Structural Engineering and Mechanics, Vol. 24, No. 1 (2006) 19-29
DOI: http://dx.doi.org/10.12989/sem.2006.24.1.019

Evolution of bone structure under axial and transverse loads

Chuanyong Qu†

Department of Mechanics, Tianjin University, Tianjin, 300072, China

Qing-Hua Qin‡

Department of Engineering, Australian National University, Canberra, Australia

(Received November 18, 2005, Accepted April 20, 2006)

Abstract. The evolution process of an initially homogeneous bone structure under axial and transverse loads is investigated in this paper. The external loads include axial and external lateral pressure, electric, magnetic and thermal loads. The theoretical predictions of evolution processes are made based on the adaptive elasticity formulation and coupled thermo-magneto-electro-elastic theory. The adaptive elastic body, which is a model for living bone diaphysis, is assumed to be homogeneous in its anisotropic properties and its density. The principal result of this paper is determination of the evolution process of the initially homogeneous body to a transversely inhomogeneous body under the influence of the inhomogeneous stress state.

Keywords: bone remodeling; piezoelectric; piezomagnetic; biomechanics; biomaterials.

1. Introduction

Adaptive bone remodelling under multi-field loads is attracting widespread attention from biological scientists and mechanical engineers. The remodelling process is the mechanisms by which bone adapts its histological structure to changes in long term loading. Since bone tissues remodel themselves without the control of the nervous system (Hert *et al.* 1971), it is quite rational to assume the presence of an unknown mechanism in bone tissues which can assess the surrounding mechanical environment and control bone formation and resorption. The bone remodelling mechanism has been investigated by many authors (Cowin and van Buskirk 1978, 1979, Cowin and Firoozbakhsh 1981, Cowin and Hegedus 1976, Gjelsvik 1973a,b). Many hypotheses as to the nature of this mechanism have been proposed, including the theory of adaptive elasticity (Cowin and van Buskirk 1978, 1979, Cowin and Firoozbakhsh 1981, Cowin and Hegedus 1976), piezoelectric theory (Gjelsvik 1973a,b), hydrostatic theory (Jendrucko *et al.* 1976), fatigue damage theory (Martin and Bur 1982, Bur *et al.* 1985, Carter 1984) and the transport of growth factors theory (Takakuda

† Ph.D. Student
‡ Professor, Corresponding author, E-mail: qinghua.qin@anu.edu.au

1993). Among these theories, the theory of adaptive elasticity is the most popular and widely applied in biomedical engineering. The adaptive elastic theory does not, however, take into account the piezoelectric or piezomagnetic properties of bone tissues. Investigation of some tissues such as living bone and collagen has shown these materials to be piezoelectric (Fukada and Yasuda 1957, 1964, Williams and Breger 1974, Guzelsu 1978, Johnson *et al.* 1980, Demiray 1983, Qin and Ye 2004) and indicates that the piezoelectric properties of bone play an important role in the development and growth of remodelling of skeletons. Evidences (Bessett *et al.* 1982, Mcleod and Rubin 1992, Giordano and Battisti 2001) showed that the pulsed extremely low frequency electromagnetic field can stimulate the bone tissue to remodel itself. This feature was widely applied to cure the skeletal disease such as osteoporosis, fracture and nonunion. However the behaviour of bone remodelling under multi-field loads is still less investigated. To the authors' knowledge, the theoretical study of bone remodelling during the past decades has been limited to elasticity. Recently, Qin and Ye (2004) and Qin *et al.* (2005) extended previous study to include piezoelectric effects.

The purpose of this paper is to extend previous results to include piezomagnetic effects and to investigate how the magnetic field and further the coupled multi-field can simultaneously affect bone remodelling and the evolution process of an initially homogeneous bone material subjected to axisymmetric external loads which generate an inhomogeneous stress, electric and magnetic fields. These external loads include axial and external lateral pressure, electric, magnetic, and thermal loads. It should be mentioned here that one of the thermal loads a person may experience is the fluctuation of body temperature. However, how this may affect bone remodelling process is still an open question. As an initial investigation, the purpose of this study is to show how a bone may response to thermal and multi-field loads and to provide information for possible use of imposed external temperature and/or electrical fields in medical treatment and controlling healing process of injured bones. The bone structure is simulated by a hollow circular cylinder composed of linearly thermomagnetoelectro-elastic materials. The theoretical predictions of evolution behaviour are based on the extended theory of adaptive elasticity and thermomagnetoelectroelastic constitutive formulation. According to the theory, an inhomogeneous thermomagnetoelectroelastic field will result in an inhomogeneous bone structure. The evolution of an initially homogeneous body to an inhomogeneous one under the influence of an inhomogeneous stress field is illustrated graphically. The values of some constants needed for the adaptive elastic model are available in the literature, for example, the elastic moduli of cortical bone and the variation of moduli with bulk density. The values of other constants, such as the remodelling rate coefficients, are not known and they are estimated in the present work by physical arguments and by imposing the restriction that the remodelling time constant be of the order of 100 days.

