and applied loads including mechanical loads, electrical loads and magnetic loads.

(4) After shear wave arrive crack tip, the energy density factor against time has no peak for concentrated combined impact loads. According to the maximum energy density factor fracture criterion, the nearer the impact loads approaching the crack tip, the easier the crack growth and propagation.

(5) Previous dynamic semi-infinite crack problems of piezoelectric, piezomagnetic, or purely elastic materials can directly be reduced from the present work.

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