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Influence of cable loosening on nonlinear parametric vibrations of inclined cables

Qingxiong Wu†

College of Civil Engineering, Fuzhou University, 523 Gongye Road, Fuzhou, Fujian, China

Kazuo Takahashi‡

Department of Civil Engineering, Faculty of Engineering, Nagasaki University, 1-14, Bunkyo-machi, Nagasaki, Japan

Baochun Chen‡†

College of Civil Engineering, Fuzhou University, 523 Gongye Road, Fuzhou, Fujian, China

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Abstract. The effect of cable loosening on the nonlinear parametric vibrations of inclined cables is discussed in this paper. In order to overcome the small-sag limitation in calculating loosening for inclined cables, it is necessary to first derive equations of motion for an inclined cable. Using these equations and the finite difference method, the effect of cable loosening on the nonlinear parametric response of inclined cables under periodic support excitation is evaluated. A new technique that takes into account flexural rigidity and damping is proposed as a solution to solve the problem of divergence. The regions of inclined cables that undergo compression are also indicated.

Keywords: inclined cable; loosening; parametric vibration.

1. Introduction

In conventional nonlinear vibration analysis of cables, the equations of motion are formulated based on two assumptions: that the cable is a continuum resisting only axial forces and that the same laws as apply to truss members are applicable. In other words, the cables are assumed to be able to resist both tensile and compressive axial forces (Irvine 1981, Yamaguchi *et al.* 1979, Yamaguchi 1997). However, this assumption is invalid when the sum of the initial and deflection-induced additional horizontal tensions results in compression, since actual cables have no resistance

† Associate Professor, E-mail: wuqingx@fzu.edu.cn
‡ Professor, Corresponding author, E-mail: takahasi@civil.nagasaki-u.ac.jp, t-kazuo@net.nagasaki-u.ac.jp
‡† Professor, E-mail: baochunchen@fzu.edu.cn

to compressive forces. This situation may be easily observed in the nonlinear vibration of cables under the influence of wind-rain action (Matsumoto *et al.* 1995, Honda *et al.* 1995), wind uplift, strong earthquakes, etc. This means it is necessary to evaluate the effect of cable loosening when dealing with the nonlinear vibration of cables. An analysis taking into account cable loosening has been carried out on the stay cables of a cable-stayed bridge subjected to strong ground motions (Wu *et al.* 2003). The nonlinear loosening effect has also been found in the hangers used for the main cables of cable suspension bridges (Lazer *et al.* 1990, Peterson 1990, Sepe *et al.* 2001). Cable loosening has been noted in low-tension single cables such as power and signal transmission lines, underwater cables and mooring lines used for offshore applications (Leonard *et al.* 1972, Triantafyllou *et al.* 1992). The nonlinear dynamic response and vibration of low-tension cables has been evaluated by introducing a small flexural rigidity to avoid the singularity at the point when cable tension becomes zero (Burgess 1993, Triantafyllou *et al.* 1994, Sun *et al.* 1998). Low-tension cable dynamics have been solved by the finite difference method using a modified box scheme (Koh *et al.* 1999).

There appear to be few published papers in which both loosening and the mass of taut cables with small sags that include cable extension are taken into account. Against this background, the authors proposed a new technique for evaluating cable loosening and also examined the effect of loosening on the nonlinear vibration of horizontal cables with small sags (Wu *et al.* 2003, Wu *et al.* 2004). The conventional nonlinear equations of motion of a cable formulated as a continuum were discretized using an explicit form of the finite difference method on the assumption that the cable has no compressive resistance. The problem of divergence was solved by a newly proposed technique that takes into account flexural rigidity and damping, two physical properties of a cable. Finally, the authors discussed the effect of cable loosening on the response and looked at the regions in which compressive forces are generated from nonlinear forced vibrations and parametric vibrations, focusing on horizontal cables with small sags. An investigation of nonlinear forced response (Wu *et al.* 2003) showed that loosening can easily occur in a cable with a sag-to-span ratio corresponding to the region in which there is a mode transition from lower mode to higher mode under periodic vertical loading. Research into nonlinear parametric response (Wu *et al.* 2004) revealed that cables with a specific sag-to-span ratio easily become loose. Within the range of cable loosening, the principal unstable region is larger than the second unstable region.

This paper discusses the effect of loosening on the nonlinear parametric vibrations of inclined cables. When a cable is inclined, there are additional properties that differ from those of a horizontal cable (Yamaguchi *et al.* 1979, Triantafyllou 1984). The crossover of natural frequencies of a symmetric mode toward the natural frequencies of an antisymmetric mode never occurs and the corresponding modes are neither symmetric nor antisymmetric. Therefore, in order to calculate loosening for an inclined cable without a small-sag limitation, it is necessary to first derive equations of motion for an inclined cable. Using these equations and then applying the proposed method, it is possible to evaluate the effect of loosening on the nonlinear parametric response of inclined cables under periodic support excitation. In order to evaluate both the common and differing properties of inclined cables and horizontal cables expressed in the cited research (Wu *et al.* 2004), this paper looks at cables with small sags. The influence of inclination angle on cable loosening is evaluated and the regions that generate compressive forces in inclined cables are shown.

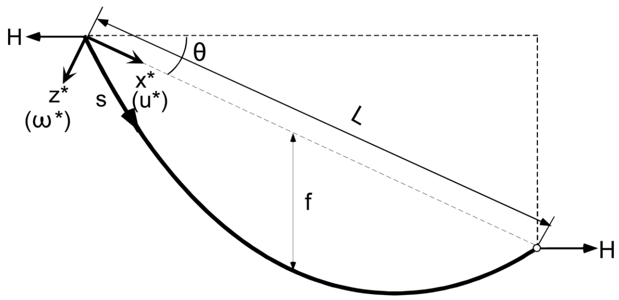


Fig. 1 Geometry of an inclined cable

2. Equations of motion for an inclined cable

A cable with a uniform cross section and uniform weigh per unit length hanging between two points and inclined at an angle θ is analyzed, as shown in Fig. 1. Throughout this paper, the longitudinal and transverse directions (x^*, z^*) of the corresponding string are denoted the local coordinate system.

In the local coordinate system (x^*, z^*) of the inclined cable, the following equations are obtained by removing the self-weight term in the equations of motion:

$$m\frac{\partial^2 u^*}{\partial t^2} - \frac{\partial}{\partial s}\left(\Delta T\frac{dx^*}{ds} + (T+\Delta T)\frac{\partial u^*}{\partial s}\right) = p_{x^*}(x^*, t) \tag{1}$$

$$m\frac{\partial^2 w^*}{\partial t^2} - \frac{\partial}{\partial s}\left(\Delta T\frac{dz^*}{ds} + (T+\Delta T)\frac{\partial w^*}{\partial s}\right) = p_{z^*}(x^*, t) \tag{2}$$

where ΔT is the additional tension generated, $T = H/(\cos\theta \cdot dx^*/ds - \sin\theta \cdot dz^*/ds)$ is the initial tension obtained from the initial horizontal tension H, u^* is the longitudinal displacement in the x^* direction, w^* is the transverse displacement in the z^* direction, $p_{x^*}(x^*, t)$ and $p_{z^*}(x^*, t)$ are the loads in the x^* and z^* directions, s is the coordinate along the cable, m is the mass per unit length of the cable and t is time.

The additional tension ΔT can be obtained from

$$\Delta T = EA\left\{\frac{dx^*}{ds}\frac{\partial u^*}{\partial s} + \frac{dz^*}{ds}\frac{\partial w^*}{\partial s} + \frac{1}{2}\left(\frac{\partial u^*}{\partial s}\right)^2 + \frac{1}{2}\left(\frac{\partial w^*}{\partial s}\right)^2\right\} \tag{3}$$

where E is Young's modulus and A is the cross-sectional area of the cable.

