Structural Engineering and Mechanics, Vol. 24, No. 5 (2006) 529-541
DOI: http://dx.doi.org/10.12989/sem.2006.24.5.529

Genetic optimization of vibrating stiffened plates

Jean Luc Marcelin†

Laboratorie Sols Solides Structures 3S, UMR CNRS C5521, Domaine Universitaire, BP n°53, 38041 Grenoble Cedex 9, France

(Received November 2, 2004, Accepted July 3, 2006)

Abstract. This work gives an application of stochastic techniques for the optimization of stiffened plates in vibration. The search strategy consists of substituting, for finite element calculations in the optimization process, an approximate response from a Rayleigh-Ritz method. More precisely, the paper describes the use of a Rayleigh-Ritz method in creating function approximations for use in computationally intensive design optimization based on genetic algorithms. Two applications are presented; their deal with the optimization of stiffeners on plates by varying their positions, in order to maximize some natural frequencies, while having well defined dimensions. In other words, this work gives the fundamental idea of using a Ritz approximation to the response of a plate in vibration instead of finite element analysis.

Keywords: optimization; stiffened plates; genetic algorithms; Rayleigh-Ritz method; vibrations.

1. Introduction

Plates are thin mechanical structures, one dimension of which (the thickness) is far smaller than the other two. Because of the very interesting ratio of rigidity/weight, as well as the good behavior that they present under different cases of loading, these structures are greatly used in industry and in civil applications. Considering their vast area of usefulness and in order to improve their performance, researchers are analyzing the problems of bending, stability and vibration of these structures. The scope of this paper is limited to optimization of the vibrations of plates.

This paper presents an application of genetic algorithms for the maximization of natural frequencies for vibrating stiffened plates, where savings in computational resources are achieved by using a Rayleigh-Ritz method as a universal function approximator. In other words, this paper addresses the problem of optimal placement of stiffeners in plates in a way that maximizes some natural frequencies. The analysis is based on classical Rayleigh-Ritz method and the optimization is performed using a genetic algorithm. The dimensions of the plate and the stiffeners are considered fixed. The examples given in this paper deal with plate-type structures with addition of beam-type stiffeners. The essential purpose consists of optimizing the position of these stiffeners, the objective function being the maximization (or minimization) of natural frequencies. In other words, we try to find the best position of a certain number of stiffeners having well defined section and length, on plates possessing well known boundary conditions. This kind of example has been dealt with in

† E-mail: marcelin@imap.ujf-grenoble.fr

statics in the past (Kallassy and Marcelin 1997, Marcelin 2001) by the author. This paper can be considered as the extension of this previous paper (Marcelin 2001) which describes the optimization of stiffened plates by genetic search in statics.

To our knowledge, there are only a few other papers which deal with similar problems. In Belblidia *et al.* (1999), topology optimization and conventional structural sizing optimization procedures are used together to obtain optimum designs for plate structures A three-layer Mindlin-Reissner plate model is first used with topology optimization to determine optimal stiffening zones. The central layer represents the unstiffened plate and the symmetrically located upper and lower layers are potential stiffening zones. A stiffening volume is specified and the objective is to minimize the strain energy. A sizing optimization procedure is then used for the stiffener dimensions. In Inoue *et al.* (2002), it is explained that dynamic excitation from gears generates vibration modes in gearboxes which causes radiation of unwanted structure-borne noise. To reduce the noise as well as the vibration, the stiffened plate construction is frequently used for the housing, where the rib stiffener layout is the key to this design. In Inoue *et al.* (2002), the most effective position of stiffeners in order to reduce the vibration and noise radiation is searched and discussed. The rib stiffener is modeled by beam elements, and its optimum layout is searched by a genetic algorithm. In Liu *et al.* (1998), the eigenvalue sensitivity of a stiffened plate with respect to stiffener location is formulated; the analysis is based on the generalized Rayleigh quotient of the combined plate-beam system. The results show that the eigenvalue sensitivity is proportional to the force between the plate and the stiffener as well as to the slope of the eigenfunction at the interface between the plate and the stiffener. In Brosowski and Ghavami (1997), the mathematical modelling of the multi-criteria optimization problem of longitudinally stiffened plates is presented. This leads to a non-linear multi-criteria optimization problem with a finite number of side-conditions. As an application, the optimal design of plates where the weight should be as small as possible and the ultimate buckling load as high as possible is considered.

