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Application of artificial neural networks to the response prediction of geometrically nonlinear truss structures

Jin Cheng†

Department of Bridge Engineering, Tongji University, Shanghai, 200092, China

C. S. Cai‡

Department of Civil and Environmental Engineering, 3418H CEBA, Louisiana State University, Baton Rouge, LA 70803, USA

Ru-Cheng Xiao‡†

Department of Bridge Engineering, Tongji University, Shanghai, 200092, China

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Abstract. This paper examines the application of artificial neural networks (ANN) to the response prediction of geometrically nonlinear truss structures. Two types of analysis (deterministic and probabilistic analyses) are considered. A three-layer feed-forward backpropagation network with three input nodes, five hidden layer nodes and two output nodes is firstly developed for the deterministic response analysis. Then a back propagation training algorithm with Bayesian regularization is used to train the network. The trained network is then successfully combined with a direct Monte Carlo Simulation (MCS) to perform a probabilistic response analysis of geometrically nonlinear truss structures. Finally, the proposed ANN is applied to predict the response of a geometrically nonlinear truss structure. It is found that the proposed ANN is very efficient and reasonable in predicting the response of geometrically nonlinear truss structures.

Keywords: artificial neural networks; geometrically nonlinear analysis; truss structures; uncertainties; response.

1. Introduction

Response prediction of geometrically nonlinear structures has been the subject of extensive studies during the past few decades. This problem is of great importance to solve the structural response of long-span and slender structures such as cable-supported bridges. Various methods have been developed for determining the response of these types of structures. These methods may be broadly divided into two categories as: (1) deterministic methods; and (2) probabilistic methods.

† Research Associate Professor, Corresponding author, E-mail: chengjin@tsinghua.org.cn
‡ Associate Professor, E-mail: cscai@lsu.edu
‡† Professor, E-mail: xiaorc@mail.tongji.edu.cn

Deterministic methods are based on the assumption of complete determinacy of structural parameters. This nonlinear finite element method is one of the most common and traditional methods for a deterministic analysis. Extensive reviews of the method are found in Bathe (1982), Crisfield (1991). However, the method involves step-by-step numerical iterations for each response analysis of geometrically nonlinear structures, and therefore requires a relatively long computation time. Manual intervention to the program may be needed since iterations do not converge for some cases.

In reality, however, there are uncertainties in design variables. These uncertainties include geometric properties (cross-sectional properties and dimensions), material mechanical properties (modulus and strength, etc.), load magnitude, distribution, etc. Thus, deterministic methods cannot provide complete information regarding the responses of geometrically nonlinear structures.

Probabilistic methods are appropriate tools for the analysis of structural systems with randomly varying material and/or geometric properties. Three methods have been used to predict the response of geometrically nonlinear structures with parametric uncertainties, namely, the Monte Carlo simulation (MCS), the first-order approximation (FORA), and the response surface method (RSM). Imai and Frangopol (2000) used the first two methods to analyze probabilistically the mean and variance of member axial forces of a truss and a suspension cable. The results show that the first-order approximation method and Monte Carlo simulation are in close agreement. However, the two methods have some drawbacks. First, for accurate results, the Monte Carlo simulation needs numerous repetitions of a deterministic analysis, thus consuming an enormously large amount of computation time. Second, the first-order approximation method needs computations of the response gradients for geometrically nonlinear structures with parametric uncertainties. Unfortunately, the existing deterministic finite element codes available to design engineers cannot compute response gradients. Therefore, to use the method, it is necessary to modify the existing deterministic finite element codes.

RSM is another method for the probabilistic analysis of structures with random system parameters. This method has been developed to perform probabilistic response analysis of geometrically nonlinear structures. Cheng *et al.* (2004) used the RSM for predicting the response of a geometrically nonlinear truss and the flutter reliability of suspension bridges (Cheng *et al.* 2005). The main limitation of the method is that when the number of random variables is increased, the number of deterministic analysis increases greatly, thus making them computationally expensive.

