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# Stochastic optimum design criterion of added viscous dampers for buildings seismic protection

Giuseppe Carlo Marano† and Francesco Trentadue‡

*Department of Environmental Engineering and Sustainable Development, Technical University of Bari, viale del Turismo, 10, 74100 Taranto, Italy-EU*

Rita Greco‡†

*Department of Civil Engineering and Architecture, Technical University of Bari, via Orabona, 4, 72126 Bari, Italy-EU*

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**Abstract.** In this study a stochastic approach for linear viscous dampers design adopted for seismic protection of buildings is developed. Devices optimal placement into the main structure and their mechanical parameters are attained by means of a reliability-based optimum design criterion, in which an objective function (O.F.) is minimized, subject to a stochastic constraint. The seismic input is modelled by a non stationary modulated Kanai Tajimi filtered stochastic process. Building is represented by means of a plane shear type frame model. The selected criterion for the optimization searches the minimum of the O.F., here assumed to be the cost of the seismic protection, i.e., assumed proportional to the sum of added dampings of each device. The stochastic constraint limits a suitable approximated measure of the structure failure probability, here associated to the maximum interstorey drift crossing over a given threshold limit, related, according with modern Technical Codes, to the required damage control.

**Keywords:** seismic protection; random vibrations; optimum design; viscous dampers.

## 1. Introduction

Recently, technologies in seismic engineering often make use of additional devices installed on the main structures. The most important aim of using these techniques is to increase the level of protection against seismic actions and to reduce potential damage levels in structural and non structural elements and to limit failure probability of the structure (Housner *et al.* 1997). Concerning linear viscous dampers, which are the main topic of this work, efforts to develop these concepts into a workable technology have recently increased significantly, and a remarkable number of these devices have now been installed in structures throughout the world. Currently, recent reports show

† Assistant Professor of Structural Design, Corresponding author, E-mail: gmarano@poliba.it
‡ Associate Professor of Structural Mechanics
‡† Assistant Professor of Structural Design

that there are over 150 structures in the United States and over 2,000 structures in Japan utilizing dampers to improve seismic response behaviour of structures (Ikeda 2004).

Dampers applications, and in the context of this study linear viscous dampers, are closely tied to *Performance Based Seismic Design* (PBSD), that is going to play an important role in future seismic engineering. Dampers applications will be increased through these design criteria, to provide safe and not very expensive earthquake protection systems.

Seismic structural projects using PBSD provide different protection levels defined by means of stated performance objectives, depending on the importance of the construction and on earthquake intensity. This is a modern philosophy in which the design criteria are expressed in terms of achieving given performance objectives, when the structure is subjected to different levels of seismic hazard.

Many efforts and works have been developed to identify, to define, and to quantify various aspects of PBSD. Both SEAOC's Vision 2000 (1995) projects and FEMA 356 NEHRP guidelines for the Seismic Rehabilitation of Buildings (2000) present the first guiding principles for multi-level performance objectives and provide the most complete work on performance-based design of buildings. In detail, with reference to VISION 2000 performance matrix, three principal structural performance levels are defined. They are, respectively, the *Operational* (eventually *Full*), the *Life Safety*, and finally the *Collapse Prevention* one. The expected structural performance shall be controlled by the assignment of a specific building to a "*Seismic hazard exposure group*" (also known as "*Seismic Use Group*"), that is defined on the basis of its occupancy and the relative consequences of earthquake induced damage. The performance objectives are progressively more conservative in order to attain minimum levels of performance, suitable to the occupancy and importance of the building. In case of *Operational* or *Full Operational* performance levels, the building is expected to sustain only minimal or no damage to structural and non structural elements and, in this case, a minimal repair is required. These protection levels must be guaranteed also under strong earthquakes, if referred to essential or safety critical facilities.

In all these situations, adding aseismic devices, such as viscous dampers, could guarantee prescribed performance levels better and cheaper than by using the traditional approaches (Constantinou and Symans 1993, Shen and Soong 1995) based on design of structures with sufficient strength capacity. Moreover, viscous dampers can be implemented also on existing buildings for retrofitting actions (Crosby *et al.* 1994, Reinhorn and Li 1995a/b) more easily and with less damage than other conventional approaches. In future, including viscous dampers in structures design, could have a wide diffusion in applications for important new and existing buildings.

For these reasons, several researches have been directed to selection of the optimal protection capacity of these devices by using different optimization methods. For seismic applications, however, only a limited number of studies have been carried out to obtain the optimal placement and design characteristics of viscous dampers, especially if referred to random vibrations approach. One of the earlier criteria introduced for optimal design of discrete passive dampers in the vibrations control of flexible systems was developed by De Silva (1981). He defined an algorithm capable to find the solution by minimizing distance between modal damping along with natural frequencies of the system and initial pre-assigned values.

A similar criterion is based on optimal distribution of the supplemental damping, able to maximize the damping ratio of the fundamental mode, as this mode contribution to the structure's overall response is often significant (Ashour and Hanson 1987). An energy minimization method was used (Gürgöze and Müller 1992) to evaluate the optimal placement and damping coefficient for a single viscous device in a MDoF system. A different study utilizes ideas of optimal control theory to find the best damping coefficients of viscoelastic devices (Gluck *et al.* 1996). Other authors (Zhang and Soong 1992, Shukla and Datta 1999) proposed and extended the sequential seismic design method to find the optimal configuration of viscous dampers for buildings with specified storey stiffness. They used an intuitive criterion to place additional dampers sequentially on the storey at which the interstorey drift response is maximum.

An interesting stochastic approach for damping devices optimum design criteria in seismic protection is proposed by Park (2004), concerning a minimization of the total building life-cycle cost. It is based on a stochastic dynamic approach for failure probability evaluation. The optimization problem is also formulated by adopting as design variables locations and amount of the viscoelastic dampers. Constraints are the maximum interstorey drifts evaluated by first crossing theory application in stationary conditions.

In this work a stochastic reliability-based optimum design is proposed for multi-storey buildings, protected by interstorey viscous dampers. The main goal is to obtain an improved seismic performance and an "*acceptable*" level of protection, depending on building importance and seismic hazard.

Assuming that on an existing building the seismic protection cost depends on the total added damping, the Objective Function (O.F.) here adopted is the sum of all added dampings. A constraint on the failure probability, associated to the crossing of an assigned interstrorey displacement, directly related to a given damage level, is imposed. Damage is in fact related to the interstorey lateral displacement, as common assumed in many scientific and practical applications. Therefore, in this optimum design criterion a deterministic O.F. and a stochastic constraint are considered.