Rayleigh-Ritz method is often used for mechanical structural analyses. In Yang and Gupta (2002), a Ritz vector approach for static and dynamic analysis of plates is proposed. In Nallim *et al.* (2002), a Rayleigh-Ritz approach to transverse vibration of isotropic polygonal plates is given. In Jaunky *et al.* (1998), a design strategy for optimal design of composite grid-stiffened cylinders subjected to global and local buckling constraints and strength constraints is developed using a discrete optimizer based on a genetic algorithm. Local buckling of skin segments are assessed using a Rayleigh-Ritz method that accounts for material anisotropy.

All these papers give an idea of possible applications of the present work.

2. The methods used

2.1 Genetic Algorithms

Considering the non-linearity, the non-convexity and the discontinuity of the optimization problems dealt with in this paper, we have chosen to use a stochastic method that is a standard genetic algorithm (GA) described in Goldberg (1989). GA are chosen because the deterministic methods of optimization known as methods of gradient require a reliable calculation of the sensitivities of the frequencies. Indeed, the calculation of the sensivities of the frequencies with respect to the design variables (the position of the stiffeners) could be a rather delicate exercice because of the discrete nature of the problem.

So, the genetic algorithm used is as described in Goldberg (1989). The author has worked extensively in GAs and published in some reputed journals on this topic (Kallassy and Marcelin 1997, Marcelin 2001, 1999a,b). As the topic of GAs is still relatively new in the structural mechanics community, we provide here some details of exactly what is used in this GA. A multiple point crossover is used rather than a single point crossover. The selection scheme used at each generation is entirely stochastic. There are no particular limitations and constraints for the use of the GA in our examples. So no penalty functions are used. Limitations are included in the coding. For our examples, the number of generations is equal to that used for convergence. The results provided for our examples were consistently reproduced by using different seeds in the GA.

2.2 The strategy used: the Rayleigh-Ritz method

It is a question here of optimizing the position of stiffeners of the beam type on plates. This application is described in the statics case in Marcelin (2001). One recalls first here the general lines of this application in statics, before passing to the case of vibrations.

We consider a plate in statics subjected to boundary conditions and given loads. The objective in statics is to find the best geometrical configuration of a number of stiffeners of the beam type on the plate. The number of stiffeners and their characteristics are given. The objective is to minimize the maximum deflection of the plate. As calculations of analysis are carried out by the finite element method, the provision of the stiffeners is inevitably dependent on the finite element mesh of the plate, since the nodes of the stiffeners must correspond to nodes of the mesh of the plate. The selected method of optimization is the genetic algorithms technique, the localization of a stiffener on the plate being coded in a binary way as indicated in Marcelin (2001) (and in next item). However, the problem is that the systematic use of the finite element method for the analysis of each element of the populations quickly becomes prohibitive. Thus the idea, proposed in certain previous applications (Marcelin 1999a,b), of replacing these finite elements calculations by the use of a neural network. This is not the approach which was adopted mainly in Marcelin (2001), where the finite element analyses were replaced by the use of a Ritz method in statics. One will find details in (Craveur 1996, Zienkiewicz 1971). More precisely, before beginning optimization itself, one evaluates by the finite element method the deformations or displacements of the plate corresponding to around fifty (for example) standard configurations of stiffener positions among those allowable at the same time by the mesh of the plate and the maximum number of stiffeners that one sets as a limit. In the Ritz method, one builds then a new solution as being a linear combination of the 50 vectors of test. Displacements corresponding to a new configuration of stiffeners then will be obtained by optimizing the 50 parameters of the linear combination using an energy principle (minimization of total potential energy). Before launching the genetic algorithm on the basis of this calculation, the validity and the precision of the procedure must be evaluated carefully. The advantage of the method is obviously its speed. Instead of seeking all displacements with the nodes of the mesh, one obtains a correct estimate by seeking only 50 parameters (this number of 50 is given here as an indication, the number of parameters used can vary according to the desired precision). The use of the genetic algorithm becomes faster at this point.

More precisely, let us suppose that we are confronted with a given configuration for the plate. The boundary conditions, the loads, and number of stiffeners to be used are known. One indicates by Ui a vector displacement of the structure (of dimension n, n being the total number of degrees of freedom of the structure) corresponding to a given distribution i of stiffeners on the plate. One determines by finite element calculations a number p of test vectors corresponding to different configurations of provisions of stiffeners. The solution U then will be sought in the form according to:

$$U = \Sigma\ (i = 1 \text{ to } p)\ aiUi \tag{1}$$

The coefficients ai are optimized to have the displacement vector corresponding to all new configurations of stiffeners whose analysis is required by the genetic algorithm.

