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Optimal design of laminated composite plates to maximise fundamental frequency using MFD method

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Abstract. This paper deals with optimal fibre orientations of symmetrically laminated fibre reinforced composite structures for maximising the fundamental frequency of small-amplitude. A set of fiber orientation angles in the layers are considered as design variable. The Modified Feasible Direction method is used in order to obtain the optimal designs. The effects of number of layers, boundary conditions, laminate thicknesses, aspect ratios and in-plane loads on the optimal designs are studied.

Keywords: laminated plates; Modified Feasible Direction method; maximization; fundamental frequency; optimal designs.

1. Introduction

Since the composite materials have higher specific modulus and strength compared to the conventional materials and due to the possibility of their design for the required mechanical properties, the plates made of fibre reinforced composite materials are being extensively used in many engineering applications, especially for light-weight structures that have tight stiffness and strength requirements. However, their analysis and design are more complicated due to anisotropy of each layer than that of conventional metal plate. Furthermore, the light-weight structures are often exposed to severe vibration circumstances and the consideration for optimizing anti-resonance performance (e.g., by maximising the fundamental frequency) becomes more important than before in composite structural design.

More recently, a vast body of literature for vibration analysis of laminated plate is available. Haldar and Sheikh (2005) studied a high precision composite plate-bending element developed by Sheikh *et al.* has been applied to the free vibration analysis of isotropic and fibre-reinforced laminated composite folded plates. The in-plane displacements, transverse displacement and rotations of the normal have been taken as independent field variables and they have approximated with polynomials of different orders. Ashour (2005) analyzed the natural frequencies of symmetrically laminated plates of variable thickness using the finite strip transition matrix

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technique. The natural frequencies of such plates are determined for edges with being elastically restrained against both rotation and transition or both. A successive conjunction of the classical finite strip method and the transition matrix method is applied to develop a new modification of the finite strip method to reduce the complexity of the problem. Ferreira *et al.* (2005) used the first-order shear deformation theory in the multiquadric radial basis function (MQRBF) procedure for predicting the free vibration behavior of moderately thick symmetrically laminated composite plates. Patel *et al.* (2005) studied the free flexural vibration behavior of bimodular laminated angle-ply composite plates. The formulation is based on the theory that accounts for the transverse shear and transverse normal deformations, and incorporates higher order through the thickness approximations of the in-plane and transverse displacements. Chen and Lu (2005) developed a semi-analytical method, which combines the state space approach with the technique of differential quadrature on the basis of the three-dimensional theory of elasticity, is developed for free vibration of a cross-ply laminated composite rectangular plate. The plate is assumed to be simply supported at one pair of opposite edges such that trigonometric functions expansion can be used to satisfy the boundary conditions precisely at these two edges. Lanhe *et al.* (2005) employed a novel numerical solution technique, the moving least squares differential quadrature (MLSDQ) method to study the free vibration problems of generally laminated composite plates based on the first order shear deformation theory. Wang and Zhang (2005) developed a layerwise B-spline finite strip method for free vibration analysis of truly thick and thin composite laminated plates within the context of a layerwise plate theory proposed by Reddy. Nallim *et al.* (2005) developed a variational approach for the study of the statical and dynamical behaviour of arbitrary quadrilateral anisotropic plates with various boundary conditions based on the classical laminated plate theory. Leung *et al.* (2005) applied a new trapezoidal *p*-element is to solve the free vibration problem of polygonal laminated composite plates subjected to in-plane stresses with various boundary conditions. Chaudhuri *et al.* (2005) presented a generalized boundary-continuous displacement based double Fourier series solution to the boundary-value problem of free vibration of thin anisotropic fiber reinforced plastic (FRP) rectangular plates. Kabir (2004) presented an analytical solution to a boundary value problem. The eigenvalues and mode shapes obtained are compared with the moderately thick plate theory based analytical and finite element solutions. Onkar and Yadav (2004) analyzed the effect of material parameter dispersion on the large amplitude free vibration of especially orthotropic laminated composite plates. The basic formulation of the problem has been developed based on the classical laminate theory and Von-Karman non-linear strain–displacement relation. Hu *et al.* (2004) proposed an analytical method for vibration of an angle-ply laminated plate with twist considering transverse strain and rotary inertia. Numayr *et al.* (2004) used the finite difference method to solve differential equations of motion of free vibration of composite plates with different boundary conditions. Also, the effects of shear deformation and rotary inertia on the natural frequencies of laminated composite plates are investigated. Rao and Desai (2004) presented a semi-analytical method to evaluate the natural frequencies for simply supported, cross-ply laminated and sandwich plates by using higher order mixed theory. More results can be found in Shi *et al.* (2004), Setoodh and Karami (2004), Liew *et al.* (2003), Gorman and Ding (2003), Aydogdu and Timarci (2003), Messina (2002), Wang *et al.* (2002), Harras *et al.* (2002), Messina and Soldatos (2002), Matsunaga (2002), Kant and Swaminathan (2001).