The artificial neural networks method (ANN) can be pursued for the response prediction of both deterministic and nondeterministic structures. The ANN has the following advantages: (1) it is able to learn and generalize from examples and experience to produce meaningful solutions to problems; (2) it is easy to map the relationship between the input and output data without knowing 'a priori' a relationship between those data; (3) it is robust in dealing with noise or incomplete input data; and (4) it can adapt to solutions over time and to compensate for changing circumstances. Due to these advantages, the ANN has found numerous applications in civil and structural engineering. A paper by Adeli (2001) presents a summary of applications of ANN in this area from 1989 to 2000. More recent development of the ANN may be found in papers by Flood *et al.* (2001), El-Kassas *et al.* (2001), Lee (2003), Oreta (2004), Giunta *et al.* (2004), Hemez (2004), Pierce *et al.* (2006), among others. Although much work has been performed in this area, to the authors' knowledge, the application of ANN to the response prediction of either deterministic or nondeterministic geometrically nonlinear truss structures has not been reported.

The purpose of this paper is to examine the feasibility of using the ANN to predict the response

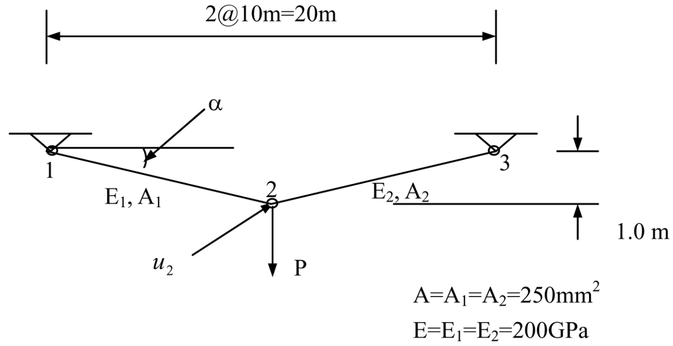

Fig. 1 Two-bar truss

of both deterministic and nondeterministic geometrically nonlinear truss structures. For this purpose, a description of the geometrically nonlinear problem is first given. Secondly, an ANN for deterministic response analysis is developed. The accuracy of the ANN is then compared with the results from nonlinear finite element method. Finally, the developed ANN, combined with the MCS method, is used to predict the response of nondeterministic geometrically nonlinear truss structures.

2. Description of problem

Consider a truss shown in Fig. 1, which was used in Imai and Frangopol (2000). The problem considered here is to determine the truss response in both deterministic and nondeterministic approaches. This can be done by mapping a functional relationship between the response and the various design variables which can be expressed mathematically as

$$u_2 = f_1(E, A, P) \tag{1}$$

$$T = f_2(E, A, P) \tag{2}$$

where u_2 = vertical displacement at Node 2; T = member axial force; f_1, f_2 = unknown approximate functions; and E, A, P = modulus of elasticity, area and load, respectively.

Note that the above design variables (E, A and P) are usually treated as a deterministic quantity for all discretized elements of the structure, thus the structural response is also deterministic. However, in reality there are uncertainties in design variables. As it is well known, due to the system uncertainties caused during the process of measurement, structural element manufacturing and erection of the truss, material properties and geometric parameters of the truss may fluctuate in the vicinity of the nominal values. Therefore, system parameters should be treated as random rather than deterministic variables. Besides, the external loads are also random variables due to their natures and/or insufficient information.

For the sake of simplicity, only three design variables (above-mentioned E, A and P) are chosen as the random variables of interest for this study. Table 1 shows the statistics of these random variables. As the objective of this study is to examine the feasibility of using the ANN to predict the response of both deterministic and nondeterministic geometrically nonlinear truss structures, all

Table 1 Statistical parameters of random variables of a two-bar truss

Variable	μ	σ	Unit	Distribution	Sources
E	200	4	GPa	Normal	Imai and Frangopol (2000)
A	250	5	mm^2	Normal	Assumed
P	18	3.6	kN	Normal	Imai and Frangopol (2000)

random parameters in the present study are based on arbitrary but typical values. On the other hand, since the determination of the correlation of the random parameters is a difficult task, using the independence assumption can greatly simplify the response analysis of nondeterministic truss structures. Therefore, all random parameters in the present study are treated as stochastically independent from each other. Without any loss of generality, three different values (i.e., 0.5, 5.7 and 10.0 degrees) of the angle in Fig. 1 are considered in this study.