Let us study now to the applications in vibrations, while starting with a simple case. One is interested, for example, in a clamped-free plate of rectangular form in vibrations (this plate is clamped along one of its small sides), and one seeks, for example, to maximize the first fundamental frequency of the plate while exploiting only the provision of a fixed number of stiffeners on the plate. The form of the mode of resonance associated with this first fundamental frequency is well known, and it is assumed that the stiffeners on the plate have dimensions such that the presence of the stiffeners does not alter the shape corresponding to this first mode. In addition, one can perfectly obtain the same kind of shape in statics, by applying adequate bending forces at the side opposed to the clamped side of the plate. The Ritz method in statics is often called the method of Rayleigh-Ritz in dynamics (Craveur 1996, Zienkiewicz 1971). For some authors, the Ritz method can also be referred to as Rayleigh-Ritz method, even in statics. One thus takes the approach of Rayleigh-Ritz while determining p vectors of test for different configurations of stiffeners, but with the loads applied to the plate with a deformation similar to the first mode in dynamics.

Let us recall that the objective is the maximization of the first frequency of resonance while exploiting the position of the stiffeners (of which the number is fixed). This distribution is coded in the form of chromosome as in Marcelin (2001) (and as indicated in next item and in Fig. 3). In the absence of use of the method of Rayleigh-Ritz, the genetic algorithm will require for all the individuals of all the successive generations the calculation of the first frequency of resonance, which implies in the calculation of the stiffness and mass matrices for each configuration; and the GA needs the calculation of the eigenvectors and the eigenvalues (even if one does not calculate them for the n degrees of freedom of the problem), which takes a very long time.

In an approach of Rayleigh-Ritz, calculations will proceed in the following way, for each analysis required by the genetic algorithm:

- knowing that the stiffness and mass matrices of the plate are constants of the problem, one calculates at each stage only the stiffness and mass matrices of the stiffeners of the beam type for the given configuration, and one assembles these last matrices with the stiffness and mass matrices of the plate to obtain the total matrices K and M. These calculations are relatively fast.
- if one indicates by U the vector of displacements of the structure (of dimension n, where n is the total number of degrees of freedom of the structure), one seeks U in the following form:

$$U = \Sigma\ (i = 1 \text{ to } p)\ aiUi \tag{2}$$

Ui values are taken as preliminary as it was previously indicated, and remain the same throughout the process of the genetic algorithm. One then puts this Rayleigh-Ritz solutions in the quadratic expressions of strain energy and kinetic energy, in a matrix form:

$$U = B\,A \tag{3}$$

where B is a matrix (n lines, p columns) whose columns are the p vectors Ui, and A is the column matrix of the p parameters.

Strain energy E is written as:

$$E = 1/2U^tKU = 1/2A^tB^tKBA \tag{4}$$

Kinetic energy T is written as:

$$T = 1/2U'^tMU' = 1/2A'^tB^tMBA' \tag{5}$$

where U' indicates the derivative of U with respect to time, and A' the derivative of the parameters with respect to time.

We take:

$$K^* = B^tKB, \text{ and } \quad M^* = B^tMB \tag{6}$$

The matrices K^* and M^* have as dimensions pxp, and the eigenvalues are obtained simply by solving:

$$\det\,(K^* - M^*\omega^2) = 0 \tag{7}$$

where ω indicates the pulsation of resonance.

3. Examples of genetic optimization of stiffened plates in vibration

3.1 Example 1

In example 1, the GA was used to optimize the position of five stiffeners of specified dimensions on a square plate of constant thickness (Fig. 1). In this academic example, all five stiffeners must start at point A and must form an unbroken chain of five consecutive links. It is an arbitrary choice for this example. It would be also possible to start somewhere else, or have two or more chains starting at different points.

So, we are going to launch the GA to optimize the position of five stiffeners of well defined dimensions on this plate. We shall be concerned with the study of a rectangular plate of dimensions 10 mm × 10 mm and thickness 1 mm. The stiffeners are formed of the same material as the plate and are totally integrated to it. Only five stiffeners are used for this very small plate. It is a purely arbitrary choice for this academic example which aims only to be an example of demonstration. We limit our study to the use of a square section for the stiffeners: 0.3 mm wide and 0.3 mm total height. The plate is discretised by F.E. (finite element) of rectangular plate type and 1 mm × 1 mm, therefore we have 10 elements following the width and 10 following the length. As a total we have 121 nodes (Fig. 1), each node with 3 degrees of freedom: a perpendicular displacement to the plane of the plate and two rotations around two perpendicular axes and parallel to the plane of the plate.

