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The effects of uncertainties in structural analysis

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Abstract. Model-based predictions of structural behavior are negatively affected by uncertainties of various type and in various stages of the structural analysis. The present paper focusses on dynamic analysis and addresses the effects of uncertainties concerning material and geometric parameters, mainly in the context of modal analysis of large-scale structures. Given the large number of uncertain parameters arising in this case, highly scalable simulation-based methods are adopted, which can deal with possibly thousands of uncertain parameters. In order to solve the reliability problem, i.e., the estimation of very small exceedance probabilities, an advanced simulation method called Line Sampling is used. In combination with an efficient algorithm for the estimation of the most important uncertain parameters, the method provides good estimates of the failure probability and enables one to quantify the error in the estimate. Another aspect here considered is the uncertainty quantification for closely-spaced eigenfrequencies. The solution here adopted represents each eigenfrequency as a weighted superposition of the full set of eigenfrequencies. In a case study performed with the FE model of a satellite it is shown that the effects of uncertain parameters can be very different in magnitude, depending on the considered response quantity. In particular, the uncertainty in the quantities of interest (eigenfrequencies) turns out to be mainly caused by very few of the uncertain parameters, which results in sharp estimates of the failure probabilities at low computational cost.

Keywords: uncertainty modeling; uncertainty quantification; model uncertainties; structural reliability; Monte Carlo simulation.

1 Introduction - Uncertainties in structural analysis and design

Favored by the unarrestable progress in the field of computer hardware, predictive models are enormously powerful tools in the hands of the structural analyst. One of the major nuisances arising in the use of these models, is the uncertainty associated with all the assumptions necessary for the construction of the model. This relates in particular to the parameters of the models, but not exclusively: fundamental assumptions of the model such as the structure of the governing equations, constitutive models constitute additional sources of uncertainty.

The unavoidable presence of uncertainties in structural engineering is particularly obvious when the loading conditions are analyzed, to which structures are exposed. Observations of the underlying natural phenomena such as wind (storms), earthquakes or sea states immediately reveal their

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inherent spatial and temporal variability. Usually with a smaller magnitude, but often equally or even more important in terms of the resulting effects, the structure itself is affected by uncertainties, which cannot be reduced significantly, either due to their inherent nature or due to practical limitations (limited access, cost, etc.).

Traditional structural analysis and design procedures by and large still rely on a deterministic view of the uncertainty problem. For instance, uncertainties in loading conditions are "enveloped" on the basis of extreme events. These envelopes are determined in a quite arbitrary way, i.e., based on extreme observations of the past. In contrast to this well-established yet possibly wasteful and sub-optimal practice stand rational approaches to the quantification of uncertainties. The probability-based approach has been proposed several decades ago (Freudenthal 1947), whereas alternative approaches have been proposed more recently, such as that based on fuzzy logic (Möller and Beer 2004). What the approaches surely have in common, is that they are active topics of research and that they imply a significantly increased computational cost, as the uncertainty expands the model in the direction of an additional "dimension".

The most widely accepted framework for modeling the uncertainties and for propagating their effects on the quantities of interest makes use of the well-developed theory of probability. It is based on describing uncertain parameters by random variables, stochastic processes (time domain), random fields (spatial domain) or random waves (time and spatial domain). Typical examples of quantities suitable for probabilistic modeling are earthquake ground motions, sea waves, wind turbulence, road roughness, imperfections of shells, fluctuating properties in random media, etc. The choice of the respective probabilistic models depends both on physical aspects and statistical evidence, respectively. Consequently, the fields of mathematical statistics and estimation theory clearly play a key role in uncertainty modeling, as they allow to move from observations to the probabilistic model. The observed data themselves are, of course, not yet associated with uncertainties. Only under the assumption that a data point constitutes a sample of an independently identically distributed random variable, it is theoretically sound to use a probabilistic model to describe the underlying uncertainties.

Independently of the approach selected to take into account the uncertainties in structural analysis and design, two aspects should be kept in mind:

(1) The accuracy and quality of the mechanical modeling should be up to the current standards of deterministic analysis - but not more. For instance, no benefits result from excessively fine discretizations, besides "fictitious" accuracy, when significant uncertainties affect some parameters of the model. On the other hand, poor mechanical models will in most cases lead to poor results of the uncertainty quantification.

(2) The procedures should allow to analyze higher dimensional problems - i.e., allow the efficient treatment of a large number of uncertain parameters - as generally encountered in engineering practice.

2. Quantification and processing of uncertainties

2.1 Introductory remarks

The basis for a realistic and possibly predictive quantification of uncertainties consists in observations of particular parameters or functions, i.e., the statistical data items of sample points or sample functions, etc. With the formalisms of probability theory and with statistical estimation, the description of the uncertainties affecting the behavior of the structural systems can then be undertaken, in the form of random variables, processes and fields. Once the uncertainties in the input (loading conditions) and in the system characteristics itself have been described in probabilistic terms, the propagation of these uncertainties through the structural system is required, in order to obtain probabilistic information on the output (response), for subsequent design purposes.

Numerous methods for uncertainty propagation have been proposed and developed. Most methods focus on capturing the mean value and the covariance function, i.e., the second moment information, of the response, rather than the full set of joint probability distributions characterizing the random response. For instance, the perturbation method (Liu *et al.* 1986, Kleiber and Hien 1992, Babuška and Chatzipantelidis 2002) approximates the stochastic response with the aid of Taylor expansions of the random quantities about their mean values. In the Neumann expansion method (Bharrucha-Reid 1959, Yamazaki *et al.* 1988) the inverse of the stochastic system matrices are approximated by its Neumann series. A comprehensive representation of the response in terms of its coordinates in a suitable Hilbert space is afforded by the chaos expansion, leading to the so-called Spectral Stochastic Finite Element method (see e.g., Ghanem and Spanos 1991).

Needless to say, the computational efforts associated with uncertainty propagation are considerably larger when compared to traditional deterministic analysis. This is particularly the case for high-dimensional problems, i.e., those involving large numbers of random variables, and when the task consists in estimating the likelihood of low-probability events.

In the present paper the focus is on simulation-based methods, mainly due to their robustness, versatility and to their fitness for dealing with high-dimensional problems, as they frequently arise in the context of large-scale structural systems.

2.2 Simulation-based uncertainty quantification and reliability estimation

Monte-Carlo simulation (MCS) procedures to assess uncertainty propagation consist in the generation of sample populations that are consistent with the probability distribution of the input data (loading) and/or of the system properties (structural parameters). Some of the main advantages of MCS have been already mentioned. From a practical and computational point of view two additional major assets are the following: (i) the non-intrusive nature of MCS facilitates its use in a black-box fashion, in combination with general-purpose finite element codes (Pellissetti and Schuëller 2006); (ii) the MCS algorithm is, due to the generation of independent samples, particularly well suited for parallel processing and can hence take full advantage of high-performance computing facilities that are becoming increasingly affordable and widespread.