3. ANN for deterministic response analysis

ANN is an information processing technique based on the way biological nervous systems, such as the human brain, process information (Adhikary and Mutsuyoshi 2004). This technique as a whole has the capability to respond to input stimuli, produce the corresponding response, and adapt to the changing environment by learning from experience. More detailed description of ANN can be found in Flood and Kartam (1994).

There are a number of ANN paradigms. A multilayer feed-forward backpropagation network, which is one of the well-known and the mostly widely used ANN paradigms, is used in this study. The neural network toolbox available in MATLAB software is utilized to construct the proposed ANN analysis. Some of the important elements of the proposed ANN are briefly discussed in the following sections.

3.1 ANN architecture and training algorithm

The proposed ANN structure consists of an input layer with 3 input nodes, a hidden layer with a different number of hidden nodes and an output layer with 2 output nodes. The ANN with 3 input nodes and 2 output nodes will be referred to as N3 models. For the sake of comparison, three N3 models are considered in this study. They are N3-1-2 (3 input nodes, 1 hidden node, and 2 output nodes), N3-3-2 (3 input nodes, 3 hidden node, and 2 output nodes) and N3-5-2 (3 input nodes, 5 hidden node, and 2 output nodes). Fig. 2 shows the architecture of a N3-3-2 model, in which the left column is the input layer, the right most column is the output layer, and the middle one is the hidden layer. In all N3 models, a hyperbolic tangent sigmoid transfer function $g(z) = 2/(1 + e^{-2z}) - 1$ is used to transfer the values of the input layer nodes to the hidden layer nodes, whereas the linear transfer function $g(z) = z$ is adopted to transfer the values from the hidden layer to the output layer.

The training phase of the N3 models is implemented by using a training algorithm. In this study, a backpropagation training algorithm is used to train all models. To make sure that these models are well trained and have the capability to generalize, Bayesian regularization is implemented in the above backpropagation training algorithm. Bayesian regularization minimizes a linear combination of squared errors on training samples and network weights. It also modifies the linear combination

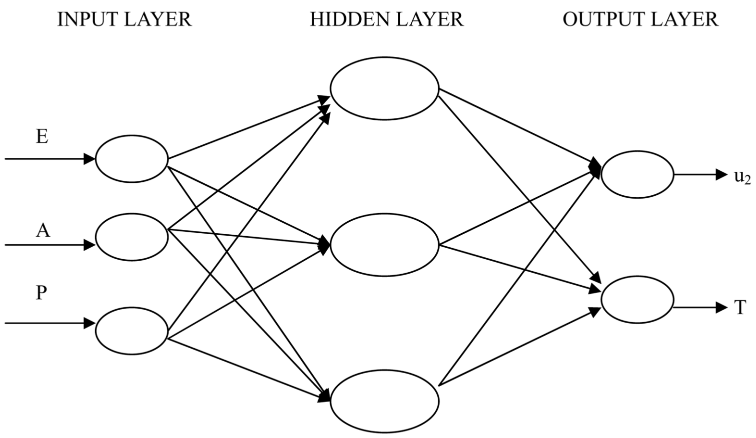

Fig. 2 Architecture of a N3-3-2 model

so that at the end of training the resulting network model has good generalization qualities. A more detailed discussion of the Bayesian regularization can be found in the literature (e.g., MacKay 2005).

The training process of the N3 models is terminated when any of the following conditions are satisfied: the maximum number of iterations (epochs) is reached; the performance gradient falls below a minimum value, or the performance is minimized to the target value.

3.2 Data preparation and processing

To train the above N3 models, 27 sets of training data were generated using a 3^3 full factorial experiment design with 3 levels in E, A and P respectively. Particulars of the levels of each design variable are as follows:

E = {188, 200, 212}(GPa)

A = {235, 250, 265}(mm^2)

P = {7.2, 18.0, 28.8}(kN)

For each point in this design space, a deterministic analysis of the truss was carried out using the nonlinear finite element method. Six new data sets were randomly generated within the above design space. These new data sets (referred to as test data sets) were be used for testing the trained N3 models to verify their prediction ability.