Structural optimization of laminated plates involving vibration are found in some papers. Narita (2003) proposed a layerwise optimization approach to determine the maximum fundamental frequency of laminated composite plates. Narita (2006) also extended the layerwise optimization approach to accommodate the finite element analysis for optimizing the free vibration behavior of laminated composite plates with discontinuities along the boundaries. Narita and Hodgkinson (2005) applied the layerwise optimization approach to point-supported, symmetrically laminated rectangular plates. The plates considered rest on some elastic or rigid point supports distributed in different arrangements. Adali and Verijenko (2001) studied the design of hybrid symmetric laminated plates consisting of high-stiffness surface and low-stiffness core layers. The maximization of the fundamental frequency and frequency seperation was performed over a discrete set of available ply angles. Correia *et al.* (1997) researched the structural optimization of multilaminated composite plate structures of arbitrary geometry and lay-up, using single layer higher order shear deformation theory discrete models. Hu and Tsai (1999) maximized the fundamental frequencies of fiber-reinforced laminated cylindrical shells with a given material system with respect to fiber orientations by using the golden section method. Kam and Lai (1995) studied the lamination arrangements of moderately thick laminated composite plates for optimal dynamic characteristics via a constrained multi-start global optimization technique. In the optimization process, the dynamical analysis of laminated composite plates was accomplished by utilizing a shear deformable laminated composite finite element, in which the exact expressions for determining shear correction factors were adopted and the modal damping model constructed based on an energy concept. Fukunaga *et al*. (1994) examined the optimal laminate configurations of symmetric laminated plates for maximizing fundamental frequencies. The coupling between bending and twisting are taken into consideration in the free vibration analysis of symmetric laminated plates. With the use of lamination parameters, the effect of bending-twisting coupling on the fundamental frequencies are discussed for the cases of simply supported or clamped edges. Duffy and Adali (1991a) maximized the fundamental frequency and the frequency separations of antisymmetric, angle-ply laminates subject to a mass constraint. Fibre orientation was considered as design variables. Bert (1977) presented a rationale method for determining the optimal laminate design for a thin plate consisting of multiple layers of equi-thickness composite material. The optimal design criterion is maximization of the fundamental frequency of small-amplitude, free flexural vibration. More studies can be found in the literature about maximization of frequency of laminated structures (Bert 1978, Reiss and Ramachandran 1987, Grenestedt 1989, Duffy and Adali 1991b, Adali 1984, Hu and Ou 2001, Sivakumar *et al*. 1999).

The current work deals with the optimum design of laminated composite plates for maximising the fundamental frequency. The objective function is maximised with respect to the fibre orientations of the layers. The Modified Feasible Direction method is used in order to obtain the optimal designs. Finally, the effects of different number of layers, boundary conditions, laminate thicknesses, aspect ratios and in-plane loads on the results are given.

2. Basic equations

Consider a symmetrically laminated rectangular plate of length a, width b and thickness h ($h = \Sigma h_i$, h_i represents thickness of a layer) which consists of N orthotropic layers with fibre angles θ_k ($k = 1,2,\ldots, N$) which is measured counterclockwise about the z-axis from the element local x-axis to the material 1-axis as shown in Fig. 1.

