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Optimal stacking sequence of laminated anisotropic cylindrical panel using genetic algorithm

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Abstract. This paper presents stacking sequence optimization of laminated angle-ply cylindrical panel based on natural frequency. Finite element method (FEM) is used to obtain the vibration characteristic of an anisotropic panel using the first order shear deformation theory(FSDT) and genetic algorithm (GA) is used to obtain the optimal stacking sequence of the layers. Cylindrical panel has finite length and arbitrary boundary conditions. The thicknesses of the layers are assumed constant and their angles are specified as design variables. The effect of the number of plies and boundary conditions in the fitness function is considered. Numerical examples are presented for four, six and eight layered anisotropic cylindrical panels.

Keywords: composite laminate; optimal design; genetic algorithm; panel; vibration.

1. Introduction

Laminated cylindrical panels are widely used in industries as structural elements, and their vibration characteristics are important in view of the current interest in designing with composite materials. Maximum frequency problems are of practical importance in the design of laminates against resonance due to external excitation. The frequency of an external excitation can be placed either between zero and the fundamental frequency or in a gap between two consecutive higher-order frequencies depending on its magnitude. In the case of a discrete set of ply angles, the optimal stacking sequence is to be determined such that the fundamental frequency or the frequency separation is maximized. The greatest advantage of laminated composite materials, in addition to high strength-to- weight properties is that they provide the designer with the ability to tailor the directional strengths and stiffnesses of a material to a given loading environment of the structure. Therefore, laminated composite structures provide ample opportunities for engineers and designers

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to optimize structures for a particular or even multiple objectives. The problems are often formulated as a continuous optimization problem with the thickness and orientation of plies, as design variable (Schmit and Farshi 1979), but for most particular problems, the thickness of the layers are fixed and orientations are limited to a set of angles, thus the design problem becomes a stacking sequence optimization. The design space usually contains many local extremum, with a number being singular. Moreover, many near optimum designs may exist. Thus, there is a need for optimization techniques that can identify multiple and singular extrema. Genetic algorithms (GAs) can be used to find the global solution of discrete optimization problems and simulate the mechanics of natural genetics for artificial systems based on operations that are the counterparts of the natural type (Goldberg 1993). GAs are fundamentally different from traditional search techniques, seeking an optimal solution through random probability methods without auxiliary information such as derivatives or an intelligently chosen starting point. In this method when a population of biological species evolves over generations, characteristics that are useful for survival tend to be passed on to future generations, as individuals carrying them acquire more chances to breed. Individual characteristics in biological populations are stored in chromosomal strings (Goldberg 1993).

Various authors have investigated the use of GAs for optimizing composite structures. A topological design of structural components using GA was studied by Sandgren and Jensen (1990). In the area of composite structural design, GAs are used to optimize the stacking sequence of laminated plates for buckling load (Riche and Haftkaa 1993). A minimum thickness design for plates with discrete ply angles subject to strength and buckling constraints was considered by Kogiso (1994), where a genetic algorithm search technique was used to achieve the optimal design. Optimum designs of laminates under various boundary conditions were investigated by Kim *et al.* (1997). Tabakov studied the multidimensional design optimization of laminated structures (Tabakov 2001), while Sivakumar researched the optimum design of laminated composite plates (Sivakumar 1998). The otimization of hybrid thick-walled cylindrical shells under external pressures was presented by Byon (1998). Genetic algorithm was used to design stiffened composite panels by Nagendra *et al.* (1996). Soremekun *et al*. (2002) used GAs to blend the stacking sequence of multiple composite laminates while Park *et al.* (2001) investigated the stacking sequence design of composite laminates for maximum strength using genetic algorithm. Chen and Karunaratne (2002) optimized the stacking sequence design of composite laminates using GAs whereas Sadagopan and Pitchumani (1998) discussed the application of GAs to the optimal tailoring of composite materials. Recently, the authors presented the optimum design of laminated cylindrical shell using GAs (Shakeri *et al.* 2005). In this investigation the maximum first natural frequency was obtained analytically and the influence of effective parameters in convergence was discussed.

In the present paper, the genetic algorithms technique and finite element analysis are used to maximize the fundamental natural frequency with several discrete design variables in laminated angle ply cylindrical panels. In the optimization problem, the layer thickness are constant and orientations are changed in a set of angles. A laminated cylindrical panel is considered anisotropic with a finite length. Different combinations of free (F), simply supported (S) and clamped (C) boundary conditions are implemented at the four edges of the panel. Free vibration analysis is based on the first order shear deformation theory.

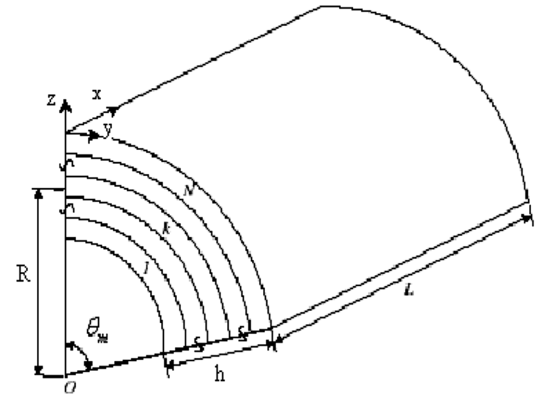


Fig. 1 Geometry and coordinate system of laminated panel

2. Free vibration analysis

Vibration characteristic of multi-layered cylindrical panel (Fig. 1) such as fundamental frequency was investigated through the use of the finite element method. For this, the first order shear deformation theory (FSDT) is used. According to the FSDT, in-plane and transverse displacements for the K_{th} layer are assumed as follows (Ganapathi *et al.* 2004)

$$
\begin{aligned}
u^k(x, y, z, t) &= u_0(x, y, t) + z\theta_x(x, y, t) \\
v^k(x, y, z, t) &= v_0(x, y, t) + z\theta_y(x, y, t) \\
w^k(x, y, z, t) &= w_0(x, y, t)
\end{aligned}
\tag{1}
$$

where u_0, v_0, w_0 are the displacements of points on the reference surface and θ_x, θ_y are the rotations of normal to the reference surface about the x- and y-axes, respectively.

