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Elastic modulus in large concrete structures by a sequential hypothesis testing procedure applied to impulse method data

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Abstract. An experimental method denoted as *Impulse Method* is proposed as a cost-effective non-destructive technique for the on-site evaluation of concrete elastic modulus in existing structures: on the basis of Hertz's quasi-static theory of elastic impact and with the aid of a simple portable testing equipment, it makes it possible to collect series of local measurements of the elastic modulus in an easy way and in a very short time. A Hypothesis Testing procedure is developed in order to provide a statistical tool for processing the data collected by means of the *Impulse Method* and assessing the possible occurrence of significant variations in the elastic modulus without exceeding some prescribed error probabilities. It is based on a particular formulation of the renowned sequential probability ratio test and reveals to be optimal with respect to the error probabilities and the required number of observations, thus further improving the time-effectiveness of the *Impulse Method*. The results of an experimental investigation on different types of plain concrete prove the validity of the *Impulse Method* in estimating the unknown value of the elastic modulus and attest the effectiveness of the proposed Hypothesis Testing procedure in identifying significant variations in the elastic modulus.

Keywords: non-destruction testing; statistical data processing; hypothesis testing; concrete, elastic modulus.

1. Introduction

Recent findings by the scientific community and modern design and verification codes have highlighted the importance of investigating the elastic properties of existing concrete structures and infrastructures. For example, a careful determination of concrete elastic modulus and constitutive relation is required by prEN 1992-1-1 (2003), whenever high accuracy in calculating structural deformations is needed (e.g., deflection control in Serviceability Limit States, etc.). A special care in

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investigating the elastic modulus of existing concrete structures is also recommended in case of restoration works: indeed, recent studies concerning the durability of repairs (Sharif *et al.* 2006, Morgan 1996, Emberson and Mays 1990, Emberson and Mays 1990) point out that serious compatibility problems may arise if noticeable mismatch in the elastic modulus and creep characteristics is observed between the original concrete and the repaired part. Hence it comes the importance of correctly characterizing the parent material, in terms of elastic modulus. Finally, the relevance of promptly observing a possible elastic modulus decay in existing structures subjected to repeated mechanical or environmental actions is apparent in Continuum Damage Mechanics based theories and other sophisticated models for material degradation (Lemaitre 1996, Alves *et al.* 2000, Baluch *et al*. 2003, Alliche 2004, Tao and Phillips 2005, Park 1990, Taliercio and Gobbi 1996), in which the elastic modulus is strictly related to the damage variables describing the irreversible deterioration process. It follows that monitoring the elastic modulus and its variations over time or point-by-point in existing structures could provide useful information in diagnostic problems.

Unfortunately, determining the elastic modulus in existing structures is a complex task, especially when dealing with large structures, such as bridges, viaducts, etc. It is in this kind of structures, indeed, that assessing the elastic modulus turns out to be particularly hard because, due to their size, a systematic use of standard methods which involve drilling a high number of cores to be subjected to laboratory tests would be very expensive and, in most cases, unfeasible. Additionally, in general it is presumable to work in a context of limited time and resources, so that rapidity in appraisal turns out to be crucial, rather than extreme accuracy in measurements.

For these reasons, in the last decades, some authors have proposed the use of correlations with other properties that could be more easily determined (Demir 2005, Persson 2004), and many others have developed experimental techniques in the attempt of obtaining rapid and cost-effective methodologies for the elastic modulus estimation. Some of these techniques are based on the measurement of ultrasonic waves propagation velocity (Qixian and Bungey 1996, Hassan *et al.* 1995) and reveal to be fast and easy-to-perform, but, on the other hand, their accuracy deeply depends on the ability of the operator and on testing conditions in general. Conversely, other mechanical techniques make it possible to achieve a substantially higher accuracy, but their use is restricted to specific applications (Karadelis 2000, Gasparetto and Giovagnoni 2000) or has the drawback of requiring a longer time and causing a little damage to the structural member under investigation (Antonaci and Bocca 2005).

Based on the foregoing considerations, the present paper deals with the problem of experimentally determining the elastic modulus in existing concrete structures and appropriately processing the experimental data by a practical statistical tool, in order to ascertain if significant elastic modulus variations are likely to have occurred over time or point-by-point in the structure under investigation.

A novel mechanical method, denoted as *Impulse Method*, is proposed for the on-site estimation of the elastic modulus: it proves to be accurate, non-destructive and sufficiently fast and easy-to-perform to make it possible to collect a suitable number of data in a very short time, thus meeting the requirement of cost-and time-effectiveness typically needed for large structures.

Subsequently, the task of assessing the possible occurrence of significant elastic modulus variations on the basis of the experimental evidence is addressed. Indeed, this could be desirable in many circumstances: for example, if in a given region of the structure under consideration a substantial decrease in the elastic modulus has occurred over time, then it could be used as an indicator of possible degradation phenomena in progress, in accordance with Lemaitre (1996), Alves *et al.* (2000), Baluch *et al.* (2003), Alliche (2004), Tao and Phillips (2005), Park (1990), Taliercio and Gobbi (1996). Conversely, if remarkable elastic modulus differences exist between two different regions of the structure, then they could indicate the presence of possible casting discontinuities, or local material degradation occurrences, or some other kind of discontinuity, resulting in a possible detrimental effect on the global structural performance. Since such variations have to be investigated by means of experimental measurements, then it is needed to suitably take into account the unavoidable variability in measurements themselves. Such a variability could be ascribed either to actual changes of the measured physical quantity, or also to reading errors, as well as to small material in-homogeneities, which could eventually alter the local measurements even though they have no influence on the whole structural behavior. For this reason, there is the need of a statistical tool able to cope with the variability in the experimental measurements. It should provide scientific support to the decision-making process, keeping the probabilities of mistaken judgments under control.