Structure stochastic response characterization, needed for performing optimization, is obtained by solving non stationary *Lyapunov* differential covariance equation, being the earthquake input acting on structure modelled by means of an uniform modulated Kanai Tajimi stochastic process. Reliability evaluation is done by means of Poisson hypothesis in determining the mean threshold crossing rate for each interstorey drift. Global reliability is, in an approximate way, assumed to be the product of each interstorey reliability, in the assumption that each of them is a rare and then independent event. Even if this hypothesis could be too poor and results tend to be excessively conservative, it is a suitable method for pre-design purpose, as in this study. As example of proposed optimum design method a generic multi-storey building under different seismic and structural conditions is analysed.

## 2. PBSD of added viscous dampers

Many applications of added viscous damping concern strategic or essential constructions, where the main aim of seismic protection is not just collapse prevention but also operational assurance, as well as after intense or severe earthquakes. As stated before, *Performance Based Seismic Design* is based on these concepts, where different structural performances are required for different levels of seismic hazard. For operational performance demand, required not only under moderate but also under strong seismic events (as prescribed for strategic facilities), the use of added viscous dampers represents an interesting solution. For new design or retrofitting of multi-storey buildings, by means of this approach, the operational performance level must be assured controlling one or more suitable seismic response parameters, characterizing not only structural but also non structural damage levels.

The maximum interstorey drifts $X_i^{\max} = |x_{i+1} - x_i|^{\max}$ reached during the whole seismic event is often used to represent damage levels. The selection of the appropriate drift associated with different levels of damage is significant in terms of economy and safety, and is an important part of *PBSD*. Then, according to the operational level required (full, partial or similar), the maximum acceptable drifts are based on structural importance.

The definition of acceptable interstorey drift for operational conditions plays a fundamental role in order to characterize not only the seismic protection level, but also the required damping characteristic. Different extreme acceptable value definitions have been proposed in new seismic codes, with the aim of guaranteeing an acceptable building functionality in post earthquake situations. For example, the European Seismic Code EC8 (2003) at point 2.2.3 provides "Damage Limitation State" as maximum interstorey drift that has not been exceeded, to limit or to prevent structural damage. Under the expected seismic action the interstorey drifts are limited according to the following values:

- for buildings having non-structural elements of brittle materials attached to the structure: $|x_{i+1} - x_i| < 0.004 h_i \upsilon$;
- for buildings having non-structural elements fixed in a way as not to interfere with structural deformations: $|x_{i+1} - x_i| < 0.006 h_i \upsilon$;

where $h_i$ is the interstorey height and $\upsilon$ is a reduction factor which takes into account of the lower return period of the seismic event associated with the serviceability limit state. Its value can be assumed according to the building importance.

A similar approach is adopted in the American FEMA 450 (2004), where the allowable storey drift is defined as the linear function of each storey height, on the basis of the building seismic group and structure characteristics.

In all these cases, *PBSD* could be suitably implemented in a stochastic way by prescribing a limitation on the probability that maximum interstorey lateral displacement will exceed the assigned limit. That means giving a lower threshold bound to the probability of crossing the maximum acceptable lateral interstorey displacement. Minimum reliability level depends on the required protection for specific limit state violation and building importance; anyway, it is high and quite closer to unit. This objective could be quite easily reached by using an added viscous damper placed between the storeys, which can represent one of the most interesting and used technology.

The proposed probabilistic approach yields to a suitable devices design, on the basis of PBSD, in order to find the best/minimum total added damping value able to reduce structural displacements and hence the damage level under the assigned required operational level.

## 3. Motion equations and covariance analysis

In this section, briefly, the stochastic response of a building, for using in the optimum design criterion, is evaluated.

In detail, a modulated Kanai Tajimi stochastic process is used to describe ground acceleration $\ddot{X}_g(t)$, expressed by:

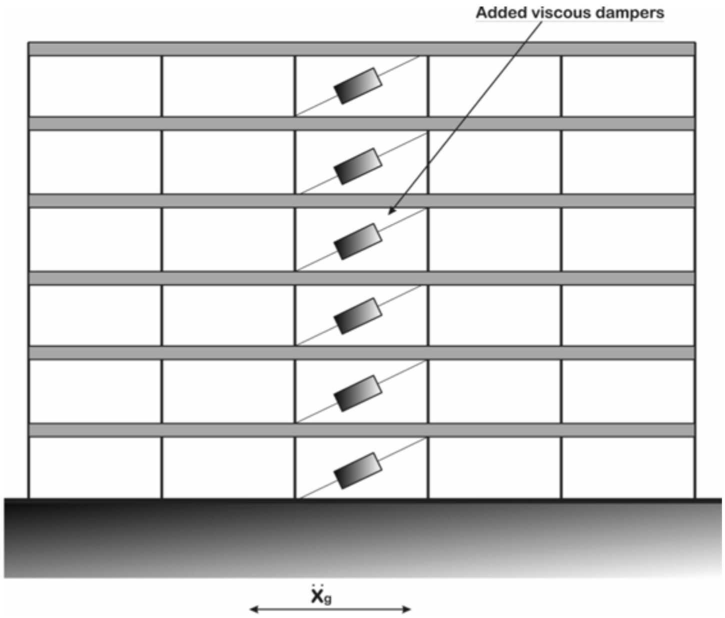


Fig. 1 Building model with added viscous dampers located at each inter-storey

$$\begin{cases} \ddot{X}_f(t) + 2\xi_g\omega_g\dot{X}_f + \omega_g^2 X_f = -w(t)\varphi(t) \\ \ddot{X}_g(t) = \ddot{X}_f(t) + w(t)\varphi(t) = -(2\xi_g\omega_g\dot{X}_f + \omega_g^2 X_f) \end{cases} \tag{1}$$

being $w(t)$ a stationary Gaussian zero mean white noise process, representing the excitation at the bed rock, $\xi_g$ and $\omega_g$, respectively, the damping and the frequency of the filter representing the ground, and finally $\varphi(t)$ the deterministic modulation function, here assumed as that proposed by Jennings (1964):

$$\varphi(t) = \begin{cases} \left(\frac{t}{t_1}\right)^2 & t < t_1 \\ 1 & t_1 \le t \le t_2 \\ e^{-\beta(t-t_2)} & t > t_2 \end{cases} \tag{2}$$

Analysis is performed on a generic shear type structural model (Fig. 1) whose mechanical behaviour is assumed linear. This is more than a simplification hypothesis, but is due to the circumstance that the maximum interstorey displacements will not exceed an admissible value, that is the main goal of design of seismic protection strategy, i.e., designed device characteristics will guarantee linearity in structural behaviour by limiting maximum displacements. By adopting these assumptions, motion equations for a generic protected building with $n$ floors is:

$$\begin{cases} \ddot{\mathbf{X}}(t) + \mathbf{M}^{-1}(\mathbf{C}^0 + \mathbf{C}^{add})\dot{\mathbf{X}}(t) + \mathbf{M}^{-1}\mathbf{K}\mathbf{X}(t) = \mathbf{r}\ddot{X}_g(t) \\ \ddot{X}_f(t) + 2\xi_g\omega_g\dot{X}_f(t) + \omega_g^2 X_f(t) = -w(t)\varphi(t) \end{cases} \tag{3}$$