By making Eqs. (1), (2) and (3) non-dimensional by means of $H\sec\theta$, the length L between supports and the first natural circular frequency ω_0 of the inclined taut string, the following equations are obtained:

$$\frac{\partial^2 \bar{u}^*}{\partial \tau^2} - \frac{1}{\pi^2}\frac{d}{d\bar{s}}\left\{\Delta\bar{T}\frac{d\bar{x}^*}{d\bar{s}} + \left(\frac{\cos\theta}{\frac{d\bar{x}^*}{d\bar{s}}\cos\theta - \frac{d\bar{z}^*}{d\bar{s}}\sin\theta} + \Delta\bar{T}\right)\frac{\partial \bar{u}^*}{\partial \bar{s}}\right\} = \frac{8\beta}{\pi^2}\cdot\frac{p_{x^*}(\bar{x}^*, \tau)}{mg} \tag{4}$$

$$\frac{\partial^2 \overline{w}^*}{\partial \tau^2} - \frac{1}{\pi^2} \frac{d}{d\overline{s}} \left\{ \Delta \overline{T} \frac{d\overline{z}^*}{d\overline{s}} + \left(\frac{\cos\theta}{\frac{d\overline{x}^*}{d\overline{s}}\cos\theta - \frac{d\overline{z}^*}{d\overline{s}}\sin\theta} + \Delta \overline{T} \right) \frac{\partial \overline{w}^*}{\partial \overline{s}} \right\} = \frac{8\beta}{\pi^2} \cdot \frac{p_{z^*}(\overline{x}^*, \tau)}{mg} \tag{5}$$

$$\Delta \overline{T} = k^2 \left\{ \frac{d\overline{z}^*}{d\overline{s}} \frac{\partial \overline{w}^*}{\partial \overline{s}} + \frac{d\overline{x}^*}{d\overline{s}} \frac{\partial \overline{u}^*}{\partial \overline{s}} + \frac{1}{2}\left(\frac{\partial \overline{w}^*}{\partial \overline{s}}\right)^2 + \frac{1}{2}\left(\frac{\partial \overline{u}^*}{\partial \overline{s}}\right)^2 \right\} \tag{6}$$

where $\Delta \overline{T} = \Delta T / H\sec\theta$ is the non-dimensional additional tension, $\beta = mgL/8H\sec\theta$, $\overline{u}^* = u^*/L$ and $\overline{w}^* = w^*/L$ are the non-dimensional displacements in the x^* and z^* directions, $\tau = \omega_0 t$ is the non-dimensional time, $\omega_0 = \pi/L\sqrt{H\sec\theta/m}$ is the first natural circular frequency of an inclined taut string, $k^2 = EA/H\sec\theta$ is the ratio of axial stiffness to longitudinal tension of the cable, H is the initial horizontal tension of the inclined cable, $\overline{\omega}_1 = \omega_1/\omega_0$ is the first natural non-dimensional circular frequency of the inclined cable, ω_1 is the first natural circular frequency of the inclined cable, g is the gravitational acceleration, $\overline{x}^* = x^*/L$, $\overline{z}^* = z^*/L$, and $\overline{s} = s/L$.

For comparison, if Eqs. (4) and (5) are rewritten in the global co-ordinate system, the results coincide with the Yamaguchi equations (Yamaguchi 1997).

3. Analytical conditions and numerical analysis method

3.1 Analytical conditions

Fig. 2 shows the small-sag ($f = \beta L$) horizontal cable is discussed in this paper. The inclination angle θ is changed in order to maintain the same span length L. The profile of the inclined cable varies with inclination angle θ.

The nonlinear parametric vibration of the cable is discussed. Parametric excitation is provided by displacement of the longitudinal support, $X(t)^*$, at the upper end, as shown in Fig. 2. Loads $p_{x^*}(x^*, t)$ and $p_{z^*}(x^*, t)$ are zero. Support excitation is given by the longitudinal displacement u^* at

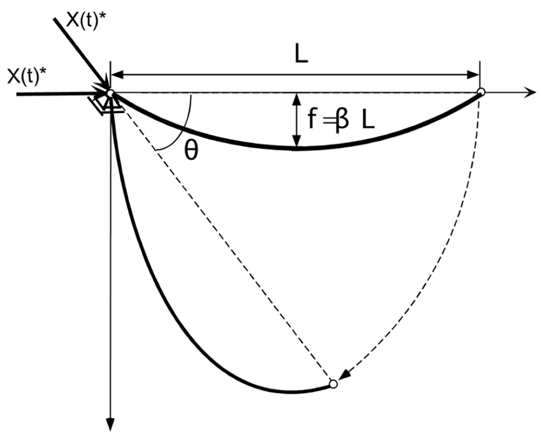


Fig. 2 Analytical model and longitudinal excitation at upper end

the upper end of the inclined cable, as described by the following equation:

$$\bar{u}^*(0,\tau) = \bar{X}(\tau)^* = \bar{X}^* \sin\bar{\Omega}\tau \tag{7}$$

where $\bar{X}(\tau)^* = X(t)^*/L$ is the non-dimensional support excitation, $\bar{X}^*$ is the non-dimensional amplitude of the support excitation and $\bar{\Omega}$ is the non-dimensional circular frequency of the parametric excitation.

Parametric vibrations of the cable are known to be most likely to appear in the region of cable lowest frequency (Takahashi 1991, Lilien *et al.* 1994). Therefore, frequency $\bar{\Omega}$ is assumed to be the lowest frequency $\bar{\omega}_1$ or twice the lowest frequency $2\bar{\omega}_1$. When $\bar{\Omega} = \bar{\omega}_1$ is used, the excitation corresponds to parametric excitation of the second unstable region. When $\bar{\Omega} = 2\bar{\omega}_1$ is used, it corresponds to that of the principal unstable region (Wu *et al.* 2004).

3.2 Numerical analysis method

When considering cable loosening in nonlinear vibration analysis, if the value of total tension is less than zero, it is considered to be zero. This is described by the following equation:

$$\frac{\cos\theta}{\frac{d\bar{x}^*}{d\bar{s}}\cos\theta - \frac{d\bar{z}^*}{d\bar{s}}\sin\theta} + \Delta\bar{T} = 0 \quad \text{when} \quad \frac{\cos\theta}{\frac{d\bar{x}^*}{d\bar{s}}\cos\theta - \frac{d\bar{z}^*}{d\bar{s}}\sin\theta} + \Delta\bar{T} < 0 \tag{8}$$

In the case of an inclined cable, loosening must be evaluated at every point along the length since the total tension is different at every point.

There is a possibility of cable loosening appearing when vibration components of higher modes with discontinuous angles occur (Wu *et al.* 2003). In order to take into account the effect of all cable modes, the direct integration method is used to solve Eqs. (4), (5) and (6) while evaluating Eq. (8). An explicit form of the finite difference method is employed (Ames 1992).

The time interval for numerical analysis should be defined so as to satisfy the stability condition of the scheme that is used. Parameter β is set to less than 1/8 for small-sag cables. The ratio of axial stiffness to horizontal tension, k^2, is set to 900. The cable is divided into 100 elements. In other words, the non-dimensional length, $\Delta\bar{x}^*$, is 0.01. In order to satisfy the stability conditions, the time interval, $\Delta\tau$, must be less than $1/4\Delta\bar{x}^{*2}$; in the case considered here, 1.0×10^{-5} is used.

In order to prevent higher mode vibrations of the cable with discontinuous angles, flexural rigidity and damping, actual properties possessed by cables, are considered in this paper. The effect of flexural rigidity on vibration of the cable becomes significant in the case of higher modes that change the curvature of the cable. This is the reason for taking flexural rigidity into consideration in the present analysis (Wu *et al.* 2003, 2004).