The strain vector in terms of middle-surface deformation and rotations of normal for K_{th} layer are

$$
\{\varepsilon\} = \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{Bmatrix} u^k_{,x} \\ (v^k_{,y} + w^k/R)/(1+z/R) \\ v^k_{,x} + u^k_{,y}/(1+z/R) \\ u^k_{,z} + w^k_{,x} \\ v^k_{,z} + (w^k_{,y} - v^k/R)/(1+z/R) \end{Bmatrix}
\tag{2}
$$

Using Eqs. (1) and (2) can be written as

$$
\{\varepsilon\} = [\bar{Z}]\{\bar{\varepsilon}\} = [\bar{Z}]\{\varepsilon_1 \;\; \varepsilon_2 \;\; \varepsilon_3\}^T
\tag{3}
$$

where

$$[\bar{Z}] = \begin{bmatrix} 1 & 0 & 0 & 0 & z & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{1+z/R} & 0 & 0 & 0 & \frac{z}{1+z/R} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{1+z/R} & 1 & 0 & 0 & \frac{z}{1+z/R} & z & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \frac{1}{1+z/R} & \frac{z}{1+z/R} \end{bmatrix}$$

$$\{\varepsilon_1\} = \begin{Bmatrix} u_{0,x} \\ v_{0,y} + w_0/R \\ u_{0,y} \\ v_{0,x} \end{Bmatrix} \quad \{\varepsilon_2\} = \begin{Bmatrix} \theta_{x,x} \\ \theta_{y,y} \\ \theta_{x,y} \\ \theta_{y,x} \end{Bmatrix} \quad \{\varepsilon_3\} = \begin{Bmatrix} \theta_x + w_{0,x} \\ \theta_y \\ w_{0,y} - v_0/\mathrm{R} \\ -\theta_y/R \end{Bmatrix} \tag{4}$$

The stress-strain relations for the K_{th} layer is expressed as

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{16} & 0 & 0 \\ C_{12} & C_{22} & C_{26} & 0 & 0 \\ C_{16} & C_{26} & C_{66} & 0 & 0 \\ 0 & 0 & 0 & C_{44} & C_{45} \\ 0 & 0 & 0 & C_{45} & C_{55} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{bmatrix} \tag{5}$$

The governing equations for the shell structures are obtained by applying Lagrangian equations of motion given by

$$d/dt[\partial(T-U)/\partial\dot{\delta}_i] - [\partial(T-U)/\partial\delta_i] = 0; \quad i = 1{:}n \tag{6}$$

where $\{\delta\} = \{\delta_1, \delta_2, \ldots, \delta_i, \ldots, \delta_n\}^T$ is the vector of generalized displacements with δ_i independent. A dot over the variables represents the partial derivative with respect to time. The kinetic energy of the panel is given by

$$T(\delta) = \frac{1}{2}\iint\left[\sum_{k=1}^{n}\int_{h_k}^{h_{k+1}}\rho_k\{\dot{u}^k \ \dot{v}^k \ \dot{w}^k\}\{\dot{u}^k \ \dot{v}^k \ \dot{w}^k\}^T\left(1+\frac{z}{R}\right)dz\right]dxdy \tag{7}$$

where h_k, h_{k+1} are the z coordinates of laminate corresponding to the inner and outer surfaces of the K_{th} layer. Using Eqs. (1) and (7) can be rewritten as

$$T(\delta) = \frac{1}{2}\iint\left[\sum_{k=1}^{n}\int_{h_k}^{h_{k+1}}\rho_k\{\dot{d}\}^T[Z]^T[Z]\{\dot{d}\}\left(1+\frac{z}{R}\right)dz\right]dxdy \tag{8}$$

where

$$\{\dot{d}\}^T = \{\dot{u}_0 \;\; \dot{v}_0 \;\; \dot{w}_0 \;\; \dot{\theta}_x \;\; \dot{\theta}_y\}, \quad \text{and} \quad [Z] = \begin{bmatrix} 1 & 0 & 0 & z & 0 \\ 0 & 1 & 0 & 0 & z \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

The strain energy functional U is given by

$$U(\delta) = \frac{1}{2}\iint\left[\sum_{k=1}^{n}\int_{h_k}^{h_{k+1}}\{\sigma\}^T\{\varepsilon\}\left(1+\frac{z}{R}\right)dz\right]dxdy \tag{9}$$

Using Eqs. (3) and (5), Eq. (9) can be written as

$$U(\delta) = \frac{1}{2}\iint\left[\sum_{k=1}^{n}\int_{h_k}^{h_{k+1}}(\{\bar{\varepsilon}\}^T[\bar{Z}]^T[\bar{C}^k][\bar{Z}]\{\bar{\varepsilon}\})\left(1+\frac{z}{R}\right)dz\right]dxdy \tag{10}$$