2. Equation for internal bone remodeling

The equations of the theory of adaptive elasticity of Cowin and Hegedus (1976) are used and extended to include piezoelectric and piezomagnetic effects in this study. The remodelling rate equation in cylindrical coordinates is

$$\begin{aligned} \dot{e} &= A^{*}(e) + A_r^E(e)E_r + A_z^E(e)E_z + G_r^E(e)H_r + G_z^E(e)H_z \\ &\quad + A_{rr}^s(e)(s_{rr} + s_{\theta\theta}) + A_{zz}^s(e)s_{zz} + A_{rz}^s(e)s_{rz} \end{aligned} \tag{1}$$

where e is a change in the volume fraction of bone matrix material from its reference value, say ξ_0; $A^*(e), A_i^E(e), G_i^E(e)$ are $A_{ij}^s(e)$ material coefficients dependent upon the volume fraction e. s_{ij}, E_i, H_i are components of strain, electric field and magnetic field, respectively.

3. Analytical solution to the remodeling equation of a homogeneous hollow circular cylindrical bone

3.1 The linear theory of thermomagnetoelectroelastic solids

Consider a thermomagnetoelectroelastic bone cylinder subjected to axial-symmetric axial and external lateral pressure, electric, magnetic, and thermal loads. For simplicity, the cylindrical coordinate system is used the analysis. The axial, circumferential and normal to the middle-surface coordinate length coordinates are denoted by z, θ and r, respectively. With the cylindrical coordinate system, the constitutive equations of a thermomagnetoelectroelastic solid can be given by Gao and Noda (2004).

$$\begin{aligned}
\sigma_{rr} &= c_{11}s_{rr} + c_{12}s_{\theta\theta} + c_{13}s_{zz} - e_{31}E_z - \alpha_{31}H_z - \beta_1 T \\
\sigma_{\theta\theta} &= c_{12}s_{rr} + c_{11}s_{\theta\theta} + c_{13}s_{zz} - e_{31}E_z - \alpha_{31}H_z - \beta_1 T \\
\sigma_{zz} &= c_{13}s_{rr} + c_{13}s_{\theta\theta} + c_{33}s_{zz} - e_{33}E_z - \alpha_{33}H_z - \beta_3 T \\
\sigma_{zr} &= c_{44}s_{zr} - e_{15}E_r - \alpha_{15}H_r \\
D_r &= e_{15}s_{zr} + \kappa_1 E_r + d_1 H_r \\
D_z &= e_{31}(s_{rr} + s_{\theta\theta}) + e_{33}s_{zz} + \kappa_3 E_z + d_3 H_z - p_3 T \\
B_r &= \alpha_{15}s_{zr} + d_1 E_r + \mu_1 H_r \\
B_z &= \alpha_{31}(s_{rr} + s_{\theta\theta}) + \alpha_{33}s_{zz} + d_3 E_z + \mu_3 H_z - m_3 T \\
h_r &= k_r q_r, \quad h_z = k_z q_z
\end{aligned} \tag{2}$$

where σ_{ij}, D_i, B_i and h_i are components of stress, electrical displacement, magnetic induction and heat flow, respectively; c_{ij} are elastic stiffness; e_{ij} are piezoelectric constants; α_{ij} are piezomagnetic constants; κ_i are dielectric permitivities; d_i are magnetoelectric constants; μ_i are magnetic permeabilities; T denotes temperature change; p_3 is a pyroelectric constant; m_3 is a pyromagnetic constant; β_i are stress-temperature coefficients; q_i are heat intensity; and k_i are heat conduction coefficients. The associated strains, electric fields, and heat intensities are respectively related to displacements u_i, electric potential φ, magnetic potential ψ, and temperature change T as

$$\begin{aligned}
&s_{rr} = u_{r,r}, \; s_{\theta\theta} = \frac{u_r}{r}, \; s_{zz} = u_{z,z}, \; s_{zr} = u_{z,r} + u_{r,z}, \; E_r = -\varphi_{,r} \\
&E_z = -\varphi_{,z}, \; H_r = -\psi_{,r}, \; H_z = -\psi_{,z}, \; q_r = -T_{,r}, \; q_z = -T_{,z}
\end{aligned} \tag{3}$$

For quasi-stationary behaviour, in the absence of heat source, free electric charge, electric current, and body forces, the thermopiezoelectricmagnetic theory of bone is completed by adding the following equations of equilibrium for heat flow, stress, electric displacements and magnetic induction:

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{\partial \sigma_{zr}}{\partial z} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = 0, \quad \frac{\partial \sigma_{zr}}{\partial r} + \frac{\partial \sigma_{zz}}{\partial z} + \frac{\sigma_{zr}}{r} = 0$$

$$\frac{\partial D_r}{\partial r} + \frac{\partial D_z}{\partial z} + \frac{D_r}{r} = 0, \quad \frac{\partial B_r}{\partial r} + \frac{\partial B_z}{\partial z} + \frac{B_r}{r} = 0 \tag{4}$$

$$\frac{\partial h_r}{\partial r} + \frac{\partial h_z}{\partial z} + \frac{h_r}{r} = 0$$

Consider now a hollow circular cylinder of bone, subjected to an external temperature change T_0, a quasi-static axial load P, an external pressure p, an electric potential load φ_a(or/and φ_b) and an magnetic potential load ψ_a(and/or ψ_b). The boundary conditions are

$$T = 0, \quad \sigma_{rr} = \sigma_{rz} = 0, \quad \varphi = \varphi_a, \quad \psi = \psi_a \quad \text{at } r = a$$

$$T = T_0, \quad \sigma_{rr} = -p, \quad \sigma_{rz} = 0, \quad \varphi = \varphi_b, \quad \psi = \psi_b \quad \text{at } r = b \tag{5}$$

and

$$\int_S \sigma_{zz} dS = -P \tag{6}$$

where a and b denote the inner and outer radii respectively of the bone, and S is the cross-sectional area. For a long bone, it is assumed that except for the axial displacement u_z, all displacements, temperatures and electrical potential are independent of the z coordinate and that u_z may have linear dependence on z.

The solution of displacements u_r, u_z, and electric potential φ to the problem above in the absence of piezoelectric magnetic field has been discussed elsewhere (Qin and Ye 2004). This work extends the results in Qin and Ye (2004) to include the piezomagnetic effect. The strains, electric field intensity, magnetic field intensity and the temperature change T can be found by introducing the boundary conditions (5) and (6) into (2) (For the reader's convenience the derivation for the corresponding u_r, u_z, φ and ψ is briefly discussed in Appendix A at the end of this paper). They are, respectively,

$$s_{rr} = \frac{1}{F_3^*}\left(c_{33}\beta_1^*[\beta_2^* T_0 + p(t)] + \varpi \frac{c_{33}c_{12}}{c_{11}} + \frac{F_2^* T_0 + P(t)}{\pi(b^2 - a^2)} c_{13} - F_1^* T_0 c_{13} \right)$$
$$- \frac{a^2 \beta_1^*[\beta_2^* T_0 + p(t)]}{r^2 (c_{11} - c_{12})} + \frac{\varpi \ln(r/a)}{c_{11}} \tag{7}$$

$$s_{\theta\theta} = \frac{1}{F_3^*}\left(c_{33}\beta_1^*[\beta_2^* T_0 + p(t)] + \varpi \frac{c_{33}c_{12}}{c_{11}} + \frac{F_2^* T_0 + P(t)}{\pi(b^2 - a^2)} c_{13} - F_1^* T_0 c_{13} \right)$$
$$+ \frac{a^2 \beta_1^*[\beta_2^* T_0 + p(t)]}{r^2 (c_{11} - c_{12})} + \frac{\varpi[\ln(r/a) - 1]}{c_{11}} \tag{8}$$

$$s_{zz} = \frac{1}{F_3^*}\left(\left[F_1^* T_0 - \frac{F_2^* T_0 + P(t)}{\pi(b^2 - a^2)}\right](c_{11} + c_{12}) - 2c_{13}\beta_1^*[\beta_2^* T_0 + p(t)] - \frac{2c_{13}c_{12}\varpi}{c_{11}}\right) \tag{9}$$

$$s_{rz} = -\frac{e_{15}(\varphi_b - \varphi_a)}{rc_{44}\ln(b/a)} - \frac{\alpha_{15}(\Psi_b - \Psi_a)}{rc_{44}\ln(b/a)} \tag{10}$$

$$E_r = -\frac{(\varphi_b - \varphi_a)}{r\ln(b/a)} \quad H_r = -\frac{(\psi_b - \psi_a)}{r\ln(b/a)} \quad T = -\frac{\ln(r/a)}{\ln(b/a)}T_0 \tag{11}$$

where

$$\varpi = \frac{\beta_1 T_0}{2\ln(b/a)} \quad F_1^* = \frac{1}{\ln(b/a)}\left(\frac{c_{13}\beta_1}{c_{11}} - \frac{\beta_3}{2}\right) \quad F_2^* = \pi b^2\left(\frac{c_{13}}{c_{11}}\beta_1 - \beta_3\right) \tag{12}$$

$$F_3^* = c_{33}(c_{11} + c_{12}) - 2c_{13}^2, \quad \beta_1^* = \frac{b^2}{(a^2 - b^2)}, \quad \beta_2^* = \frac{\beta_1}{2}\left(\frac{c_{12}}{c_{11}} - 1\right) \tag{13}$$