The approach suggested in this paper in order to develop such a statistical tool is based on a Sequential Hypothesis Testing procedure. It is formulated in such a way that a null hypothesis and an alternative one are made, taking into account the random nature of the elastic modulus data, collected by means of the *Impulse Method*; then accepting the null hypothesis implies that the elastic modulus is to be considered as sensibly constant, while rejecting it implies that systematic significant variations are likely to exist. The proposed solution is such that the probabilities to make wrong decisions (i.e., accepting the null hypothesis while it is false, or rejecting it while it is in force) can be kept both under control. In addition, the number of observations required to make a decision is minimized under the constraint that some arbitrary levels of these error probabilities are not exceeded. In this way, an increased time-effectiveness of the *Impulse Method* is obtained, thus achieving a relevant result in the framework of the elastic modulus evaluation problem.

An extensive experimental investigation on four different types of plain concrete has been conducted, with the aim of assessing the effectiveness and the possible drawbacks of the *Impulse Method* and the proposed Sequential Hypothesis Testing procedure: the results achieved proved to be fairly accurate and reliable. Future developments are expected, in view of the integration with other experimental techniques, the final aim being the minimization of the total number of tests to be performed for a complete material characterization.

2. Experimental determination of the elastic modulus: *Impulse method*

2.1 Theoretical foundations

The main theoretical aspects concerning the proposed method are derived from Hertz's theories of contact and impact between elastic solids in the absence of friction (Johnson 1987), which make it possible to easily determine the material's elastic modulus locally, on the basis of the response of the material itself to the impact of a mass having known geometric and elastic properties.

As known, Hertz approached and solved the problem of describing the mechanics of two elastic non-conforming bodies brought into contact and subject to a compressive force based on the following assumptions:

- the surface of each body is considered to be topographically smooth on both micro and macro scales

- each body is regarded as an elastic half-space loaded over a small elliptical region of its plane surface: in order for this simplification to be justifiable it is needed that the significant dimension of the contact area is small with respect to the dimension of each body and the relative radii of curvature of their surfaces
- the surfaces are assumed to be frictionless so that only a normal pressure is transmitted between them.

According to these assumptions, it is possible to calculate the distribution of the mutual pressure acting over the surfaces of the bodies that gives rise to the elastic displacements of the surfaces themselves. Once the distribution of mutual pressure is determined, it is possible to find out the expression of the mutual approach of two distant points in the two solids, η. In the particular case of contact between solids of revolution, η turns out to be

$$\eta = \left(\frac{9P^2}{16RE^{*2}}\right)^{1/3} \tag{1}$$

being P the total load compressing the solids and R and E such that

$$\frac{1}{R} = \frac{1}{R_0} + \frac{1}{R_1} \tag{2}$$

$$\frac{1}{E^*} = \frac{1-\nu_0^2}{E_0} + \frac{1-\nu^2}{E} \tag{3}$$

R_0 and R_1 are the radii of curvature of the surfaces in contact and E_0, ν_0, E and ν are the Young's modulus and the Poisson's ratio related to each body, respectively.

Hertz's theory of elastic contact can be extended to include cases of dynamic loading: quasi-static theory of elastic impact, indeed, is directly derived from the static theory of elastic contact and provides a solution to the problem of describing the mechanics of two elastic bodies impacting against each other. Further assumptions are required, namely:

- deformation is assumed to be restricted to the vicinity of the contact area and given by the static theory: the effect of elastic waves caused by the dynamic loading is thus disregarded
- the total mass of each body is assumed to be moving at any instant with the velocity of its centre of mass.

Under these assumptions, let us consider the impact of a body of mass m_0 with a hemispherical surface against the plane surface of a semi-infinite elastic space. Let us assume that the direction of the impact is perfectly perpendicular to such a surface and in addition that the geometric characteristics (radius R_0) and the elastic properties (Young's modulus E_0 and Poisson's ratio ν_0) of the mass m_0 are known, while the elastic properties E and ν related to the half-space are considered to be unknown. The velocity of the mass m_0 at the moment when it gets in contact with the surface of the plane is denoted by v_0, while the velocity of the half-space is nil.

During the impact, due to elastic deformation, the centres of the bodies approach each other by the displacement η given by Eq. (1): as a result, their relative velocity can be expressed as the time derivative of the elastic displacement η. Therefore, the application of the second principle of dynamics gives a differential equation in η that, after an integration with respect to η, provides the expression of the maximum compression η^*, which takes place at the instant when the relative velocity becomes nil. By a further integration of η^* with respect to time, the compression-time curve can be finally obtained: this makes it possible to determine the value of E^* according to the following expression

$$E^* = \sqrt{\left(\frac{2.87}{T}\right)^5 \cdot \frac{m_0^3}{R_0 A}} \tag{4}$$

in which T is the total time of contact between the two bodies and A turns out to be the variation in the momentum from the beginning of the impact and the instant when the maximum compression takes place

$$A = m_0 \cdot v_0 \tag{5}$$

A can be regarded as the area subtended by the force vs. time curve describing the impact from the moment when the bodies get in contact to the instant of maximum compression. The procedure to be followed to obtain Eq. (4) is described in greater detail in Johnson (1987), Bocca *et al.* (1991).

Once the value of E^* has been determined through Eq. (4), the unknown value of the elastic modulus E related to the half-space can be easily obtained recalling Eq. (3). In this connection, it should be remarked that the value of ν is unknown; however, since it is known to have but little bearing on the value of E, it can be assigned a nominal value, which amounts to introducing a negligible error.

On the basis of the above-described model, a simple experimental procedure can be set up for the indirect measurement of the elastic modulus. It consists of the following steps:

- producing the impact of a hemispherical body having known mass and radius (m_0 and R_0 respectively) against the surface of the material to be examined
- tracing the force vs. time curve describing the impact
- computing the parameters T and A from the force vs. time curve, so as to determine the elastic modulus according to Eqs. (3) and (4).