$\dot{\mathbf{X}}(t)$, $\dot{\mathbf{X}}(t)$ and $\mathbf{X}(t)$ are respectively acceleration, velocity and displacement $nx1$ vectors, $\mathbf{r} = [1, 1, 1, \ldots, 1, 1]^T$ is the $nx1$ drag vector, $\mathbf{M}$, $\mathbf{C}^0$ and $\mathbf{K}$ are, respectively, the deterministic mass, viscous and stiffness $nxn$ principal structure matrices. Devices mechanical characteristics are introduced in matrix $\mathbf{C}^{add}$, which describes the added viscous damping system.

In the state space, with well known symbols notations, motion equations can be written as;

$$\dot{\mathbf{Z}}(t) = \mathbf{A}\mathbf{Z}(t) + \mathbf{F}(t) \tag{4}$$

Covariance analysis for the analysed system needs the solution of the so called *Lyapunov Equation* (see for example [Lutes and Sarkani 2001 or Soong and Grigoriu 1993]), whose expression in the present case is:

$$\dot{\mathbf{R}}_{\mathbf{ZZ}}(t) = \mathbf{A}\mathbf{R}_{\mathbf{ZZ}}(t) + \mathbf{R}_{\mathbf{ZZ}}(t)\mathbf{A}^T + \mathbf{B}(t) \tag{5}$$

where $\mathbf{A}$ is the state matrix, $\mathbf{B}(t)$ has all elements equal zero except the last one $2\pi S_0 \varphi^2(t)$, being $S_0$ the white noise intensity.

### *3.1 Stochastic optimization criterion for added viscous dampers*

In the proposed design method the optimum solution represents the added dampings which minimize the seismic protection total cost. Since during the optimization process masses $m_i$ and stiffnesses $k_i$ are constant, the cost of added dampings for unit mass could be evaluated as the ratio of total added damping over the given total mass $M_T$, and the O.F. is defined as:

$$O.F. = \frac{\sum_{i=1}^{n} c_i^{add}}{M_T} = \frac{\sum_{i=1}^{n} c_i^{add}}{\sum_{i}^{n} m_i} \tag{6}$$

In dimensionless terms it could be expressed as:

$$O.F. = 2\sum_{i=1}^{n} [\omega_i \psi_i(\bar{\mu})]\xi_i^{add} \tag{7}$$

where

$$\omega_i = \sqrt{\frac{k_i}{m_i}}; \quad \xi_i^{add} = \frac{c_i^{add}}{2\omega_i m_i}; \quad \psi_i(\bar{\mu}) = \frac{m_i}{M_T} = \frac{\prod_{j=0}^{i-1} \mu_j}{\sum_{k=1}^{n} \prod_{j=1}^{k-1} \mu_j} \tag{8}$$

and where $\begin{cases} \mu_0 = 1 \\ \mu_j = \dfrac{m_{j+1}}{m_j} \qquad J = 1 \ldots (n-1) \end{cases}$

Thus, the design vector **b** is defined as the collection of each interstorey damping ratio:

$$\mathbf{b} = [\xi_1^{add}, \xi_2^{add}, \ldots, \xi_n^{add}]^T \tag{9}$$

Moreover, each feasible design vector **b** must satisfy a probabilistic constrain limiting the failure probability $P_f$ under a given value $P_f \leq 1 - r_{\min}$, being $r_{\min}$ the minimum level of requested reliability. Failure is associated to the first excursion of a structural response vector **U** from a *safe domain D*, whose equation is $G(\mathbf{U}(\mathbf{b}, t), \boldsymbol{\beta}) \geq 0$ and where the vector $\boldsymbol{\beta}$ collects the threshold values defining the safe domain. Since in the present work failure is associated to a threshold crossing of an interstorey drift, here vector **U** collects these structural responses.

Under Poisson assumption, the stochastic constraint on reliability can be written as:

$$r(T, \boldsymbol{\beta}, \mathbf{b}) = \Pr(G(\mathbf{U}(\mathbf{b}, \tau), \boldsymbol{\beta}) \geq 0 \mid \tau < T) = \exp\left\{-2\int_0^T \nu_{\mathbf{U}}^{+}(\mathbf{R}_{\mathbf{Z}_U\mathbf{Z}_U}, \boldsymbol{\beta}, \mathbf{b}, \tau) d\tau\right\} \geq r_{\min} \tag{10}$$

The covariance matrix $\mathbf{R}_{\mathbf{Z}_U\mathbf{Z}_U}(t)$ is related to $\mathbf{R}_{\mathbf{ZZ}}(t)$ by means the relation $\mathbf{R}_{\mathbf{Z}_U\mathbf{Z}_U}(t) = \mathbf{T}\mathbf{R}_{\mathbf{ZZ}}(t)\mathbf{T}^T$, where **T** is a suitable transformation matrix. Moreover, further constraints impose positive values to the added damping ratios

$$\mathbf{b} \geq \mathbf{0} \tag{11}$$

With reference to the proposed problem, it is required to evaluate the probability $P_{f,h}$ that each interstorey drift $u_h$ of the $h^{th}$ floor exceeds the thresholds $\beta_h$ at least once in a given earthquake duration.

Then, for each $h^{th}$ level this failure event is associated with the condition $|x_{h+1} - x_h| = |u_h| = \beta_h$. For each level $h \in [1, n]$, $r_h(T)$ is defined as $r_h(T) = P[|U_h(t)| \leq \beta_h;\ t \in [0, T]]$, and under Poisson hypothesis we get (Lutes and Sarkani 2003):

$$\begin{aligned} r_h(T) &= \exp\left\{-2\int_0^T \nu_{U_h}^{+}(\beta_h, \tau) d\tau\right\} \\ &= \exp\left\{-\frac{1}{\pi}\int_0^T \left(\frac{\sigma_{\dot{U}_h}(\tau)}{\sigma_{U_h}(\tau)}\sqrt{1 - \rho_{U_h\dot{U}_h}^2(t)}\exp\left\{-\frac{1}{2}\eta_h^2(\tau)\right\}\chi[d_{U_h}(t)]\right) d\tau\right\} \end{aligned} \tag{12}$$

where:

$$\eta(t) = \frac{\beta}{\sigma_X(t)}; \quad \chi(y) = \exp\left(-\frac{y^2}{2}\right) + \sqrt{2\pi}y\Phi(y); \quad \Phi(y) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{y} \exp\left(-\frac{u^2}{2}\right) du;$$

$$d_X(t) = \frac{1}{\sqrt{2}}\eta(t)\left(\frac{\rho_{X\dot{X}}(t)}{\Delta_X(t)}\right) \text{ and } \Delta_X = \sqrt{1 - \rho_{X\dot{X}}^2} \quad \text{(Lutes and Sarkani 2001).}$$