Substituting (7)-(11) into (1) yields

$$\begin{aligned}\dot{e} = {} & A^*(e) + \frac{2A_{rr}^s}{F_3^*}\left(c_{33}\beta_1^*[\beta_2^* T_0 + p(t)] + \varpi\frac{c_{33}c_{12}}{c_{11}} + \frac{F_2^* T_0 + P(t)}{\pi(b^2 - a^2)}c_{13} - F_1^* T_0 c_{13}\right) \\ & + \frac{A_{rr}^s\varpi[2\ln(r/a) - 1]}{c_{11}} + \frac{A_{zz}^s}{F_3^*}\left(\left[F_1^* T_0 - \frac{F_2^* T_0 + P(t)}{\pi(b^2 - a^2)}\right](c_{11} + c_{12})\right. \\ & \left. - 2c_{13}\beta_1^*[\beta_2^* T_0 + p(t)] - \frac{2c_{13}c_{12}\varpi}{c_{11}}\right) - \frac{\varphi_b - \varphi_a}{r\ln(b/a)}\left(A_r^E + \frac{e_{15}}{c_{44}}A_{zr}^s\right) \\ & - \frac{\psi_b - \psi_a}{r\ln(b/a)}\left(G_r^E + \frac{\alpha_{15}}{c_{44}}A_{zr}^s\right)\end{aligned} \tag{14}$$

Since we do not know the exact expressions of the material functions $A^*(e), A_i^E(e), A_{ij}^s(e)$, c_{ij}, e_{ij}, α_{ij} and β_i the following approximate forms of those functions, as proposed by Cowin and van Buskirk (1978) for small value of e, are used here:

$$A^*(e) = C_0 + C_1 e + C_2 e^2, \; A_i^E(e) = A_i^{E0} + eA_i^{E1}, \; G_i^E(e) = G_i^{E0} + eG_i^{E1}, \; A_{ij}^s(e) = A_{ij}^{s0} + eA_{ij}^{s1} \tag{15}$$

and

$$\begin{aligned} c_{ij}(e) &= c_{ij}^0 + \frac{e}{\xi_0}(c_{ij}^1 - c_{ij}^0) \quad e_{ij}(e) = e_{ij}^0 + \frac{e}{\xi_0}(e_{ij}^1 - e_{ij}^0) \\ \alpha_{ij}(e) &= \alpha_{ij}^0 + \frac{e}{\xi_0}(\alpha_{ij}^1 - \alpha_{ij}^0) \quad \beta_i(e) = \beta_i^0 + \frac{e}{\xi_0}(\beta_i^1 - \beta_i^0) \end{aligned} \tag{16}$$

where C_0, C_1, C_2, $A_i^{E0}, A_i^{E1}, A_{ij}^{s0}, A_{ij}^{s1}$, c_{ij}^0, c_{ij}^1, e_{ij}^0, e_{ij}^1, α_{ij}^0, α_{ij}^1, β_i^0, β_i^1 are material constants.

To simplify the writing and analysis, but without loss of generality, constant p and P are assumed in this paper, which means that both of them don't change with time. Using these approximations the remodelling rate Eq. (14) can be simplified as

$$\dot{e} = \alpha(e^2 - 2\beta e + \gamma) \tag{17}$$

by neglecting terms of e^3 and its higher orders, where α, β and γ are independent of time t (Qin and Ye 2004). The solution to (17) is straightforward and has been discussed by Cowin and Hegedus (1976). For the reader's benefit, the solution process is briefly described here. Let e_1 and e_2 denote solutions to $e^2 - 2\beta e + \gamma = 0$, i.e.,

$$e_{1,2} = \beta \pm (\beta^2 - \gamma)^{1/2} \tag{18}$$

When $\beta^2 < \gamma$, e_1 and e_2 are a pair of complex conjugates, the solution of (15) is

$$e(t) = \beta + \sqrt{(\gamma - \beta^2)}\tan\left(\alpha t\sqrt{(\gamma-\beta^2)} + \arctan\frac{e_0 - \beta}{\sqrt{(\gamma-\beta^2)}}\right) \tag{19}$$

where $e = e_0$ is initial condition.

When $\beta^2 = \gamma$ the solution is

$$e(t) = e_1 - \frac{e_1 - e_0}{1 + \alpha(e_1 - e_0)t} \tag{20}$$

Finally, when $\beta^2 > \gamma$ we have

$$e(t) = \frac{e_1(e_0 - e_2) + e_2(e_1 - e_0)\exp(\alpha(e_1 - e_2)t)}{(e_0 - e_2) + (e_1 - e_0)\exp(\alpha(e_1 - e_2)t)} \tag{21}$$

Since it has been proved that both solutions (19) and (20) are physically unlikely (Cowin and van Buskirk 1978), we use solution (21) in the following numerical analysis.