Fig. 1 Impulse hammer

2.2 Experimental equipment

For the purposes of a practical implementation of the theoretical concepts recalled in section 2.1, an appropriate experimental equipment is needed: it should be able to produce the impact of a mass against the surface of the material under investigation in controlled conditions and subsequently to trace the resulting force vs. time curve.

The testing equipment employed in the present study is the following:

- a special impulse hammer, that is a completely self-contained instrument including an electrically activated mechanism that produces the impact of a hemispherical tip of bonded steel (20 mm in diameter) with controlled energy, a piezoelectric force sensor, a signal conditioner with an amplifier and a remote trigger hook-up (see Fig. 1)
- a portable data acquisition system which makes it possible to acquire and digitize the impulse hammer signals in a software-controlled manner. Digitization incorporates techniques that combine an ultra-low noise floor with fully linear 24-bit system performance at up to 204.8 kHz.
- a post-processing system for subsequent data elaboration.

3. Hypothesis testing procedure

3.1 Assumptions

It is desired to approach the problem of assessing the possible occurrence of significant elastic modulus variations, on the basis of some experimental evidence. Accordingly, let us suppose to acquire N measurements of the elastic modulus related to N points belonging to a certain region of the structure, by means of the *Impulse Method.* Let us denote each measurement by x_k, with $k = 1, 2, \cdots N$. Due to the unavoidable variability in the experimental measurements, each value x_k could be regarded as a realization or "observation" of a random variables X_k[1]. Similarly, N measurements of the elastic modulus related to N points belonging to another region of the structure can be acquired. They will be denoted by y_k, with $k = 1, 2, \cdots N$, and will be be considered as realizations of random variables Y_k[2].

By analogy with recent studies concerning concrete strength (Bazant and Novak 2003, Bartlett 1997, Bocca and Indelicato 1988), it will be assumed that the random variables X_k and Y_k belong to two independent sets of independent identically distributed (i.i.d.) random variables.

For the sake of simplicity, it will be supposed that the variances σ_X^2 and σ_Y^2 related to the distributions of the variables X_k and Y_k respectively, are the same, so that $\sigma_X^2 = \sigma_Y^2 = \sigma^2$.

Finally, the value of σ^2 will be considered as known since it could be effectively estimated on the basis of a statistically relevant number of measurements, collected *una tantum* with the apparatus described in Section 2.2.

[1]Random variables will be denoted by capital letters, while the values that they assume will be denoted by the corresponding small letters.

[2]Alternatively, the measurements y_k can be taken in the same region as x_k but at a different time.

3.2 Hypotheses

No information is available about the actual distribution of the two populations of i.i.d random variables X_k and Y_k representing the elastic modulus related to the two regions of the structure. However, it is known that their expectations, μ_X and μ_Y respectively, provide a synthetic indication which is representative of the whole distribution they refer to. Accordingly, their relative distance, referred to as δ

$$\delta \triangleq \mu_X - \mu_Y \tag{6}$$

turns out to be a crucial parameter.

In particular, it can be assumed that no significant elastic modulus differences exist between the two regions whenever the absolute value of the parameter δ does not exceed a given reference value, ε_m. Such a reference value must be appropriately selected so as to correctly account for the natural heterogeneity of the material: indeed, concrete being macroscopically heterogeneous due to the typological and dimensional variety of its components, small differences between the expectations μ_X and μ_Y may exist with no substantial variations in the behavior of the structure as a whole. In the present study ε_m was set to 2000 N/mm^2.

Conversely, it is advisable to choose a further reference value, $\varepsilon_M > \varepsilon_m$, and assume that significant differences in the elastic modulus exist whenever the absolute value of the parameter δ exceeds ε_M. Such a reference value must be appropriately selected, according to the care required in distinguishing differences. In the present study ε_M was set to 4000 N/mm^2.

The above considerations represent the formulation of a Hypothesis Testing problem, in which the hypotheses

$$\begin{aligned} \mathcal{H}_0&: \quad |\delta| \le \varepsilon_m \\ \mathcal{H}_1&: \quad |\delta| \ge \varepsilon_M \end{aligned} \tag{7}$$

must be statistically tested so that a decision on which hypothesis is in force may be made on the basis of the experimental data, with the constraint that some prescribed levels of the error probabilities are not exceeded.

3.3 Performance requirements

Due to the symmetry of the problem, it is desired that the error probabilities of the first and second kind related to such a test are both constrained not to exceed some prescribed values, denoted by α_d and β_d respectively

$$\alpha \triangleq \mathbb{P}\ \{\text{accept } \mathcal{H}_1 \mid \mathcal{H}_0 \text{ is in force}\} \le \alpha_d \quad \text{and} \quad \beta \triangleq \mathbb{P}\ \{\text{accept } \mathcal{H}_0 \mid \mathcal{H}_1 \text{ is in force}\} \le \beta_d \tag{8}$$

Alternatively, the performance requirements may be expressed in terms of the power function $\pi(\delta)$ related to the test, which must satisfy the conditions

$$\begin{aligned} \pi(\delta) &\triangleq \mathbb{P}\quad \{\text{accept } \mathcal{H}_1 \mid \delta\} \le \alpha_d \quad &\text{if} \quad |\delta| \le \varepsilon_m \\ \pi(\delta) &\triangleq \mathbb{P}\quad \{\text{accept } \mathcal{H}_1 \mid \delta\} \ge 1 - \beta_d \quad &\text{if} \quad |\delta| \ge \varepsilon_M \end{aligned} \tag{9}$$

3.4 Solution

In order to provide a solution for the problem stated in the preceding sections, it is useful to introduce the additional random variables Z_k, defined as the difference between X_k and Y_k. The same way as X_k and Y_k, they turn out to be i.i.d. random variables, with expectation $\mathbb{E}\{Z_k\} = \mu_X - \mu_Y = \delta$ and variance $\sigma_Z^2 \triangleq \mathbb{E}\{(Z_k - \delta)^2\} = \sigma_X^2 + \sigma_Y^2 = 2\sigma^2$.