If the rotary inertia deformation is neglected, dynamic equilibrium of the infinitesimal element yields the following equations:

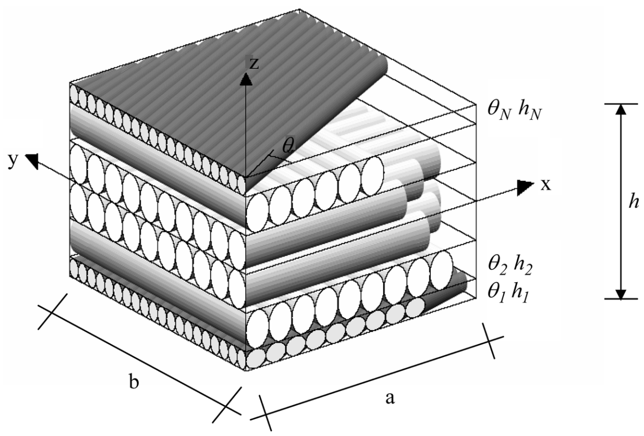

Fig. 1 Structure of a layered laminate plate

$$D_{11}\frac{\partial^2 \varphi_x}{\partial x^2} + (D_{12} + D_{16})\frac{\partial^2 \varphi_y}{\partial x \partial y} + D_{66}\frac{\partial^2 \varphi_x}{\partial y^2} - \alpha A_{55}\left(\varphi_x + \frac{\partial w}{\partial x}\right) = 0 \tag{1}$$

$$(D_{12} + D_{66})\frac{\partial^2 \varphi_y}{\partial x \partial y} + D_{66}\frac{\partial^2 \varphi_y}{\partial x^2} + D_{22}\frac{\partial^2 \varphi_y}{\partial y^2} - \alpha A_{44}\left(\varphi_y + \frac{\partial w}{\partial y}\right) = 0 \tag{2}$$

$$\alpha A_{55}\left(\frac{\partial \varphi_x}{\partial x} + \frac{\partial^2 w}{\partial x^2}\right) + \alpha A_{44}\left(\frac{\partial \varphi_y}{\partial y} + \frac{\partial^2 w}{\partial y^2}\right) = \rho\frac{\partial^2 w}{\partial t^2} \tag{3}$$

where D_{ij} is the out-of-plane stiffnesses, A_{44} and A_{55} are the shear rigidities, α is shear correction factor, ρ is the average mass density of all laminates, w is the displacement in the z direction and φ_x and φ_y are the rotations in the x and y directions. D_{ij}, A_{44}, A_{55} can be calculated as

$$D_{ij} = \int_{-h/2}^{h/2} \overline{Q}_{ij}^{(k)} z^2 dz, \quad A_{ml} = \int_{-h/2}^{h/z} \overline{Q}_{ml}^{(k)} dz \quad i, j = 1, 2, 6;\ m, l = 4, 5 \tag{4}$$

where $\overline{Q}_{ij}^{(k)}$ are components of the transformed reduced stiffness matrix for the kth layer.

For the finite element analysis, if the damping is neglected, the equation of motion of the structure for free vibration can be written as

$$[M]\{\ddot{D}\} + [K]\{D\} = \{0\} \tag{5}$$

where $\{D\}$ is a vector containing the unrestrained nodal degrees of freedoms, $[M]$ is a structural mass matrix, $[K]$ is a structural stiffness matrix. Since $\{D\}$ undergoes harmonic motion, the vectors $\{D\}$ and $\{\ddot{D}\}$ become

$$\{D\} = \{\overline{D}\}\sin\omega t, \quad \{\ddot{D}\} = -\omega^2\{\overline{D}\}\sin\omega t \tag{6}$$

where $\{\overline{D}\}$ vector contains the amplitudes of $\{D\}$ vector and ω is the frequency. Therefore, Eq. (5) can be written in as

$$([K] - \lambda[M])\{\bar{D}\} = 0 \tag{7}$$

where $\lambda = \omega^2$ is the eigenvalue and $\{\bar{D}\}$ becomes the eigenvector. Finite element solution is done to solve for the eigenvalues, the natural frequency, and the eigenvectors. The obtained smallest natural frequency (fundamental frequency) is then the objective function for maximization.