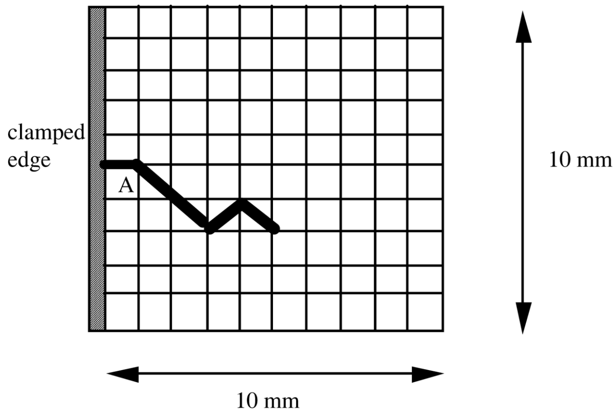

Fig. 1 The mesh of the plate

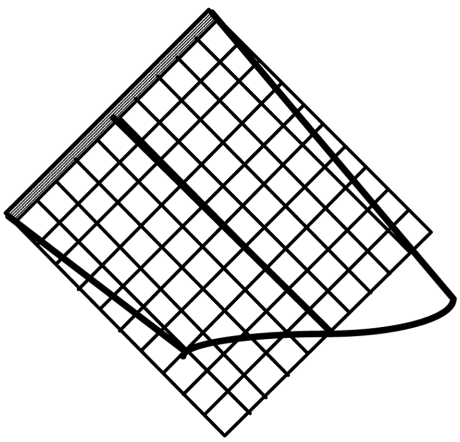

Fig. 2 The second mode (torsion mode)

The stiffeners can act between two consecutive nodes of the meshed plate. The displacement field assumed for the stiffeners is Hermite-type polynomials, and the displacement field assumed for the plate finite element is compatible with the displacement field assumed for the stiffener, so that the stiffener and the plate deform together. For this academic example, we assumed that stiffeners consist of straight segments. Obviously, it would be possible to introduce curvilinear stiffeners. For this example, the five stiffeners are put end to end starting from node A as indicated on Fig. 1. It is only a choice. Obviously, a single string of consecutive stiffeners is not a practical solution, and it is not always enough to control a single vibration mode.

Next, we present results obtained by the application of the GA on the stiffened plate. This horizontally maintained plate is fixed on the left end; the plate is studied in free-free vibration, and the second mode is considered for the optimization (Fig. 2). It is a torsion mode. Let's recall that it is supposed that the stiffeners on the plate have dimensions such that their presence does not alter the shape of the deformation corresponding to this second mode. Our objective is to maximize the second natural frequency of the plate by finding the optimal distribution of the five stiffeners.

The plate and the stiffeners are made of the same material: Young's modulus of 3.E7N/mm^2 and Poisson ratio of 0.3, and mass density 7800 kg/m^3.

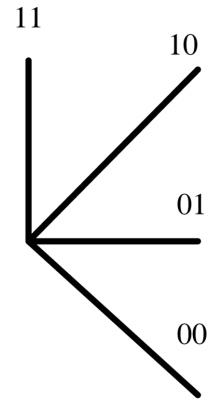

Fig. 3 The coding for the chromosomes

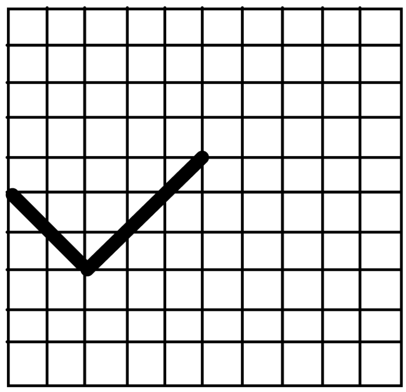

Fig. 4 The optimal solution

With regard to coding in the form of chromosoms, it is the orientation of each stiffener which is coded in the way indicated on Fig. 3. Thus, the provision of the stiffeners indicated on Fig. 1 corresponds to the chromosome 01 00 00 10 00. Each chromosome has a size of 10 binary digits. A standard and simple genetic algorithm is used. A one-point crossover is used. The selection method and the mutation operator are standard, and are given in Goldberg (1989). The probability of crossover Pc is 0.8, and the probability of mutation Pm is 0.05.