A Lagrange shell element with nine second-order nodes is used, in which there are five degrees of freedom $\{\delta_i^e\} = \{u_0^i, v_0^i, w_0^i, \theta_x^i, \theta_y^i\}^T$ per node. The interpolation function of displacement field is defined as

$$\{\dot{d}\}_{5\times 1} = [H]_{5\times 45}\{\dot{\delta}^e\}_{45\times 1}; \quad \{\bar{\varepsilon}\}_{12\times 1} = [B]_{12\times 45}\{\delta^e\}_{45\times 1} \tag{11}$$

where $\quad \{\delta^e\} = \{\{\delta_1^e\}^T \; \{\delta_2^e\}^T \; \ldots \; \{\delta_9^e\}^T\}$

The kinetic and strain energy expressions, using Eq. (11), are given respectively as

$$T(\delta^e) = \frac{1}{2}\{\dot{\delta}^e\}^T[M^e]\{\dot{\delta}^e\} \tag{12}$$

$$U(\delta^e) = \frac{1}{2}\{\delta^e\}[K^e]\{\delta^e\} \tag{13}$$

The elemental governing equations, obtained by substituting Eqs. (12) and (13) in Eq. (6), are

$$[M^e]\{\ddot{\delta}^e\} + [K^e]\{\delta^e\} = \{0\} \tag{14}$$

where the elemental mass $[M^e]$ and stiffness $[K^e]$ matrices can be expressed as

$$[M^e] = \iint\left[\sum_{k=1}^{n}\int_{h_k}^{h_{k+1}}\rho_k\{H\}^T[Z]^T[Z]\{H\}\times\left(1+\frac{z}{R}\right)dz\right]dxdy$$

$$[K^e] = \iint\left[\sum_{k=1}^{n}\int_{h_k}^{h_{k+1}}(\{B\}^T[\bar{Z}]^T[\bar{C}^k][\bar{Z}]\{B\})\left(1+\frac{z}{R}\right)dz\right]dxdy \tag{15}$$

The coefficient of mass and stiffness matrices involved in governing Eq. (14) can be rewritten as the product of the term have only a thickness coordinate z and the term containing x and y. In the present study, terms having thickness coordinate z are explicitly integrated whereas the terms containing x and y are evaluated using full integration with the 2×2 points Gauss integration rule.

After assembling the elemental mass and stiffness matrices in a general matrix, Eq. (14) is converted to the following general governing equation

$$[M]^G\{\ddot{\delta}\}^G + [K]^G\{\delta\}^G = \{0\} \tag{16a}$$

or

$$[K]^G\{\delta\}^G = \omega^2[M]^G\{\delta\}^G \tag{16b}$$

Solving Eq. (16b) which is the eigenvalue problem, the natural frequencies 'ω' are obtained.

3. Optimization by genetic algorithm

Genetic algorithm is a search algorithm based on the mechanism of natural selection and natural genetics (Jianqlao and Soldatos 1994). In this approach, one starts with a set of designs (population). From this set, new and better designs are reproduced using the fittest members of the set. Each design must be represented by a finite length string (chromosome). Usually, binary strings have been used for this purpose. In this paper binary decoding is used to increase the convergence rate. In the genetic algorithm the reproduction, crossover and mutation operations are used. The primary objective of the reproduction is to make duplicates of good solutions and eliminate bad solutions in a population. The roulette wheel method is widely used for the implementation of reproduction; for this purpose, the wheel is divided into M (population size) slices, where the size of each is marked in proportion to the fitness of each population member. When the wheel is spun (simulated by using a random number generator between 0 and 1, where the circumference of the wheel is normalized to be 1), those solutions that occupy larger slices of the wheel have a better chance to be chosen as parent designs (Walker and Smith 2003). The tournament method is an alternative for reproduction. In this way string populations are compared with each other, and the string with the high fitness value is chosen and inserted into the parent population instead of the worst one. After reproducing the parent, a cross over operation is used to produce the new population. In almost all crossover operators, two strings are chosen from the population at random and some portions of the strings are exchanged between the strings to create two new strings. In a single-point crossover operator, this is performed by randomly choosing a crossing site along the string and by exchanging all bits on the right side of the crossing site, In two points crossover, two point along the string are selected and the other portions of these two points are replaced with each other. In uniform crossover, some portion along the string is selected randomly and the gens of these two points are replaced with each other. As shown in Fig. 2, in this paper uniform crossover is used. The crossover takes place with probability between 0.7-1.

The need for mutation is to maintain diversity in the population, and prevent the existing population from seeking local maxima. Without mutation operation it is not possible to generate a

Parent 1=0 1 1 0 0 | 0 1 0 Child 1 =0 1 1 0 0 1 1 0
Parent 2=1 1 0 1 1 | 1 1 0 Child 2 =1 1 0 1 1 0 1 0.