3. The Modified Feasible Direction method

A general optimization problem may be defined as below

Minimize	$f(\underline{x})$		(8)
subject to	$g_i(\underline{x}) \le 0$	$i = 1, \ldots, r$	(9)
	$h_j(\underline{x}) = 0$	$j = r + 1, \ldots, m$	(10)
	$p_k \le x_k \le q_k$	$k = 1, \ldots, n$	(11)

where $f(\underline{x})$ is an objective function, $g_i(\underline{x})$ are inequality constraints, $h_j(\underline{x})$ are equality constraints, and $\underline{x} = (x_1, x_2, \ldots, x_n)^T$ is a vector of design variables.

The Modified Feasible Direction (MFD) method is a powerful general method, and can be applied to most constrained nonlinear problems. MFD is a modification of the classical steepest descent method. It takes into account not only the gradients of objective function and the retained active and/or violated constraints, but also the search direction in the former iteration. Let x_o be an initial x vector. The design is updated according to the following equation:

$$x_q = x_{q-1} + \lambda S_q \tag{12}$$

MFD

1. q=0, $X^q = X^m$
2. q=q+1
3. Evaluate objective function $F(X_i)$ and behavior constraints $g_j(X_j) \le 0$ where j=1, 2, …, N_c
4. Identify critical and potentially critical constraints, $\bar{N}_c$
5. Calculate gradient of the objective function $-F(X_i)$ and behavior constraints $\nabla g_k(X_i)$ where k=1, 2, …, $\bar{N}_c$
6. Find the usable-feasible direction S_q
7. Perform a one-dimensional search $X^q = X^{q-1} + \alpha\, S_q$
8. Check convergence. If satisfied, go to 9. Otherwise go to 2
9. $X^{m+1} = X^q$

Fig. 2 The Modified Feasible Direction method where S_q is the search direction. λ is a scalar whose value is determined through a onedimensional search. Different optimization methods are characterized by different methods to determine the search direction S_q. The search direction S_q in MFD is determined using the Fletcher-Reeves conjugate direction method when there is no active or violated constraint.

$$S_q = -\nabla f(\underline{x}_{q-1}) + \beta S_{q-1} \tag{13}$$

where

$$\beta = \frac{|\nabla f(\underline{x})_{q-1}|^2}{|\nabla f(\underline{x})_{q-2}|^2} \tag{14}$$

Fig. 2 shows the iterative process within each optimization process.

Based on the Modified Feasible Direction method, the mathematical expression for such an optimization problem is written as

$$\begin{aligned} &\omega_1 = \omega_1(\theta) \rightarrow \text{Max} && \text{(objective function)} \\ &\theta = (\theta_1, \theta_2, \dots, \theta_k) && \text{(design variable)} \\ &0^\circ \le \theta_k \le 90^\circ && \text{(constraint)} \end{aligned}$$

4. Numerical results and discussions

Numerical results are given for T300/5208 graphite/epoxy material with $E_1 = 181$ GPa, $E_2 = 10.3$ GPa, $G_{12} = 7.17$ GPa, $\nu_{12} = 0.28$, $\rho = 1600$ kg/m^3. The symmetric laminated plate is constructed of equal thickness layers with the continuous case for which the stacking sequence is taken as $(\theta/-\theta/\theta/-\theta/\dots)_{sym}$ and the thickness ratio is specified as $h/b = 0.01$. Frequencies are obtained for the first five natural modes of vibration.

