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A numerical method for buckling analysis of built-up columns with stay plates

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Abstract. A new numerical model based on the spline finite strip method is presented here for the analysis of buckling of built-up columns with and without end stay plates. The channels are modelled with spline finite strips while the connecting elements are represented by a 3D beam finite element, for which the stiffness matrix is modified in order to ensure complete compatibility with the strips. This numerical model has the advantage to give all possible failure modes of built-up columns for different boundary conditions. The end stay plates are also taken into account in this method. To validate the model a comparative study was carried out. First, a general procedure was chosen and adopted. For each numerical analysis, the lowest buckling loads and modes were calculated. The basic or "pure" buckling modes were identified and their critical loads were compared with solutions obtained using analytical methods and/or other numerical methods. The results showed that the proposed numerical model can be used in practice to study the elastic buckling of built-up columns. This model is considered accurate and efficient for the local buckling of short columns and global buckling for slender columns.

Keywords: batten plates; built-up columns; elastic buckling; spline finite strip; stay plates.

1. Introduction

Built-up columns are widely used in steel structures especially when effective lengths are important and the compressive forces are low. These columns are formed by two or more principal elements assembled by lacing or batten plates (see Fig. 1). The moment of inertia of the cross section of the column, and thus its flexural stiffness in the plane of the connecting element, increases with the distance between the chord axes. Nevertheless, this saving may be

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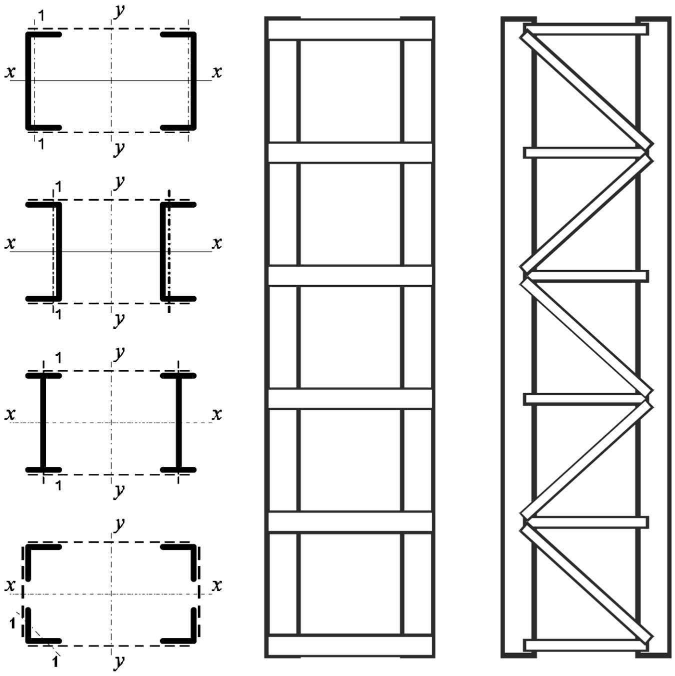

Fig. 1 Built-up columns

counterbalanced by the increase in the weight and the cost of the connecting element. It should be noted that for equal inertia, the built-up columns (especially the battened ones) are more flexible than solid columns (Niazi 1993).

Generally, to estimate the bearing capacity of steel built-up columns with doubly symmetric cross-sections under compressive loads, one should study the following modes of elastic instability in addition to verifying the connecting element and their connections:

- The flexural buckling of the built-up column in a plane perpendicular to the battens. This is classical Euler buckling.
- The flexural buckling of the built-up column in a plane parallel to the battens. In this case, one should take into account the flexibility of the connecting element. These are usually replaced by a continuous web with an equivalent shear stiffness *K*. One of the earliest theories is that of Engesser (1889, 1891 and 1909) and more recently, Paul's theory (1995). The latter has brought two new innovations: it can account for the effect of stay plates at the ends of the column and the equivalent web-chord attachment line is not restricted to be at the chord's centroidal lines. A more recent buckling solution (Wang 2002) considers the built-up column as a solid Timoshenko column. The erosive effect of transverse shear deformation on the buckling capacity is clearly shown by this method.
- The global buckling of the chords which can be flexural, torsional or torsional-flexural. This is an instability that involves translation and/or rotation of the entire cross section of the chord. Rational analysis hand solutions to long column buckling are available (Yu 2000). The partial built-in of the chords into the battens may however cause some additional difficulty. The adequate effective length factors could be evaluated by procedures used for columns in multi-storey unbraced frames (Liu 2005).

For these three modes, the cross sections of the chords move as a rigid body without any distortion. In the case of built-up columns where the thickness of the chords are small, such as for the cold-formed steel structural members, the local buckling of the plate elements forming the section may occur and should be taken into account. Sometimes, these types of sections may fail in a mode of distortional buckling which takes place as a consequence of distortion of the cross section. These two modes which modify the cross section can be considered as "sectional" modes. They can interact with each other as well as with the three global modes cited below (Dubina 1999). The interaction phenomena can be covered theoretically by asymptotic theory (Teter 2004), but numerical analyses such as Finite Element Method, Finite Strip Methods and Generalised Beam Theory are more practical to study these problems.

In the last decades, the finite strip methods have been widely used to study the stability of thin walled sections taking into account all possible failure modes: local, distortional, overall instabilities (flexural, flexural-torsional, lateral) and their combinations. They have been accurately applied by many researchers (Lau and Hancock 1988, Key and Hancock 1993, Cheung *et al.* 1999, Ovesy *et al.* 2005) to linear and non-linear buckling analyses allowing for various types of non-linearities. However, these numerical methods have never been used to study the stability of built-up columns. The aim of this work is, precisely, to prove that this can be accomplished by combining adequate beam finite elements with spline finite strips.