To improve the training process of the model, all training data sets need to be scaled before presenting them to these models. A scaling equation (Alqedra and Ashour 2005) for a design variable P that is limited to the minimum ($P_{\min}$) and maximum ($P_{\max}$) values given in Table 2 was used and is described as $P_{scaled} = 2 \times (P - P_{\min})/(P_{\max} - P_{\min}) - 1$.

Table 2 Values for scaling data

Variable	Minimum	Maximum
E	188 GPa	212 GPa
A	235 mm^2	265 mm^2
P	7.2 kN	28.8 kN

3.3 Evaluation of the ANN performance

Once the above N3 models are trained, the relationship between the response and the various design variables in Eqs. (1) and (2) can be readily retrieved by using the N3 models. The next step is to validate and evaluate the trained N3 models. This can be done by using common error metrics such as the mean absolute error (MAE) or root-mean-squared error (RSME). The two error functions can be expressed as

$$\mathrm{MAE} = \frac{\sum_{i=1}^{n}\sum_{j=1}^{m}|P_{ij}-T_{ij}|}{n\cdot m} \tag{3}$$

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{i=1}^{n}\sum_{j=1}^{m}(P_{ij}-T_{ij})^2}{n\cdot m}} \tag{4}$$

where n = the numbers of patterns in the validation data set (i.e., test data set); m = the number of components in the output vector; P = the output vector from the N3 models; and T = the desired output vector from the nonlinear finite element method. It should be noted that the above-mentioned scaling equation is used in computing the MAE and RMSE.

4. ANN-based MCS method for probabilistic response analysis

Direct MCS is a commonly used traditional method for a probabilistic response analysis. Extensive reviews of the method are found in Melchers (1999), Haldar and Mahadevan (2000). In brief, the method uses randomly generated samples of the input variables for each deterministic analysis, and estimates response statistics after numerous repetitions of the deterministic response analysis (Haldar and Mahadevan 2000). The main advantages of the method are: (1) engineers with only a basic working knowledge of probability and statistics can use it; and (2) it always provides correct results when a sufficiently large number of simulation cycles are performed (one simulation cycle represents a deterministic analysis). However, the method has one drawback: it may need an enormously large amount of computation time. In this paper, an ANN-based MCS method is used to overcome this drawback. The procedures of the method can be summarized as follows:

(1) Construct a database of deterministic analysis results (27 training data sets and 6 test data sets).
(2) Determine the architecture of the ANN model and training algorithm.
(3) Train the ANN model with the training data sets.
(4) Evaluate and validate the ANN model with the test data sets.
(5) Based on the trained ANN model, apply direct MCS to obtain the probabilistic results for structural response.

5. Results and discussion

5.1 Performance of N3 models

Table 3 shows the comparison of the performance of the N3 models for MAE and RMSE, both for the training and test data. It can be seen that the N3-5-2 model has the least error for both MAE and RMSE.

When the output data of the N3 model are processed or converted to their physical attributes, the maximum, mean, and minimum prediction errors (i.e., the difference between the predicted values and the nonlinear finite element analysis results) of the N3 models for the training and test data for u_2 and T may be compared as listed in Tables 4 and 5, respectively. The N3-5-2 model performs better than the other two models based on the maximum and mean values of the errors of the predicted u_2 and T, and it is thus selected for further validation.