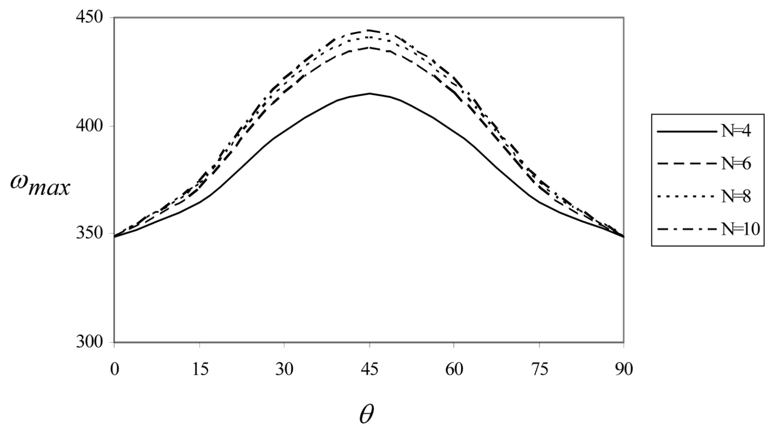

Fig. 3 The fundamental frequency (ω_{max}) versus θ for different number of layers for simply supported laminated plate with $a/b = 1$, $h = 0.01$ m

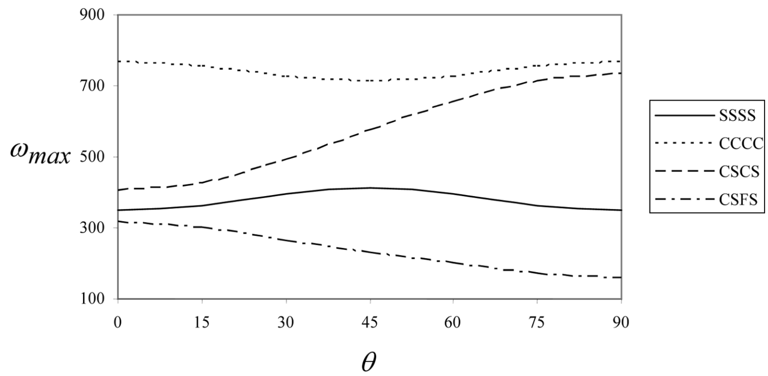

Fig. 4 Effect of the boundary conditions on the fundamental frequency of laminated plates with $a/b = 1$, $N = 4$, $h = 0.01$ m

4.1 Laminated plates with different number of layers

Fig. 3 shows the fundamental frequency ($\omega_{\max}$) versus θ for different number of layers for simply supported laminated plates. It can be seen that, for laminates with symmetric lay-up, there is a gradual increase in the value of the optimal fundamental frequency with increase in the number of layers. Also, the optimal design with $\theta = 45^{\circ}$ is outstanding for square plate. The optimal fundamental frequency leads to an increase around 5%, 1%, 0.7% between $N = 4$-6, $N = 6$-8 and $N = 8$-10 layered laminated plates, respectively. That is, differences of the optimal fundamental frequency decrease, as the number of layer increases.

4.2 Laminated plates with different boundary conditions

The different combinations of free (F), simply supported (S) and clamped (C) boundary conditions are considered, viz. (SSSS), (CCCC), (CSCS) and (CSFS). The symbolism (CSCS), for example, identifies a rectangular plate with edges clamped, simply supported, clamped and simply supported; start counting anticlockwise from the left edge of the plate. As can be seen from Fig. 4, (CSFS) boundary condition gives the smallest fundamental frequency, on the other hand, (CCCC) boundary condition gives the largest values.

The optimum fibre angles are (0°, 90°), (90°) and (0°) for (CCCC), (CSCS) and (CSFS) boundary conditions, respectively. The optimal fundamental frequency leads to an increase around 85%, 77% between (SSSS)-(CCCC) and (SSSS)-(CSCS) boundary conditions, respectively. On the other hand, the optimal fundamental frequency leads to a decrease around 24% between (SSSS)-(CSFS) boundary conditions.