The first spline finite strip method (SFSM) for buckling analysis (Lau and Hancock 1985) used the uniform B3-spline functions in the longitudinal direction and conventional interpolation functions in the transverse direction to represent the displacement field in a strip. This interpolation allowed the finite strip method to take into account intermediate supports, various boundary conditions and arbitrary loading. Recent improvement of the SFSM introduced non-symmetrically spaced knots in the longitudinal direction (Kim 2004). This allows the selective local refinement to improve the accuracy of solution at the location of concentrated effects (loads, connections, restraints).

It is obvious that the discrete effect of the batten plates or lacing bars in the built-up columns could be better modelled by this last technique, however, in this study, uniform B3-spline is used for simplicity. The purpose of this study is to develop a powerful, or at least a sufficient, numerical tool to determine the linear local buckling load and mode of failure of built-up columns taking into account the complete range of behaviour. This tool should serve as a better approach, or replace the classical one based on a "guess" of all possible instabilities.

2. Spline finite strip method for the buckling analysis of built-up columns

2.1 The Spline Finite Strip Method (SFSM)

In the spline finite strip method, structures are divided into longitudinal strips with "n" nodal lines parallel to the Z axis and having the same length L, as shown in Fig. 2. The use of uniform B3-spline aims to "perfectly" fit the displacement function in the longitudinal direction, which will have C^2 continuity. Each nodal line is then divided into "m" equal sections by $m+1$ section knots. Two additional section knots are, however, required to completely define the spline function over the length of the strip. These "$m+3$" section knots are numbered from -1 to $m+1$ as shown in Fig. 3 (Lau and Hancock 1985). A section knot "k" has four degrees of freedom u_k, v_k, w_k and θ_k. The

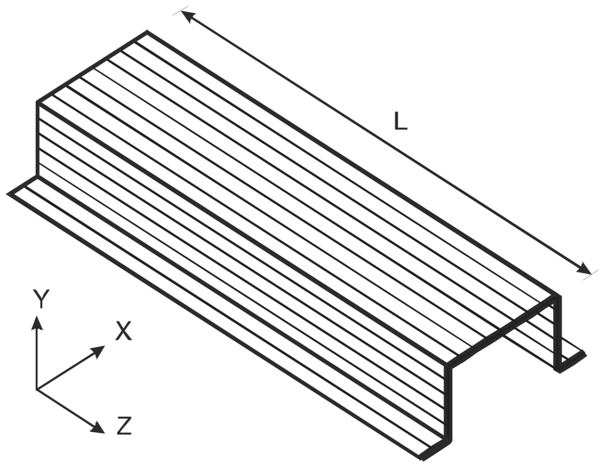

Fig. 2 Strip subdivision of a thin-walled structure

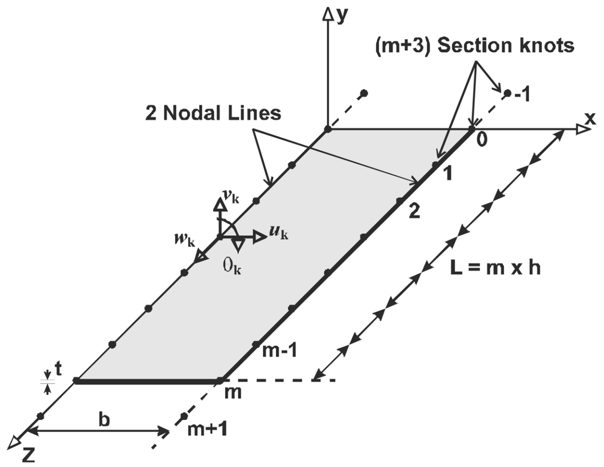

Fig. 3 A typical B3-spline strip

total number of degrees of freedom in a spline finite strip model is then equal to $4n(m+3)$. The four degrees of freedom cited are used to define four displacement functions of the nodal line: the three translations parallel to X, Y and Z axes and the rotation about the Z-axis. The longitudinal variation of any displacement function is given by the summation of $(m+3)$ local uniform B3-splines

$$f(Z) = \sum_{k=-1}^{m+1} \alpha_k \phi_k(Z) \tag{1}$$

Where $\phi_k(Z)$ is a local uniform B3-spline function as shown in Fig. 4 and α_k is the degree of freedom corresponding to the displacement function. α_k may be considered as the amplitude of the spline function. These displacement functions can take into account all possible end boundary conditions and intermediate supports.

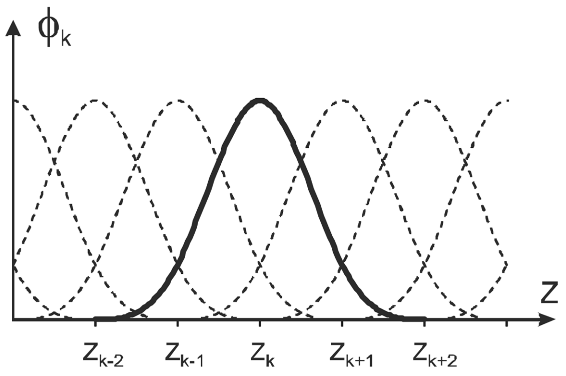

Fig. 4(a) A local B3-spline

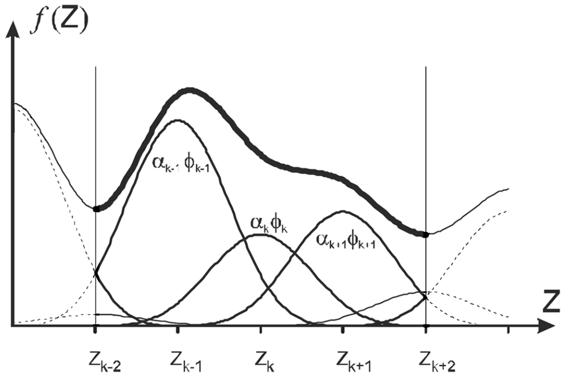

Fig. 4(b) A linear combination of B3-splines

For a single strip, the vector of general displacement fields in local coordinates: $\{u\} = [u\ v\ w]^T$ is approximated by the displacement functions at its two nodal lines and selected transverse shape functions (i.e., in the x direction). For the in-plane, or membrane, displacements u and v linear shape functions are employed. Out-of-plane displacement, w, is approximated by a cubic polynomial in accordance to the Classical Plate Theory of plates. Using energy methods, the linear stability eigenproblem can be obtained (Lau and Hancock 1985)