Subsequently, an appropriate statistic should be constructed as a function of the experimental data. In the present study, the statistic $\hat{\Delta}_N$ defined as

$$\hat{\Delta}_N \triangleq \frac{1}{N}\sum_{k=1}^{N} Z_k \tag{10}$$

has been used, supposing that it would be a sufficient statistic for the parameter δ (Sellone 2005). The number of observations N is imposed to be sufficiently high, i.e., $N \geq 30$, so that the statistic $\hat{\Delta}_N$ is asymptotically normally distributed

$$\hat{\Delta}_N \sim N\left(\delta, \frac{2\sigma^2}{N}\right) \tag{11}$$

Some observations are required before a solution for the stated problem can be given. Let us preliminarily consider simple hypotheses of the form

$$\begin{aligned} \mathcal{H}_0^s: &\quad \delta = \delta_0 \\ \mathcal{H}_1^s: &\quad \delta = \delta_1 \end{aligned} \tag{12}$$

It is known from the Neyman-Pearson fundamental lemma (Lehmann 1994) that for a fixed number of observations $N = n$, the best procedure for testing the simple hypothesis $\mathcal{H}_0^s$ against the simple alternative $\mathcal{H}_1^s$ is such that $\mathcal{H}_0^s$ is accepted or rejected as the likelihood ratio

$$\lambda_N = \frac{f_{\hat{\Delta}_N}(\delta_N;\ \delta = \delta_1)}{f_{\hat{\Delta}_N}(\delta_N;\ \delta = \delta_0)} \tag{13}$$

is less or greater than a suitable constant κ, correctly chosen so as to guarantee a given performance of the test. Generally, it is determined in such a way that the error probability of the first kind does not exceed a prescribed value α_d. Such a test proves to be optimal in the sense that it has the maximum power $\pi(\delta_1)$ with respect to any other test having the same fixed number of observations n and error level α_d. The value of the power $\pi(\delta_1)$, however, cannot be controlled without changing the number of observations to be taken into account.

Although optimal in the above specified sense, the test based on the Neyman-Pearson fundamental lemma can be furthermore improved if the number of observations is not fixed in advance but is permitted to depend on the previous observations (Sellone 2005, Lehmann 1994). Then, a sequential probability ratio test can be worked out in such a way that the number of observations, N, is gradually increased as long as the sequential probability ratio (13) satisfies the condition $\kappa_0 < \lambda_N < \kappa_1$ and the hypothesis $\mathcal{H}_0^s$ is then accepted or rejected at first violation of this condition, with $\lambda_N \leq \kappa_0$ or $\lambda_N \geq \kappa_1$. The thresholds κ_0 and κ_1 should be determined as a function of both the error probabilities of the first and second kind that shall not be exceeded. In particular, due to the difficulty of exactly calculating κ_0 and κ_1 as a function of the prescribed values α_d and β_d, they can be determined in an approximate way, according to Wald's findings (Lehmann 1994)

$$\kappa_0 \cong \kappa_0' = \frac{\beta_d}{1-\alpha_d} \quad \text{and} \quad \kappa_1 \cong \kappa_1' = \frac{1-\beta_d}{\alpha_d} \tag{14}$$

It can be demonstrated (Lehmann 1994) that the sequential probability ratio test with error probabilities α_d and β_d minimizes both $\mathbb{E}\{N \mid \mathcal{H}_0^s\}$ and $\mathbb{E}\{N \mid \mathcal{H}_1^s\}$ with respect to any fixed sample size test that controls the errors at the same levels. Therefore, the sequential probability ratio test appears to be optimum when the optimality criterion consists in the minimization of the expected number of observations required to guarantee the prescribed error levels.

Let us now recall the composite hypotheses (7) and the conditions (9) that define the original Hypothesis Testing problem to be solved in this paper.

If the analysis is restricted to positive values of the parameter δ, it is easy to verify that $\forall \delta_0 \le \varepsilon_m$ and $\delta_1 \ge \varepsilon_m$, the likelihood ratio (13) is a monotone non-decreasing function of $\hat{\delta}_N$. Consequently, it can be demonstrated (Lehmann 1994) that any sequential probability ratio test for testing $\delta = \delta_0$ against $\delta = \delta_1$ ($\delta_0 < \delta_1$) has a non-decreasing power function.

Conversely, if the analysis is restricted to negative values of the parameter δ, it can be easily verified that $\forall \delta_0 \ge -\varepsilon_m$ and $\delta_1 \le -\varepsilon_M$, the likelihood ratio (13) is a monotone non-increasing function of $\hat{\delta}_N$. Therefore, according to Lehmann (1994), any sequential probability ratio test for testing $\delta = \delta_0$ against $\delta = \delta_1$ ($\delta_0 > \delta_1$) has a non-increasing power function.