4.3 Laminated plates with different plate thickness

Fig. 5 shows $\omega_{\max}$ versus θ for different plate thickness for simply supported laminated square composite plates. As expected, as the thickness of plate increases, the fundamental frequency increases. Also, the optimum fibre angle is 45° for all plate thicknesses. The optimal fundamental frequency leads to an increase around 95%, 48%, 32% between $h = 0.01$-0.02 m, $h = 0.02$-0.03 m

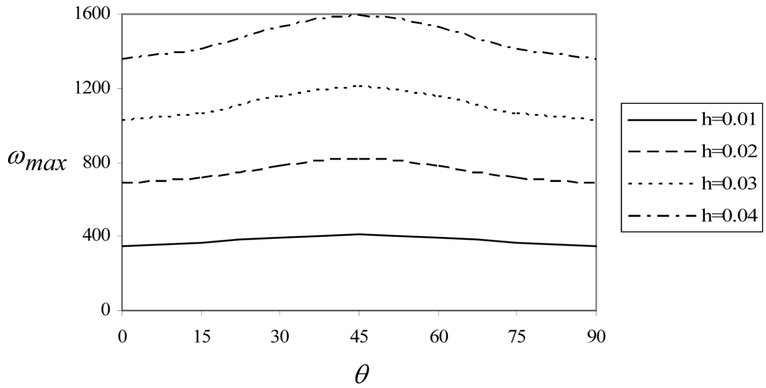

Fig. 5 Effect of the plate thickness on the fundamental frequency of simply supported laminated plates with $a/b = 1$, $N = 4$

and $h = 0.03$-0.04 m laminated plates, respectively. That is, differences of the fundamental frequency decrease, as the thickness of plate increases.

4.4 Laminated plates with different plate aspect ratio

Fig. 6 shows ω_{max} versus θ for different aspect ratios for different boundary conditions of laminated plates.

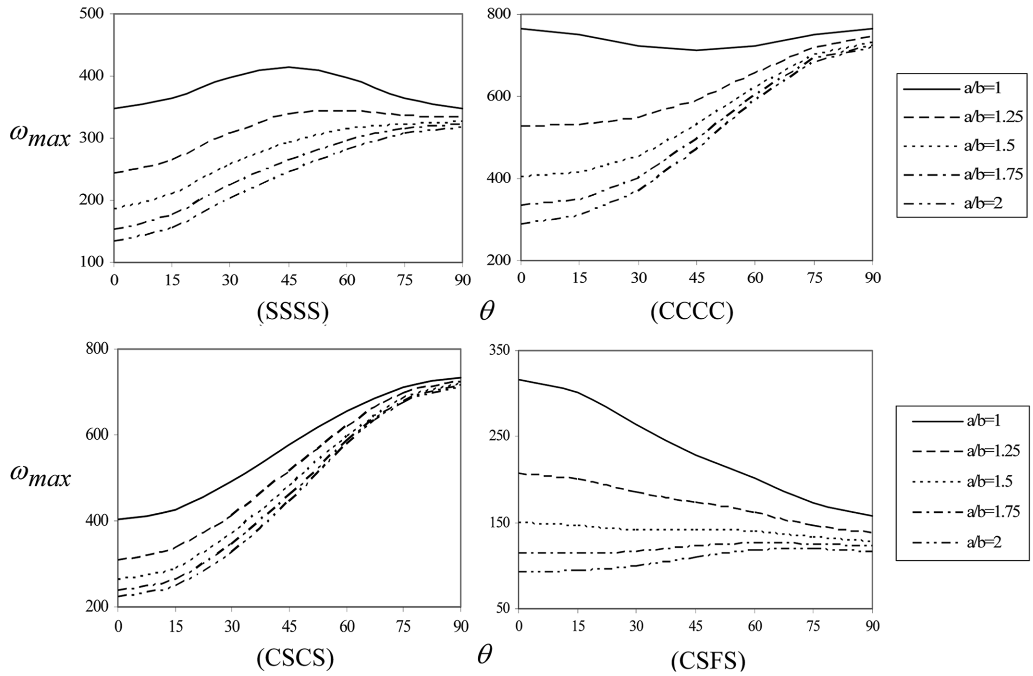

Fig. 6 ω_{max} versus θ for different aspect ratios for different boundary conditions for laminated plates with $N = 4$, $h = 0.01$ m