$$([K] - \lambda[G])\{\Delta\} = \{0\} \tag{2}$$

where $[K]$ and $[G]$ are the stiffness and stability matrices for the overall system, respectively. They are obtained by assembling the elementary strip matrices. These matrices are first formulated in a local strip coordinate system, and then transformed to the global coordinate system (for more details see Lau and Hancock 1985). $\{\Delta\}$ is a vector of all degrees of freedom defined in the global coordinate system. It is described as a vector composed of the degrees of freedom of the "n" nodal lines

$$\{\Delta\} = \langle \{d_1\}^T\ \{d_2\}^T\ \ldots\ \{d_q\}^T\ \ldots\ \{d_n\}^T \rangle^T \tag{3}$$

Each nodal line "q" has four vectors; each one describes one displacement type

$$\{d_q\} = \langle \{u_q\}^T\ \{v_q\}^T\ \{w_q\}^T\ \{\theta_q\}^T \rangle^T \tag{4}$$

Each one has $m + 3$ components, the amplitudes of the displacement at the $m + 3$ section knots

$$\{v_q\}^T = \langle v_{-1}\ v_0\ \ldots\ v_k\ \ldots\ v_{m+1} \rangle^T \tag{5}$$

It is now established that the spline finite strip method is, with finite element method, the numerical method that offers the largest number of possibilities for studying the buckling of thin walled structures. It is sometimes recommended for practical use as in the Australian Standard AS1538 (SAA 1988).

2.2 The SFSM for built-up columns

The aim of this work is to extend the SFSM for the stability analysis of built-up columns. The main idea is to incorporate beam elements into the SFSM in order to be able to model the batten plates or the lacing bars of built-up columns.

A classical Bernoulli finite element beam has two end nodes, i and j, with 6 DOF per node, three translations and three rotations (see Fig. 5) giving a displacement vector $\{\delta_b^L\}$ with 12 components. Its stiffness matrix $[K_b^L]$ is widely given in many finite element (Bathe 1982). The superscript L denotes local coordinate system while the subscript b denotes the beam element.

In the proposed method, the beam finite element may have any orientation in space. The only condition is that the connection with the finite strip must be at nodal lines (see Fig. 6). Then, the joint i must be on a nodal line, lets say q, at a coordinate Z_i; its two other coordinates are given by those of the nodal line: X_q and Y_q. The second node j is on the nodal line r at Z_j position. Its other coordinates are X_r and Y_r. At this level it should be noticed that one can have the two beam joints on the same nodal line and that one can have a beam joint on a common nodal line to many strips.

With the coordinates of the two nodes i and j, plus a parameter defining the orientation of one of the two principal axis of the beam section, one can easily define a (12 × 12) rotation matrix $[R]$ necessary to transform the DOF and the elementary matrix of the beam element from local to global

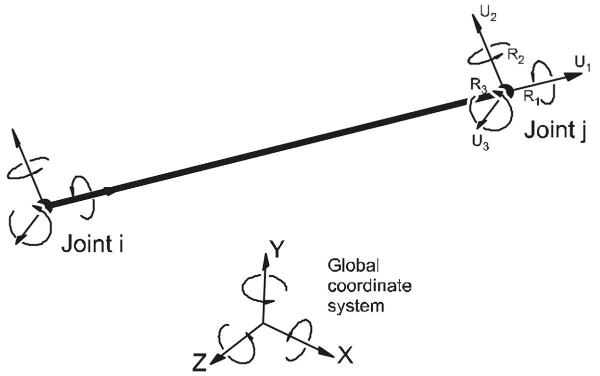

Fig. 5 A beam finite element with local and global coordinate systems

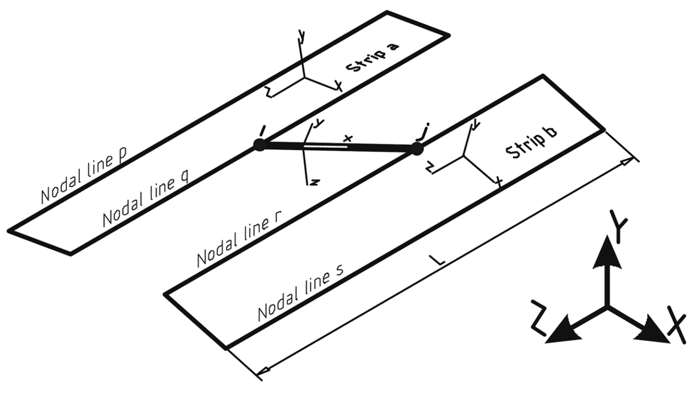

Fig. 6 Two strips connected by a beam element

coordinate systems (Bathe 1982) as follows

$$\{\delta_b^L\} = [R]\{\delta_b^G\} \tag{6}$$

$$[K_b^G] = [R]^T[K_b^L][R] \tag{7}$$

where $[K_b^G]$ is the beam element stiffness matrix in the global coordinate system. $\{\delta_b^G\}$ is the beam DOF vector in the global coordinate system; it contains the DOF of nodes i and j

$$\{\delta_b^G\} = \langle \{\delta_i\}^T \ \{\delta_j\}^T \rangle^T \tag{8a}$$

Those of node i are

$$\{\delta_i\} = \langle U_i \ V_i \ W_i \ \theta x_i \ \theta y_i \ \theta z_i \rangle^T \tag{8b}$$

The beam element stiffness matrix (Eq. (7)) cannot be directly assembled to the global system of Eq. (2). The vector $\{\Delta\}$ contains exclusive amplitudes of the B3-spline functions defining the four displacement functions per nodal line.