4. Numerical example

As numerical illustration of the bone evolution process, we consider a femur with $a = 25$ mm and $b = 35$ mm. The material properties assumed for the bone are Qin and Ye (2004)

$$c_{11} = 15(1+e)\text{GPa}, \quad c_{12} = c_{13} = 6.6(1+e)\text{GPa}, \quad c_{33} = 12(1+e)\text{GPa}, \quad c_{44} = 4.4(1+e)\text{GPa}$$

$$\beta_1 = 0.621(1+e)\times 10^5\text{NK}^{-1}\text{m}^{-2}, \quad \beta_3 = 0.551(1+e)\times 10^5\text{NK}^{-1}\text{m}^{-2}$$

$$e_{15} = 1.14(1+e)\text{C/m}^2, \quad \alpha_{15} = 500(1+e)\text{N/Am}$$

The remodelling rate coefficients are assumed to be Qin and Ye (2004)

$$C_0 = 0.002\,\text{day}^{-1}, \quad C_1 = -0.05\,\text{day}^{-1}, \quad C_2 = 10^{-6}\,\text{day}^{-1}$$

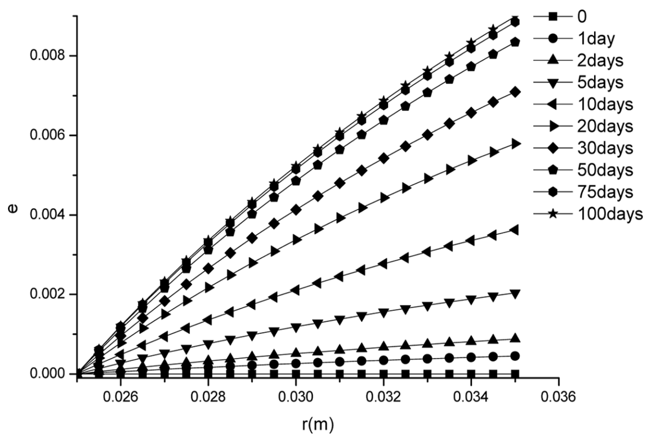

Fig. 1 Variation of e with time t along the radii for electric load

and

$$A_{rr}^{s0} = A_{rr}^{s1} = A_{zz}^{s0} = A_{rz}^{s1} = A_{rz}^{s0} = A_{rz}^{s1} = 4.4\,\mathrm{day}^{-1}$$

$$A_r^{E0} = A_r^{E1} = 10^{-7}\mathrm{m/(V \cdot day)}, \quad G_r^{E0} = G_r^{E1} = 1.5 \times 10^{-8}\mathrm{m/(A \cdot day)}$$

The initial inner and outer radii are assumed to be

$$a_0 = 25 \text{ mm}, \quad b_0 = 35 \text{ mm}$$

and $e_0 = 0$ is assumed. In the calculation, $u_r(t) << a_0$ has been assumed for the sake of simplicity, i.e., $a(t)$ and $b(t)$ may be approximated by a_0 and b_0.

To illustrate the evolution process, we investigate the change of the volume fraction of bone matrix material from its reference value, which is denoted by e, in the transverse direction at several specific times. We also distinguish the following three loading cases to investigate the influence of electric, magnetic and thermal loads on the bone structure. Finally effect on the bone of coupling loads of electric and mechanical loads is studied.

(1) $p(t) = 0, P = 1500N, T_0(t) = 0^{\circ}\mathrm{C}, \varphi_b - \varphi_a = 30\mathrm{V}, \psi_b - \psi_a = 0$

Fig. 1 shows the variation of e with time t in the transverse direction of bone when the loading case is $p(t) = 0, P = 1500N, T_0(t) = 0^{\circ}\mathrm{C}, \varphi_b - \varphi_a = 30\mathrm{V}, \psi_b - \psi_a = 0$.

It can be seen from Fig. 1 that as the time passes, the initially homogeneous bone structure gradually becomes inhomogeneous. The change in the volume fraction of bone matrix material on its inner surface is less than that on its outer surface. This means the bone tissue near the outer surface is less porous and thus denser than that near the inner surface, which means it is stronger.