As can be seen from Fig. 6, the optimal fundamental frequency occurs at 90^o at aspect ratios rather than a/b = 1.25 for (SSSS) boundary condition. The optimal fundamental frequency occurs at 90^o for all aspect ratios for (CCCC) (also 0^o for a/b = 1) and (CSCS) boundary conditions. The optimal fundamental frequency occurs at 0^o for aspect ratios lower than a/b = 1.75 for (CSFS) boundary condition.

The optimal fundamental frequency leads to a decrease around 17%, 6%, 1.5%, and 1% between for a/b = 1-1.25, a/b = 1.25-1.5, a/b = 1.5-1.75 and a/b = 1.75-2, respectively for (SSSS) boundary condition. The optimal fundamental frequency leads to a decrease around 3.5%, 1.5%, 1%, and 0.6% between for a/b = 1-1.25, a/b = 1.25-1.5, a/b = 1.5-1.75 and a/b = 1.75-2, respectively for (CCCC) boundary condition. The optimal fundamental frequency leads to a decrease around 3%, 0.8%, 0.5%, and 0.35% between for a/b = 1-1.25, a/b = 1.25-1.5, a/b = 1.5-1.75 and a/b = 1.75-2, respectively for (CSCS) boundary condition. The optimal fundamental frequency leads to a decrease in the fundamental frequency around 35%, 30%, 15%, and 10% between for a/b = 1-1.25, a/b = 1.25-1.5, a/b = 1.5-1.75 and a/b = 1.75-2, respectively for (CSFS) boundary condition. That is, differences of the fundamental frequency decrease, as the aspect ratio increases.

In Table 1, the optimum ply angles are given for different aspect ratios for different boundary conditions.

Fig. 7 shows ω_{max} versus a/b for different boundary conditions for laminated plates. As expected, as the plate aspect ratio increases, the fundamental frequency decreases. The optimal fundamental frequency leads to an increase around 115%, 110% between (SSSS)-(CCCC) and (SSSS)-(CSCS)

Table 1 The optimum ply angles for different aspect ratios for different boundary conditions for laminated plates with N = 4, h = 0.01 m

Aspect ratio (a/b)	(SSSS)	(CCCC)	(CSCS)	(CSFS)
		θ_{opt} (^{o})		
1	45	0 and 90	90	0
1.25	57.694	90	90	0
1.5	90	90	90	0
1.75	90	90	90	61.806
2	90	90	90	74.192

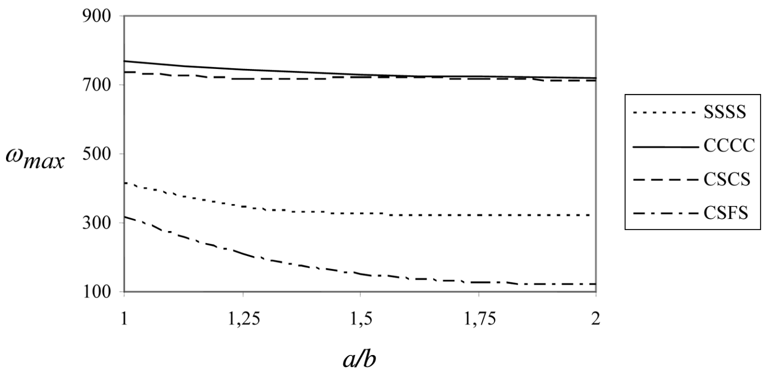

Fig. 7 ω_{max} versus a/b for different boundary conditions of laminated plates with N = 4, h = 0.01 m