The objective is then to find a transformation allowing the passage from the beam DOF to those in the $\{\Delta\}$ vector. The idea is to use the compatibility equations: the displacements of node i are equal to those of nodal line q at Z_i coordinate. For the four displacements of the SFSM, one can directly write the four following equations

$$U_i = U_q(Zi), \ V_i = V_q(Zi), \ W_i = W_q(Zi), \ \theta z_i = \theta z_q(Zi) \tag{9}$$

The two remaining DOF of joint i, the rotations about X and Y axes, can be found from the derivatives of the displacement functions of the nodal line q, which have C^2 continuity, as follows

$$\theta x_i = -\left.\frac{dV_q(Z)}{dZ}\right|_{Z = Zi} \quad \text{and} \quad \theta y_i = \left.\frac{dU_q(Z)}{dZ}\right|_{Z = Zi} \tag{10}$$

Using Eq. (1) and the notations of Eqs. (4), (5) and (8), it is possible to write Eqs. (9) and (10) in matrix form as follows

$$\begin{Bmatrix} U_i \\ V_i \\ W_i \\ \theta x_i \\ \theta y_i \\ \theta z_i \end{Bmatrix} = \begin{bmatrix} \langle \phi_u(Zi)\rangle_q & & & \\ & \langle \phi_v(Zi)\rangle_q & & \\ & & \langle \phi_w(Zi)\rangle_q & \\ & -\langle \phi_v'(Zi)\rangle_q & & \\ \langle \phi_u'(Zi)\rangle_q & & & \\ & & & \langle \phi_\theta(Zi)\rangle_q \end{bmatrix} \begin{Bmatrix} \{u_q\} \\ \{v_q\} \\ \{w_q\} \\ \{\theta_q\} \end{Bmatrix} \tag{11}$$

The elements $\langle \phi_u(Zi)\rangle_q$, $\langle \phi_v(Zi)\rangle_q$, $\langle \phi_w(Zi)\rangle_q$, $\langle \phi_\theta(Zi)\rangle_q$ of the matrix are line vectors composed of $m + 3$ values of the B3-spline functions evaluated at the position Z_i of nodal line q, as defined by the following equation

$$\langle \phi(Zi) \rangle = \langle \phi_{-1}(Zi) \ \ \phi_0(Zi) \ \dots \ \phi_k(Zi) \ \dots \ \phi_{m+1}(Zi) \rangle \tag{12}$$

The vectors $\langle \phi_u'(Zi) \rangle_q$ and $\langle \phi_v'(Zi) \rangle_q$ contain the first derivatives of the B3-spline functions at the coordinate Z_i of the nodal line q.

Then, the 6 displacements of node i are related to the $4(m+3)$ DOF of nodal line q by the Eq. (11), which can be written in a condensed form

$$\{\delta_i\} = [Tr_{iq}]\{d_q\} \tag{13}$$

A similar equation can be obtained between the displacements of node j and the DOF of the nodal line r where it is fixed at coordinate Z_j

$$\{\delta_j\} = [Tr_{jr}]\{d_r\} \tag{14}$$

Eqs. (13) and (14) give a general relation between the 12 DOF of the beam element and degrees of freedom of the spline finite strip model, those of nodal lines q and r

$$\begin{Bmatrix} \{\delta_i\} \\ \{\delta_j\} \end{Bmatrix} = \begin{bmatrix} [Tr_{iq}] & \\ & [Tr_{jr}] \end{bmatrix} \begin{Bmatrix} \{d_q\} \\ \{d_r\} \end{Bmatrix} \tag{15a}$$

$$\{\delta_b^G\} = [T]\{d_b^{SFS}\} \tag{15b}$$

The strain energy of the beam element is given by

$$\frac{1}{2}\{\delta_b^G\}^T[K_b^G]\{\delta_b^G\} \tag{16}$$

Using Eq. (15b), this energy can be expressed in DOF of the spline finite strip model, and can then be added to the total energy of strips. Consequently, the stiffness matrix of the beam element in the SFSM model is

$$[K_b^{SFS}] = [T]^T[K_b^G][T] \tag{17}$$

This matrix must be assembled into Eq. (2) at the DOF of the nodal lines q and r. Because of the localized nature of the B3-spline, most of the terms of the transformation matrix are nulls. The computing time can then be reduced by bandwidth minimisation.

Based on the proposed method, a computer program has been developed to calculate the lowest buckling loads using the sub-space technique and then draw the corresponding failure modes (Djafour *et al.* 1999, 2001).

3. Numerical corroboration

The developed program is used for stability analysis of built-up columns. A comparative study is done and results are presented to validate the model. A U-battened built-up column is selected for the study. The analysis is conducted by varying the number of the connecting elements and the length of the column. Results are then compared to those obtained by other existing numerical and/ or theoretical methods. A 5 meter long U built-up column with 9 batten plates was selected as a particular example to explain the adopted approach. For all the analyses the cross section of the built-up column and its boundary conditions remain unchanged.

3.1 Analysis of built-up column and finite strip mesh

The studied column is composed of two equal U sections with a web of 90 mm and two flanges of 30 mm (see Fig. 7). To characterize the behaviour of an isolated U section under a compressive load an analysis by the semi-analytical finite strip method is performed first (Kerdal 1995). In this method, the column's ends are simply supported and instability occurs due to developing harmonic waves in the longitudinal direction. The result of this work is given in Fig. 8 showing the variation of the critical stresses and their corresponding shapes versus the harmonic half-wave length. Under the action of compressive load, the channel section buckles locally into a number of half-waves, for