The time of each launching of the algorithm for 70 generations, each containing 50 individuals, without using Rayleigh-Ritz method, can become very critical for slightly complicated structures. The optimal solution found by the GA is the chromosome 0000101010, represented graphically in Fig. 4. The value of the corresponding function cost is 833,91 Hz.

The results described here were obtained using only FE analyses. Optimization using the Rayleigh-Ritz approximations begins in the following.

We then carried out optimization by using the calculations based on Rayleigh-Ritz method. In Amabili and Garziera (1999), an other simple and systematic choice of admissible functions in the Rayleigh-Ritz method, which are the eigenfunctions of the closest, simple problem extracted from the one considered, is proposed. It is not the approach retained for the present example. In this example, we use finite element analyis for static loads to evaluate the displacements shapes that are then used as Ritz functions in the Rayleigh-Ritz method for approximating the eigensolution.

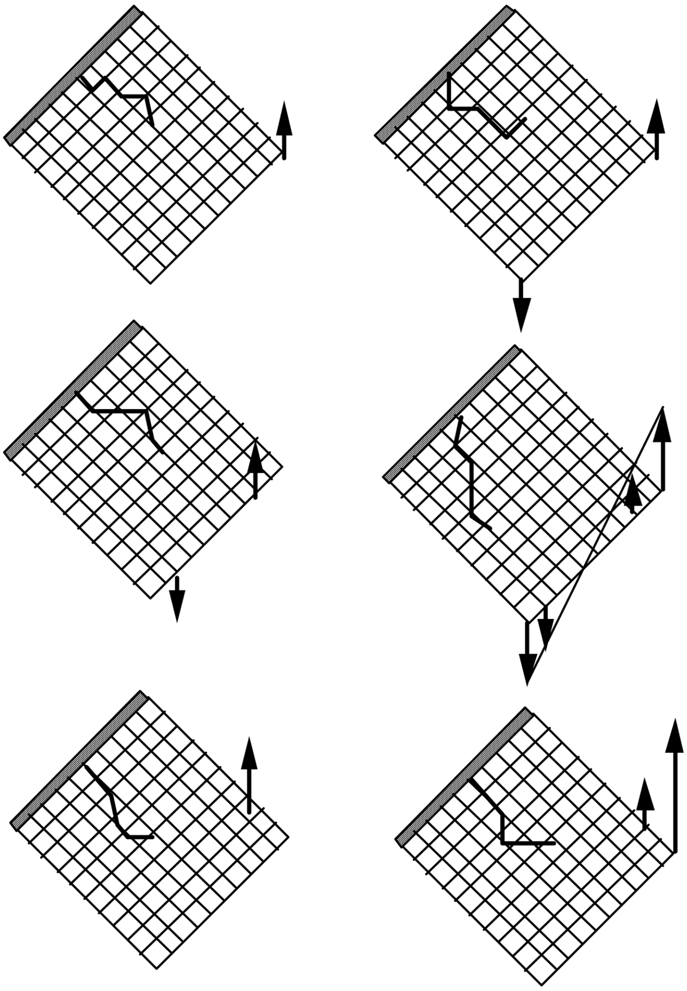

Fig. 5 Some of the statics tests used for the Rayleigh-Ritz method

To implement the Rayleigh-Ritz method, we used 20 results calculated by finite elements in statics. These 20 static test vectors are calculated for various cases of applied external forces, leading a priori to deformations similar to the second mode. Fig. 5 gives a sample of these 20 vectors. The arrows on Fig. 5 represent the forces applied to determine the statics solutions. These 20 static test vectors are moreover associated to random distributions of stiffeners. Let us recall that the interest in using the method of Rayleigh-Ritz is to reduce the number of unknown factors (the degrees of freedom) which is close to 360 for the finite element method, as compared to a number of unknown factors of 20 by the Rayleigh-Ritz method. Then, we tested the precision of calculations in dynamics by using 20 other vectors of test. Fig. 6 gives 9 samples of these 20 vectors of test with the results obtained by the Rayleigh-Ritz method compared with calculations by finite elements. By Rayleigh-Ritz, 15 solutions of test (on the 20) present an absolute error lower than 0.05%. 5 solutions of test present an absolute error ranging between 0.05% and 0.15%. It is thus noted that the Rayleigh-Ritz method gives good results for the present problem. We then