When the effects of flexural rigidity and damping are included, Eqs. (4) and (5) become the following non-dimensional equations:

$$\frac{\partial^2\bar{u}^*}{\partial\tau^2} + 2h\bar{\omega}_1\frac{\partial\bar{u}^*}{\partial\tau} + \frac{k^2\delta\partial^4\bar{u}^*}{\pi^2\ \partial\bar{s}^4} - \frac{1}{\pi^2}\frac{d}{d\bar{s}}\left\{\Delta\bar{T}\frac{d\bar{x}^*}{d\bar{s}} + \left(\frac{\cos\theta}{\frac{d\bar{x}^*}{d\bar{s}}\cos\theta - \frac{d\bar{z}^*}{d\bar{s}}\sin\theta} + \Delta\bar{T}\right)\frac{\partial\bar{u}^*}{\partial\bar{s}}\right\}$$

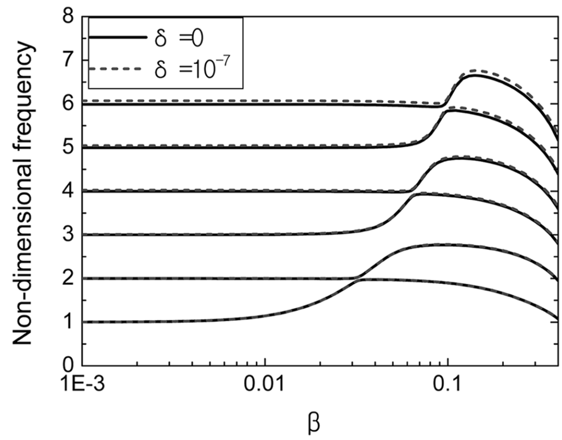
$$= \frac{8\beta}{\pi^2}\cdot\frac{p_{x^*}(\overline{x}^*,\ \tau)}{mg} \tag{9}$$

$$\frac{\partial^2\overline{w}^*}{\partial\tau^2} + 2h\overline{\omega}_1\frac{\partial\overline{w}^*}{\partial\tau} + \frac{k^2\delta}{\pi^2}\frac{\partial^4\overline{w}^*}{\partial\overline{s}^4} - \frac{1}{\pi^2}\frac{d}{d\overline{s}}\left\{\Delta\overline{T}\frac{d\overline{z}^*}{d\overline{s}} + \left(\frac{\cos\theta}{\frac{d\overline{x}^*}{d\overline{s}}\cos\theta - \frac{d\overline{z}^*}{d\overline{s}}\sin\theta} + \Delta\overline{T}\right)\frac{\partial\overline{w}^*}{\partial\overline{s}}\right\}$$

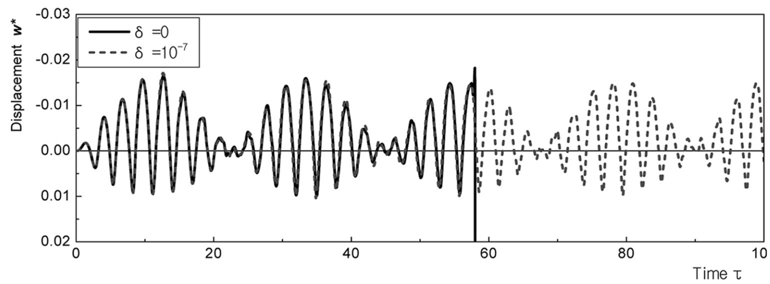
$$= \frac{8\beta}{\pi^2}\cdot\frac{p_{z^*}(\overline{x}^*,\ \tau)}{mg} \tag{10}$$

where $\delta = EI/L^2EA$ is the ratio of the flexural rigidity to axial stiffness and h is the damping constant.

In the analysis described here, the flexural rigidity parameter is assumed to be linear and without relation to excitation amplitude. At large values of support excitation amplitude, this parameter would need to be reset. This paper, however, discusses the characteristics of occurrence of cable loosening for inclined cables and support excitation displacement is not very large. So parameters d and h, which are needed to solve the divergence problem, are set to $\delta = 10^{-7}$ and $h = 0.001$, respectively (Wu *et al.* 2003, 2004).



(a) Natural frequency



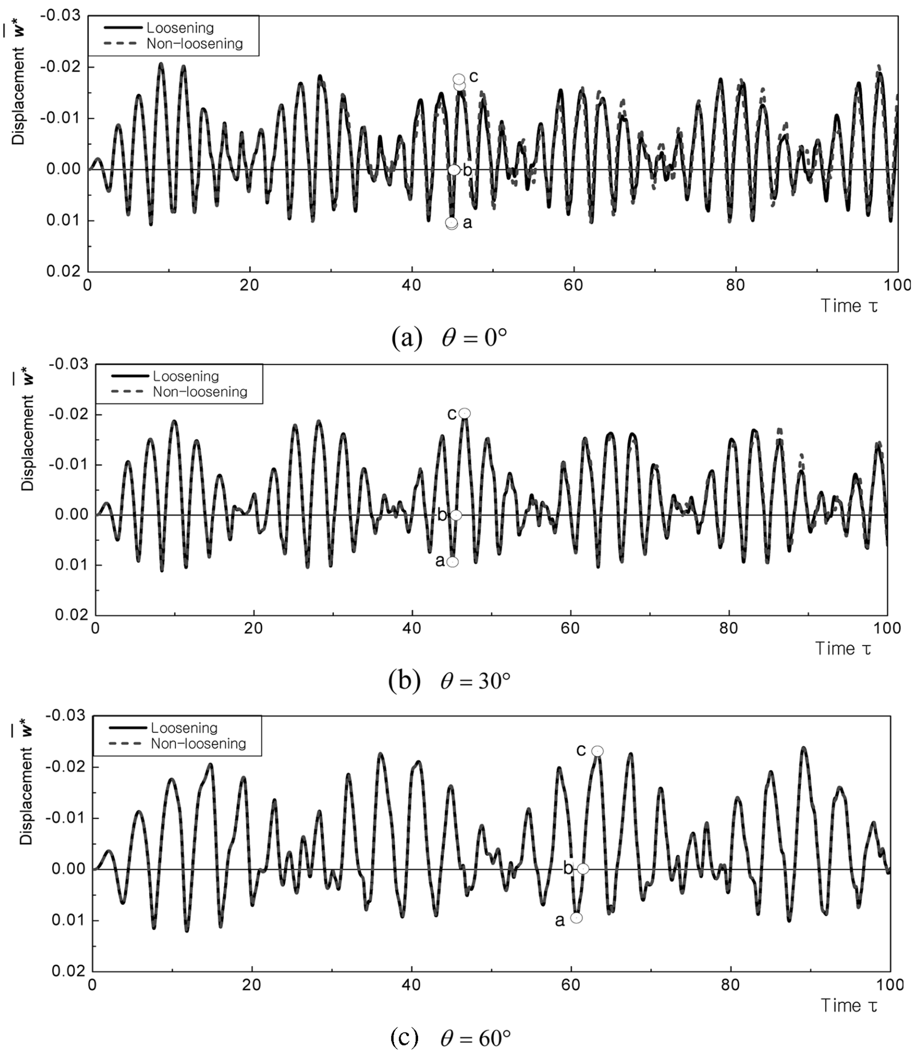
(b) Time history for β=0.04, $\overline{x}$*=0.00026

Fig. 3 Effect of δ on natural frequency and time history for inclined cables with inclination angle θ= 30°

Fig. 3 shows the effect of flexural rigidly parameter δ on natural frequency and time history when δ is 10^{-7}. For the natural frequencies shown in Fig. 3(a), the effect of flexural rigidity on the lower frequencies is very small. As regards the time history, shown in Fig. 3(b), responses using $\delta = 10^{-7}$ do not readily diverge and can be calculated even if compressive forces appear in the cable. Therefore, the flexural rigidly parameter δ is set to 10^{-7} in the discussion that follows.