Based on the property stated above, it appears that the solution for the problem under consideration is represented by a sequential probability ratio test with error probabilities α_d and β_d, evaluated for the simple hypotheses $\delta = \varepsilon_m$ against $\delta = \varepsilon_M$ and $\delta = -\varepsilon_m$ against $\delta = -\varepsilon_M$, i.e., in the extremes of the composite hypotheses $\mathcal{H}_0$ and $\mathcal{H}_1$ reported in Eq. (7). Indeed, due to the peculiar trend of its power function, the conditions (9) are satisfied if

$$\pi(\varepsilon_m) = \alpha_d \quad \text{and} \quad \pi(\varepsilon_M) = 1-\beta_d \tag{15}$$

or

$$\pi(-\varepsilon_m) = \alpha_d \quad \text{and} \quad \pi(-\varepsilon_M) = 1-\beta_d \tag{16}$$

From a practical point of view, once the thresholds to be used to ensure the prescribed performances of the test are determined according to (14), it is possible to transform the sequential probability ratio test into a sequential test based directly on the sufficient statistic $\hat{\Delta}_N$. Recalling general properties of logarithms, indeed, it can be found that the hypothesis $\mathcal{H}_0$ in (7) must be accepted when the realization $\hat{\delta}_N$ of the statistic $\hat{\Delta}_N$ is such that $|\hat{\delta}_N| \le \eta_{m,N}$, while the alternative hypothesis $\mathcal{H}_1$ must be accepted when $|\hat{\delta}_N| \ge \eta_{M,N}$, with

$$\eta_{m,N} \triangleq \frac{\frac{4\sigma^2}{N} \cdot \lg\left(\frac{\beta_d}{1-\alpha_d}\right) + (\varepsilon_M^2 - \varepsilon_m^2)}{2(\varepsilon_M - \varepsilon_m)} \quad \text{and} \quad \eta_{M,N} \triangleq \frac{\frac{4\sigma^2}{N} \cdot \lg\left(\frac{1-\beta_d}{\alpha_d}\right) + (\varepsilon_M^2 - \varepsilon_m^2)}{2(\varepsilon_M - \varepsilon_m)} \tag{17}$$

On the contrary, if the condition $\eta_{m,N} < |\hat{\delta}_N| < \eta_{M,N}$ is verified, then no decision can be made, due to ambiguity in the experimental data. Consequently, additional information is needed and the number of observations is increased, as schematically illustrated in Algorithm 1.

The advantages offered by such a test are apparent: not only it is possible to minimize the effort required during the experimental stage, since only the strictly necessary number of measurements is performed, but, in addition, both error probabilities of the first and second kind can be controlled.

Algorithm 1 Sequential hypothesis testing procedure

Require: σ^2; ε_m and ε_M; α_d and β_d

$$\eta_{m,0} \leftarrow \frac{4\sigma^2 \log \frac{\beta_d}{1-\alpha_d}}{2(\varepsilon_M - \varepsilon_m)}, \quad \eta_{M,0} \leftarrow \frac{4\sigma^2 \log \frac{1-\beta_d}{\alpha_d}}{2(\varepsilon_M - \varepsilon_m)}, \quad \bar{\varepsilon} \leftarrow \frac{\varepsilon_M + \varepsilon_m}{2}$$

Acquire $N = n_{\min} - 1$ measurements $z_1, z_2, \ldots, z_N$ via the *Impulse Method*

$$s_N \leftarrow \sum_{k=1}^{N} z_k$$

loop

$N \leftarrow N + 1$

$$\eta_{m,N} \leftarrow \frac{\eta_{m,0}}{N} + \bar{\varepsilon}, \quad \eta_{M,N} \leftarrow \frac{\eta_{M,0}}{N} + \bar{\varepsilon}$$

Acquire 1 measurement z_N via the *Impulse Method*

$s_N \leftarrow s_{N-1} + z_N$

$$\hat{\delta}_N \leftarrow \frac{s_N}{N}$$

if $|\hat{\delta}_N| \leq \eta_{m,N}$ **then**

  return Accept $\mathcal{H}_0$

else if $|\hat{\delta}_N| \geq \eta_{M,N}$ **then**

  return Accept $\mathcal{H}_1$

end if

end loop

4. Experimental investigation

4.1 Materials and specimens

Four different types of plain concrete were tested in the present experimental campaign: they differ from each other in cement and aggregate proportions and in water/cement ratio, resulting in a strength range that is fairly representative of the values typically encountered in existing structures. Their compositions were designed in order to obtain couples of concretes whose properties are comparable (couple $A - B$ and couple $C - D$) or sufficiently dissimilar (couples $A - C$, $A - D$, $B - C$ and $B - D$). Table 1 summarizes the composition of the four mixes.

Table 1 Mixes compositions

	Mix A	Mix B	Mix C	Mix D
Cement type	CEM II A/L 42.5 R	CEM II A/L 42.5 R	CEM II A/L 42.5 R	CEM II A/L 42.5 R
Cement proportions	270 kg/m^3	340 kg/m^3	410 kg/m^3	450 kg/m^3
Sand proportions	1063 kg/m^3	957 kg/m^3	876 kg/m^3	825 kg/m^3
Gravel proportions	799 kg/m^3	846 kg/m^3	839 kg/m^3	856 kg/m^3
Maximum aggregate size	15 mm	15 mm	15 mm	15 mm
Water/Cement ratio	0.74	0.59	0.51	0.47

Four slabs sized 100 × 100 × 20 cm^3, i.e., one slab per each type of mix, were produced in order to simulate different areas belonging to an imaginary structure to be investigated. In addition, three prism-shaped specimens sized 15 × 15 × 45 cm^3 and three cubic specimens of 15 cm side were produced per each type of mix to be subject to preliminary laboratory tests for material characterization. All these specimens were water-cured for 28 days and subsequently air-cured for approximately 8 months before testing.

4.2 Preliminary characterization tests

The cubic specimens were preliminarily subject to crushing tests performed according to the standard UNI EN 12390-3 using a 250 kN servo-controlled testing machine in order to determine the material static compressive strength. At the same time, the prism-shaped specimens were statically tested in compression according to the standard UNI 6556, so as to determine the reference values of the elastic modulus related to each concrete.