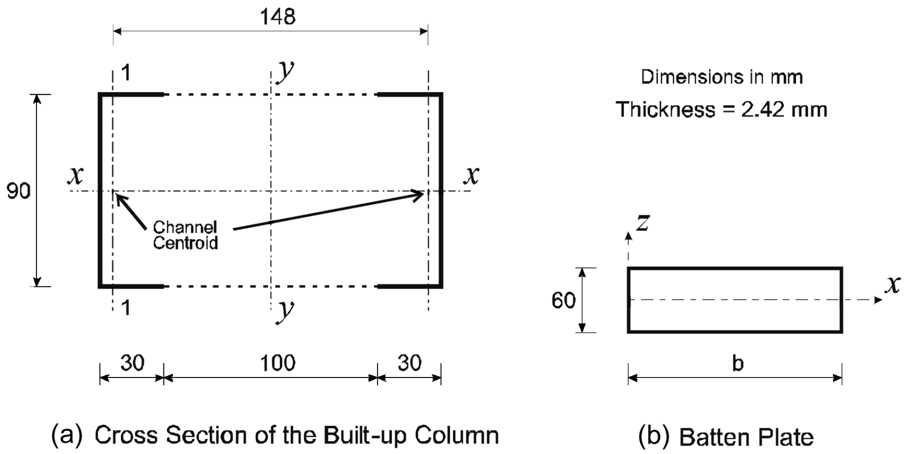

Fig. 7 Geometry of analysed column

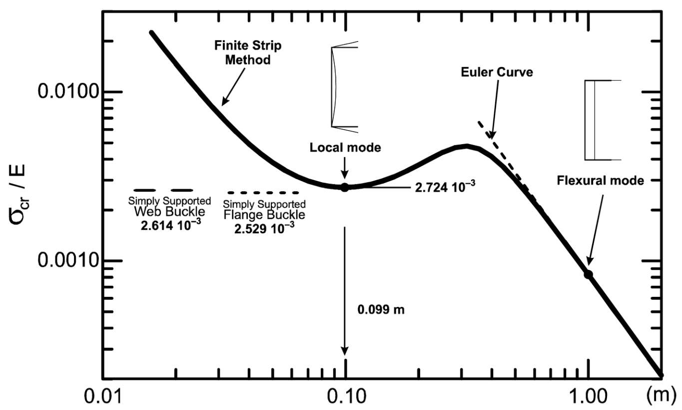

Fig. 8 Single channel: Buckling stress versus half-wavelength

length less than 0.532 m, and globally (Euler mode), for length greater than 1 m. Between these limits, mixed modes can be observed.

To obtain the built-up column, the two channel sections are inter-connected by rectangular battens having a width of 60 mm. The thickness of the chords and the battens is equal to 2.42 mm. The distance between the ends of the flanges is kept constant and equal to 100 mm. The distance between centroidal axes is 148 mm. Fig. 7 gives all the dimensions of the analysed built-up column. Other geometrical data such as cross sections and moments of inertia of the components of the built-up column are given in Table 1. The model used in the analysis is elastic linear with Young's modulus E and a Poisson's ratio of 0.3.

In the spline finite strip method, a big variety of boundary conditions can be modelled (Lau and Hancock 1985). Because most of the built-up columns have stay plates at their ends in order to suppress the supports shear deformation (Paul 1995), they are assumed hinged in the numerical model by restraining U and V displacements in all nodal lines at sections $Z = 0$ and $Z = L$ while

Table 1 Geometrical characteristics of the built-up column

Structural component	Area (mm^2)	Moment of inertia with respect to centroid: Minor (mm^4)	Moment of inertia with respect to centroid: Major (mm^4)
Single channel	363,0	30640,8	441434,7
One batten plate	145,2	70,9	43560,0
Built-up column	726,0	882869,5	4037906,4

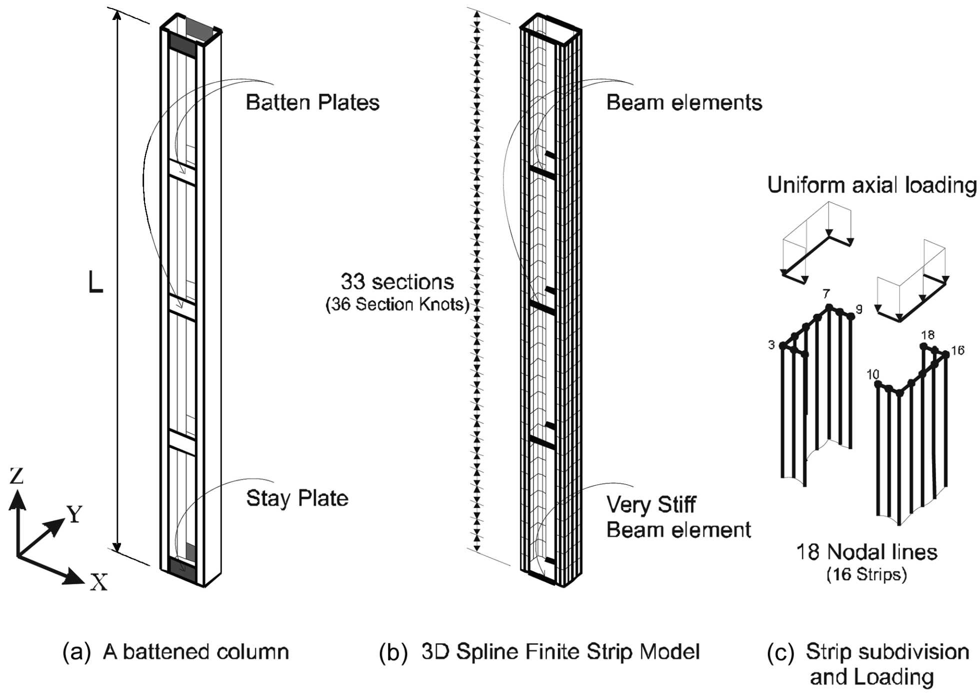

Fig. 9 A typical battened column and its modelisation (this example has "3 batten plates")

displacement W is set free. The stay plates are modelled with very stiff beam elements (10 times the stiffness of batten plates).