4. Parametric responses in the second unstable region

Figs. 4, 5, 6 and 7 show the nonlinear parametric responses of the second unstable region ($\overline{\Omega} = \overline{\omega}_1$) when the cable is subjected to support excitation at the upper end ($\overline{X}^* = 0.000338$).

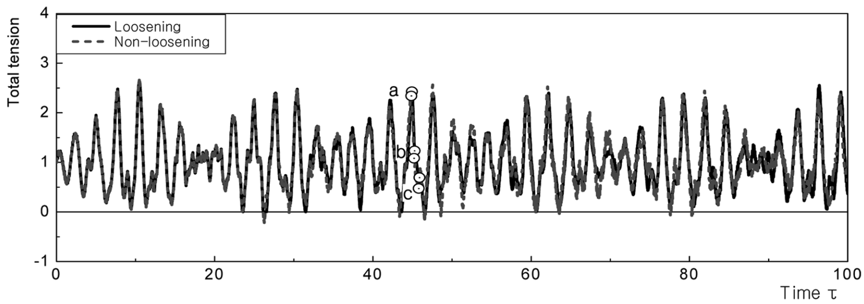


(a) $\theta = 0°$

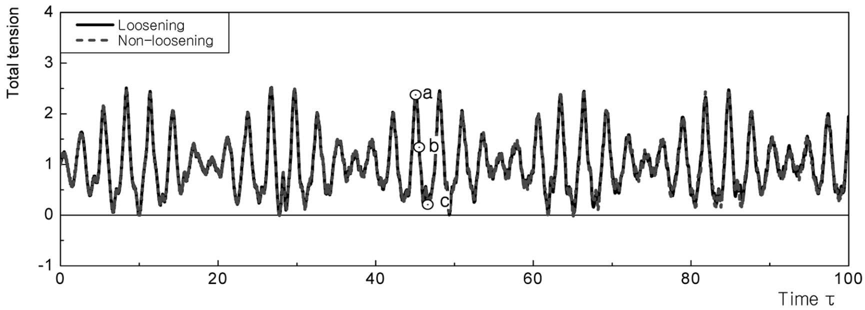
(b) $\theta = 30°$

(c) $\theta = 60°$

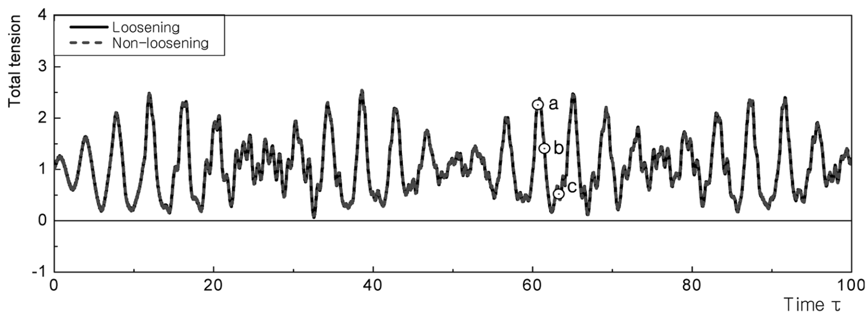
Fig. 4 Time history of transverse displacement in the second unstable region ($\beta = 0.04$, $\overline{X}^* = 0.000338$)



(a) $\theta = 0°$



(b) $\theta = 30°$

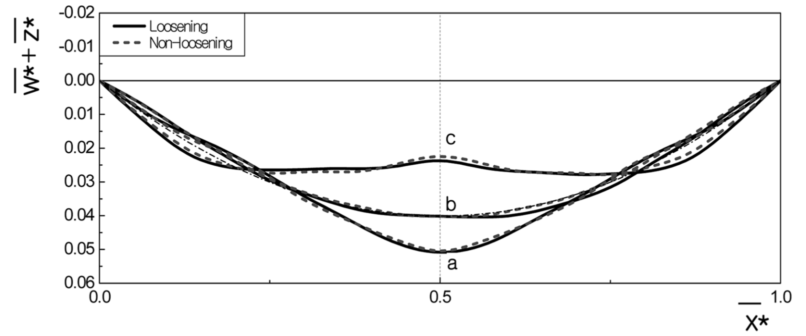


(c) $\theta = 60°$

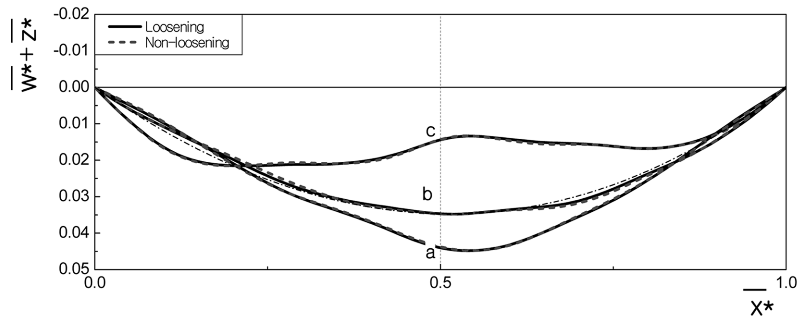
Fig. 5 Time history of total tension in the second unstable region ($\beta = 0.04$, $\overline{X}^* = 0.000338$)

Figs. 4 and 5 are the time histories of transverse displacement and total tension at the center of the inclined cable. Fig. 6 gives the space shapes of the three cables. The corresponding maximum transverse displacement and total tension during nonlinear parametric vibration are shown in Fig. 7. Notations *a*, *b* and *c* correspond to the maximum, zero, and minimum displacements at the center of the horizontal span.

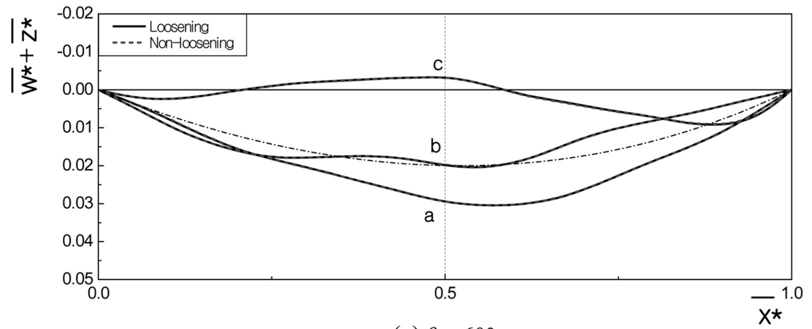
Fig. 4 demonstrates that the effect of loosening on the response of an inclined cable under parametric excitation in the second unstable region is small because the regions that undergo compression are narrow. This is also true for horizontal cables (Wu *et al.* 2003, 2004). However, the loosening appears at a different point in an inclined cable than in a horizontal cable. The initial tension of an inclined cable decreases from the upper end to the lower end and the minimum initial tension appears at the lower end of the inclined cable (see Fig. 7). Therefore, in the case of inclined