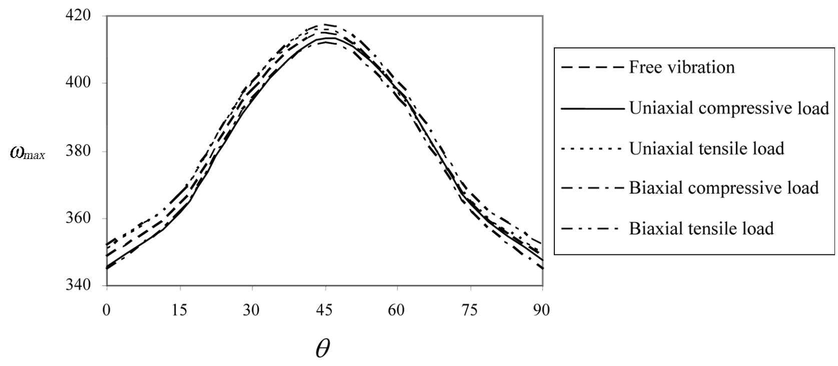

Fig. 8 Effect of in-plane loads on the optimal designs

boundary conditions, respectively, while leading to a decrease around 40% between (SSSS)-(CSFS) boundary conditions for a/b = 1.25. The optimal fundamental frequency leads to an increase around 125%, 120% between (SSSS)-(CCCC) and (SSSS)-(CSCS) boundary conditions, respectively, while leading to a decrease around 55% between (SSSS)-(CSFS) boundary conditions for a/b = 1.5. The optimal fundamental frequency leads to an increase around 125%, 125% between (SSSS)-(CCCC) and (SSSS)-(CSCS) boundary conditions, respectively, while leading to a decrease around 60% between (SSSS)-(CSFS) boundary conditions for a/b = 1.75. The optimal fundamental frequency leads to an increase around 125%, 125% between (SSSS)-(CCCC) and (SSSS)-(CSCS) boundary conditions, respectively, while leading to a decrease around 65% between (SSSS)-(CSFS) boundary conditions for a/b = 2.

4.5 Effect of in-plane loads

The effect of in-plane loads on the optimal design is given in Fig. 8. The laminated plate subjected to uniaxial compressive load, uniaxial tensile load, biaxial compressive load and biaxial tensile load is considered. $N_x = -3 \times 10^5$ N for uniaxial compressive load, $N_x = 3 \times 10^5$ for uniaxial tensile load, $N_x = N_y = -3 \times 10^5$ N for biaxial compressive loads and $N_x = N_y = 3 \times 10^5$ N for biaxial tensile loads are considered. As can be seen, the optimal fundamental frequency is the highest for biaxial tensile loads and increases about 0.6%. On the other hand, the optimal fundamental frequency is the lowest for biaxial compressive loads and decreases about 0.6%. The optimal fibre angle is $\theta_{opt} = 45^{\circ}$ for all cases.

5. Conclusions

For the optimal free vibration analysis of laminated plates with various plate layers, boundary conditions, plate thicknesses, aspect ratios and in-plane loads, the following conclusions may be drawn

- The optimal fundamental frequencies of simply supported laminates plates increase with the increasing the number of layer and optimal angle is 45° for all layers. However, the differences of the fundamental frequency decrease, as the number of layer increases.
- (CCCC) boundary condition gives the largest optimal fundamental frequency, on the other hand,

(CSFS) boundary condition gives the smallest values.

- The optimal fundamental frequency increases with the increasing the thickness of plate. The optimal angle is 45^o for simply supported laminates for all plate thicknesses. However, differences of the fundamental frequency decrease, as the thickness of plate increases.
- For (CCCC) and (CSCS) boundary conditions, the optimal fiber angles are the same (90^o) for all aspect ratios (also 0^o for a/b = 1 for (CCCC)). On the other hand, the optimal fiber angles are different for other boundary conditions. The optimal fundamental frequencies lead to decrease with the increasing the aspect ratio for all boundary conditions. Also, the differences of the fundamental frequencies decrease, as the aspect ratio increases.
- The optimal fibre angle is not effected by in-plane loads for simply supported laminates. On the other hand, the optimal fundamental frequency gives the highest values for biaxial tensile loads, while it gives the smallest values for biaxial compressive loads.

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