Assuming that all failure events associated to the crossing of the threshold $\beta_h$ by interstorey displacements are independent, the whole structural reliability can be determined as the product of reliabilities of each storey:

$$r(T, \boldsymbol{\beta}, \mathbf{b}) = \prod_{i=1}^{n} \Pr(|U_i| \leq \beta_i | \mathbf{b}) \tag{13}$$

and the reliability constraint can be written as:

$$\begin{aligned} r(T,\boldsymbol{\beta},\mathbf{b}) &= \prod_{i=1}^{n}\Pr(|U_i| \le \beta_i|\mathbf{b}) \\ &= \prod_{i=1}^{n}\exp\left\{-2\int_0^T \nu_{U_i}^{+}(\mathbf{R}_{\mathbf{Z}_U\mathbf{Z}_U},\beta_i,\mathbf{b},\tau)d\tau\right\} \\ &= \exp\left\{-2\sum_{i=1}^{n}\int_0^T \nu_{U_i}^{+}(\mathbf{R}_{\mathbf{Z}_U\mathbf{Z}_U},\beta_i,\mathbf{b},\tau)d\tau\right\} \ge r_{\min} \end{aligned} \tag{14}$$

from which we find:

$$2\sum_{i=1}^{n}\int_0^T \nu_{U_i}^{+}(\mathbf{R}_{\mathbf{Z}_U\mathbf{Z}_U},\beta_i,\mathbf{b},\tau)d\tau \le \log(1/r_{\min}) \tag{15}$$

Although exact analytical solutions for the whole structural reliability are generally unavailable, it is known that Eq. (13) provides an approximate, upper-bound estimate of the global reliability (Lutes and Sarkani 2001) which can be used for design and pre-design purposes in a practical viewpoint, as in this study.

The optimal solution, in the general case is performed by standard gradient approach. In details, it has been carried out by using the standard *Matlab* optimization toolbox, using the minimization tool

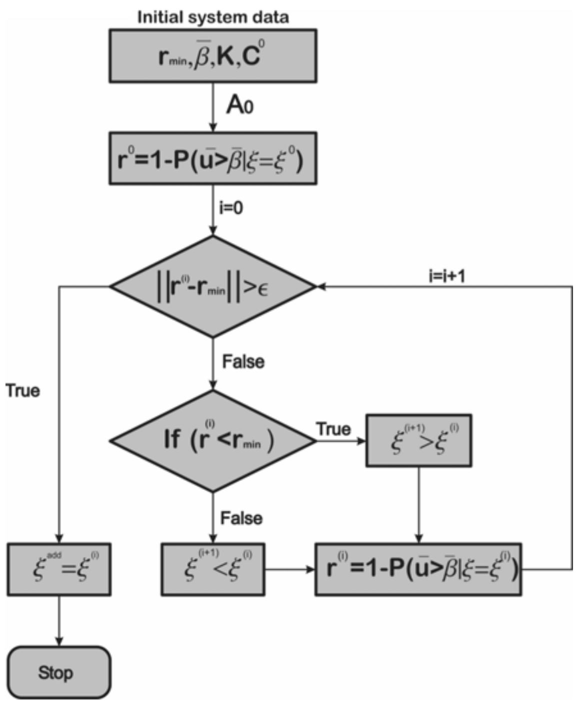


Fig. 2 Optimization flow chart for constant damping at each floor. The parameter $\varepsilon$ is the assigned numerical tolerance for the convergence of the procedure.

for constraint conditions. It has been implemented using as starting point the solution of the simplified case, where the design vector is represented by a single scalar variable. This is an important point, due to the fact that standard optimization tools are able in general to find a local minimum of O.F. near the initial point proposed, and therefore they are typically sensitive to these parameters. The simplified optimization problem is representative of a situation where the added damping devices have all the same characteristics and are placed on all the interstoreys. In this case, optimal solution is obtained by the reduction of the above stated optimization criterion to a single parameter problem, in which a constant damping is assumed to each floor. Its evaluation could be performed by standard optimization algorithms or by a simplified iterative approach shown in Fig. 2. Once obtained the optimum $\xi_0^{add}$, the vector $\mathbf{b}_0 = [\xi_0^{add}, \xi_0^{add}, \ldots, \xi_0^{add}]^T$ is thus inserted in the *fmincon* function as initial point, and general solution of (6) and (10) is thus obtained.

## 4. Results analysis

Several numerical analyses have been carried out as applications of the proposed optimum design method. Efficiency under different seismic and structural conditions has been also assessed. Seismic action is represented by means of the peak ground acceleration $a_g^{\max}$, which is assumed varying between 0.20 (g) and 0.50 (g). It is assumed that ground parameters in (1) correspond to a firm soil condition: $\omega_g$ = 25 (rad/sec) and $\xi_g$ = 0.4; moreover, the modulation function parameters in (2) are:

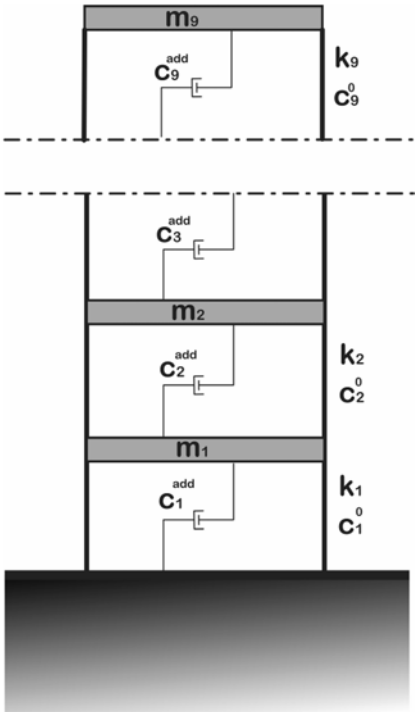


Fig. 3 Schematic model of a 9 storeys building, here analysed to obtain the best added interstorey damping distribution under different base acceleration intensities

$t_1 = 3$ (sec), $t_d = t_2 - t_1 = 20$ (sec), $\beta = 0.4$, able to represent a wide range of real seismic events. The plane shear frame here considered is a 9 storeys building (Fig. 3). The initial interstorey damping values are defined by the parameters $\xi_0^i = c_i^0/2\sqrt{k_i m_i} = 0.02$. Four different interstorey stiffness distributions are assumed, defined by the values $\omega_i = \sqrt{k_i/m_i}$ (reported in Fig. 4), whereas masses are assumed to be the same for all floors.