Fig. 2 One point crossover

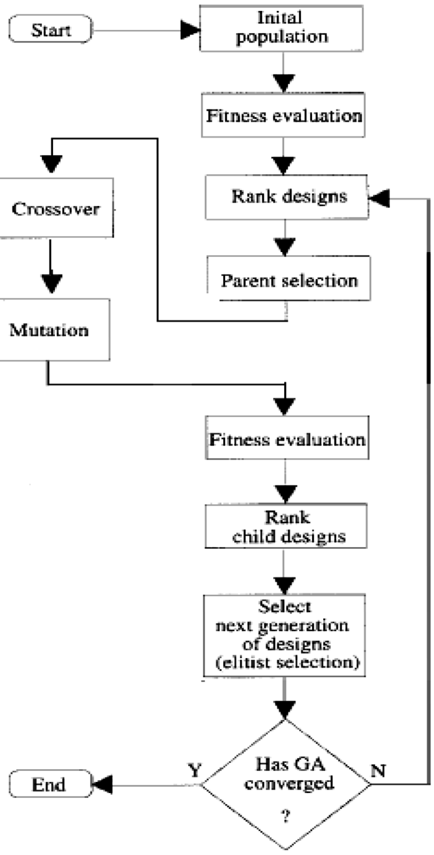


Fig. 3 Genetic algorithm flowchart

new population. Mutation operation selects a position along the string randomly and replaces it with a randomly selected number. The algorithm genetic process is described by the flowchart shown in Fig. 3.

The aim of this study is to maximize the fundamental frequency 'ω' of the laminated composite panel by altering the ply orientation for a given thickness of a layer. The ply orientation angles of the laminates are taken as design variables.

4. Numerical results

An optimum design of a symmetric laminated anisotropic cylindrical panel with simply supported edges using GA was performed. The GA parameters are given in Table 1. The material properties and geometric dimensions of the panel are

$$E_{11} = 132.5 \text{ GPa} \quad E_{22} = E_{33} = 10.8 \text{ GPa} \quad G_{12} = G_{13} = 5.7 \text{ GPa} \quad G_{23} = 3.4 \text{ GPa}$$

$$\nu_{12} = \nu_{13} = 0.24 \quad \rho = 1540 \text{ kg·m}^{-3} \quad R = 1 \quad \theta_m = \frac{\pi}{3} \quad Lb = \frac{L}{R \times \theta_m} = 1$$

Table 1 GA parameters

Population size: 8	Mutation probability : 0.5	Swap probability: 0.8
Selection strategy: Rolette Wheel (elitist)	Crossover strategy: Uniform crossover with 0.5 probability	

Table 2 First natural frequency parameter

h/L	θ_m	[0/90/90/0]		[90/0/0/90]	
		Present	Jianqlao and Soldatos (1994)	Present	Jianqlao and Soldatos (1994)
0.1	30	0.0630	0.0625	0.0615	0.0609
	60	0.0648	0.0646	0.0599	0.0591
	90	0.0680	0.0678	0.0597	0.0589
0.2	30	0.1770	0.1740	0.1744	0.1706
	60	0.1728	0.1708	0.1637	0.1588
	90	0.1692	0.1685	0.1525	0.1472
0.3	30	0.2985	0.2933	0.2985	0.2890
	60	0.2862	0.2837	0.2793	0.2682
	90	0.2706	0.2741	0.2576	0.2455

Governing differential equations are solved using FEM and the first natural frequency is obtained as needed in the GA process. The fundamental frequency and ply angle are the objective function and design variable, respectively. A discrete set of ply angles is specified between −90 and 90 deg. with an increment of five degrees. Therefore, 37 different chances are available for each layer and each lamina is coded with two digits from 01 up to 37. Thus, there are $(90-(-90)/5)^{N/2}$ possible design conditions in the optimum design of a multi-layered panel with N symmetric layers. The optimum stacking sequence will be achieved after 100 iterations of the GA process without any change in the objective function quantity.

Table 2 shows the first natural frequency parameter, $\omega^* = \omega h(\rho/\pi^2 C_{66})^{1/2}$, which is obtained by using the FEM. According to this table, as the thickness-to-length ratio increases, the stiffness of the panel will increase and consequently cause an increase in the fundamental frequency. Increasing the span angle of the panel alone causes a slight increase in the natural frequency. The results of the present discussion are also compared with the analytical results obtained in Uemura and Fukunaga

Table 3 Initial population and the optimum stacking sequence for four-layered panel

Group initial population	Best angles	Best fitness
First	$[80/40]_s$, $[-20/60]_s$, $[-70/-10]_s$, $[25/-20]_s$ $[15/-80]_s$, $[-30/5]_s$, $[-25/-90]_s$, $[-20/45]_s$	4252.1
Second	$[20/45]_s$, $[85/-20]_s$, $[35/-15]_s$, $[-90/30]_s$ $[-85/30]_s$, $[55/20]_s$, $[-70/-45]_s$, $[-90/-50]_s$	4252.1
Third	$[55/5]_s$, $[-35/-15]_s$, $[5/-45]_s$, $[0/-15]_s$ $[40/-30]_s$, $[-55/-35]_s$, $[0/-60]_s$, $[-65/90]_s$	4252.1

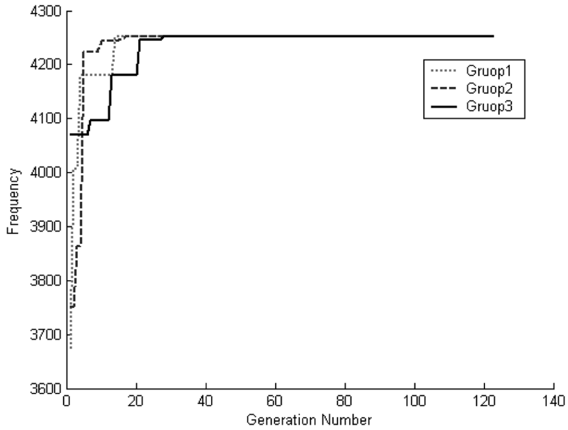


Fig. 4 Influence of initial population in optimum design for a four-layered simply supported panel

(1981). According to this table, the results are nearly identical, and the results with FEM differ from the analytical results about five percent.