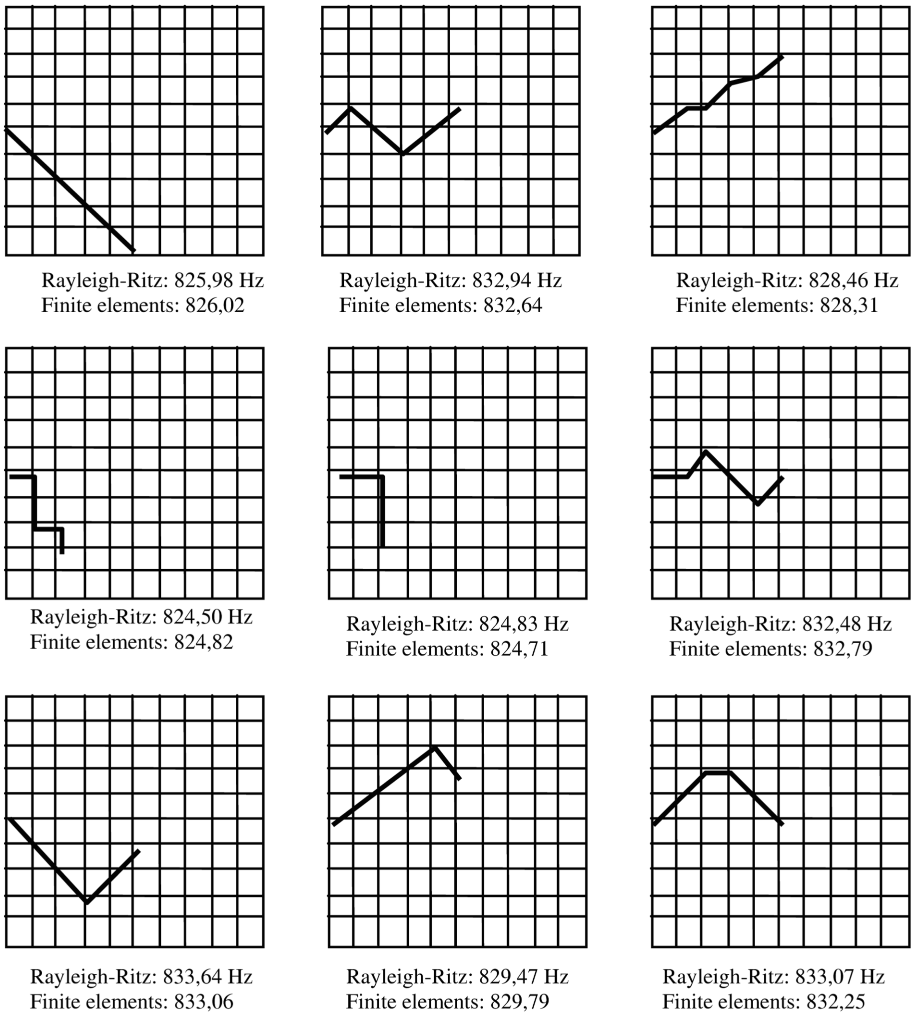

Fig. 6 Some of the test solutions

carried out 50 generations with the GA by using only the calculations approached by the Rayleigh-Ritz method.

The successive stages of the suggested strategy are as follows:

1- evaluation of p statics solutions with loads allowing to find a deformation similar to that of the considered mode,
2- checking the precision of the method of the Rayleigh-Ritz method, and this for various positions of stiffeners,
3- optimization by genetic algorithms, analyses being carried out by the Rayleigh-Ritz method.

We obtain finally the optimal solution of Fig. 4. The errors made on the evaluation of the objective function do not penalize optimization because one finds the optimum obtained previously by using finite elements calculations throughout the process of optimization. On the other hand,

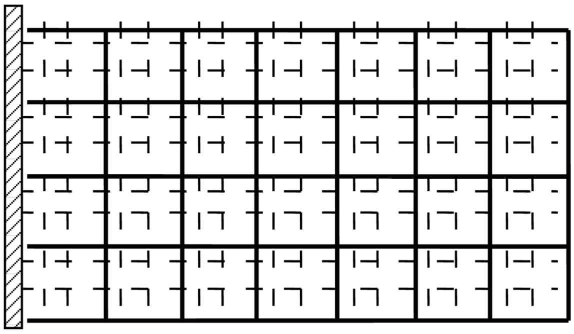

Fig. 7 The plate of example 2

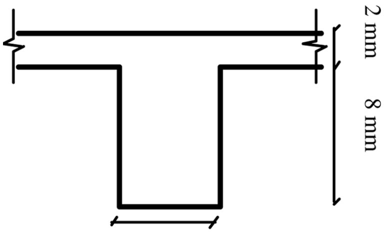

Fig. 8 Cross-section of the plate and stiffener

total time for optimization (including the phase of pre-calculation of p statics solutions, and calculation of K and M matrices of the plate) resulted in a decrease in processing time by a factor of about 12.