Fig. 9 shows the finite strip discretization used for the stability analysis of a U built-up column under compressive load. Each web is divided into 4 strips while each flange is divided into 2 strips giving a total of 16 strips and 18 nodal lines. The length of the column is divided into 33 sections giving a total of 36 section knots. The batten plates are modelled using beam elements. If, for example, the column is divided into four strips as shown in Fig. 9, the 3D model should have 6 beam elements for the batten plates and 4 more stiff elements for the stay plates. For all of the following models the number of batten plates means the number of intermediate batten plate pairs. Thus, Fig. 9 shows a model with "3 batten plates". It necessitates $3 \times 2 + 4$ beam elements.

3.2 Adopted procedures

Table 2 gives the ten lowest critical stresses with their corresponding mode shapes presented in two planes (face and profile views) for a 5 meter long U built-up column and 9 intermediate batten plates. It is clear from the table that modes 1 and 3 correspond to the first two Euler buckling modes in the YZ plane with one and two half-waves, respectively. The values of critical stresses of these two modes are compared with theoretical ones. It is clear from Table 3 that the difference is less than 1%.

The second mode in Table 2 is the flexural buckling in the batten plates plane (plane XZ) causing

Table 2 The first ten buckling modes for a built-up column connected by "9 batten plates" and having $L = 5$ m

Mode number	1	2	3	4	5	6	7	8	9	10
σ_{cr}/E ($\times 10^{-3}$)	0,479	1,538	1,901	2,809	3,042	3,097	3,113	3,218	3,221	3,241

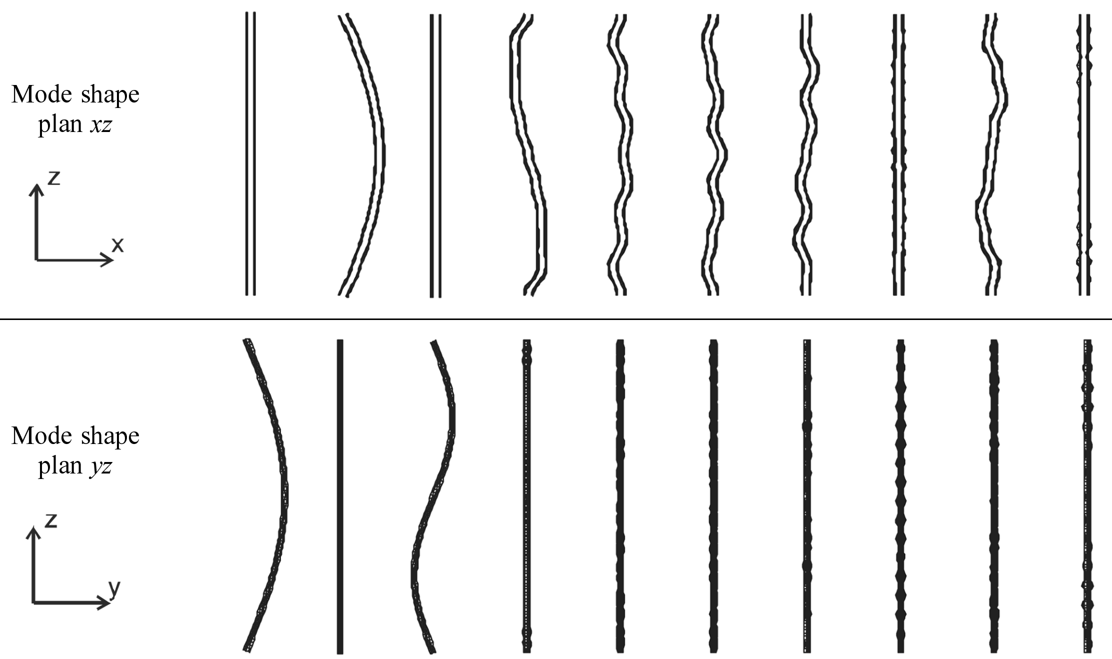

Table 3 The two first Euler buckling loads in *YZ* plan

	$\sigma_{cr,\,num}/E$ ($\times 10^{-3}$)	$\sigma_{cr,\,the}/E$ ($\times 10^{-3}$)	$\sigma_{cr,\,num}/\sigma_{cr,\,the}$
Mode 1	0.479	0.48	0.998
Mode 3	1.901	1.92	0.99

their deformability. The analytical methods (Engesser 1909, Paul 1995) have treated this problem with the assumption that the discontinuous effect of batten plates is replaced by a fictitious continuous web having a shear stiffness *K*. Among various formulas (Timoshenko 1961), the following expression is used

$$\frac{1}{K} = \frac{a \cdot b}{12EI_b} + \frac{a^2}{24EI_c} \tag{18}$$

In this expression, the batten plates are assumed to be Bernoulli beam elements, which conform to the beam element of the developed program. "a" is the batten plates spacing, "b" is the length of the plates taken between the centroidal axes of the chords, "E" is the material Young's modulus, "I_c" and "I_b" are the flexural moments of inertia of the channels and the plates, respectively. In this example, $a = 0.5$ m and $b = 0.148$ m. These numerical values and those from Table 1 are used for theoretical predictions from Engesser's formula and Paul's theory. Moreover, as in the latter method the equivalent web-chord attachment line is not restricted to be at the chord's centroidal lines, a second Paul's prediction is made. The batten plates are assumed to be attached at the ends of the flanges. Thus, their flexible length is equal to 0.1 m. Table 4 gives the critical stresses of the 2nd mode from different theories. The numerical value obtained is between Engesser's value and the second prediction value of Paul.

Modes 8 and 10 in Table 2 correspond to local buckling of the plate elements forming the U shape. A lower bound of the corresponding critical stress can be obtained from the local buckling theory of thin plates, supposing simply supported connections between plate elements (as recommended by most of current design guidelines and codes). A better prediction of this stress can be calculated by a rational elastic buckling analysis of thin walled structures, such as the semi-analytical finite strip method (see Fig. 8). If the local buckling stress obtained by the program ($\sigma_{cr}/E = 3.218 \times 10^{-3}$) is compared with the result of the finite strip method ($\sigma_{cr}/E = 2.724 \times 10^{-3}$) an 18% of difference is found.