Table 3 and Fig. 4 show the influence of the initial selected population on the optimal design of the panel. According to the figure, after 100 generations the frequency as the fitness function , 4252.1 Hz, does not change, and the related optimum stacking sequence is $[-40, 40]_s$. The number and type of initial population can only changes the number of the local maximum and consequently the time of convergence for the objective function but does not change its magnitude. The influence of the number of layers in the optimum design is shown in Figs. 5-7. According to these figures, increasing the number of layers with a constant thickness and with S = 10, causes an increase in the stiffness of the panel and consequently increases the natural frequencies. In addition, when the number of layers in an individual lay-up increases, the local maximum also increases and causes an increase in the number of iterations for convergence of the fitness function.

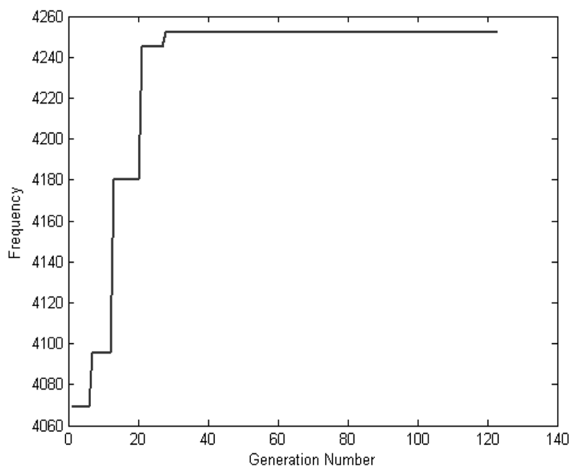


Fig. 5 Convergence of fitness function for four layers $[40/-40]_s$

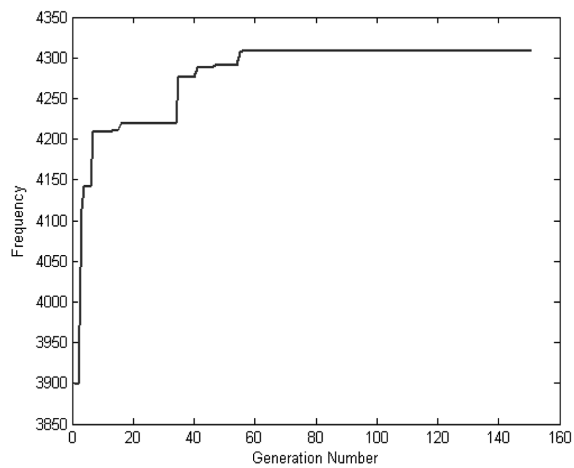


Fig. 6 Convergence of fitness function for six layers $[-40/40/45]_s$

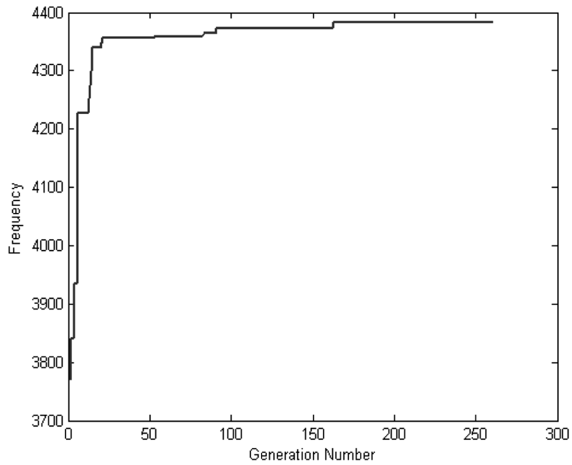
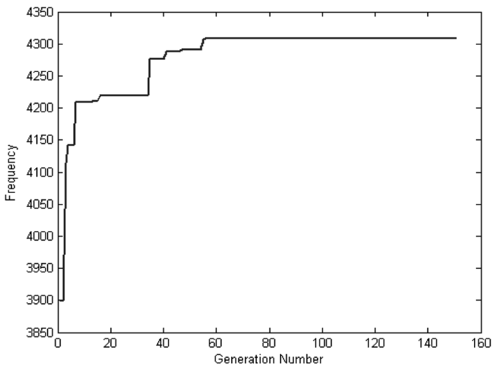


Fig. 7 Convergence of fitness function for eight layers $[40/-40/-45/40]_s$

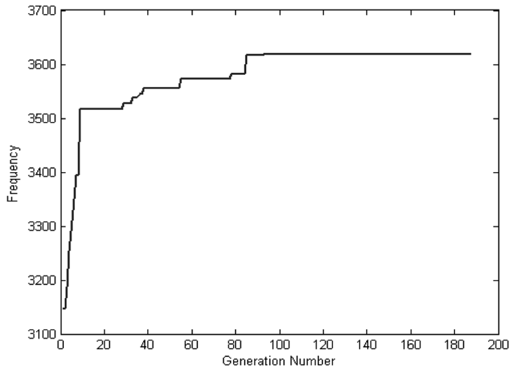
Table 4 Influence of mid radius to thickness '*S*' in the fitness function (simply supported)