Leaving aside low-probability events for the moment, the estimation of the response distributions with Direct MCS method, in particular of the response cumulative distribution function (CDF), $F_{\mathbf{R}}(\mathbf{r})$, is very efficient. Samples of the input random vector $\mathbf{X}$ are generated, such that the ensemble $\{\mathbf{X}^{(k)}\}_{k=1}^{N}$ matches the required CDF of the input vector, $F_{\mathbf{X}}(\mathbf{x})$. For each sample $\mathbf{X}^{(k)}$ of the input vector the corresponding sample response $R^{(k)}$ is evaluated and from the ensemble the desired response CDF can be estimated, typically in the form of fractiles R_p, which indicate the response level associated with a given probability level p. In other words, $P[R < R_p] = p$. These fractiles can be approximated as follows:

Let $\{R^{(k)}\}_{k=1}^{N}$ be the ensemble of responses obtained by performing N MCS runs and let

$\{\hat{R}^{(k)}\}_{k=1}^{N}$ contain the elements of $\{R^{(k)}\}$, but in increasing order,

$$\{\hat{R}^{(k)}\}_{k=1}^{N}, \quad \hat{R}^{(1)} \leq \hat{R}^{(2)} \leq \ldots \leq \hat{R}^{(N)} \tag{1}$$

Then,

$$R_p \approx \hat{R}^{(N_p)}, \qquad N_p = \mathrm{round}[p \cdot (N+1)], \qquad 1 \leq N_p \leq N \tag{2}$$

where the operator round $[\cdot]$ rounds the argument to the nearest integer.

The efficiency of Direct MCS deteriorates dramatically when the CDF is to be estimated accurately in the tails, i.e., when low failure probabilities are to be estimated. The estimator of the failure probability $\hat{p}_F$ is then given by the ratio of the number of samples leading to failure over the total number of samples N,

$$\hat{p}_F = \frac{1}{N}\sum_{i=1}^{N} 1_F(\mathbf{X}^{(i)}) \tag{3}$$

where the binary indicator function $1_F(\mathbf{X}^{(i)})$ evaluates to 1 in case the i-th sample leads to failure and to 0 otherwise. The key to the poor efficiency consists in the form for the variance of this estimator, expressed by the coefficient of variation of $\hat{p}_F$,

$$\delta_{MC} = \sqrt{Var[\hat{p}_F]}/p_F = \sqrt{(1-p_F)/Np_F} \tag{4}$$

which is independent of the number of uncertain parameters, but implies that a very large number (proportional to $1/p_F$) of samples is required for an accurate estimate of small failure probabilities p_F. In order to elude this drawback, variance-reduction techniques have been proposed, that get by with significantly smaller (and usually feasible) numbers of samples (see e.g., Schuëller *et al.* 2003).

2.3 Uncertainties in modal analysis of structures

The generalized eigenvalue problem associated with the linear elasto-dynamic equation of motion is given by,

$$(-\omega_j^2 \mathbf{M} + \mathbf{K})\phi_j = 0 \tag{5}$$

where $\{\omega_j\}_{j=1}^{N}$ and $\{\phi_j\}_{j=1}^{N}$ are the sets of eigenfrequencies and normal modes, respectively, resulting from the solution of the above eigenvalue problem.

Modal analysis, i.e., the act of determining the eigenfrequencies and the associated modes of deformation, is of fundamental importance in the construction of predictive FE-models of structures subjected to dynamic excitation. Indeed, the validation of such FE-models is typically based on the correlation of experimental and numerical results. More specifically, the modal properties predicted by the FE model and those emerged in experimental testing campaigns are correlated (see e.g., Ewins 2000, Calvi 2005).

The representation of the uncertainties in the structural parameters in a probabilistic framework, results obviously in the propagation of these uncertainties (according to Eq. (5)) to the eigenfrequencies and eigenmodes. When attempting to correctly capture the uncertainties in the modal content by analyzing the modal scatter observed in the MCS, it is indispensable to avoid the intermixing of eigenfrequencies from one simulation to the next, which correspond to modes that are physically unrelated. One way to accomplish this is to express the eigenvalues of each simulation as a weighted superposition,

$$\hat{\lambda}_i^{(k)} = \sum_j \lambda_j^{(k)} [\psi_{ij}^{(k)}]^2 \tag{6}$$

where $\hat{\lambda}_i^{(k)}$ denotes the modified eigenvalue associated with the i-th eigenmode and $\{\lambda_j^{(k)}\}$ is the set of sample eigenvalues. Qualitatively, the weight $[\psi_{ij}^{(k)}]$ may be viewed as a measure of the correlation between the j-th eigenvector of sample k with the i-th eigenmode of the nominal system. Eq. (6) constructs a population for the eigenvalue $\hat{\lambda}_i$ associated with the i-th eigenmode of the nominal system, with the following property: for each sample k, the strongest contribution to the sample eigenvalue $\hat{\lambda}_i^{(k)}$ does not necessarily come from the eigenvalue associated with the i-th eigenmode of this sample, but rather from the eigenvalue associated with the eigenmode that resembles most the i-th eigenmode of the nominal system.

In mathematical terms, the weights $\{\psi_{ij}\}$ are given by the following expression,

$$\Psi^{(k)} = \Phi^{0^T} M^0 \Phi^{(k)}, \qquad \Psi^{(k)^T} \Psi^{(k)} = \mathbf{I} \tag{7}$$

where the superscript 0 indicates the nominal case. The meaning of the matrix $\Psi^{(k)}$ may also be grasped by considering that,

$$\Phi^{(k)} = \Phi^0 \Psi^{(k)}, \quad \Phi^{(k)}, \Phi^0 \in M_{n,m}, \quad \Psi^{(k)} \in M_{m,m} \tag{8}$$

where n is the number of DOFs of the FE model and m is the dimension of the modal basis, i.e., the number of modes that are retained. Hence, the matrix $\Psi^{(k)}$ transforms (approximately) the nominal eigenvectors Φ^0 to the eigenvectors of the sample k.

It should be noted that the described methodology is very much related to the so-called *Modal Assurance Criterion* (MAC), commonly used in aerospace and mechanical engineering for comparing pairs of mode shapes (Ewins 2000). For instance, the MAC is used to correlate numerically predicted modes with experimentally measured ones.