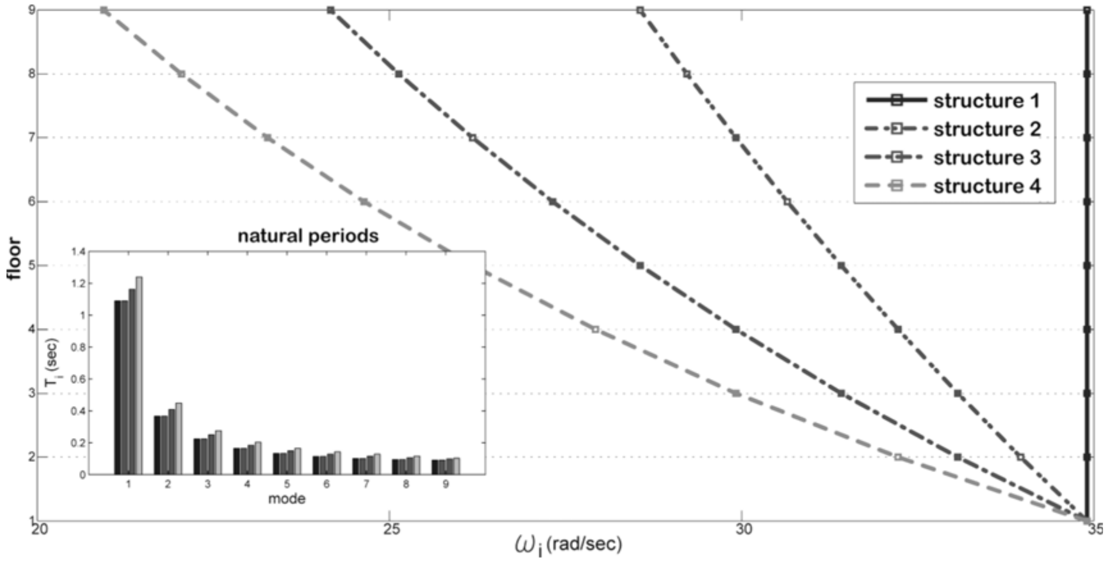


Fig. 4 Different distributions of interstorey stiffness, expressed by means of $\omega_i = \sqrt{k_i/m_i}$. In subfigure the 9 natural periods of the unprotected structure in each of the 4 structural configurations analysed are reported.

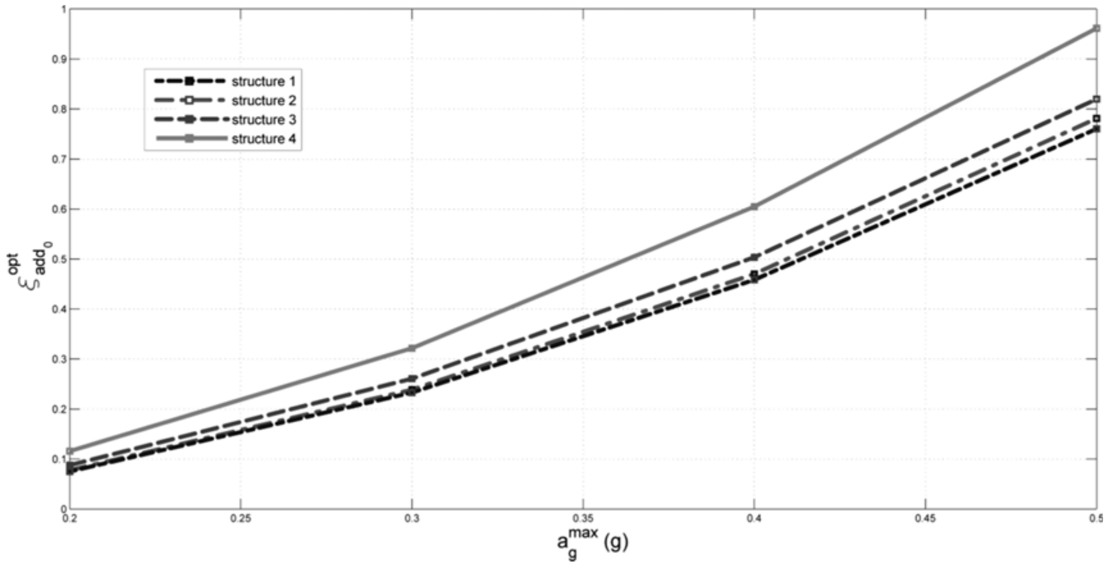


Fig. 5 Optimal added damping evaluated for equal device characteristics used at each floor. Different line represent different structural configurations.

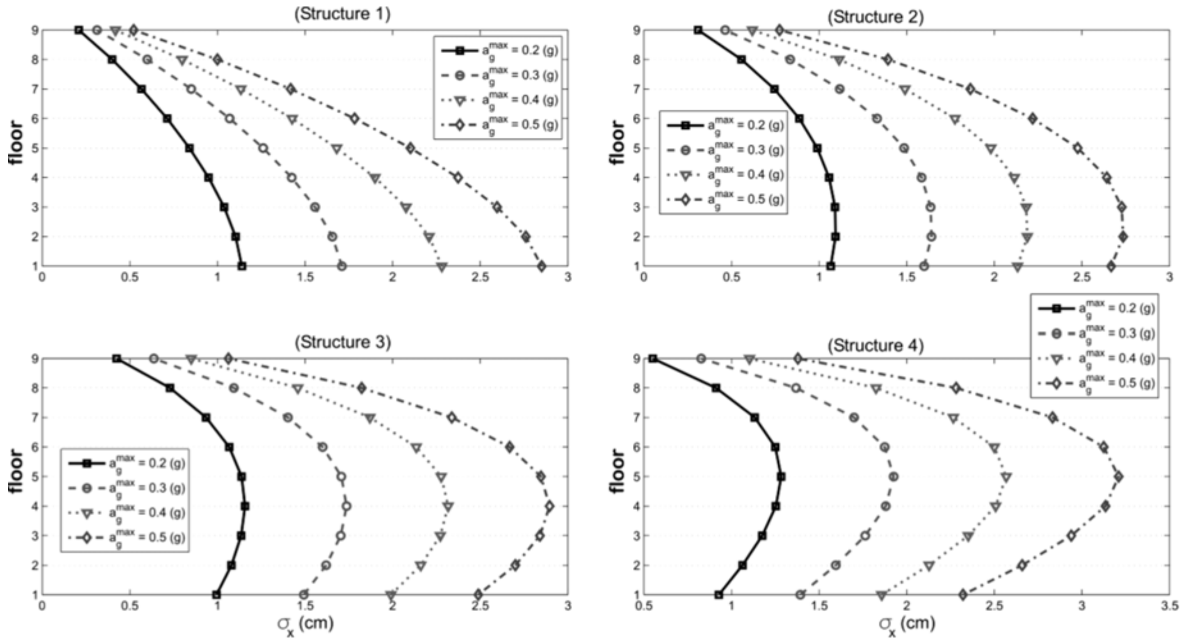


Fig. 6 Displacement maximum covariances of unprotected structures for different seismic intensities

At the first storey, in all cases, equal values $\omega_1$ are assumed, while in the upper stories different values $\omega_i$ are considered. They are representative of different reductions of lateral interstorey stiffness with the height of the building.