In this regard, it must be specified that a very low stress level was used: the maximum load applied for the determination of secant elastic modulus according to the standard UNI 6556 was set to 20% of the failure load. In this way, a negligible discrepancy is achieved between the secant values thus obtained and the actual tangent elastic modulus of the material at the origin. Accordingly, the reference static modulus and the elastic modulus estimated by means of the *Impulse Method* are made comparable, since the latter has to be regarded as a dynamic modulus value, which substantially coincides with the tangent modulus at the origin (Collepardi 1980).

The reference values of the compressive strength and the Young's modulus for each material were eventually obtained as average over three tests.

The results obtained during the material characterization stage are reported in Table 2.

4.3 Results and discussion

With the aid of the experimental equipment illustrated in section 2.2, the *Impulse Method* was applied to evaluate the elastic modulus related to the previously described types of concrete.

Grids of 10 × 10 points and arrays of 20 points were traced on each slab and on each prism-shaped specimen respectively, so as to obtain global sets of approximately 160 experimental measurements per mix as illustrated in Fig. 3. Special care was devoted to ensure that the experimental system could be as close as possible to the model assumptions reported in section 2.1. In particular:

- The hammer has been equipped with adjustable spacers able to make the direction of the impact always perpendicular to the surface tested (see Fig. 2).
- An appropriate pressure has been applied to the hammer so as to prevent possible vibrations which could affect the response of the system.

Table 2 Mixes mechanical and elastic properties

	Mix A	Mix B	Mix C	Mix D
Compressive strength [N/mm^2]	35	40	49	56
Secant elastic modulus [N/mm^2]	25800	27200	31300	33100

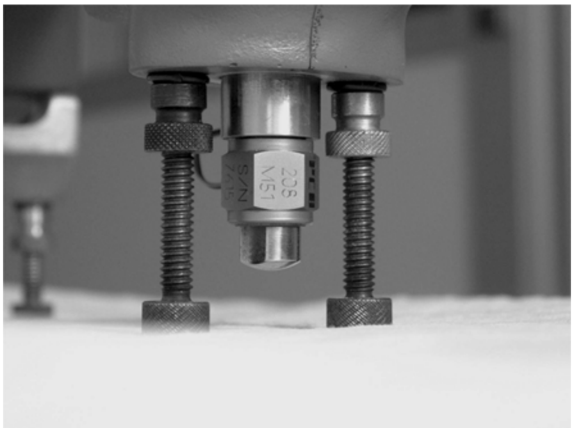

Fig. 2 Impulse hammer: Detail of the spacers

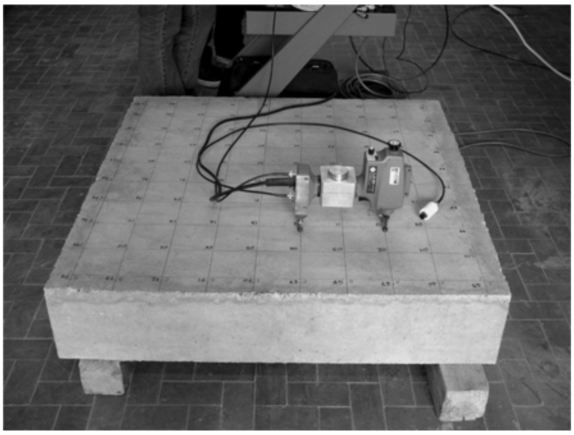

Fig. 3 One of the slabs under test

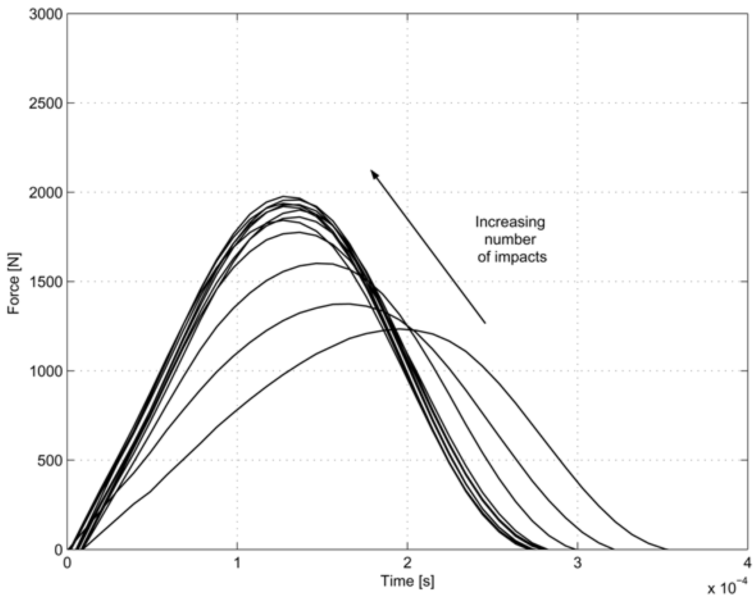

Fig. 4 Evolution of Force vs. Time curves with increasing number of impacts

- The energy supplied during the impact shall was sufficiently low to enable boundary conditions and the shape of the specimen being tested to be disregarded: in such conditions, indeed, in keeping with the assumptions adopted, the item tested could be considered as part of a semi-infinite space.
- In order to achieve a suitable measurement stabilization, series of multiple constant energy impacts have been preliminarily applied on each point being tested, without considering the initial data and basing calculations on the results of the subsequent impacts (see Fig. 4). Indeed, it can be observed that the response of the system is not stable from the very beginning: indeed, the Force vs. Time curves obtained during the initial impacts significantly vary from each other and in addition they are not symmetrical. This is due to dissipation of energy, which is essentially spent to smooth the surface roughness of the concrete. However, in accordance with

(Bocca *et al.* 1991, Antonaci and Bocca 2004), it may be remarked that these dissipation phenomena tend to decrease with increasing number of impacts: as a matter of fact, the impulse curves become almost symmetrical in shape and they tend to overlap, thus indicating a substantial stabilization, which is reflected by a corresponding stabilization in the elastic modulus estimates. In general, from a certain impact on, variations with the increase in hitting numbers may be rated as negligible: under these conditions, no significant energy is dissipated and the surface of the concrete, after undergoing local stiffening, takes on the elastic behavior of the underlying layers, in keeping with Hertz's model assumptions. Consequently, the initial values shall not be considered and calculations shall be based only on the impulse curves recorded after stabilization.