Layer	S	Best frequency	Best angle	Worst frequency	Worst angle	Best to worst requency ratio
4	10	4252.1	$[40/-40]_s$	2597	$[90/90]_s$	1.64
	20	3532.7	$[45/-45]_s$	1922	$[90/90]_s$	1.84
	50	1998.3	$[-90/30]_s$	1240	$[0/0]_s$	1.61
	100	1429.5	$[40/-15]_s$	964.8	$[0/0]_s$	1.48
6	10	4309	$[40/-40/45]_s$	2597	$[90/90/90]_s$	166
	20	3618.4	$[45/-45/40]_s$	1922	$[90/90/90]_s$	1.88
	50	2109.4	$[85/-40/25]_s$	1240	$[0/0/0]_s$	1.70
	100	1520	$[45/-35/10]_s$	964.8	$[0/0/0]_s$	1.58
8	10	4382.9	$[40/-40/-45/45]_s$	2597	$[90/90/90/90]_s$	1.69
	20	3676.7	$[-40/45/-45/40]_s$	1922	$[90/90/90/90]_s$	1.91
	50	2123.6	$[80/-60/35/-10]_s$	1240	$[0/0/0/0]_s$	1.71
	100	1524.1	$[-50/40/-20/10]_s$	964.8	$[0/0/0/0]_s$	1.58

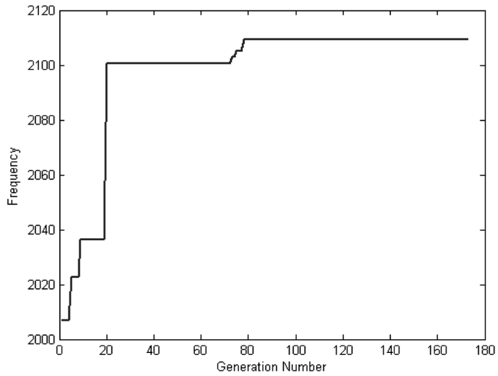
The influence of the mid radius-to-thickness ratio, $S = R/h$, in the optimal stacking sequence for a panel with simply supported edges is shown in Table 4. To demonstrate the validity of the method used in this paper, the worst solutions are evaluated as well as the optimum solutions. This table shows that increasing S causes a decrease in the stiffness of the panel, consequently decreasing the best fitness function. In this table, the best and worst stacking sequences for four-, six- and eight-layered panel are shown for various values of S. According to this table, best to worst frequency ratios in a moderately thick pane '$S = 20$', are greater than those of a thin panel '$S = 100$'. As the table shows, the optimum stacking sequence can increase the fundamental frequency by approximately 1.84 times greater than the worst selection in a thick four-layered panel. The process of obtaining the result of Table 4 for a symmetric six-layered panel is presented in Figs. 8(a)-(d). According to these figures, changing S can cause a change in the number of iterations for



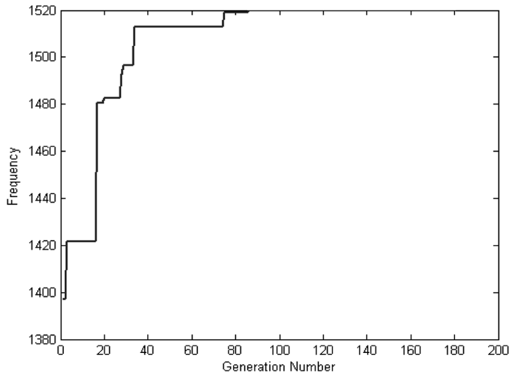
a) Optimum stacking sequence $[40/-40/45]_s$ (S = 10)



b) Optimum stacking sequence $[45/-45/40]_s$ (S = 20)



c) Optimum stacking sequence $[85/-40/25]_s$ (S = 50)



d) Optimum stacking sequence $[45/-35/10]_s$ (S = 100)

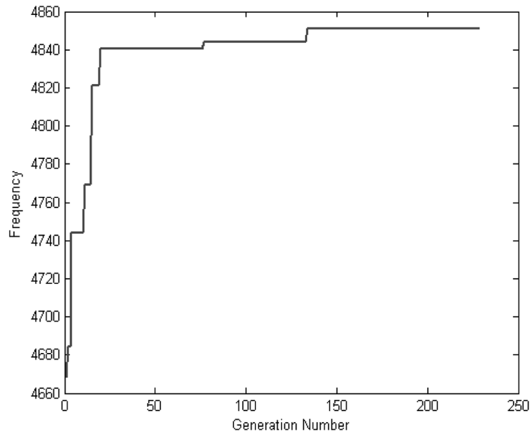
Fig. 8 Influence of *S* in the convergence of fitness function for six-layered panel (simply supported)

Table 5 Influence of mid radius to thickness '*S*' in the fitness function (clamped)