First analysis concerns a building seismically protected by adopting constant added damping devices placed at each of the nine storeys. Fig. 5 shows, for each of the 4 structural configurations, the damping needed to satisfy the required performance level. More precisely, seismic performance is defined by means of a fixed failure probability, equal to $10^{-2}$, that the admissible interstorey drift limit, here assumed $\Delta_{\text{lim}} = 3$ (cm), will be exceeded. Fig. 5 plots, for all the structural configurations here considered, the optimal added damping versus the peak ground acceleration. It could be noted that the required damping increases more than linearly with respect to ground acceleration. This is due to the non linearity of the constraint imposing the required level of seismic protection.

As predictable, the added damping increases when the lateral structural stiffness decreases from Structure 1 to Structure 4. Moreover, it should be noticed that this increasing is more than linear with the lowering of the lateral stiffness.

The peak values of interstorey displacement and velocity variances of the unprotected buildings are shown in Figs. 6 and 7, respectively. It could be noted that in case of uniform stiffness distribution the maximum displacement variance is attained at the base of the building, while in other cases this maximum is attained at intermediate storeys.

A similar feature could be noticed in Fig. 7, which shows the velocity variance distributions for all structural configurations here considered. In this case too it can be noticed that the maximum value of velocity variance is attained at the frame base in case of constant lateral stiffness (Structure 1) and, by decreasing the lateral stiffness of the upper storeys (Structures 2, 3 and 4), it tends to move toward the frame top.

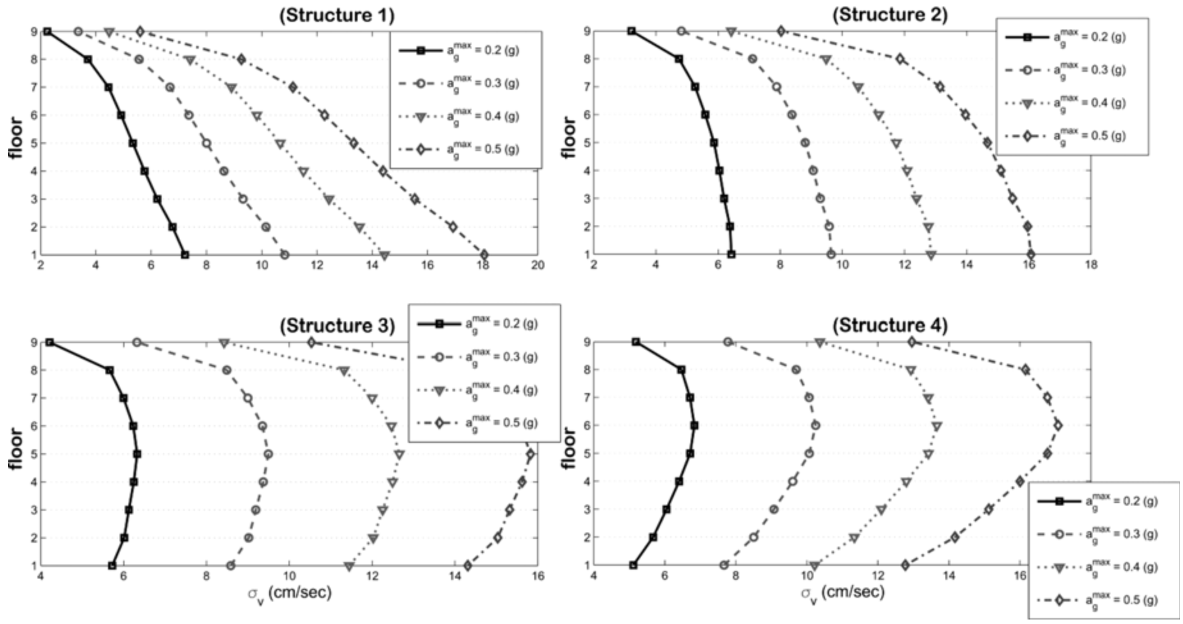


Fig. 7 Velocity maximum covariances of unprotected structures for different seismic intensities