This can be illustrated by the theory of adaptive elasticity. After the transverse electric field is loaded, an inhomogeneous stress field is generated. Then the stress of the inner surface is smaller than that of the outer one. As the bone remodelling process is ongoing, the strain field is becoming homogeneous. To achieve this, the bone tissue must change to a state with more porous endosteum

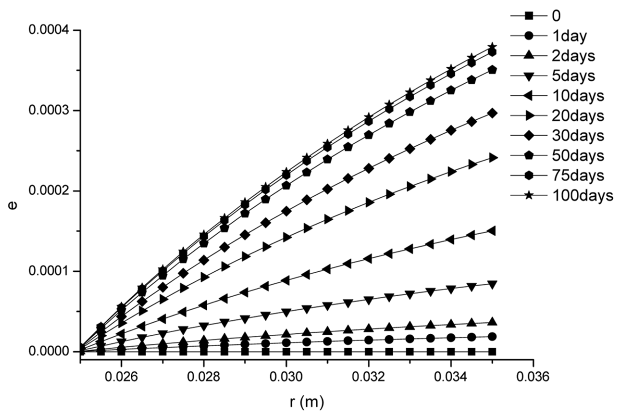

Fig. 2 Variation of e with time t along the radii for magnetic load

and less porous periosteum, which results in an inhomogeneous bone structure. Although the value of e is very small, transverse electric loads can indeed change the bone structure. If real remodelling rate coefficients are attained by experimental means, then we can evaluate the effect of electric field on the bone structure.

It is also found that as time approaches infinity, the value of e becomes less and less. This indicates that the bone structure stabilizes itself at a relatively steady state, which can be accepted as the end of the remodelling process.

(2) $p(t) = 0, P = 1500N, T_0(t) = 0^{\circ}\mathrm{C}, \varphi_b - \varphi_a = 0, \psi_b - \psi_a = 1\,\mathrm{A}$

Fig. 2 shows the variation of e with time t along the radii of bone when the loading case is $p(t) = 0, P = 1500N, T_0(t) = 0^{\circ}\mathrm{C}, \varphi_b - \varphi_a = 0, \psi_b - \psi_a = 1\,\mathrm{A}$.

It can be seen from Fig. 2 that a magnetic load has a similar influence on bone structure to an electric load. A magnetic load can also inhomogenize an initially homogeneous bone structure through the bone remodeling process. But essentially further experimental and theoretical investigations need to be developed to obtain the exact remodeling rate coefficients and to discover the importance of the role played by magnetic stimuli.

(3) $p(t) = 0, P = 1500N, T_0(t) = 0.1^{\circ}\mathrm{C}, \varphi_b - \varphi_a = 0, \psi_b - \psi_a = 0\mathrm{A}$

Fig. 3 shows the variation of e with time t along the radii of bone when the loading case is $p(t) = 0, P = 1500N, T_0(t) = 0.1^{\circ}\mathrm{C}, \varphi_b - \varphi_a = 0, \psi_b - \psi_a = 0\mathrm{A}$.

A similar phenomenon to that of Fig. 2 is found in Fig. 3, which indicates that a warmer environment may improve the remodelling process with a less porous bone structure, and change of temperature can also result in an inhomogeneous bone structure. As mentioned in Qin and Ye (2004), the process by which temperature change may affect bone remodelling is still an open question. An initial purpose of this study is to show how a bone may response to thermal, magnetic, and electric loads and to provide information for possible use of imposed external temperature and/ or magnetic-electrical fields in medical treatment and in controlling the healing process of injured bones. Further investigations are undoubtedly needed.

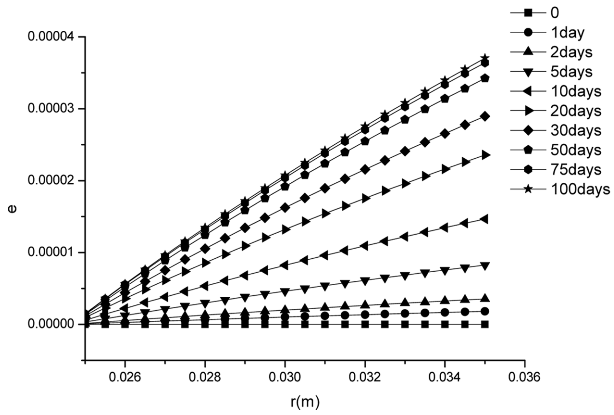

Fig. 3 Variation of e with time t along the radii for thermal load

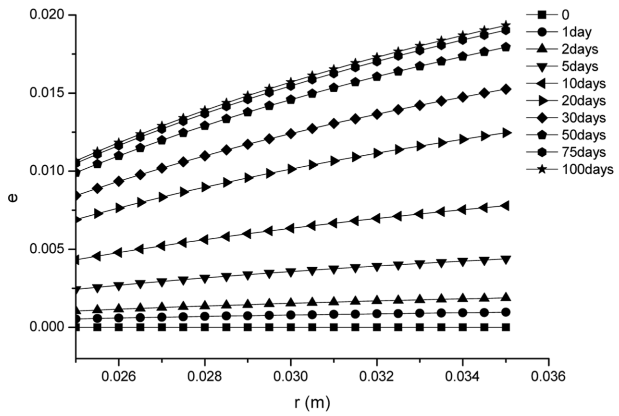

Fig. 4 Variation of e with time t along the radii for coupling loads

(4) $p(t) = 1\,\mathrm{MPa}, P = 1500N, T_0(t) = 0.1^\circ\mathrm{C}, \varphi_b - \varphi_a = 30\mathrm{V}, \psi_b - \psi_a = 1\mathrm{A}$