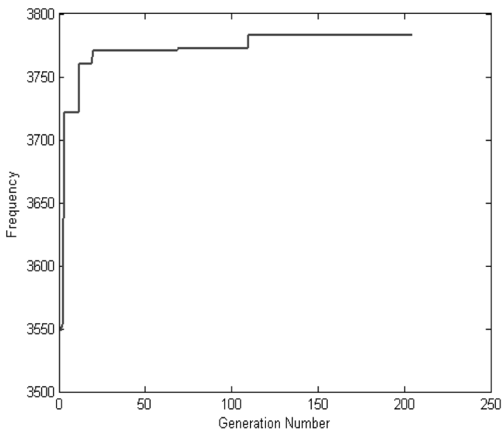
Layer	S	Best frequency	Best angle	Worst frequency	Worst angle	Best to worst requency ratio
4	10	4685	$[55/-65]_s$	2962	$[0/0]_s$	1.59
	20	3709.8	$[55/-65]_s$	2772	$[0/0]_s$	1.34
	50	2879.3	$[60/-65]_s$	1758	$[0/0]_s$	1.64
	100	2058.4	$[55/-50]_s$	1174	$[0/0]_s$	1.75
6	10	4851.2	$[30/-60/65]_s$	2962	$[0/0/0]_s$	1.64
	20	3782.6	$[40/-55/75]_s$	2772	$[0/0/0]_s$	1.40
	50	2888	$[-55/90/55]_s$	1758	$[0/0/0]_s$	1.64
	100	2125.4	$[85/-45/40]_s$	1174	$[0/0/0]_s$	1.81
8	10	4873.6	$[30/-50/-85/50]_s$	2962	$[0/0/0/0]_s$	1.65
	20	3797.4	$[45/-45/70/-70]_s$	2772	$[0/0/0/0]_s$	1.37
	50	2915.9	$[70/-40/45/-70]_s$	1758	$[0/0/0/0]_s$	1.66
	100	2128.3	$[70/-55/40/-40]_s$	1174	$[0/0/0/0]_s$	1.81

convergence as well as the type of ply. The optimization process in a thin panel converges much quicker than in a thick panel.

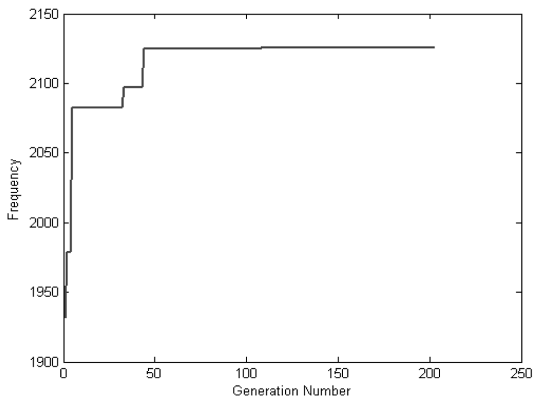
The results of optimum stacking sequence for four-, six- and eight-layered panels with clamped edge boundary conditions are presented in Table 5. In this case the optimized laminate has a maximum first natural frequency of 1.81 and is 1.37 times greater than the worst designed laminate of the eight thin- and thick- layered panels, respectively, with a related optimum stacking sequence of $[75/-55/40/-40]_s$ and $[45/-45/70/-70]_s$. As the table shows in contrast to the simply supported panel, in the panel with clamped end boundary condition the best-to-worst frequency ratio in the thin panel 'S = 100' is greater than that of thick panel 'S = 20'. Furthermore, according to this table, the optimum sequence layup in the thin panel with the clamped boundary is more efficient compared to the simply supported panel. The influence of the mid radius-to-thickness ratio 'S' with clamped boundary conditions in the optimal behavior of multi-layered panel is shown in Figs. 9(a)-(d). As all of the figures show, the local maximum decreases as the value of S increases; consequently



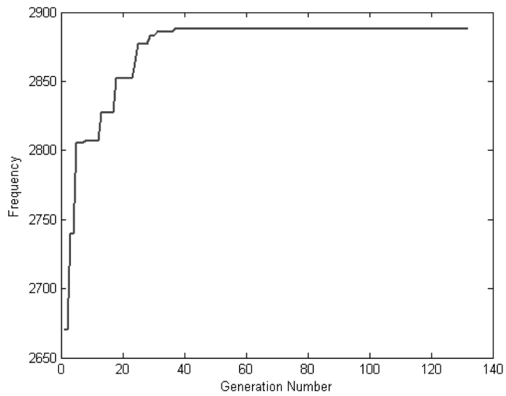
a) Optimum stacking sequence $[30/-60/65]_s$ (S = 10)



b) Optimum stacking sequence $[40/-55/75]_s$ (S = 20)



c) Optimum stacking sequence $[-55/90/55]_s$ (S = 50)

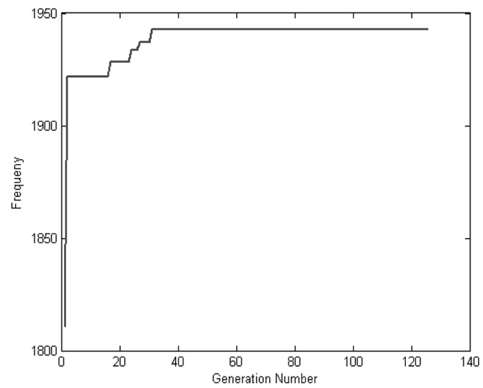


d) Optimum stacking sequence $[85/-45/40]_s$ (S = 100)

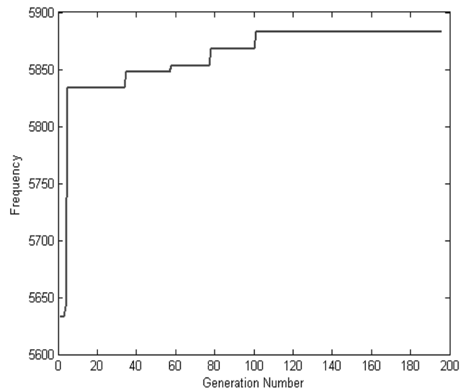
Fig. 9 Influence of S in the convergence of fitness function (clamped)

Table 6 Influence of *S* on the behavior of the panel with arbitrary boundary condition