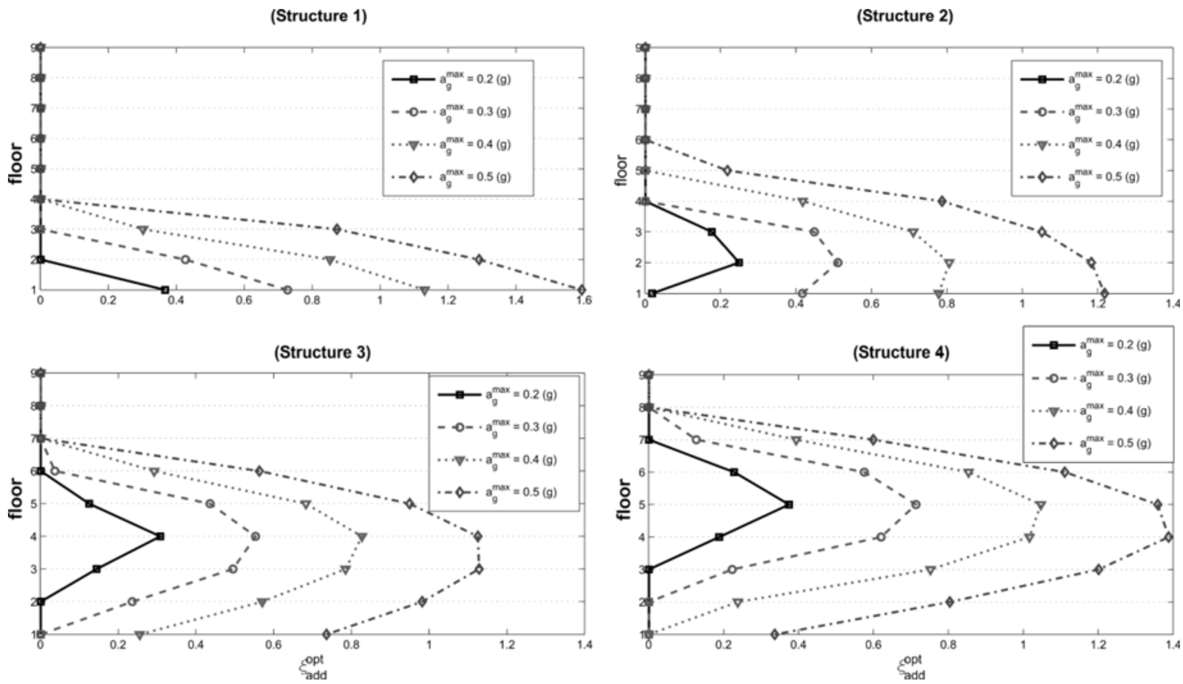


Fig. 8 Optimal added damping ratios at each floor for different seismic intensities

The solutions obtained in the hypothesis of variable damping at each floor are shown in Fig. 8. Results regard the four different examined lateral stiffness distributions for several PGAs. As reasonably related to previous considerations, results can be explained in view of the displacement

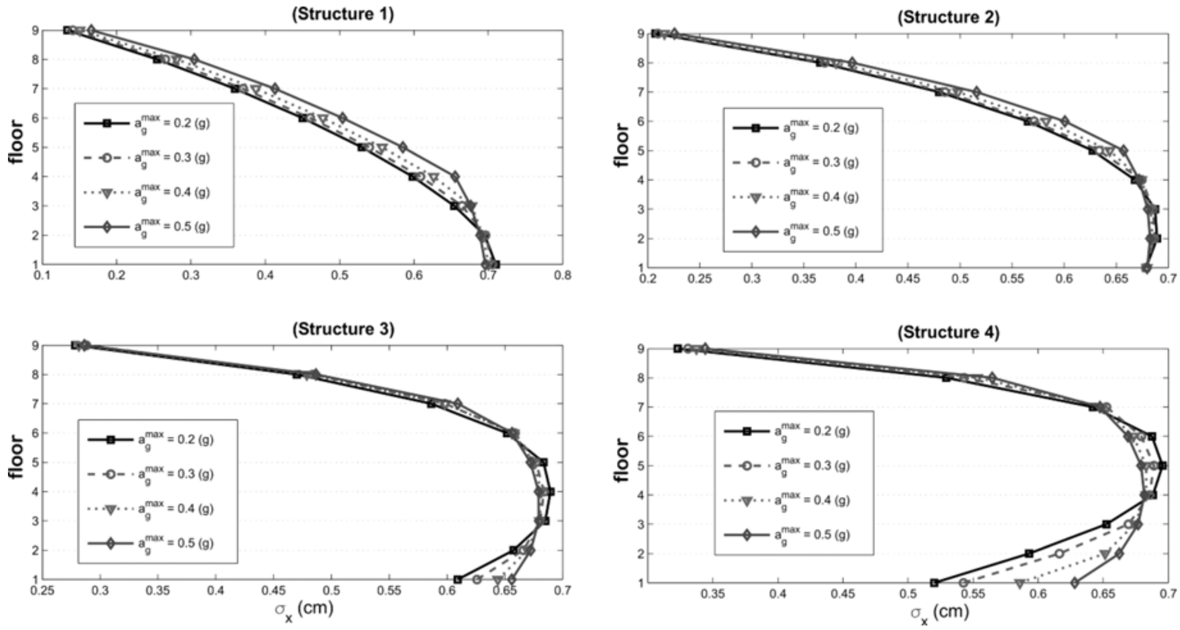


Fig. 9 Displacement maximum covariances of protected structures (as reported in Fig. 8) for different seismic intensities (line types) and stiffness distributions

and velocity covariance distributions of the unprotected structures. First, it must be noted that the more relevant damping is required where the constraint on interstorey drift is more severe, so that, where the initial displacement variance is greater. Then, a general very simple rule could indicate the placement of the maximum damping at those floors where interstorey displacements are larger. As numerical solutions show, in all cases added damping is required, essentially, at those floors where the interstorey displacement covariance attains its maximum value, as before exposed. On the contrary, in floors with a very small displacement covariance, in some cases no added damping is required at all.

So, for each of the four structural configurations, quite different optimal solutions for added damping distribution are obtained but, in every case, these can be explained in view of above considerations.

By increasing the PGA, the optimal placement of added dampings are similar, but larger values of added damping are needed and a greater number of floors are involved.

More in detail, for constant lateral stiffness distribution, the larger added damping is required at the base of the building (Structure 1) and, by decreasing the lateral stiffness of the upper storeys (Structures 2, 3 and 4) the larger added damping tends to move toward the top of building.

The optimal responses of the protected buildings in terms of interstorey displacement variance are shown in Fig. 9. It could be noticed that, for each structural configuration, displacement covariance distributions are reasonably similar for all PGAs considered.

In case of constant lateral distribution (Structure 1), the interstorey displacement variance presents an almost linear distribution along the height of building (with the maximum at the first floor). It tends to become more parabolic, with the maximum located at an intermediate floor for the other three configurations.