Figs. 3 and 4 compared the predictions of the N 3-5-2 model and the nonlinear finite element method for the training and test data for u_2 and T, respectively. As shown, a good correlation can be observed between the predictions of the N 3-5-2 model and the nonlinear finite element analysis

Table 3 Summary of mean errors of N3 models

Angle α (degrees)	ANN model	Training data		Test data	
		MAE	RMSE	MAE	RMSE
0.5	N 3-1-2	0.061	0.0762	0.069	0.0806
	N 3-3-2	0.0011	0.0015	0.0676	0.0825
	N 3-5-2	9.7987e-4	0.0013	0.0472	0.0566
5.7	N 3-1-2	0.0453	0.0632	0.0534	0.0640
	N 3-3-2	0.0013	0.0019	0.0322	0.0387
	N 3-5-2	0.0013	0.0019	0.0284	0.0346
10.0	N 3-1-2	0.0414	0.0624	0.0462	0.0557
	N 3-3-2	6.9657e-4	0.0012	0.0442	0.0510
	N 3-5-2	6.4861e-4	0.0012	0.0245	0.0287

Table 4 Summary of prediction errors of N3 models for training data

Angle α (degrees)	ANN model	u_2 (mm)			T (kN)		
		Maximum	Mean	Minimum	Maximum	Mean	Minimum
0.5	N 3-1-2	33.9	11.8	2.7	9.0744	3.1237	0.4955
	N 3-3-2	0.7	0.2556	0.0	0.1511	0.0453	0.0019
	N 3-5-2	0.7	0.2296	0.0	0.1085	0.0350	6.0e-4
5.7	N 3-1-2	17.9	4.8	0.4	5.096	1.6468	0.024
	N 3-3-2	0.4	0.2037	0.1	0.0663	0.019	0.0034
	N 3-5-2	0.4	0.2037	0.0	0.0566	0.016	0.0024
10.0	N 3-1-2	8.9	1.967	0.0	3.1862	1.0325	0.0254
	N 3-3-2	0.2	0.0407	0.0	0.0222	0.0060	3.0e-4
	N 3-5-2	0.2	0.0407	0.0	0.0094	0.0034	6.0e-4

Table 5 Summary of prediction errors of N3 models for test data

Angle α (degrees)	ANN model	u_2 (mm)			T (kN)		
		Maximum	Mean	Minimum	Maximum	Mean	Minimum
0.5	N 3-1-2	21.1	11.75	1.0	7.8126	4.0744	1.7129
	N 3-3-2	25.0	12.167	3.0	7.982	3.78	0.578
	N 3-5-2	19.0	9.833	2.0	4.2	2.2197	0.364
5.7	N 3-1-2	7.8	4.2	0.1	5.3634	2.6461	0.6413
	N 3-3-2	6.0	2.667	0.0	2.974	1.584	0.299
	N 3-5-2	5.0	2.667	0.0	2.492	1.356	0.255
10.0	N 3-1-2	3.3	1.667	0.5	3.2622	1.5499	0.1675
	N 3-3-2	3.0	1.50	0.0	2.541	1.484	0.308
	N 3-5-2	1.0	0.667	0.0	1.507	0.8702	0.177

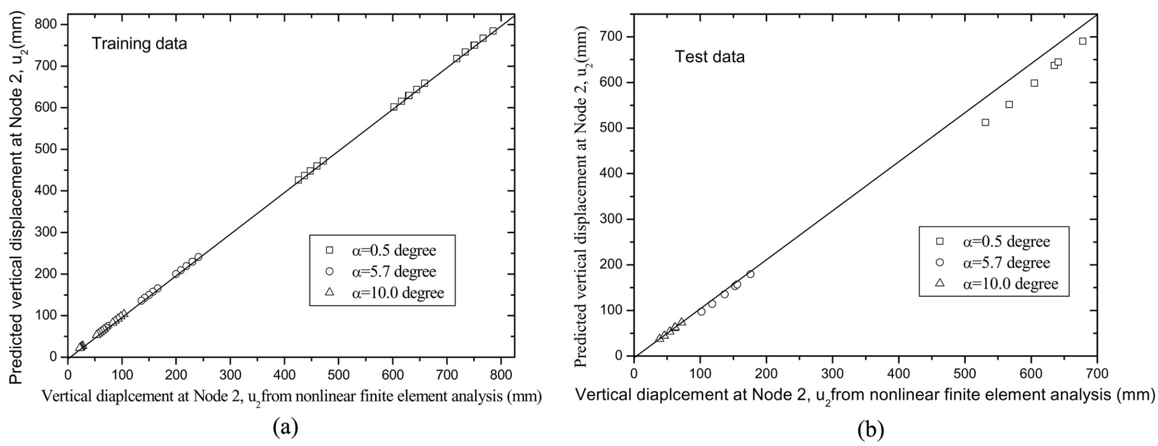

Fig. 3 Comparison between N3-5-2 model and nonlinear finite element analysis, u_2 using: (a) training data, (b) test data