2.4 Reliability of structures with large numbers of uncertain parameters

2.4.1 General remarks

The main purpose of uncertainty analysis is on one hand to be able to assess the effects of uncertainties on the analysis in a rational way and, on the other hand, to quantify the reliability of the analyzed structures. As mentioned in the introduction, the ability to analyze high-dimensional reliability problems bears remarkable importance for a productive use of stochastic procedures in engineering design. Recently, important developments have taken place in this respect (Schuëller and Pradlwarter 2006, Schuëller *et al.* 2005) with the introduction of novel methods such as Subset Simulation (Au and Beck 2001) and Line Sampling (Schuëller *et al.* 2003, 2004).

2.4.2 Line sampling

The Line Sampling method is a robust sampling technique particularly suitable for high-dimensional reliability problems, in which the considered response quantity exhibits moderate non-linearity. The key step consists in the identification of a direction in the high-dimensional input parameter space, pointing to regions which strongly contribute to the overall failure probability. An excellent candidate for this direction is typically the gradient at the nominal point (usually the mean) of the random input parameter space. It should be noted that in this context of uncertainty analysis the components of the used gradient definition are not simply the partial derivatives, but the partial derivatives scaled by the standard deviation of the considered uncertain parameter (see Eq. (10) in section 3.4). Recently introduced procedures for efficient gradient estimation (Pradlwarter *et al.* 2005a) are most useful in this respect, as they allow to identify the most important components of the gradient with a highly reduced number of response evaluations.

Once such an important direction has been identified, samples are then evaluated along this direction from randomly selected starting points and the intersection of each of these lines with the failure region is determined. The intersection points then lead to the desired estimate of the failure probability. This is visualized in Fig. 1, where the important direction is denoted by $\mathbf{e}_\alpha$ and the failure region is shaded. The intersection of each line $l^{(j)}$ with the failure region, denoted by $\bar{c}^{(j)}$, supplies a sample for the failure probability $p_F^{(j)} = \Phi(-\bar{c}^{(j)})$, where Φ denotes the Gaussian cumulative distribution function. Repeating this procedure for a number N of lines, the estimator $\bar{p}_F$ of the probability of failure and the associated variance are then,

$$\bar{p}_F = \frac{1}{N}\sum_{j=1}^{N} p_F^{(j)}, \qquad \sigma^2_{\bar{p}_F} = \frac{1}{N-1}\sum_{j=1}^{N}\left(p_F^{(j)} - \bar{p}_F\right)^2 \tag{9}$$

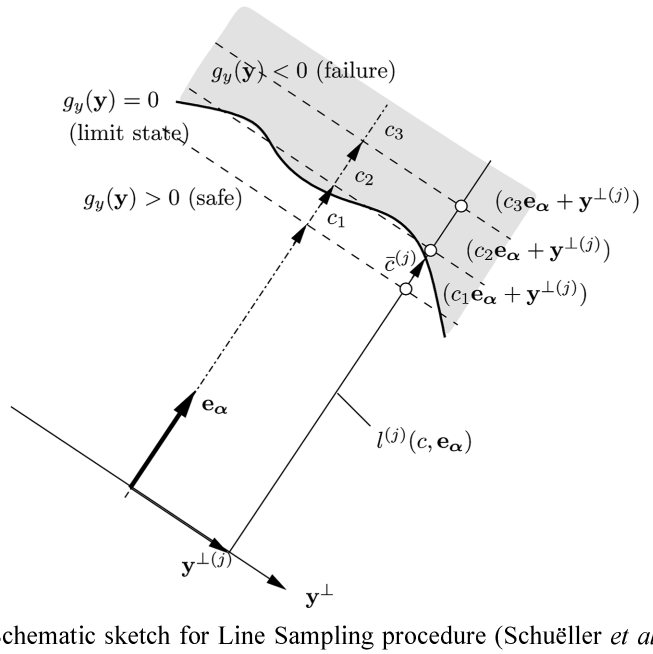

Fig. 1 Schematic sketch for Line Sampling procedure (Schuëller *et al.* 2003)

With the above approach the variance of the estimator of the probability of failure $\overline{p}_F$ can be considerably reduced. In general a relatively low number N of lines have to be sampled to obtain a sufficiently accurate estimate.

2.5 Non-parametric approach for assessment of model and data uncertainties

The consideration of uncertainties in the parameters of a given numerical model of a structural system greatly improves the credibility of the predictive model. A different source of uncertainty is however constituted by the numerical model itself and by the underlying assumptions with respect to the governing equations, boundary conditions, etc. This type of uncertainties are usually referred to as *model uncertainties* and their effect on the predicted response can be massive (Menezes and Schuëller 1997, Oden *et al.* 2003, Babuska and Oden 2004).

Recently, a so-called non-parametric approach has been proposed by Soize (2000, 2001, 2005), which allows the introduction and propagation of model uncertainties in dynamical systems. The mathematical foundation of the method is the construction of ensembles of random matrices with given properties; clearly the model uncertainties that can be captured by the methodology must be compatible with the underlying ensemble. A detailed review of the method is beyond the scope of the paper and the interested reader is referred to (Soize 2000, 2001, 2005, Capiez-Lernout *et al.* 2006).

3. Applications

3.1 Introductory remarks

In the present section it is intended to show the application of the methodologies described thus far to the analysis of real-life, large-scale problems of engineering interest. The main goal is to

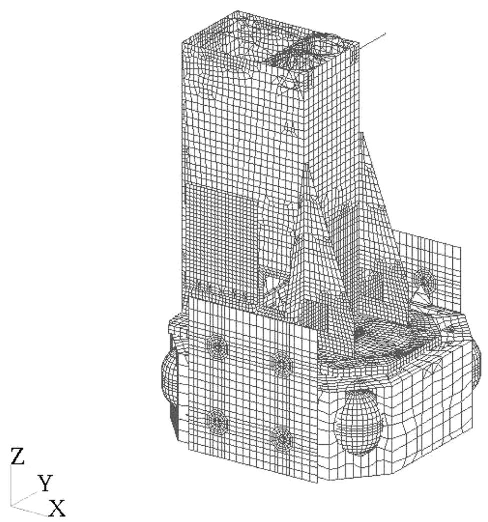

Fig. 2 Satellite finite element model (courtesy of ESA/ESTEC)

demonstrate that the methods are indeed scalable with respect to the size and the complexity of an analysis task.

More specifically, the effects of uncertainties on complex structural systems are analyzed in the context of the INTEGRAL satellite of ESA (European Space Agency). Due to its complexity the studied structure is representative of large-scale problems of industrial interest. The FE mesh is shown in Fig. 2. In total, the FE model of the satellite involves 120,000 DOFs. (For details on the modeling aspects it is referred to (Notarnicola *et al.* 1998, Moreno 1998, Oxfort 1997))

In this example, the quantity of interest under consideration are the eigenfrequencies of the structure, resulting from the modal analysis. The importance of modal analysis - and consequently of the associated uncertainties - in the finite element modeling process has been already emphasized in section 2.3.