Fig. 4 shows the variation of e with time t in the transverse direction when subjected to coupling loads. The above loading case is considered to study the coupling effect of electric-magnetic and mechanical loads on bone structure. It can be seen from Fig. 4 that the function of coupled loads is the superposition of the single loads. But they are not simply linearly superposed. Further, the properties of bone tissue change more sharply under coupled loads than when it is subjected to only one load. The combination of the magnetic, electric, thermal and mechanical loads results in significant change in bone structure and properties of bone tissues. This indicates that loading coupled fields is more effective in modifying bone structure than loading only one kind of field.

5. Conclusions

The problem of thermopiezomagneticelectric bone remodelling was addressed within the framework of adaptive elastic theory. The thermomagneticelectroelastic solution for adaptive elastic bone materials was derived through the use of adaptive elastic theory. By assuming a homogeneous bone material, the evolution process of bone structure was investigated both theoretically and numerically.

Numerical studies were carried out to verify the analytical solution for the bone remodelling process. In the numerical analysis, various load conditions were considered, including axial pressure, transverse thermal, magnetic, electric and pressure loads. The evolution process of an initially homogeneous bone structure to an inhomogeneous one was simulated.

The numerical results showed that apart from mechanical loads, electric field, magnetic field and thermal load can also affect the bone remodelling process. All of these can result in an inhomogeneous bone structure. This feature may be considered and utilized in controlling the healing process of injured bones. It must be mentioned that the model proposed above is a general one for bone remodelling. The detailed process of how the bone tissues evaluate the environment and response to it need further investigation.

References

Bessett, C.A., Valdes, M.G. and Hernandez, E. (1982), "Modification of fracture repair with selected pulsing electromagnetic fields", *J. Bone and Joint Surgery—American Volume*, **64**, 888-895.

Bur, B.B., Martin, R.B., Schaffler, M.B. and Radin, E.L. (1985), "Bone remodeling in response to *in vivo* fatigue microdamage", *J. Biomechanics*, **18**, 189.

Carter, D.R. (1984), "Mechanical loading histories and cortical bone remodeling", *Calcified Tissue International.* 36-S1:S19.

Cowin, S.C. and Firoozbakhsh, K. (1981), "Bone remodelling of diaphysial surfaces under constant load: Theoretical predictions", *J. Biomechanics*, **14**, 471-484.

Cowin, S.C. and Hegedus, D.M. (1976), "Bone remodelling I: Theory of adaptive elasticity", *J. Elasticity*, **6**, 313-326.

Cowin, S.C. and van Buskirk, W.C. (1978), "Internal bone remodelling induced by a medullary pin", *J. Biomechanics*, **11**, 269-275.

Cowin, S.C. and van Buskirk, W.C. (1979), "Surface bone remodelling induced by a medullary pin", *J. Biomechanics*, **12**, 269-276.

Demiray, H. (1983), "Electro-mechanical remodelling of bones", *Int. J. Eng. Sci.,* **21**, 1117-1126.

Fukada, E. and Yasuda, I. (1957), "On the piezoelectric effect of bone", *J. Phys. Soc. Japan*, **12**, 1158-1162.

Fukada, E. and Yasuda, I. (1964), "Piezoelectric effects in Collagen", *Jap. J. Appl. Phys.*, **3**, 117-121.

Gao, C.F. and Noda, N. (2004), "Thermal-induced interfacial cracking of magneto-electroelastic materials", *Int. J. Eng. Sci.,* **42**, 1347-1360.

Giordano, N. and Battisti, E. (2001), "Effect of electromagnetic fields on bone mineral density and biochemical markers of bone turnover in osteoporosis: A single-blind, randomized pilot study", *Current Therapeutic Research*, **62**, 187-193.

Gjelsvik, A. (1973a), "Bone remodeling and piezoelectricity — I", *J. Biomechanics*, **6**, 69-77.

Gjelsvik, A. (1973b), "Bone remodeling and piezoelectricity — II", *J. Biomechanics*, **6**, 187.

Guzelsu, N. (1978), "A piezoelectric model for dry bone tissue", *J. Biomechanics*, **11**, 257-267.