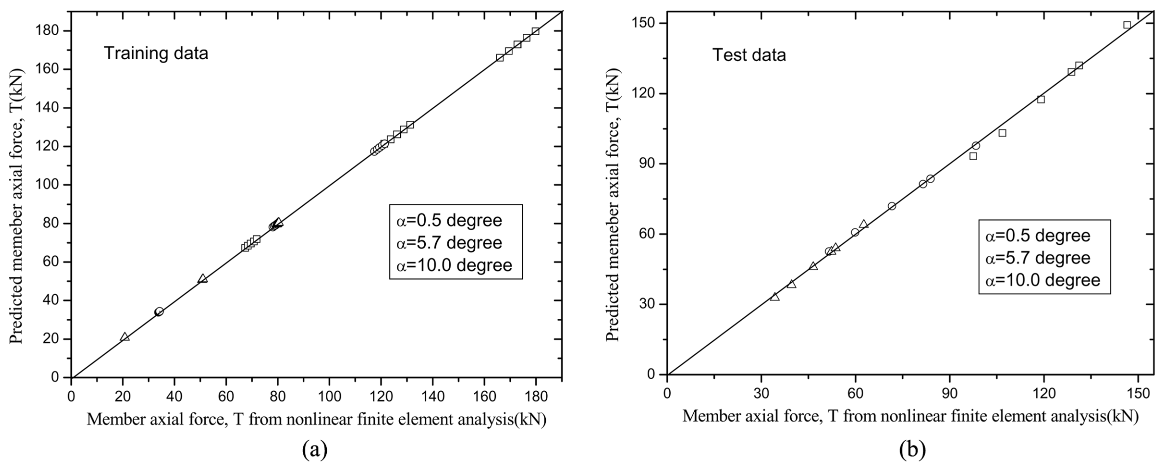

Fig. 4 Comparison between N3-5-2 model and nonlinear finite element analysis, T using: (a) training data, (b) test data

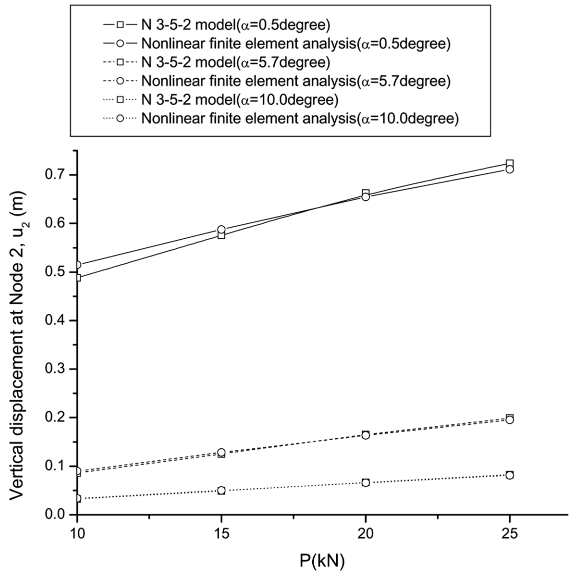

Fig. 5 Comparison between N3-5-2 model and nonlinear finite element analysis-u_2 versus P