(a) $\theta = 0°$



(b) $\theta = 30°$



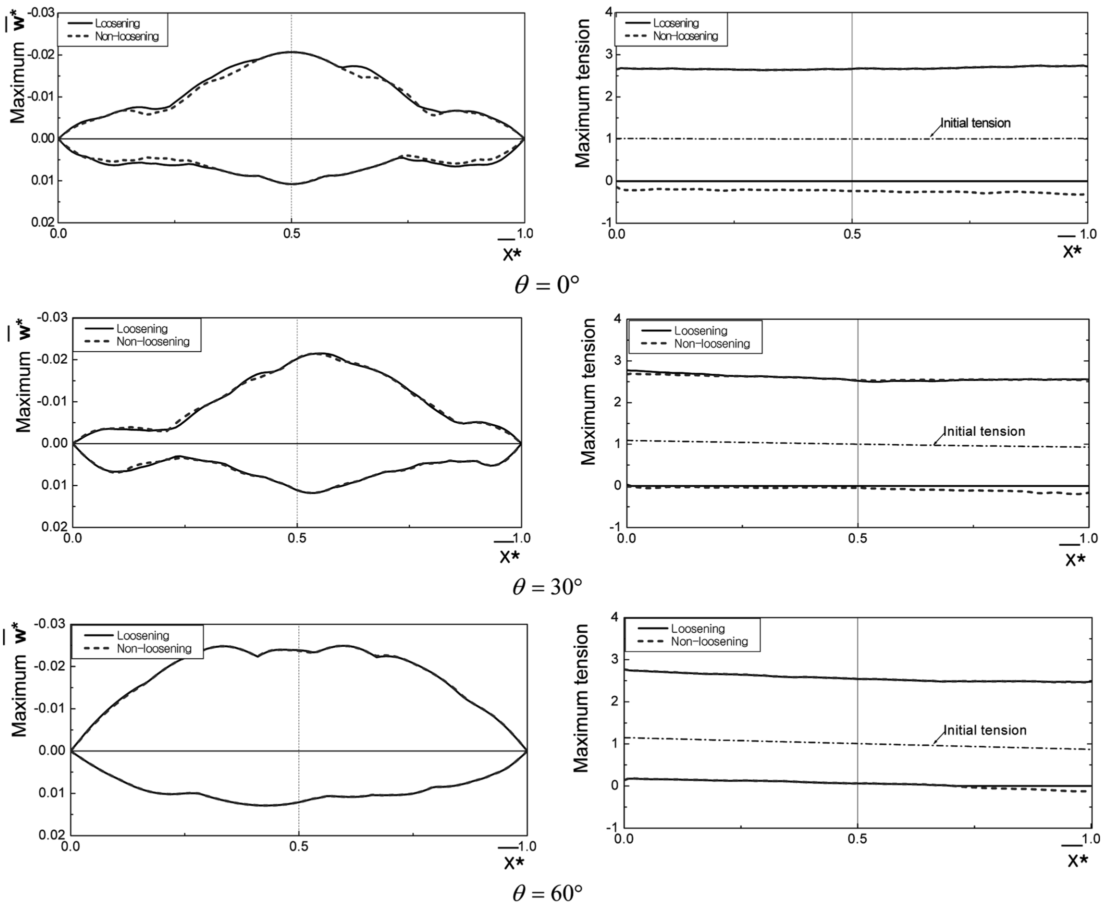
(c) $\theta = 60°$

Fig. 6 Space shape in the second unstable region ($\beta = 0.04$, $\overline{X}^* = 0.000338$)

cables with small sags, the compressive force is arises at the lower end of the cable and loosening appears first at the lower end.

Figs. 8 and 9 show the time histories, space shapes, and maximum responses when the parametric excitation is about 1.5 times the amplitude of that used to generate Figs. 4-7. Comparing Fig. 8(a) with Fig. 6(b), the cable has a higher-order modal shape in the compressive force region and maintains space shapes that do not easily generate compressive forces when loosening appears. This is also characteristic of horizontal cables (Wu *et al.* 2003, 2004).

Fig. 8(c) shows the frequency spectrum of transverse displacement. The non-dimensional frequency corresponds to the ratio of the calculated frequency to the first frequency of the inclined cable, $\omega_1/2\pi$. The predominant frequency with cable loosening taken into consideration is lower



(a) Transverse displacement (b) Total tension

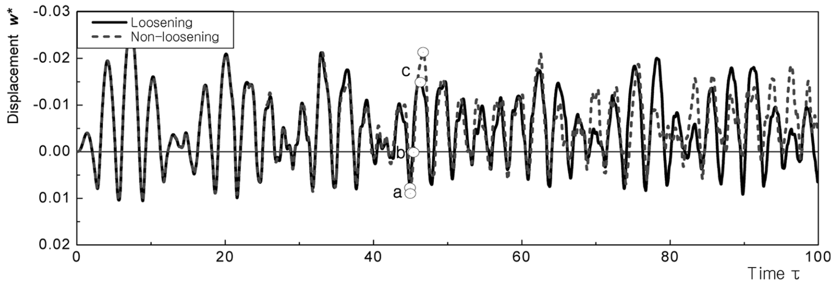
Fig. 7 Maximum response in the second unstable region ($\beta = 0.04$, $\overline{X}^* = 0.000338$)

than that when there is no loosening. Since axial stiffness decreases after a cable loosens, the predominant frequency falls and displacement increases. This figure showing the frequency spectrum of cable displacement further demonstrates that no other modes are involved in the resonance via other auto-parametric energy transfers.

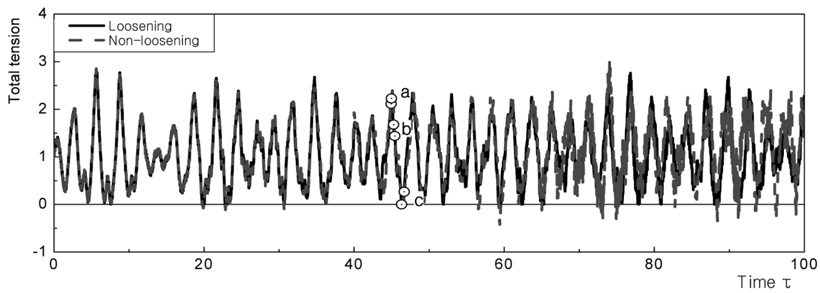
Fig. 9(b) illustrates how loosening affects the negative transverse displacement of an inclined cable and how the effect of cable loosening on transverse displacement changes with position. Loosening affects the negative maximum transverse response, but has scarcely any effect on the positive maximum transverse response. These are also characteristics of horizontal cables (Wu *et al.* 2004).

Fig. 10 shows the relationship between the minimum amplitude $\overline{X}^*$ of the support excitation that generates compressive forces in the cable and inclination angle θ in the second unstable region for the three different cables. The corresponding first natural non-dimensional frequencies of the same cables are shown in Fig. 11.

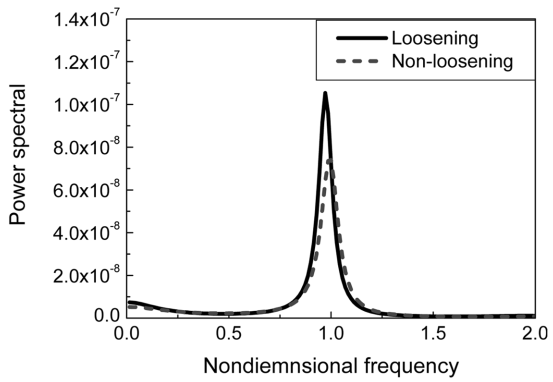
When the inclination angle $\theta = 0°\sim 30°$, cable loosening readily occurs small support excitation



(a) Transverse displacement



(b) Total tension

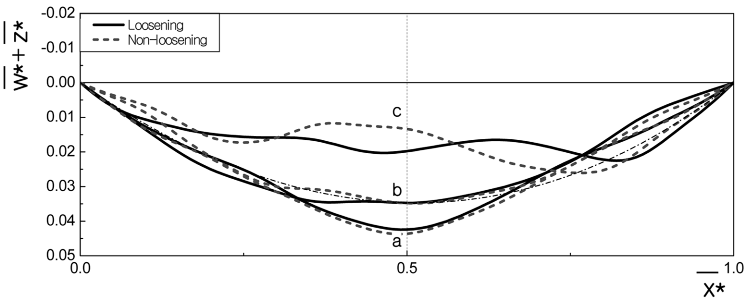


(c) Power spectrum of transverse displacement

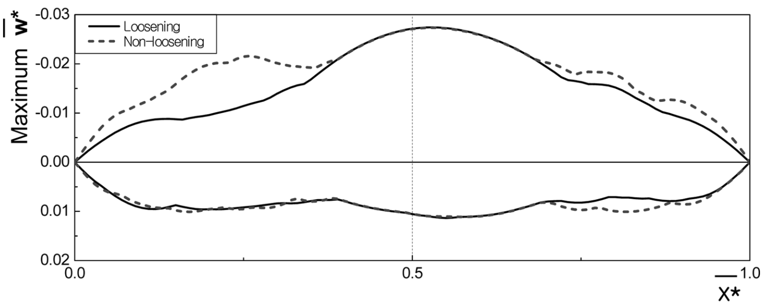
Fig. 8 Time history in the second unstable region ($\theta = 30^{\circ}$, $\beta = 0.04$, $\overline{X}^{*} = 0.000557$)

amplitudes and the effect of parameter β is small. When the inclination angle $\theta > 30^{\circ}$, the minimum amplitude of support excitation increases, except when parameter $\beta = 0.04$ and the inclination angle $\theta = 30^{\circ}$~65°, as the inclination angle becomes large and the effect of parameter β becomes conspicuous. This means that it is harder to induce loosening in inclined cables with large angles than in horizontal cables.