3.2 Uncertainty modeling

A frequently adopted simplifying assumption consists in considering a limited number of parameters as sources of uncertainty. This choice is often dictated by the lack of robustness or efficiency of many stochastic methods in the presence of large numbers of uncertain parameters. Unfortunately for complex structural systems the distinction between essential parameters and less important ones is usually not obvious or intuitive. Restricting the uncertainty modeling to a limited subset of the parameters could lead to the underestimation of the response uncertainty, particularly if parameters with an unexpected, strong influence on the response are treated as deterministic.

To avoid this hazard, no effort has been made in the applications reported here to limit the number of uncertain parameters. On the contrary, the goal pursued in the modeling was to capture as much uncertainty in the structural properties as possible. Practically speaking, this has been achieved by treating most of the *parameter types* arising in the input files of the FE model as uncertain, i.e., in a *systematic fashion*. For instance, *all* Young's moduli specified in any of the input files are considered as uncertain (see also Pradlwarter *et al.* 2005b). Here it should suffice to mention that the assumed coefficients of variation (σ/μ) range from 4% to 12%, with the mean values set equal to the nominal values of the deterministic FE model and that the uncertain parameters are assumed to be Gaussian distributed. It should be noted here that the assumptions about the type and magnitude of the uncertainty in the FE model parameters are based on data available in the literature (Esnault and Klein 1996, Klein *et al.* 1994, Székely *et al.* 1998, Simonian 1987). Due to the scarcity of statistical information on the spatial correlation of the uncertainties, this effect has not been considered at this stage, i.e., the various uncertain parameters have been assumed to be mutually uncorrelated.

3.3 Direct Monte-Carlo simulation of the eigenfrequencies

3.3.1 Introductory remarks

The first step taken in this case study on the effects of uncertainties consists in a direct Monte-Carlo simulation of the eigenfrequencies of the satellite structure. The associated results serve as the reference solution for the analysis and discussions presented in the remaining sections.

3.3.2 Separated modes

Based on the discussion in section 2.3, a straightforward synthesis of the simulation results is

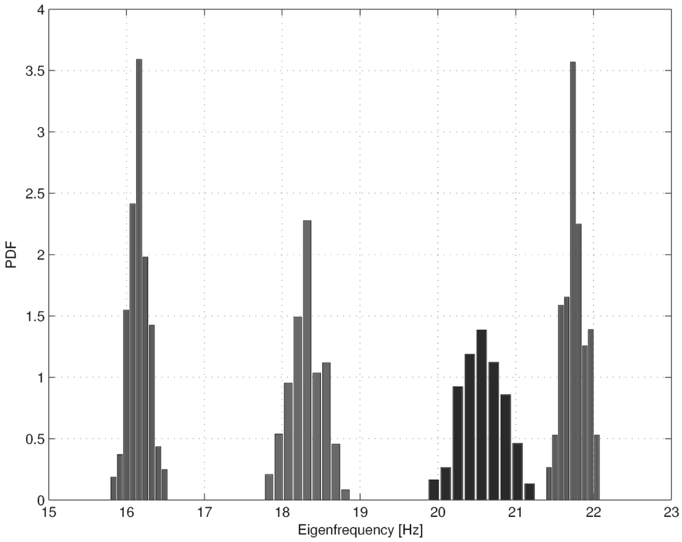

Fig. 3 Histograms of eigenfrequencies, modes 1-4, based on Direct MCS (200 samples)