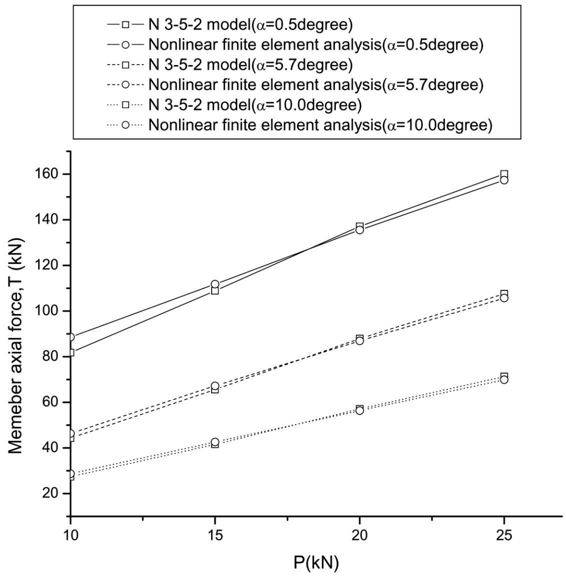

Fig. 6 Comparison between N3-5-2 model and nonlinear finite element analysis-T versus P

results in the majority of the cases. This good correlation implies that the N3-5-2 model was successful in capturing the underlying relationship between the structural response and different parameters used in the model.

To validate the performance of the N3-5-2 model in simulating the behavior of physical processes, a parametric study is performed by simply varying one input parameter at a time (e.g., load parameter P), while all other input parameters (e.g., E and A) are set to constant values. The u_2 and T predictions of the N3-5-2 model as compared to the nonlinear finite element analysis results are shown in Figs. 5 and 6, respectively. From these figures it can be seen that the N3-5-2 model prediction is almost the same as that of the nonlinear finite element analysis.

It is concluded from these comparisons that the N3-5-2 model can confidently be used as a substitute of the nonlinear finite element analysis method. Hence the N3-5-2 model is considered as an acceptable ANN model and will be combined with a direct MCS method to predict the response of nondeterministic geometrically nonlinear truss structures. It should be noted that although N3-5-2 model provides the best performance, it does not mean that this is the well-optimized configuration if one considers computation and training penalties. Also, different structures may require different sets of configuration.

5.2 Probabilistic response analysis

The MCS combined with the nonlinear finite element analysis (FEM-based MCS, simply call direct MCS) is conducted for probabilistic response analysis of the truss in Fig. 1. As described in the previous sections, the direct MCS has one drawback: it needs an enormously large amount of computation time. To overcome the drawback of the direct MCS, we use Monte Carlo simulation together with an ANN model. The ANN is used to construct approximation response functions. The

Table 6 Comparisons of different methods for the means (μ) and standard deviations (σ) for u_2

α (Degrees)	ANN-based MCS		MCS	
	μ (m)	σ (m)	μ (m)	σ (m)
0.5	0.6253	0.0577	0.6266	0.0485
5.7	0.1490	0.0276	0.1490	0.0253
10.0	0.0595	0.0122	0.0594	0.0114

Table 7 Comparisons of different methods for the means (μ) and standard deviations (σ) for T

α (Degrees)	ANN-based MCS		MCS	
	μ (m)	σ (m)	μ (m)	σ (m)
0.5	125.396	19.010	125.858	16.942
5.7	78.974	15.325	78.945	14.146
10.0	50.965	10.654	50.902	9.858

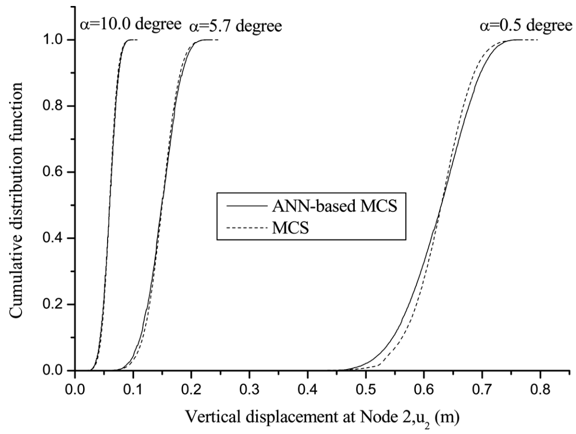

Fig. 7 Cumulative distribution function of u_2-comparison between ANN-based MCS and MCS