3.2 Example 2

A GA and Rayleigh-Ritz method for reanalyses are going to be used to optimize the position of a series of stiffeners of equal lengths on a plate in vibration. The first mode in bending is considered. The plate studied in this example has dimensions of 2.1 m long by 1.2 m wide. The plate is completely embedded along one of it's width and is made of steel of elastic modulus $E = 2.1 \times 10^4$ DaN/mm^2 and Poisson's ratio 0.3. It is 2 mm thick (Fig. 7). The stiffeners are made from the same material as the plate. They have a drop height of 8 mm and a width of 5 mm (Fig. 8).

The objective here is to maximize the first frequency of the plate while optimizing the position of a given number of stiffeners. There are 525 possible positions for the stiffeners (corresponding to the elements distribution), each 100 mm long. Such a large domain of research requires a huge calculation time, so in order to reduce this, the number of possible positions is reduced and regrouped such that a stiffener is made of three aligned consecutively to a length of 300 mm. This process means that there are only 63 possible positions.

A traditional coding technique in a genetic algorithm with such an example consists of building a chromosome with 63 genes, with each one taking a binary number (0 or 1) and decoded as follows: if a gene of position 'i' carries binary number 1, stiffener n°1 exists but in the opposite case it doesn't exist. If the genetic algorithm is run with such a coding it will obviously lead towards a chromosome where all the genes carry a binary number 1, because the stiffer the plate is, the greater the first frequence will be. Nonetheless, the aim is to limit the number of stiffeners distributed on the plate and in this study it is limited to 14.

In our study, another type of coding has been used that builds chromosomes that are of equal length to the total number of stiffeners hoped for, and only containing the number of positions of the different stiffeners that must exist in the configuration. This works so that the chromosomes built are 14 genes long with each gene taking an integer between 1 and 63 inclusive.

So with this coding technique we have individuals with one or several genes carrying the same value which can be interpreted and then decoded in several different ways following the desired design. If there are ‘n’ genes carrying the same value ‘m’, we can consider for example that at position ‘m’, we have:

- just one stiffener 5 mm wide and drop height 8 mm,
- one stiffener having a drop height of 8 mm and width n5 mm,
- one stiffener having a width of 5 mm and drop height n8 mm,
- effectively ‘n’ identical stiffeners of width 5 mm and drop height 8 mm.

Here we have adopted the fourth possibility. It is important to note that the individuals obtained by permutation of the positions of a chromosome have the same distribution configuration, so the same cost functional value and therefore the program considers them as being the same individual.

In Amabili and Garziera (1999), an other simple and systematic choice of admissible functions in the Rayleigh-Ritz method, which are the eigenfunctions of the closest, simple problem extracted from the one considered, is proposed. It is the approach retained for the present example.

Initiating a Rayleigh-Ritz approximation with 100 vectors of test for different configurations of stiffeners and admissible functions which are the eigenfunctions of the closest problem extracted from the one considered, we arrive at a relatively effective Rayleigh-Ritz approximation, since the total errors made are no greater than 7% as shown on Table 1, with, in the following order, the

Table 1

Chromosome	First frequency	Error
3 12 12 27 29 31 34 36 39 40 50 51 54 57	20.4 Hz	0.09%
3 4 5 19 19 21 22 30 31 34 53 54 55 59	20.9 Hz	0.13%
2 3 5 17 24 42 43 47 47 48 51 56 59 60	21.1 Hz	0.8%
1 13 14 21 32 38 39 44 49 53 55 56 62 63	20.2 Hz	0.85%
1 13 14 16 18 19 23 25 29 39 42 45 52 55	21.7 Hz	0.77%
2 7 9 10 12 19 34 34 34 40 43 44 53 59	20.7 Hz	6.1%
8 12 14 15 17 24 24 35 37 41 52 56 57 58	22.1 Hz	6.3%
12 31 32 36 40 42 44 44 50 50 50 55 59 61	20.6 Hz	6.9%

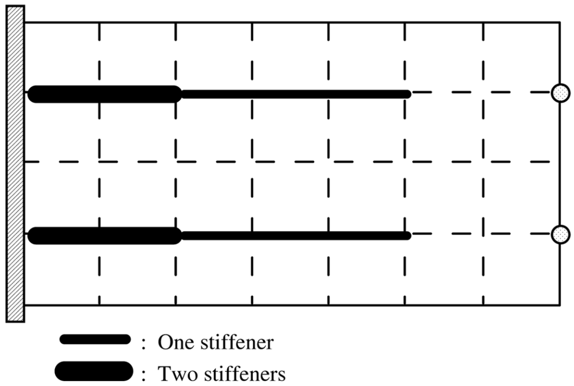

Fig. 9 Optimum position of stiffeners

chromosome, the first frequency evaluated by the Rayleigh-Ritz approximation, and the percentage error with respect to calculations from the finite elements. These errors are tolerated by the genetic algorithm (so much so that they are always of the same sign).