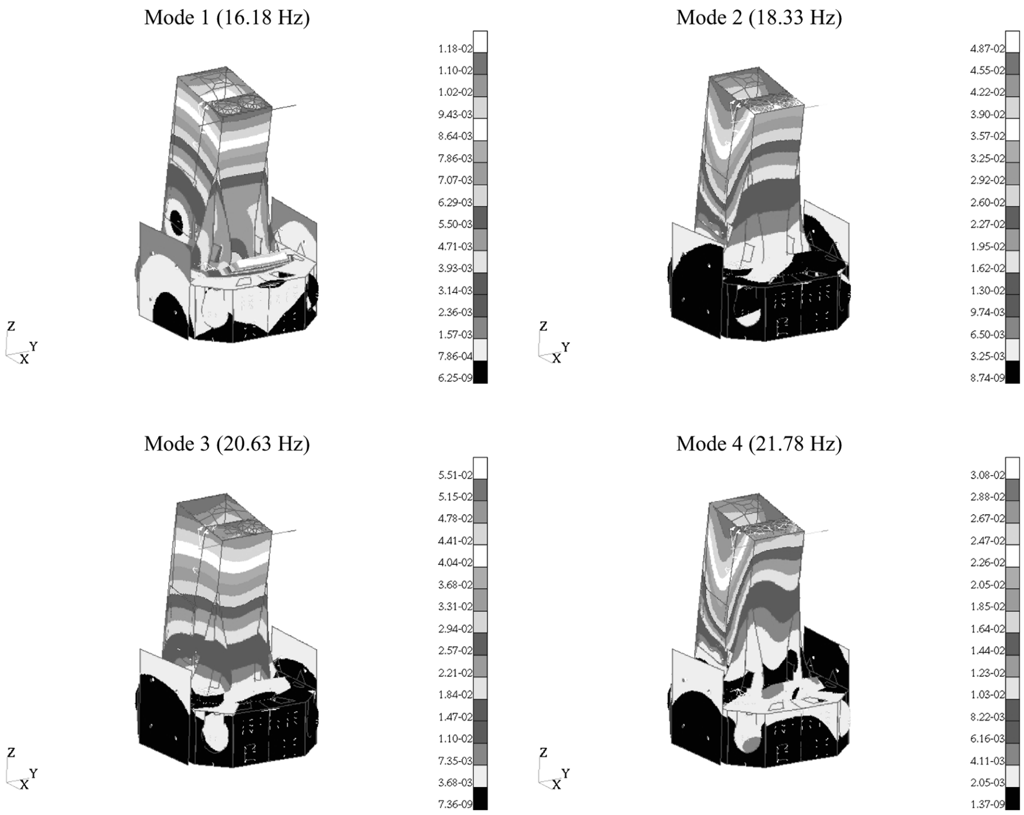

Fig. 4 Mode shapes with fringe plot of displacements, modes 1 through 4

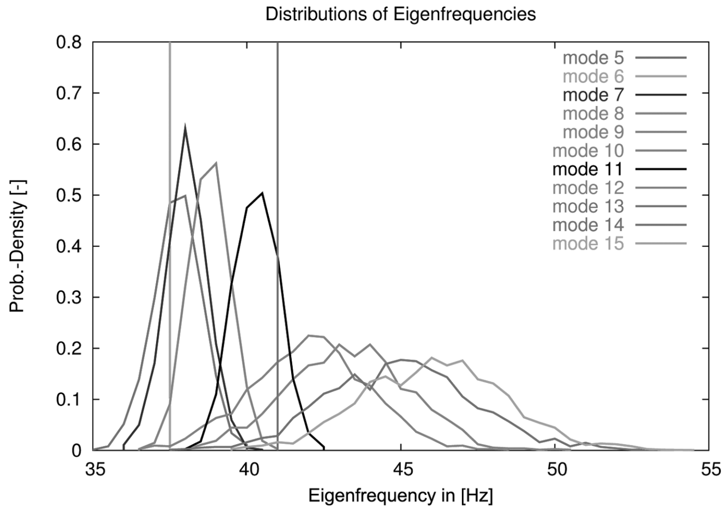

Fig. 5 Approximate Probability Density Functions (PDF's) of the eigenfrequencies, modes 5-15 (Schuëller 2006)

possible only if the eigenfrequencies are well separated from each other. This is the case for the first four modes, as the histograms of the first four eigenfrequencies in Fig. 3 show. Hence, in this case the order of these eigenvalues can be assumed not to vary among different realizations and no reordering on the basis of the modal assurance criterion is necessary.

The mode shapes associated with the first four eigenfrequencies are depicted in Fig. 4. The modes represent essentially bending modes in x-direction (modes 1 and 3) and y-direction (modes 2 and 4). In fact, modes 3 and 4 exhibit a slight diagonal component, which distinguishes them from modes 1 and 2, respectively.

3.3.3 Modal assurance

A different, more complex scenario manifests itself in the frequency band [35,55] Hz (Fig. 5), where the PDF's of different modes overlap with each other. In this range the population of the modal frequencies must not be obtained by simply ordering the modes based on their eigenfrequency. Instead, for a given mode number, which is assigned on the basis of the nominal system, the population of the eigen-frequencies must be assembled by applying the procedure described in section 2.3. This ensures the correspondence of modes of the same population.

The main observation suggested by Fig. 5 is that the effect of the uncertainties in the structural properties on the eigenfrequency associated with the various natural modes is highly variable in magnitude. Indeed, some modes are completely insensitive to the uncertainties, such as mode 6, while others (e.g., mode 8) exhibit significant variability. This information on the effect of the various uncertainties on the modal properties is of great interest for the analyst, at least in two respects: (i) in the assessment of the structural design (strong sensitivities may be undesired) and (ii) in the assessment of the FE model itself (unexpectedly strong or weak sensitivities of the modes may indicate inappropriate modeling).

3.4 Relative importance of uncertainties

3.4.1 Introductory remarks - relative importance profile

The scatter in the eigenfrequencies, represented graphically in Fig. 3, is clearly the result of the simultaneous scatter in all the parameters modelled as uncertain quantities (cf. Pradlwarter *et al.* (2005b) for the details on the uncertainty modelling). In the great majority of the problems arising in practice, the influence of each parameter on a given output quantity of interest can be quite different. A recently introduced methodology (Pradlwarter 2006, Pellissetti *et al.* 2006a), aiming at the quantification of the relative importance of the uncertain parameters has been shown to be extremely efficient and scalable and hence applicable to large-scale structural systems. The methodology is based on the following definition of the relative importance s_k of the k-th uncertain parameter,

$$s_k = \frac{\partial g(\mathbf{x})}{\partial x_k}\sigma_{x_k} \tag{10}$$

where $\mathbf{x} = [x_1\ x_2\ ...\ x_n]$ is the vector containing the uncertain parameters, $g(\mathbf{x})$ is the response quantity of interest, with respect to which the relative importance is sought, σ_{x_k} is the standard deviation of the k-th uncertain parameter. Basically, the measure corresponds to the first order approximation of the standard deviation of the quantity of interest, if only the k-th parameter is uncertain. As mentioned in section 2.4.2, the vector of the relative importance measures $\mathbf{s} = [s_1\, s_2\ ...\ s_n]$ is referred to as gradient in this uncertainty-based setting.

Fig. 6 shows the set of relative importances $\{s_k\}_{k=1319}^{1}$ of the uncertain parameters for the first

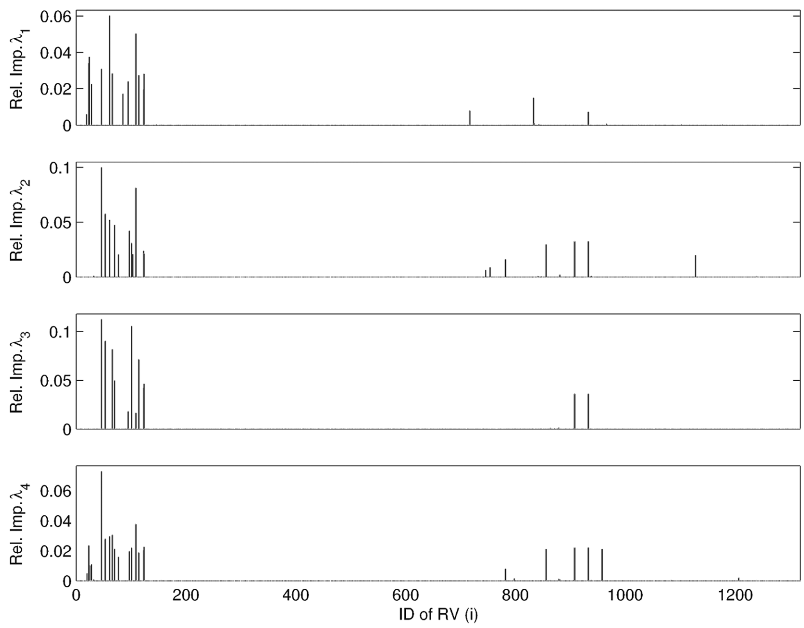

Fig. 6 Relative importance profile of the full set of uncertain parameters (1319), for the first four eigenfrequencies $\{\lambda_i\}_{i=1}^{4}$