The other modes (4, 5, 6, 7 and 9) in Table 2 are more difficult to classify. They almost represent the flexural buckling of chords between the batten plates, interacting, for some of them, with local buckling (for modes 6, 7 and 9). These modes show clearly the discrete action of the 9 batten plates, particularly mode 5 that develops 9 half-waves along the column. Supposing an effective length of $a = 0.5$ m, the Euler prediction for weak-axis flexure (see Table 1) of the chord gives $\sigma_{cr}/E = 3.332 \times 10^{-3}$. This is slightly greater than the numerical predictions for these modes. The

Table 4 The first Euler buckling load in *XZ* plan

	$\sigma_{cr,\,num}/E$ ($\times 10^{-3}$)	$\sigma_{cr,\,Engesser}/E$ ($\times 10^{-3}$)	$\sigma_{cr,\,Paul1}{*}/E$ ($\times 10^{-3}$)	$\sigma_{cr,\,Paul2}{**}/E$ ($\times 10^{-3}$)
Mode 2	1.538	1.327	1.403	1.605

*Paul 1 with "b" calculated between centroidal axes of the chords.
**Paul 2 with "b" calculated from the ends of the flanges of the chords

flexibility of the batten plates increases the effective length.

It is clear that the critical stress is given by the lowest value of the load. However, the knowledge of higher mode responses has some benefits. In a parametrical study, the position of some modes may change. The fact to calculate and draw more than one mode permits to follow, as a function of the parameter, the behaviour of a mode which becomes non critical. In this study, the results of the first ten modes are used in the buckling mode identification.

To identify and classify the modes according to their shapes constitutes the inverse work (simpler) as compared to the one asked of the engineer who should predict all possible instability modes, calculate the critical loads and retain only the smallest one. At this point, the developed program will be a very helpful tool for the engineer.

3.3 Variation of batten plates number

The analysis is first conducted with a constant length of 5 meters and a variable number of batten plates ranging from 1 to 29 in order to validate the proposed model. The built-up column has stay plates at its ends. Different buckling modes have been observed, only four of them (say the classical or pure ones) are considered in this study: the two first flexural modes in the *YZ* plane, the first flexural mode in the *XZ* plane and the local buckling mode. Fig. 10 shows the variation of the stresses corresponding to these four modes with the number of batten plates.

It is clear, from the figure that for the two first flexural buckling modes in the *YZ* plane, the values obtained by the proposed numerical method are almost the same as those obtained by the

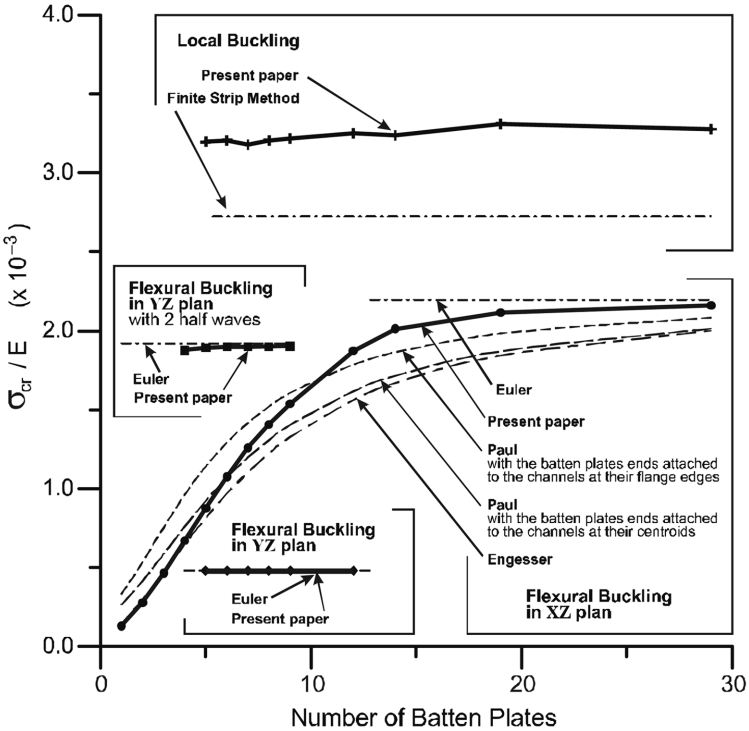

Fig. 10 Comparison of bucking stresses for a battened column (L = 5 m)

Euler theoretical method. For the first flexural buckling mode in the XZ plane (batten plates plane), the obtained results are compared with four other methods where three of them take into account the deformability of the batten plates and the fourth one supposes an Euler type buckling of a solid column. For less than 10 batten plates, the numerical results are within those obtained by Engesser's method, and those obtained by Paul's method where the batten plate's ends are supposed to be attached to the channels at their flange edges. Beyond this point, the numerical values are greater than the results of all other methods and converge rapidly to the Euler value. It is possible to say that for this type of behaviour, the curve obtained by the proposed numerical method is quite good. Also, this type of mode (the first flexural buckling in the XZ plane) has changed its position while the number of batten plates has changed. It was in the first position until 3 battens, it then passed to the second one with 4 to 12 battens, and finally to the third position from 13 battens. This is due exclusively to the flexural buckling modes in the YZ plane i.e., if the 5 meter built-up column is restrained in the YZ plane, the flexural buckling mode in the XZ plane is always critical.

For the local buckling mode, the numerical values obtained by the proposed method are 18% off from the estimation of the semi-analytical finite strip method (see Fig. 8). However, it is well known that if a plate is very long, the critical stress and the local buckling wave length are constants and independents of the boundary conditions in the longitudinal direction (Timoshenko 1961). In other words, since the distance between batten plates is, for all examples, relatively high compared to the channels' dimensions, the program should give a value near the one obtained by the finite strip method ($\sigma_{cr}/E = 2.724 \times 10^{-3}$).