Summary: the objective is to maximize the first frequency, the problem is uniquely constrained by a maximum number of stiffeners arranged on the plate (14 stiffeners).

The genetic algorithm for the plate is settled at 50 generations. The number of individuals per population is 50, Pc = 0.6 and Pm = 0.03 (Pc and Pm are probabilities of crossover and mutation respectively).

The optimum solution found by the genetic algorithm is represented graphically on Fig. 9. The corresponding cost function value is f max = 22,4 Hz.

4. Conclusions

A new approach has been introduced to perform optimization of stiffened plates in vibration. To remedy the handicap of GA, we have substituted the F.E. calculation by Rayleigh-Ritz method. The strategy suggested allows to reduce considerably the objective function calculation time and does not affect the convergence of the GA if the error due to the Rayleigh-Ritz method is relatively minimal. So, the concept of submitting for F.E. calculations an approximate response from Rayleigh Ritz method to reduce the time of the fitness function evaluation seems very interesting for the future of genetic algorithms. The only limitations imposed by the method is the need to code the stiffener geometries in some way.

References

Amabili, M. and Garziera, R. (1999), “A technique for the systematic choice of admissible functions in the Rayleigh-Ritz method”, *J. Sound Vib.,* **224**(3), 519-539.

Belblidia, F., Afonso, S.M.B., Hinton, E. and Antonino, G.C.R. (1999), “Integrated design optimization of stiffened plate structures”, *Eng. Computations*., **16**(8), 934-951.

Brosowski, B. and Ghavami, K. (1997), “Multi-criteria optimal design of stiffened Plates - II. Mathematical modelling of the optimal design of longitudinally stiffened plates”, *Thin-Walled Struct*., **28**(2), 179-198.

Craveur, J.C. (1996), *Modélisation des structures, Calcul par éléments finis*. Masson, Paris.
Goldberg, D.E. (1989), *Genetic Algorithm in Search, Optimization, and Machine Learning,* Addison-Wesley.
Inoue, K., Yamanaka, M. and Kihara, M. (2002), "Optimum stiffener layout for the reduction of vibration and noise of gearbox housing", *J. Mechanical Design*, **124**(3), 518-523.
Jaunky, N., Knight, N.F. and Ambur, D.R. (1998), "Optimal design of general stiffened composite circular cylinders for global buckling with strength constraints", *Compos. Struct.,* **41**(3-4), 243-252.
Kallassy, A. and Marcelin, J.L. (1997), "Optimization of stiffened plates by genetic search", *Struct. Optimiz.*, **13**, 134-141.
Liu, Z.S., Hansen, J.S. and Oguamanam, D.C.D. (1998), "Eigenvalue sensitivity analysis of stiffened plates with respect to the location of stiffeners", *Struct. Optimiz.*, **16**(2-3), 155-161.
Marcelin, J.L. (1999a), "Evolutionary optimization of mechanical structures", *Eng. Optimiz*., **31**(5), 571-588.
Marcelin, J.L. (1999b), "Evolutionary optimisation of mechanical structures: Towards an integrated optimization", *Eng. Comput*., **15**, 326-333.
Marcelin, J.L. (2001), "Genetic optimization of stiffened plates and shells", *Int. J. Numer. Meth. Eng*., **51**(9), 1079-1088.
Nallim, L.G., Luccioni, B.M. and Grossi, R.O. (2002), "A Rayleigh-Ritz approach to transverse vibration of isotropic polygonal plates with variable thickness", *Proc. of the Institution of Mechanical Engineers*- Part K., **216**(3), 213-222.
Yang, J. and Gupta, A. (2002), "Ritz vector approach for static and dynamic analysis of plates with edge beams", *J. Sound Vib.,* **253**(2), 373-388.
Zienkiewicz, O.C. (1971), *The Finite Element Method in Engineering Science*, McGraw-Hill, London.