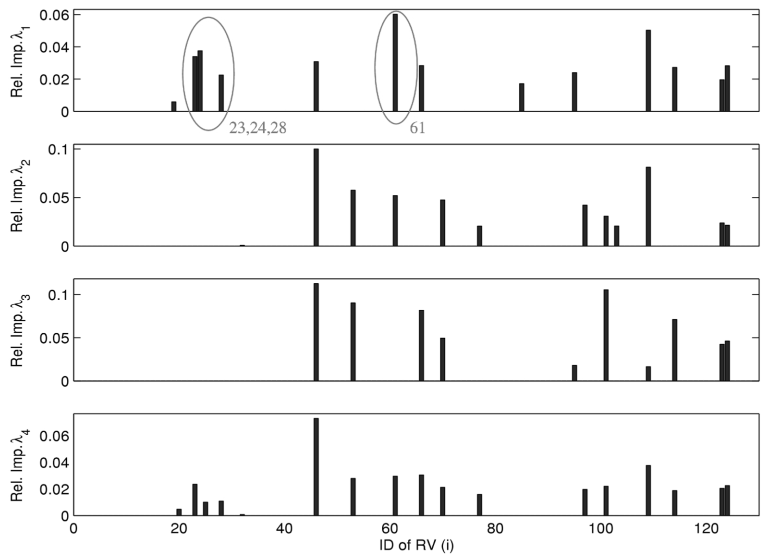

Fig. 7 Relative importance profile of the first 130 uncertain parameters, for the first four eigenfrequencies $\{\lambda_i\}_{i=1}^{4}$

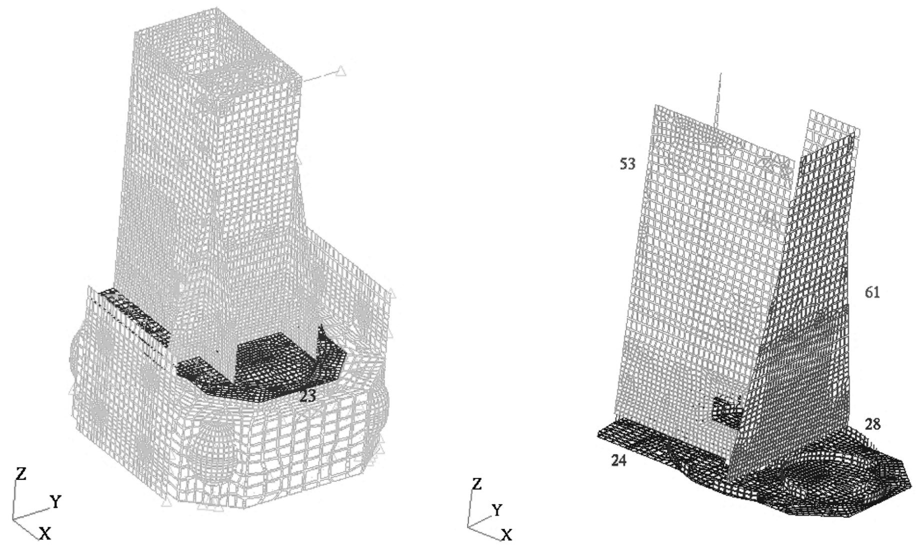

Fig. 8 Components of the FE model to which selected parameters (23, 24, 28, 53, 61) apply

four eigenfrequencies $\{\lambda_i\}_{i=1}^{4}$. For each parameter, indexed on the abscissa, the length of the vertical bar indicates the relative importance. While these "profiles" of the relative importance are somewhat similar for the various eigen-frequencies λ_1 through λ_4, the figure confirms the fact that the relative importance changes with the quantity of interest.

Table 1 Physical meaning of some of the most important parameters

Parameter #	Physical property
23	Young's modulus of aluminum thermal doubler (isotropic)
24	Young's modulus of aluminum bottom skin (isotropic)
28	Young's modulus of aluminum (isotropic)
53	Young's modulus (longitudinal) of orthotropic panel material (CFRP)
61	Young's modulus (longitudinal) of orthotropic panel material (CFRP)

3.4.2 Physical properties associated with the most important parameters

Given the large number of parameters of the satellite model, the indexing of the uncertain parameters has been performed in an automated fashion, independently of the physical properties corresponding to the various parameters.

The aim of this subsection is to convey the physical meaning of some of the most relevant parameters, namely those highlighted in Fig. 7. This figure shows the left-most portion of the relative importance profile in Fig. 6, in which an accumulation of important parameters can be noticed. The highlighted parameters 23, 24, 28 and 61 are particularly important for the first eigenfrequency, whereas parameter 51 refers to a property that is important for the second, third and fourth eigenfre-quency, but not for the first one.

To reveal the physical meaning of these parameters, the components of the FE model, to which each one of these parameters applies, are indicated in Fig. 8. Furthermore, Table 1 specifies the physical properties corresponding to the selected parameters.

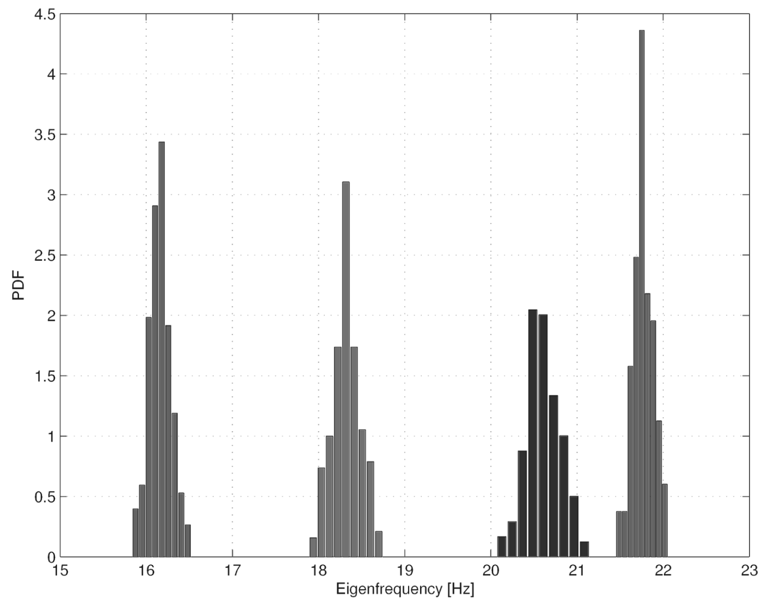

Fig. 9 Histograms of the eigenfrequencies, modes 1-4, uncertainty limited to 25 most important parameters

3.5 Stochastic model reduction

3.5.1 Introductory remarks

The information on the relative importance of the various uncertain parameters can be used to perform a reduction of the stochastic model, i.e., to reduce the number of uncertain parameters. More specifically, the uncertainty modelling can be limited to those parameters which turn out to contribute significantly to the uncertainty in the quantity of interest.