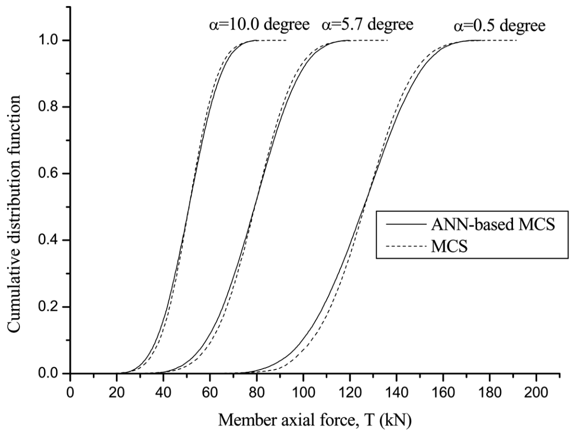

Fig. 8 Cumulative distribution function of T-comparison between ANN-based MCS and MCS

proposed method will be called as ANN-based MCS. Because the fluctuation of the means (μ) and standard deviations (σ) of the structural responses (u_2 and T) is negligibly small after 50,000 simulations, the number of simulations is fixed at 50,000. The means (μ) and standard deviations (σ) for u_2 and T obtained with both direct MCS and the proposed method are listed in Tables 6 and 7, respectively. Figs. 7 and 8 show the comparison between the cumulative distribution function obtained by the direct MCS and the proposed method for u_2 and T, respectively. From these tables and figures it can be seen that: (1) the mean and standard deviations of the structural respones (u_2 and T) from the proposed method are very similar to the results from direct MCS, indicating that the proposed method for predicting the response of the nondeterministic truss is adequate from a practical point of view; (2) the value of the angle α has a significant effect on the mean and standard deviations of the structural responses (u_2 and T). The mean and standard deviations of the structural responses (u_2 and T) decreases with the increase in the value of the angle α. A similar conclusion can be found in Imai and Frangopol (2000) and (3) once the approximation response function is found, we can use it directly instead of conducting deterministic response analysis. Performing a deterministic response analysis may require several minutes or hours of computation time; whereas the evaluation of such a response function requires only a fraction of a second. Hence, the computing time can be saved greatly, particularly when a deterministic response analysis requires a large amount of computation time or the number of response calculations is large.

6. Conclusions

This paper presents an application of artificial neural networks (ANN) to the response prediction of both deterministic and nondeterministic geometrically nonlinear truss structures. A three layer feed-forward backpropagation network with three input nodes, five hidden layer nodes, and two output nodes was firstly developed for the deterministic response analysis. A back propagation training algorithm with Bayesian regularization is used to train the ANN. The predicted structural responses obtained by utilizing the trained ANN were compared with the nonlinear finite element analysis results. The results of the ANN show good agreement with the nonlinear finite element analysis results. The proposed ANN approach has the following advantages: (1) it is easy to map the relationship between the input and output data without knowing 'a priori' a relationship between those data; and (2) it reduces the overall time required for implementations by a significant amount when compared with the existing nonlinear finite element method. This is due to the fact that the proposed ANN is used for the solution of all kinds of problems instead of step-by-step numerical iteration procedures, typically in the nonlinear finite element method.

The proposed ANN is then successfully combined with a direct MCS to predict the response of nondeterministic geometrically nonlinear truss structures. The ANN-based MCS algorithm gives similar response statistics compared to the response statistics for the FEM-based MCS (direct MCS). Also, the algorithm can practically eliminate any limitation on the scale of the problem and the sample size used for the direct MCS, provided that the predicted response statistics fall within acceptable tolerances.

In conclusion, ANN can be effectively used as a supplementary technique to conventional numerical procedures in response prediction of geometrically nonlinear truss structures. While the use of the proposed ANN should be limited within the range of the input parameters covered in this study, the proposed ANN can always be updated to obtain better results by presenting new training patterns as new data become available. However more research is required to address the limitations and the restrictions of the proposed ANN. For instance, how to obtain or how to intelligently select training data is always an open problem with the proposed ANN. Also, the intensive computation (especially for certain applications in the training stage) is possibly one limitation of the proposed ANN.

Although emphasis in this study was placed on the response prediction of geometrically nonlinear truss structures, the proposed ANN algorithm offers immediate applications to other geometrically nonlinear structures, such as cable-supported bridges.

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