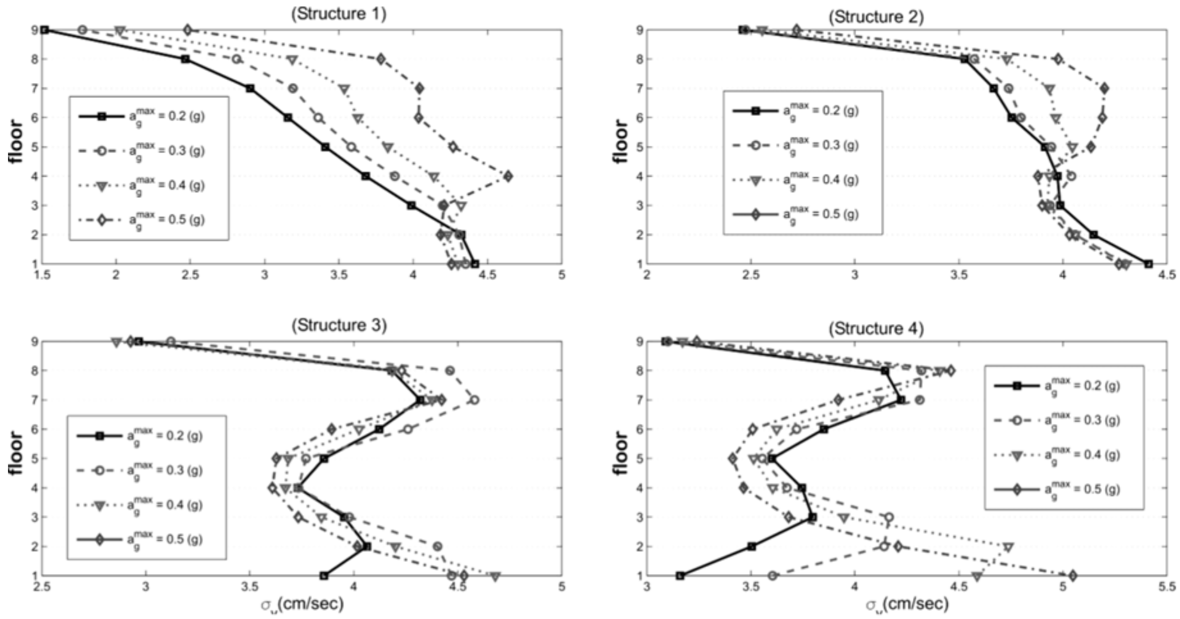


Fig. 10 Velocity maximum covariances of protected structure for different seismic intensities (type lines) and stiffness distributions

The distributions of interstorey velocity covariances are quite more complicated (Fig. 10), and shapes are more irregular, especially for Structures 3 and 4. However, it must be noticed that in this case too a correlation exists between optimal added damping distributions and velocity covariance distributions. Indeed, the main goal of added dampers is to protect structures by dissipating a relevant part of seismic energy, and in the adopted modelling this dissipation is proportional to the interstorey velocity.

Finally, a comparison between results obtained with the proposed method and those achieved by means of a deterministic approach is assessed, in order to evaluate the robustness of proposed procedure, with reference to parameters variation.

For this aim, the optimal added damping for a three floors building, which is modelled with a non-uniform stiffness distribution, has been developed, by assuming a constant added damping at each floor. The maximum interstorey drifts for each added damping obtained by means of proposed method have been compared with those obtained with the same $\xi_{add}^{const}$ by adopting four generated time histories compatible with a stiff soil spectrum, and with the same peak value (0.5 g) and duration used in the stochastic procedure. Results are shown in Fig. 11.

The optimal solutions by deterministic approaches are obtained directly working by time integration of system motion equations under assigned time histories. One can notice a serious results dispersion of added dampings able to guarantee the required limitation of maximum interstorey displacements. It depends on the variations and differences between each time history adopted, even if they have been generated for the same seismic event. This is a direct consequence of differences existing between different ground motion generated time histories that are so transmitted to structural responses obtained by their direct integration. On the contrary, the proposed stochastic approach is able to give solutions in terms of required added damping that can be

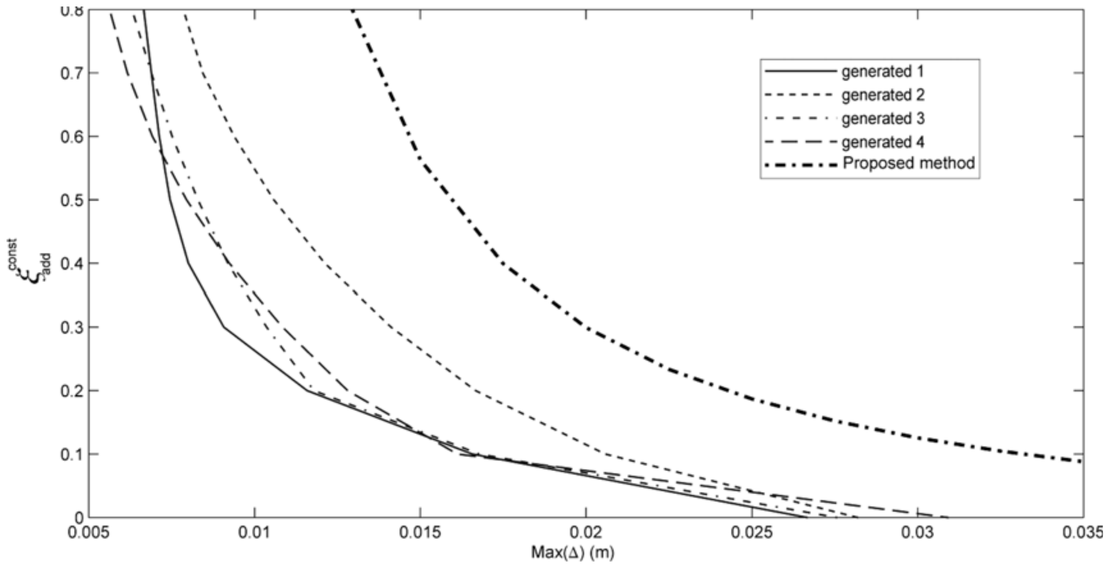


Fig. 11 Constant added damping versus maximum interstorey drift for a tree levels shear type structure. Results are obtained by means of a deterministic approach (by adopting four different time histories), and by the proposed stochastic method.

considered as an *envelope* of all deterministic solutions, because its stochastic input is able to represent a wide class of real deterministic time histories. The required added damping is quite greater than those obtained by deterministic procedure. This depends on stochastic optimal solutions are able to guarantee the required performance with a given quite great confident level, that means it is able to take into account also extreme events.

So that looking at all deterministic solutions together, it must be noticed that they present a significant dispersion; therefore, stochastic optimum solutions are able to overcome this limitation, being obtained for more general input definition. Finally, it can be concluded that this approach has reduced sensitivities to input variations if compared with direct deterministic time histories integrations approaches.

## 5. Conclusions

A stochastic optimum design criterion for added viscous damper devices for building seismic protection has been proposed, aimed to minimize the sum of added dampings for unit mass, here assumed to be proportional to the cost of seismic protection. A stochastic constraint related to interstorey drifts has been imposed to control damage level. Numerical analyses have been developed on plane shear type building models and seismic input has been represented by a non stationary modulated Kanai Tajimi filtered stochastic process.

Results indicate that simple general rules concerning devices optimal placement can be suggested. In particular, it has been shown that the most effective placement of added damping devices depends on the displacement and velocity distributions along the height of the unprotected building. Several distributions of lateral stiffness have been considered, showing that a decreasing of lateral stiffness along the height of building moves the maximum velocity and displacement covariances toward the building top. Then, at the same time, the maximum value of requested added damping moves from the building base toward the top.

The proposed stochastic modelling of environmental load actions seems to be a natural way to produce a robust optimum design, so that, a design whose performance is not strongly influenced by small variations of the given data, that is an important drawback of classical deterministic optimization procedures. Moreover, it is interesting to notice that the proposed seismic protection strategy is naturally robust, since based on energy dissipation.

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