3.5.2 Monte-Carlo Simulation with most important parameters

In order to demonstrate the equivalence of a reduced stochastic model, a Direct Monte-Carlo simulation has been performed, in which only the 25 most important parameters (according to Fig. 6) have been used. It is important to stress that the importance assessment refers to the *first* eigenfrequency as quantity of interest.

Fig. 9 shows the histograms of the first four eigenfrequencies resulting from these MC-simulations. Compared with Fig. 3, it is clearly seen that not only the histogram of the first eigenfrequencies matches very well, but also the remaining ones. This is clearly due to the fact, that the relative importance profiles of the first four eigenfrequencies are not too dissimilar, as Fig. 6 revealed. In other words, the various eigenfrequencies share some of the most important parameters.

3.5.3 Monte-Carlo Simulation with arbitrary parameters

In contrast, Fig. 10 shows the histograms associated with Monte-Carlo simulations based on confining the uncertainty to a completely arbitrary set of 25 parameters. In other words, the parameters are selected completely independently of their relative importance. Clearly, the resulting

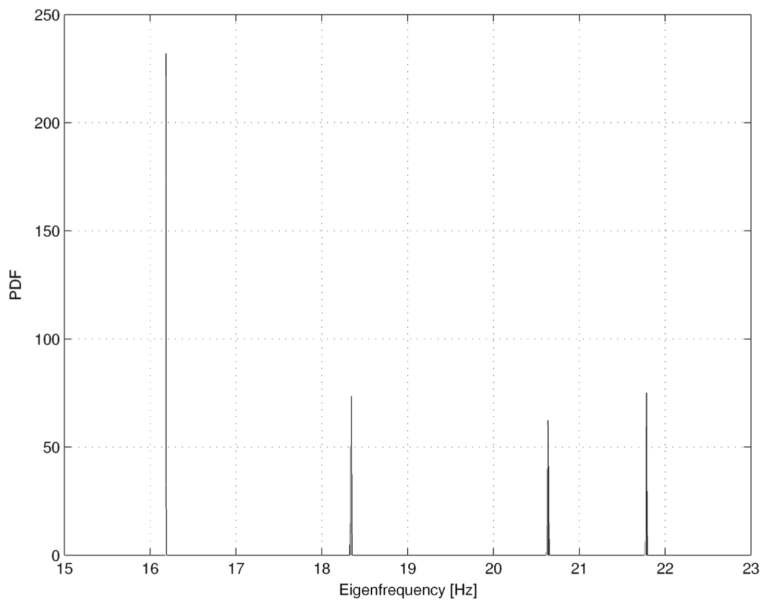

Fig. 10 Histograms of the eigenfrequencies, modes 1-4, uncertainty limited to an arbitrary set of 25 parameters

scatter in the eigenfrequencies is basically negligible, not surprisingly since by randomly picking a small set of parameters the odds of picking some of the important ones are very low.

These results also emphasize the importance to assess the effects of the uncertainty in as many parameters as possible. Expressed in another way, they show that limiting the uncertainty assessment to few parameters may indicate a fictitious robustness of the model to uncertainties, with possibly dangerous consequences.

3.6 Reliability estimation

As mentioned in section 2.4, the assessment of the reliability of structures with many uncertain parameters is a particularly arduous task. Analyzing this aspect - perhaps the most critical one - of the effects of uncertainties on the structural response, namely the unlikely but possible scenario of failure of the structure to meet one of its requirements, is particularly challenging from the computational point of view. One of the few robust methods applicable to this kind of problems is the Line Sampling method briefly reviewed in section 2.4.2.

In the present section the application of the line sampling to the reliability problem, i.e., the estimation of small failure or exceedance probabilities is discussed for the satellite structure introduced previously. The quantity of interest consists again in the first eigenfrequency of the satellite. In this case the common practice of using the relative importance measures to define the important direction, denoted by $\mathbf{e}_\alpha$ in Fig. 1, has been adopted. In other words, the important direction is defined as follows,

$$\mathbf{e}_\alpha = [s_1 s_2 \ldots s_n]/\|\mathbf{s}\| \tag{11}$$

where the importance measures s_k are defined as in Eq. (10) (see also Pradlwarter 2006, Pellissetti *et al.* 2006a). Considering the first eigenfrequency as the quantity of interest, the corresponding relative importance profile is the top portion of Fig. 6.

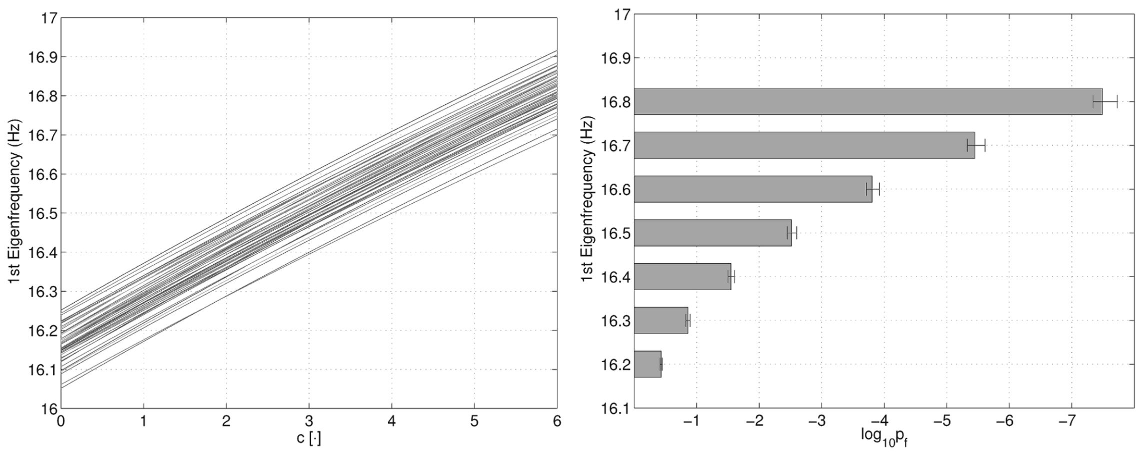

Fig. 11 Left: Line Sampling; Right: Exceedance probability estimates $\hat{p}_F$ and error bars ($\pm\ \hat{\sigma}_{\hat{p}_F}$) vs. threshold level

Fig. 11 shows the results of the line sampling procedure. The left portion depicts the trend of the first eigenfrequency for the collection of lines. Each curve corresponds to one line $l^{(j)}$ in Fig. 1. For each line the response (i.e., the first eigenfrequency) has been evaluated at five support points (denoted by c_1, c_2, etc. in Fig. 1). Between the support points, the response has been interpolated using cubic splines. A total of 48 lines have been sampled.