The same work as presented above has been performed for two more lengths of the built-up

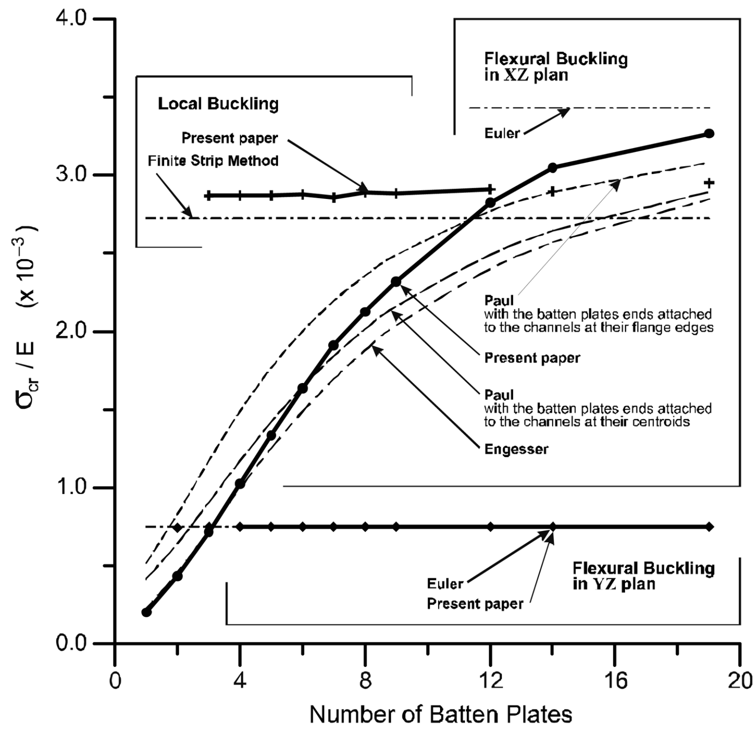

Fig. 11 Comparison of bucking stresses for a battened column ($L = 4$ m)

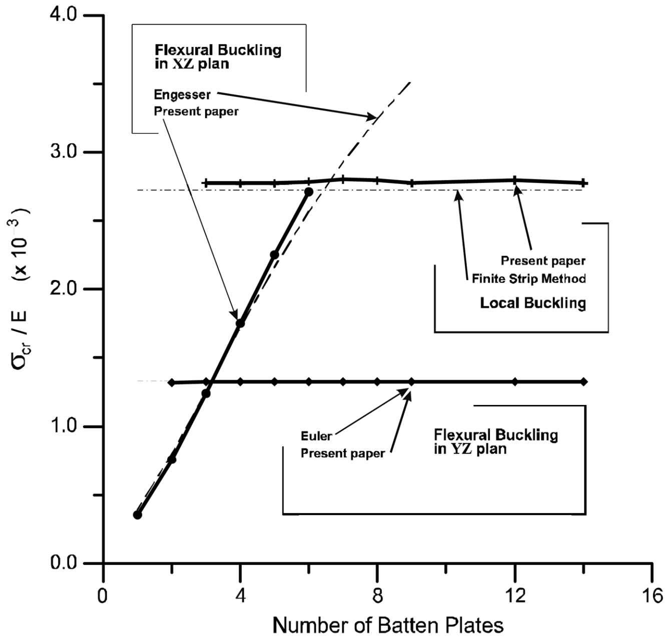

Fig. 12 Comparison of bucking stresses for a battened column (L = 3 m)

column. Figs. 11 and 12 show the variation of the four stresses (corresponding to the four considered buckling modes) with the number of batten plates for a 4 meter and 3 meter column length, respectively. It is clear that almost the same conclusions and remarks can be drawn as for the 5 meter length, except that it was difficult to monitor the buckling stress in the *XZ* plane beyond the local buckling value. This is due to the great multiplicity of local buckling eigenvalues. In fact, for longer plates, different instability shapes corresponding to different numbers of half-waves may have very close local buckling stress values.

One can also conclude from Figs. 10 through 12 that when the length of the built-up column decreases, the local buckling stress obtained by the proposed numerical method will converge to the value calculated by the semi-analytical finite strip method. This can have two explanations. First, for longer columns, the local buckling mode occupies a higher order (8th position for a 5 meter column), whereas for a shorter column it can occupy the first position. It is well known that in an eigenproblem, the precision of an eigenvalue decreases with its order. The second explanation is due to the number of sections used in the spline finite strip model. This number is maintained constant (33) for all the lengths studied. The distance between the nodal knots, which defines the finesse of the model, is then proportional to the column length. However, for an accurate representation of the local buckling, one needs to represent the local buckling wave whose length is approximately constant. So, for greater column lengths, it becomes difficult to represent this wave precisely. The semi-analytical finite strip method estimates the local buckling wave length to 0.099 m for the studied U shaped section, while for a 5 meter length the interval between the section knots is 0.152 m. This causes the 18% difference in the results. With a 2 meter built-up columns and 4 batten plates, the proposed numerical method gives $\sigma_{cr}/E = 2.750 \times 10^{-3}$ which represents a difference of less than 1%.

Finally, the proposed model based on the spline finite strip method can be applied to elastic buckling analysis of built-up columns and is considered to be accurate and efficient for the local buckling of short columns and global buckling for slender columns.

4. Conclusions

A new technique based on the spline finite strip method is proposed for the elastic stability analysis of built-up columns. The channels can have any arbitrary cross section and possess any boundary conditions. The method is able to predict the buckling load incorporating all the possible failure modes (local, overall and mixed modes). The numerical results necessitates buckling mode identification which constitutes inverse work as compared to the one asked of the engineer who should predict all possible instability modes, calculate the critical loads and retain only the smallest one. At this point, the developed program will be a very helpful tool for the engineer and to a certain extent, researchers. The obtained results are compared with the predictions of some theoretical equations. Finally, the proposed model is considered to be accurate and efficient in predicting local buckling of short built-up columns and global buckling of slender built-up columns.

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