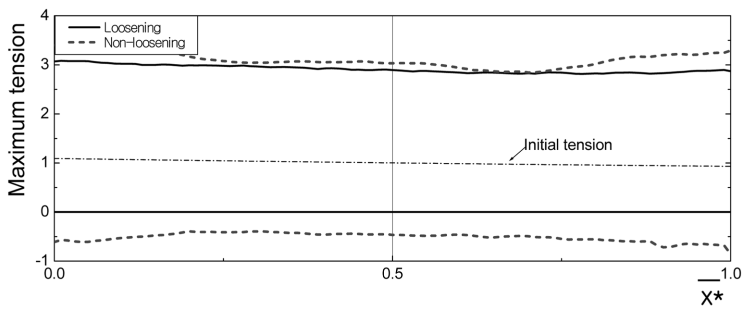
The effect of inclination angle for the parameter $\beta = 0.04$ cable differs from the other cables. This may be explained by the fact that the modal shape of the inclined cable varies with the magnitude of inclination angle, as shown as Fig. 11.



(a) Space shape



(b) Transverse displacement



(c) Total tension

Fig. 9 Space shape and maximum response in the second unstable region ($\theta = 30^{\circ}, \beta = 0.04$, $\overline{X}^* = 0.000557$)

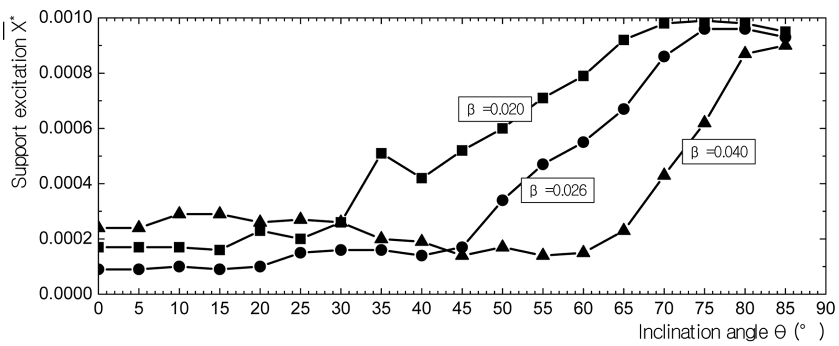


Fig. 10 Relationship between minimal amplitude of support excitation and inclination angle in the second unstable region

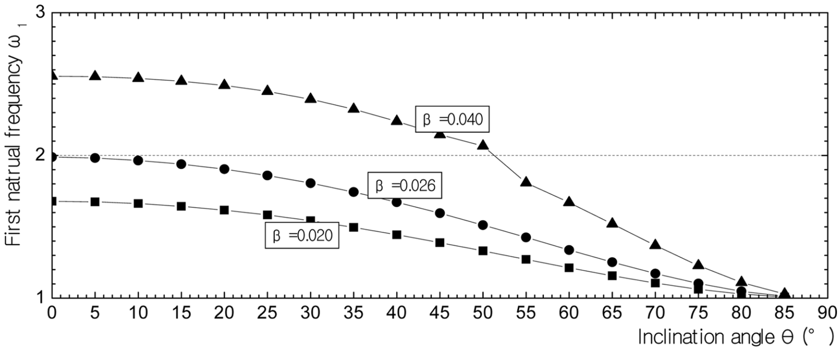
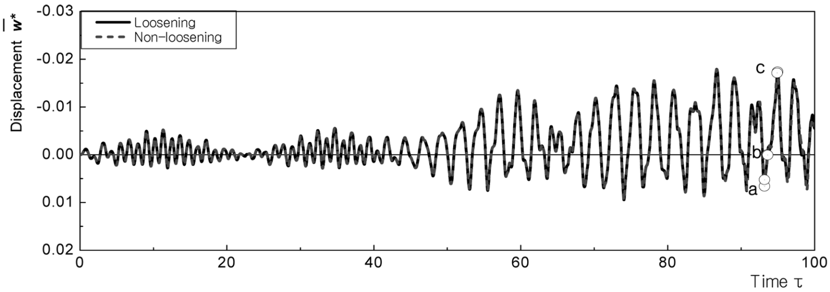
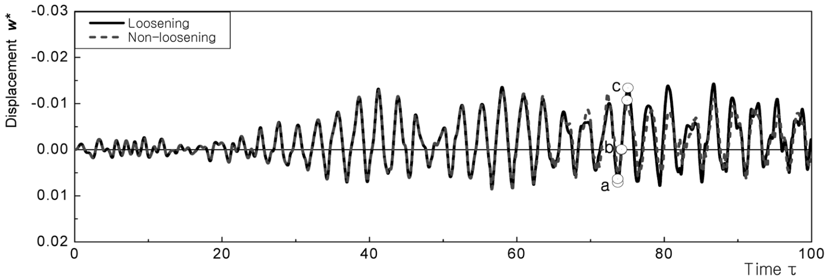


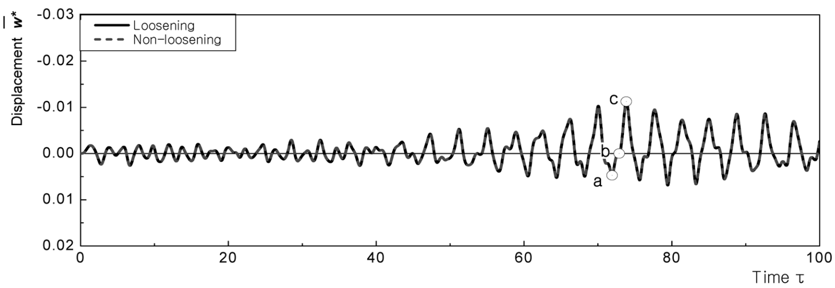
Fig. 11 First natural non-dimensional frequency of inclined cables