Histograms of the results achieved after response stabilization was obtained are reported in Figs. 5 to 8, that provide a synthetic representation of the huge amount of experimental data. The estimated value of the elastic modulus was obtained by averaging the corresponding measurements for each mix.

The reliability of the *Impulse Method* is clearly assessed through the comparison with the values resulting from standard UNI 6556 tests, which provide reference values for the elastic modulus of each mix: the discrepancy observed between such reference values and the corresponding estimates does not exceed 6.25%, as reported in Table 3. Therefore, the accuracy achieved proves to be sufficient for the purpose of a first approximate estimation; consequently, the *Impulse Method* can be rated as a suitable technique to be used on-site, especially on account of the substantial quickness in collecting measurements, a single test requiring just a few seconds to be completed.

Finally, the sequential Hypothesis Testing procedure described in section 3 has been successfully applied to the sets of experimental data obtained during the testing investigation, as shown in the following.

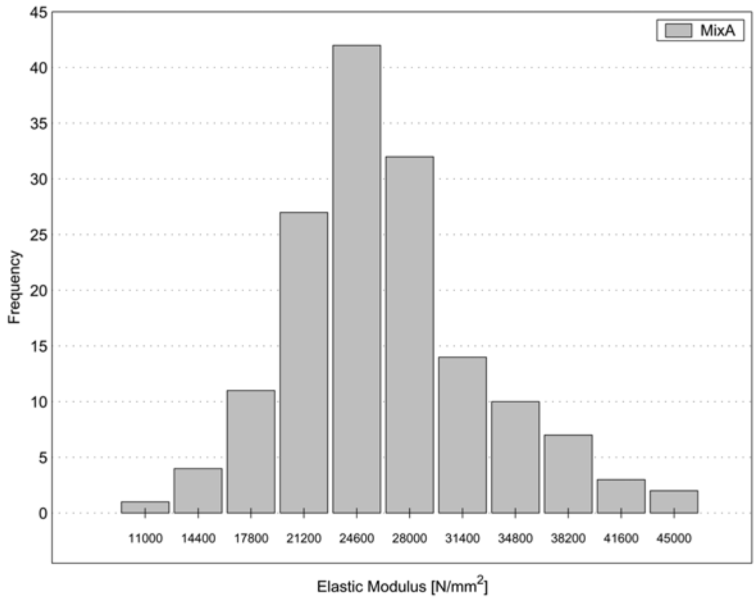

Fig. 5 Histogram of the experimental measurements on mix A

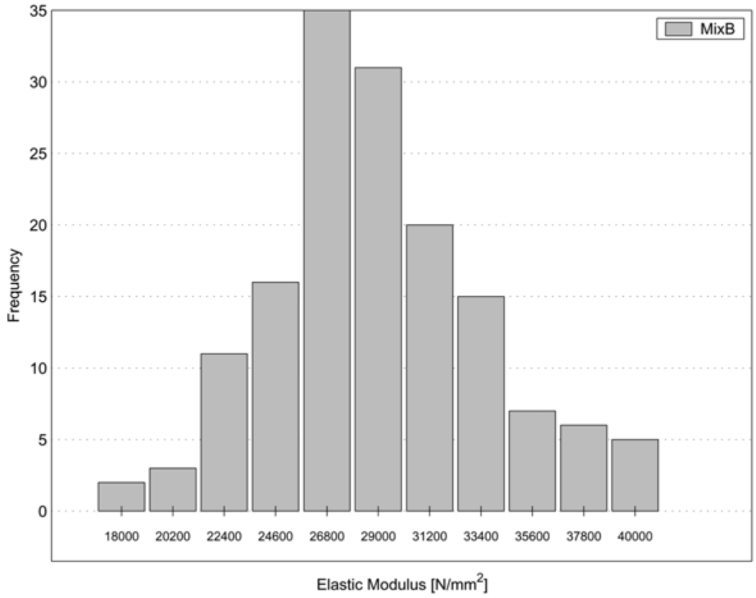

Fig. 6 Histogram of the experimental measurements on mix B

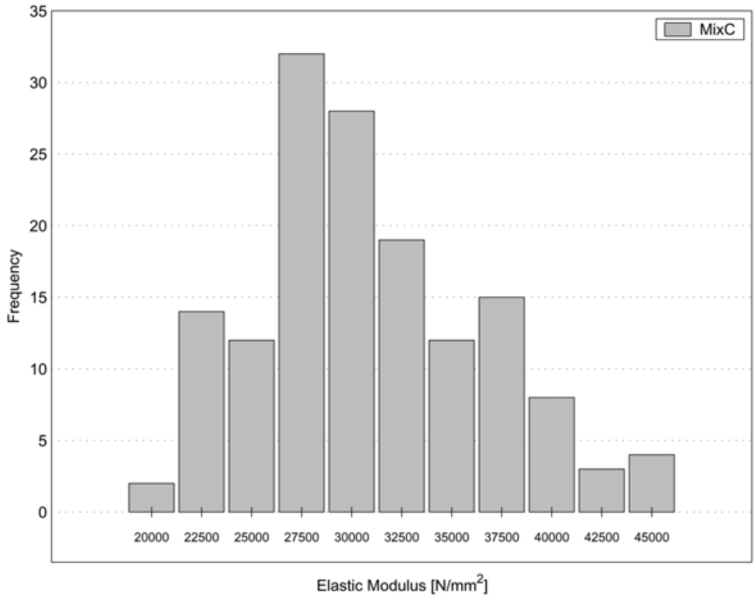

Fig. 7 Histogram of the experimental measurements on mix C

Firstly, the Hypothesis Testing procedure was employed in order to check for significant differences between mixes which were a-priori known to be substantially dissimilar in terms of composition and elastic properties, as pointed out in section 4.1. In line with the symmetry of the problem, the error probabilities of the first and second kind, α_d and β_d, were set to the same value. The number of experimental observations required to make a decision was determined as a function