Hert, J., Sklenska, M. and Liskova, M. (1971), "Reaction of bone to mechanical stimuli, part 5. Effect of intermittent stress on the rabbit tibia after resection of the peripheral nerves", *Folia Morphologica*, **19**, 397.

Jendrucko, R., Hyman, W.A., Newell, P.H. and Chakraborty, B.K. (1976), "Theoretical evidence for the generation of high pressure in bone cells", *J. Biomechanics*, **9**, 87.

Johnson, M.W., Williams, W.S. and Gross, D. (1980), "Ceramic models for piezoelectricity in dry bone", *J. Biomechanics*, **13**, 565-573.

Martin, R.B. and Bur, B.B. (1982), "A hypothetical mechanism for the stimulation of osteonal remodeling by fatigue damage", *J. Biomechanics*, **15**, 137.

Mcleod, K.J. and Rubin, C.T. (1992), "The effect of low-frequency electrical field on osteogenesis", *J. Bone and*

Joint Surgery—American Volume, **74**, 920-929.
Qin, Q.H. and Ye, J.Q. (2004), "Thermolectroelastic solutions for internal bone remodeling under axial and transverse loads", *Int. J. Solids Struct.,* **41**, 2447-2460.
Qin, Q.H., Qu, C.Y. and Ye, J.Q. (2005), "Thermolectroelastic solutions for surface bone remodeling under axial and transverse loads", *Biomaterials*, **26**, 6798-6810.
Takakuda, K. (1993), "A hypothetical regulation mechanism of adaptive bone remodeling (transport of growth factors by mechanical loads)", *JSME Int. J.,* **36**(4), 417-424.
Williams, W.S. and Breger, L. (1974), "Analysis of stress distribution and piezoelectric response in cantilever bending of bone and tendon", *Ann N.Y. Acad Sci.*, **238**, 121-130.

APPENDIX A: The solution of displacements u_r, u_z and electric φ, magnetic field ψ

Using (2) and (3), differential Eq. (4) can be written as

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)T = 0, \quad c_{11}\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - \frac{1}{r^2}\right)u_r = \beta_1\frac{\partial T}{\partial r} \tag{A1}$$

$$c_{44}\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)u_z + e_{15}\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)\varphi + \alpha_{15}\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)\psi = 0 \tag{A2}$$

$$e_{15}\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)u_z - \kappa_1\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)\varphi - \mu_1\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)\psi = 0 \tag{A3}$$

The solution to the above equations satisfying boundary conditions (5)-(6) is given by

$$u_r = \frac{r}{F_3^*}\left(c_{33}\beta_1^*[\beta_2^*T_0 + p(t)] + \varpi\frac{c_{33}c_{12}}{c_{11}} + \frac{F_2^*T_0 + P(t)}{\pi(b^2 - a^2)}c_{13} - F_1^*T_0c_{13}\right) + \frac{a^2\beta_1^*[\beta_2^*T_0 + p(t)]}{r(c_{11} - c_{12})} + \frac{\varpi r[\ln(r/a) - 1]}{c_{11}} \tag{A4}$$

$$u_z = \frac{z}{F_3^*}\left(\left[F_1^*T_0 - \frac{F_2^*T_0 + P(t)}{\pi(b^2 - a^2)}\right](c_{11} + c_{12}) - 2c_{13}\beta_1^*[\beta_2^*T_0 + p(t)] - \frac{2c_{13}c_{12}\varpi}{c_{11}}\right) - \frac{e_{15}(\varphi_b - \varphi_a)\ln(r/a)}{c_{44}\ln(b/a)} - \frac{\alpha_{15}(\psi_b - \psi_a)\ln(r/a)}{c_{44}\ln(b/a)} \tag{A5}$$

$$\varphi = \frac{\ln(r/a)}{\ln(b/a)}(\varphi_b - \varphi_a) + \varphi_a \qquad \psi = \frac{\ln(r/a)}{\ln(b/a)}(\psi_b - \psi_a) + \psi_a \tag{A6}$$

$$T = \frac{\ln(r/a)}{\ln(b/a)}T_0 \tag{A7}$$

where

$$\varpi = \frac{\beta_1 T_0}{2\ln(b/a)} \quad F_1^* = \frac{1}{\ln(b/a)}\left(\frac{c_{13}\beta_1}{c_{11}} - \frac{\beta_3}{2}\right) \quad F_2^* = \pi b^2\left(\frac{c_{13}}{c_{11}}\beta_1 - \beta_3\right) \tag{A8}$$

$$F_3^* = c_{33}(c_{11} + c_{12}) - 2c_{13}^2, \quad \beta_1^* = \frac{b^2}{(a^2 - b^2)}, \quad \beta_2^* = \frac{\beta_1}{2}\left(\frac{c_{12}}{c_{11}} - 1\right) \tag{A9}$$