(a) $\theta = 0°$

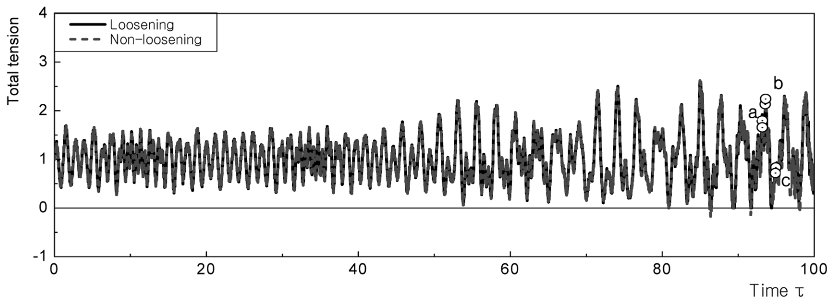


(b) $\theta = 30°$

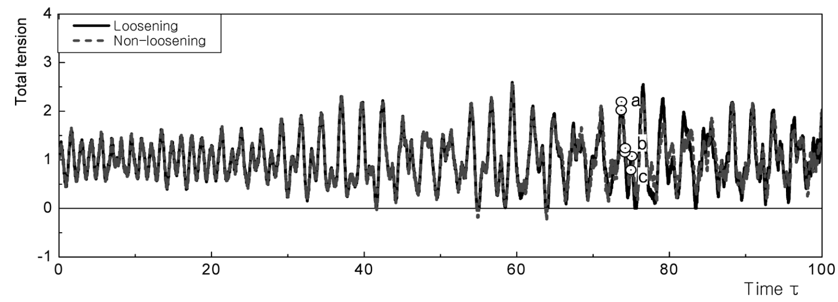


(c) $\theta = 60°$

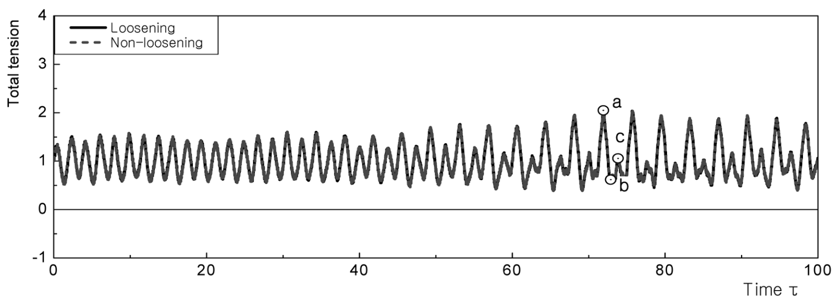
Fig. 12 Time history of transverse displacement in the principal unstable region ($\beta = 0.04$, $\overline{X}^* = 0.000338$)



(a) $\theta = 0°$



(b) $\theta = 30°$



(c) $\theta = 60°$

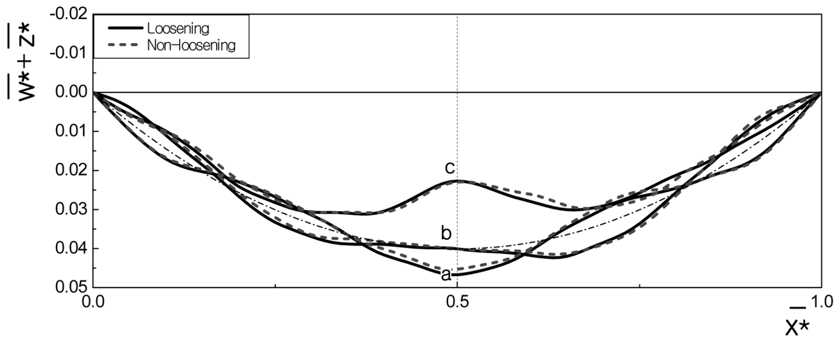
Fig. 13 Time history of total tension in the principal unstable region ($\beta = 0.04$, $\overline{X}^* = 0.000338$)

5. Parametric responses in the principal unstable region

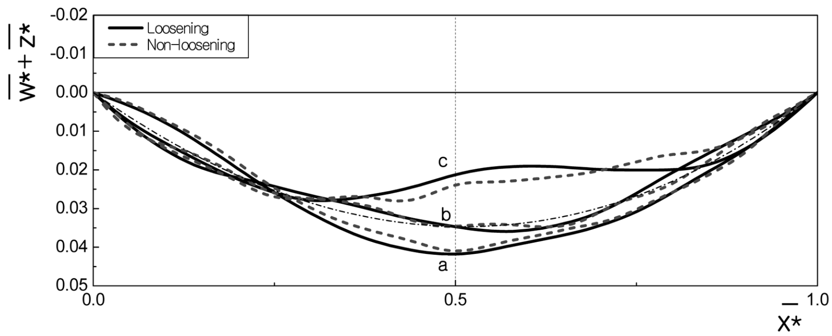
Figs. 12, 13, 14, and 15 show the nonlinear parametric responses of the principal unstable region ($\overline{\Omega} = 2\overline{\omega}_1$) when the cable is subjected to the support excitation at the upper end ($\overline{X}^* = 0.000338$). Figs. 16 and 17 are the time histories, space shapes, and maximum responses when the amplitude of support excitation $\overline{X}^* = 0.000557$. Comparing Fig. 13(b) with Fig. 16(a), the cable maintains space shapes that do not readily generate compressive forces when loosening occurs, which is the same as the result in the second unstable region.

Loosening affects the negative maximum response but scarcely affects the positive maximum response, as shown in Figs. 15 and 17(b). The results are similar to those obtained for the second unstable region.

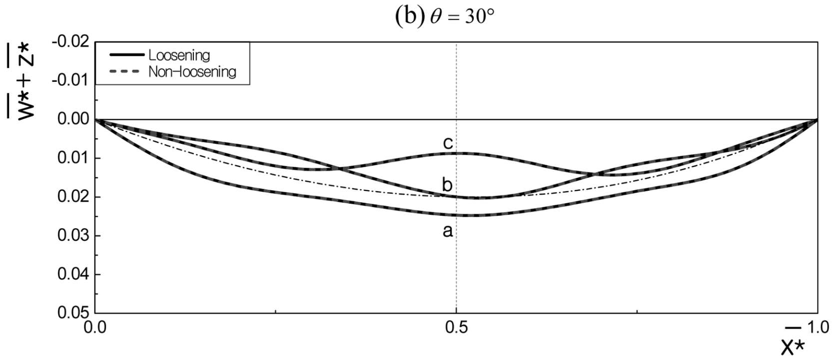
The relationship between the minimum amplitude $\overline{X}^*$ of parametric excitation that generates



(a) $\theta = 0°$



(b) $\theta = 30°$

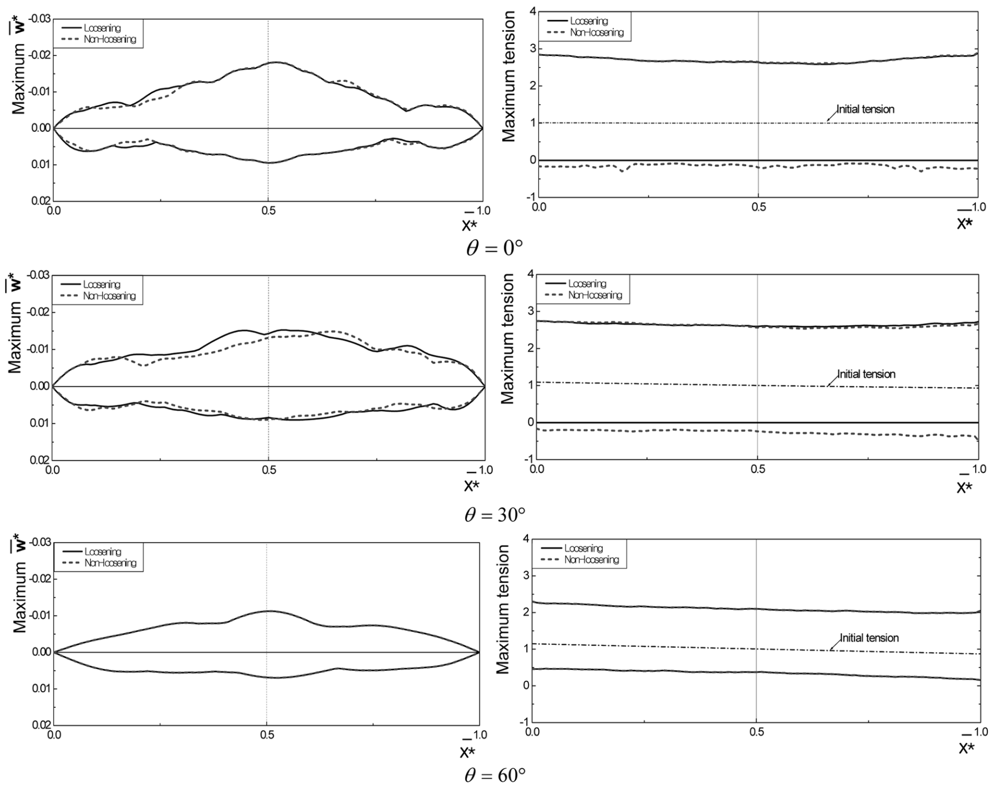


(c) $\theta = 60°$

Fig. 14 Space shape in the principal unstable region ($\beta = 0.04$, $\overline{X}^* = 0.000338$)