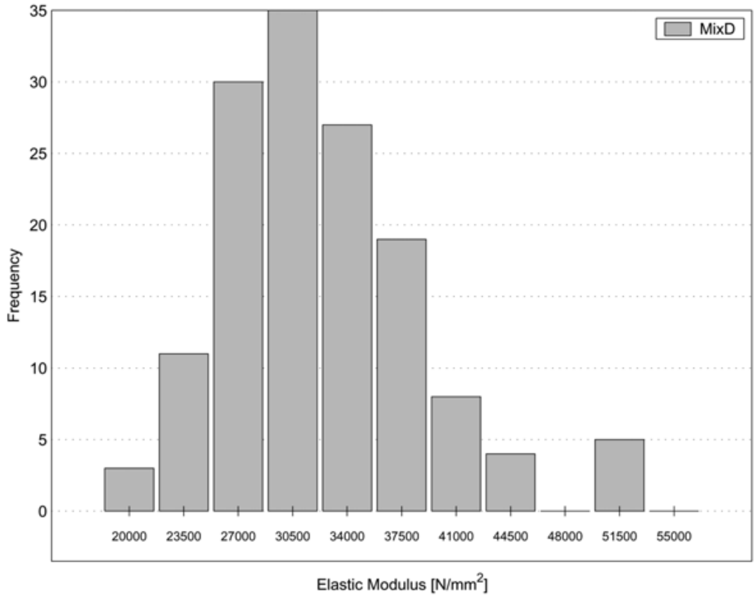

Fig. 8 Histogram of the experimental measurements on mix D

Table 3 Estimates obtained through the *Impulse Method* compared to the reference values resulting from standard UNI 6556 tests

	Elastic modulus [N/mm^2]		Discrepancy
	UNI 6556	Impulse method	
Mix A	25800	26400	2.33%
Mix B	27200	28900	6.25%
Mix C	31300	30800	−1.60%
Mix D	33100	32400	−2.12%

Table 4 *Experimental Results*. Checking for significant differences between mixes i and j: number of observations required to accept $\mathcal{H}_1$ with prescribed error probabilities α_d and β_d

Mixes		Error probability $\alpha_d = \beta_d$									
i	j	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
A	C	65	61	59	58	58	57	57	56	56	56
A	D	37	32	30	30	30	30	30	30	30	30
B	C	79	77	76	75	75	75	73	73	71	67
B	D	39	35	30	30	30	30	30	30	30	30

of α_d and β_d. In order to point out the performances of the Hypothesis Testing procedure as a function of the prescribed error probabilities, different values of α_d and β_d were used, ranging from 0.01 to 0.1.

The results are reported in Table 4. The Hypothesis Testing procedure correctly suggested to

Table 5 *Experimental Results*. Checking for significant differences between mixes i and j: number of observations required to accept $\mathcal{H}_0$ with prescribed error probabilities α_d and β_d

Mixes		Error probability $\alpha_d = \beta_d$									
i	j	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
A	B	58	51	50	47	45	45	41	35	35	30
C	D	100	93	92	81	80	79	79	73	73	73

choose the hypothesis $\mathcal{H}_1$ at every confidence level. Besides, as it can be seen, the required number of observations obviously increases with decreasing error level, but on the whole, it turns out to be quite small, meaning that the proposed procedure is capable to distinguish differences with a reduced number of observations.

Afterward, the Hypothesis Testing procedure was employed in order to assess the similarity between mixes which were a-priori known to be comparable in terms of composition and elastic properties. Also in this case, the error probabilities α_d and β_d were set to the same value, ranging from 0.01 to 0.1 and the number of experimental observations required to make a decision was determined as a function of them. The results are reported in Table 5. The hypothesis $\mathcal{H}_0$ is correctly indicated to be in force at every confidence level and the required number of observations keeps small.

5. Conclusions

As pointed out in the course of an experimental investigation, the values of the elastic modulus related to different types of plain concrete estimated by the *Impulse Method* are in good agreement with those obtained by means of standard UNI 6556 tests performed on the same materials. The discrepancy observed, indeed, does not exceed 6.25%. This finding suggests that the *Impulse Method* can be effectively employed for a first approximation estimation of the elastic modulus, especially when dealing with large existing structures for which, on account of their size, rapidity in appraisal is required rather than extreme accuracy in measurements.

In addition, being non-destructive, fast and easy-to-perform, this technique makes it possible to collect relevant numbers of experimental measurements with no difficulty and also to repeat them over time. Statistical tools may therefore be used to process those data and get a more complete information about the structure under investigation. In particular, the sequential Hypothesis Testing procedure presented in this paper reveals to be a useful instrument for the identification of possible significant variations in the elastic modulus and is optimum in the sense that it minimizes the number of experimental measurements required to make a decision with prescribed confidence levels.

The use of such a Hypothesis Testing procedure to process the data resulting from the application of the *Impulse Method* could provide important indications within the process of structural diagnosis and monitoring, since it makes it possible to investigate the variations in the elastic properties of an existing structure in a cost-effective way and hence to identify potential deteriorated regions within the structure itself or detect possible damage phenomena in progress.

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