The resulting estimates for the exceedance probabilities are shown in the right portion of the figure. Each bar shows the exceedance probability (on the abscissa, in logarithmic scale) for a given threshold level of the first eigenfrequency (on the ordinate). For instance, the probability that the first eigenfrequency will exceed the value of 16.6 Hz is approximately one in ten thousand ($p_F = 10^{-4}$). On the vertices of the bars the error bars are indicated, which delimit the interval $\hat{p}_F \pm \hat{\sigma}_{\hat{p}_F}$, where $\sigma_{\hat{p}_F}$ is the standard deviation of the estimate $\hat{p}_F$ of the exceedance probability. The small interval size shows that in this case a highly accurate estimate of the exceedance (failure) probability could be obtained, at a modest computational cost (240 simulations), which demonstrates the high efficiency of the Line Sampling procedure in the present case.

In summary, the results show that the described reliability problem is amenable to the analysis with Line Sampling, thanks to the moderate degree of non-linearity of the quantity of interest (first eigenvalue), with respect to the uncertain input parameters. Clearly, this situation is not always encountered in practical applications - in such cases alternative methods may perform better. The interested reader is referred to Schuëller *et al.* (2004) for a critical appraisal and to Schuëller and Pradlwarter (2006) for a comparative studies of various analysis procedures, as well as to Pellissetti *et al.* (2006b) for a more detailed discussion of the application of line sampling to large scale problems.

3.7 Effects of data and model uncertainties on the frequency response analysis

The assessment of the effects of uncertainties on the modal analysis of the satellite, presented in the previous section, covers exclusively *uncertainties in the parameters of the given model.*

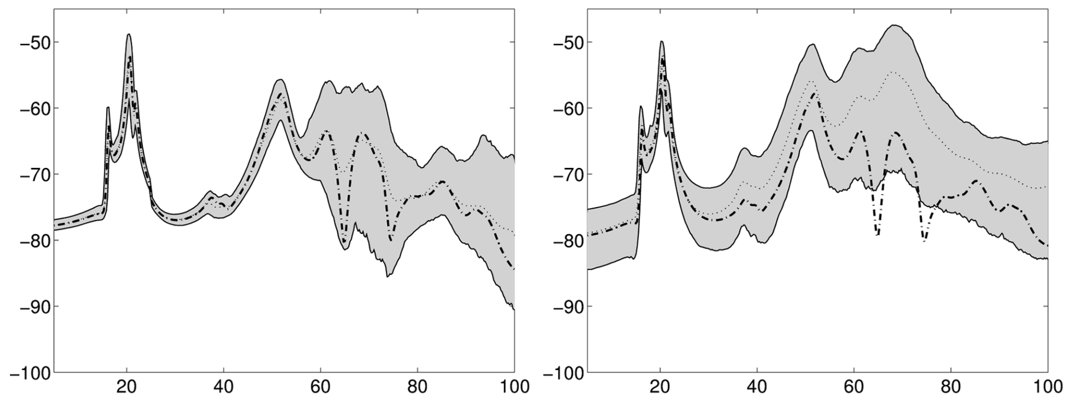

Fig. 12 Uncertain frequency response (displacement, in dB) in low-frequency band [5, 100]Hz. Left: *parametric probabilistic model*. Right: *non-parametric probabilistic model*. Dash-dotted line: full-size, nominal model. Dotted line: Mean of probabilistic model. Gray region: 96%-confidence region (Capiez-Lernout *et al.* 2006)

As mentioned in section 2.5, the effects of *model uncertainties* are often significant, in addition to the parametric (or "data"-) uncertainties. This is visualized in Fig. 12 (see also Capiez-Lernout *et al.* 2006), which compares the uncertainty in the frequency response considering: i) only parametric uncertainties (left portion of Fig. 12), and ii) parametric and model uncertainties (right portion of Fig. 12). The main observations related to the parametric model, are that the 96%-confidence interval (gray area) is very narrow over the first half of the frequency range and that the mean stochastic response (weak dotted line) shows generally little deviation from the response predicted by the full size, nominal FE model (strong dash-dotted line). In contrast, the frequency response ensemble predicted by the non-parametric model matches that of the parametric one only in the frequency range of the first few eigen-frequencies, i.e., [15,25] Hz. In this range the structure is clearly not sensitive to model uncertainties, as the confidence interval is very narrow. In the remaining frequency ranges the effect of model uncertainties is felt quite heavily, as indicated by the following two facts: (i) the significant width of the confidence intervals, and (ii) the strong deviation of the mean stochastic response from the nominal frequency response, especially in the upper half of the spectrum.

In summary, the present example shows that significant additional uncertainty propagates to the frequency response when model uncertainties are considered in addition to parametric uncertainties, by means of a non-parametric model.

4. Conclusions

The present paper addressed from various different angles the effects of uncertainties on the analysis of structures, with special attention to the modal dynamic analysis. A frequently arising problem in this case is how to deal with the uncertainty in modes that are closely spaced. The approach here presented, based on the weighted superposition of the eigenfrequencies, turns out to be capable to avoid the problem of mode intermixing and hence to adequately propagate the uncertainty from the parameters to the eigenfrequencies.

Concerning the estimation of small exceedance probabilities, advanced simulation based methods, such as the Line Sampling described in this paper, are well suited for the reliability analysis of large scale structures. This is due to their excellent scalability, which preserves the performance also in the presence of large numbers of uncertain parameters.

One more aspect that this paper has touched is that of model uncertainties. Clearly, this type of uncertainty has an important effect, too, on the structural analysis. The non-parametric approach, briefly reviewed in this paper, can be used to analyse the effect of the model uncertainties, in the context of structural dynamics. The appealing feature of this approach is that it has been shown to match well with experimental observations, which can indeed be used in its calibration.

The described methodologies have been demonstrated in the context of a complex satellite structure, leading to the following conclusions:

- With the exception of the first few modes, which are very well separated, the analysis revealed the necessity of postprocessing the population of modal quantities. In this case this has been done using a weighted expansion that is reminiscent of the modal assurance criterion used for comparing measured and predicted modes.
- The relative importance of the various uncertain input parameters varies greatly, depending both on the considered parameter and on the quantity of interest. In general it could be observed, that

the effects on the quantities of interest (eigen-frequencies in this case) are typically induced by few of the uncertain parameters.

- The challenging problem of estimating low-probability events - for the sake of reliability analysis - has been tackled with the help of the Line Sampling method. The results confirm that using this procedure the effect of uncertainties in the structural properties on the failure probability can be analysed accurately and hence controlled.
- The analyses performed in this study were limited to uncertainties in the parameters of a given model. In this respect it should be noted that recent analyses, based on a non-parametric approach, have revealed a significant effect of model uncertainties, in particular for the higher frequencies of the considered band (5-100 Hz).

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