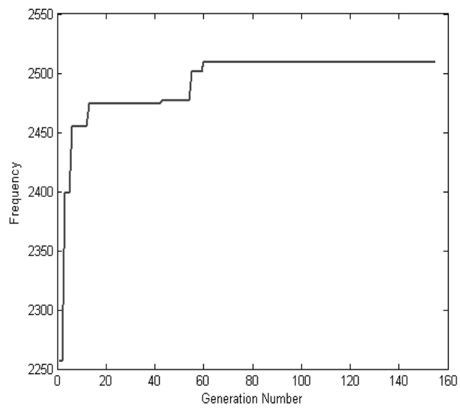
Boundry condition	*S*	Best frequency	Best angle	Worst frequency	Worst angle	Best to worst frequencyratio
SSFF	10	3031.6	$[5/-45/60]_s$	1563.8	$[90/90/90]_s$	2.12
	20	1942.9	$[10/-30/40]_s$	914.47	$[90/90/90]_s$	2.12
	50	1051.5	$[35/-35/0]_s$	513.51	$[90/90/90]_s$	2.05
	100	687.14	$[45/-40/5]_s$	415.18	$[90/-85/90]_s$	1.66
SSCC	10	7409.4	$[75/-60/65]_s$	3822.2	$[5/5/5]_s$	1.94
	20	5869.9	$[80/-55/40]_s$	2958.6	$[10/10/10]_s$	1.98
	50	3521.1	$[85/-50/45]_s$	1691.8	$[0/0/0]_s$	2.08
	100	2585.7	$[85/-45/40]_s$	1195.1	$[0/0/0]_s$	2.16
CCFF	10	3664.3	$[10/-40/75]_s$	2042	$[90/90/90]_s$	1.80
	20	2510.8	$[5/-25/40]_s$	1164	$[90/90/90]_s$	2.16
	50	1325.5	$[10/-30/35]_s$	658.4	$[90/90/90]_s$	2.01
	100	827.02	$[35/40/0]_s$	524.8	$[90/90/90]_s$	1.58



a) SSFF, S = 20 $[10/-30/40]_s$



b) SSCC, S = 20 $[80/-55/40]_s$



c) CCFF, S = 20 $[5/-25/40]_s$

Fig. 10 Influence of edge boundary conditions on GAs convergence decreasing the time of convergence. Optimum designs of laminates under various boundary conditions are presented in Table 6. For the SSCC boundary condition, the optimal stacking sequence of 6-ply thick laminate with the maximum first natural frequency 1942.9 Hz, is $[10/-30/40]_s$. The maximum first natural frequency for this ply is between the one for the clamped and simply supported panels. The optimum design for the SSFF and CCFF boundary conditions are $[80/-55/40]_s$ and $[5/-25/40]_s$ respectively. According to this table clamping two opposite edges of panel, in comparison with the all edges simply supported, causes an increase in the stiffness of the panel leading to an increase in the natural frequency. In addition if the two opposite edges of the panel are free, with respect to the simply supported panel, the stiffness will be decreased, leading to a decrease in the fundamental frequency. The process of GAs for obtaining the optimum stacking sequence versus the end boundary conditions is shown in Figs. 10(a)-(c). According to these figures, the optimum design and time of convergence depends on the end boundary condition. As the figures show, the number of generations in the SSFF boundary condition is greater than that of the SSSS and smaller than that of the SSCC boundary condition.

5. Conclusions

A combination of genetic algorithms and FEM to maximize the fundamental frequency of fiber reinforced panel with the orientation of fiber as the design variable is described in this paper. Formulation used is based on first order shear deformation theory. Results are presented for different ‘S’ and various combinations of clamped, simply supported and free boundary conditions. In this study the acceptable final result is defined as the result which is repeated for one hundred generations. From the numerical results the following can be concluded

- The optimum frequency ratio in a clamped thin panel is greater than that of a thick panel, whereas in the simply supported boundary condition, the best-to-worst frequency ratio in a thick panel is greater than in a thin panel.
- The total iterated number of generations depends on the number and type of the initial selected population. A suitable selection can reduce the number of generations.
- The maximum natural frequency for the optimal design in a clamped boundary condition is higher than that of other arbitrary boundary conditions.
- The mid radius-to-thickness ratios and different types of edges boundary conditions, influence the optimal behavior of multi-layered panel.

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Notation

$C_{ij}(i, j = 1, 2, \ldots, 6)$	: elastic constants
h	: thickness of the panel
$[H]$, $[B]$	: interpolation and strain displacement matrices
$[K]^G_{45 \times 45}$	: general stiffness matrix
$[F]^G_{45 \times 1}$	: general force matrix
L	: axial length of the panel
L_b	: aspect ratio
$[M]^G_{45 \times 1}$	: general mass matrix
R	: mid radius
$S = R/h$	: radius to thickness ratio
T	: kinetic energy
U	: strain energy
u, v, w	: displacements in x, y, z directions respectively
u_0, v_0, w_0	: displacements of a point on the reference surface
$\{\delta\}^G$	: general nodal displacement
θ_x, θ_y	: rotations of normal to the reference surface about the x- and y-axes
γ_{zy}, γ_{xz}, γ_{xy}	: shear strains
$\sigma_i(i = x, y, z)$	: normal stresses
τ_{zy}, τ_{xz}, τ_{xy}	: shear stresses

$\varepsilon_i (i = x, y, z)$	: normal strains
θ_m	: span angle of panel
ρ_k	: mass density of K_{th} layer