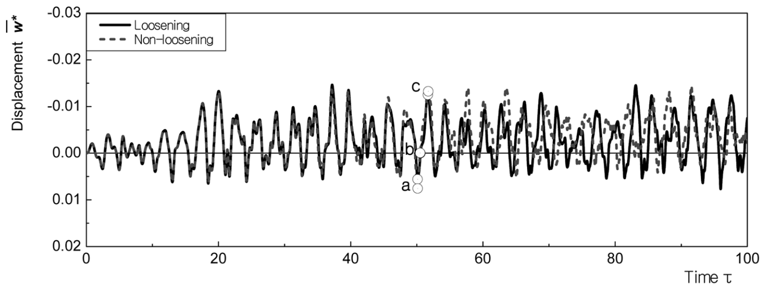
compressive forces in the cable and inclination angle θ in the principal unstable region is shown in Fig. 18.

Comparing Fig. 10 with Fig. 18, the minimum amplitudes of support excitation in the principal unstable region are higher than those in the second unstable region for inclination angles in the range $\theta = 0^\circ \sim 20^\circ$. Unlike in the second unstable region, the minimum amplitude of excitation in the principal unstable region decreases when the inclination angle $\theta = 20^\circ \sim 60^\circ$, except when $\beta = 0.04$. The minimum amplitude of excitation increases thereafter and approaches the same value as in the second unstable region when inclination angle θ increases.

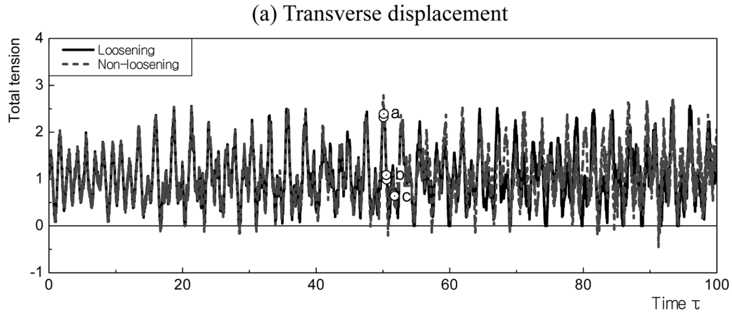


(a) Transverse displacement (b) Total tension

Fig. 15 Maximum response in the principal unstable region ($\beta = 0.04$, $\overline{X}^* = 0.000338$)



(a) Transverse displacement



(b) Total tension

Fig. 16 Time history in the principal unstable region ($\theta = 30^{o}$, $\beta = 0.04$, $\overline{X}^* = 0.000557$)

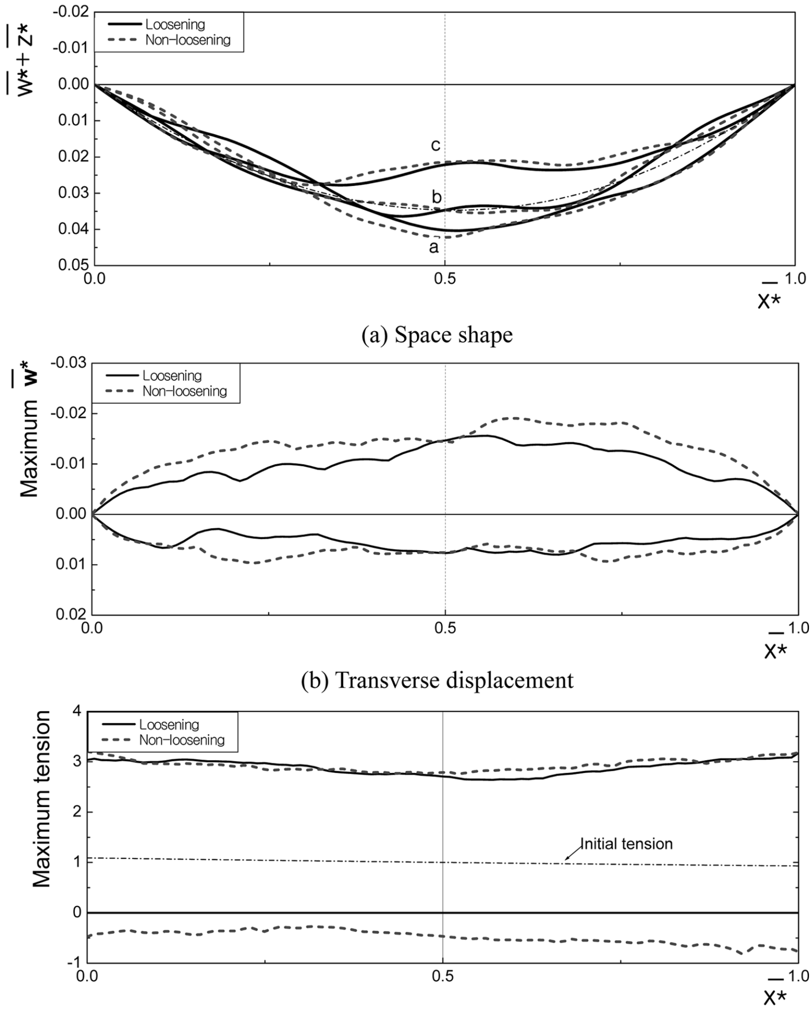


Fig. 17 Space shape and maximum response in the principal unstable region ($\theta = 30^o$, $\beta = 0.04$, $\overline{X}^* =$ 0.000557)

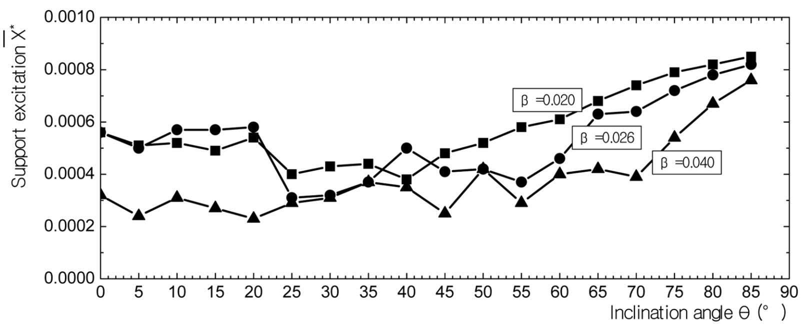


Fig. 18 Relationship between minimum amplitude of support excitation and inclination angle in the principal unstable region

6. Conclusions

This paper examined the effect of cable loosening on the nonlinear parametric vibration of inclined cables subjected to periodic support excitation. In order to calculate the loosening of inclined cables without a small-sag limitation, it was necessary to first derive new equations of motion for an inclined cable. Regarding the effect of loosening on the nonlinear parametric vibrations of inclined cables with small sags, the main findings are as follows:

1. The total tension in an inclined cable during nonlinear parametric vibration is not constant and cable loosening arises first at the lower end.
2. The minimum level of support excitation that generates a compressive force in the cable varies with the inclination angle. The influence of inclination angle depends on the initial profile of the cable.
3. Cable loosening affects the negative maximum transverse response but scarcely affects the positive maximum transverse response.
4. Cable loosening readily occurs in a cable with a small inclination angle under small amplitudes of parametric excitation in the second unstable region.

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