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Nonlinear analysis of reinforced concrete beams strengthened with polymer composites

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Abstract. Strengthening of existing old structures has traditionally been accomplished by using conventional materials and techniques, viz., externally bonded steel plates, steel or concrete jackets, etc. Alternatively, fibre reinforced polymer composite (FRPC) products started being used to overcome problems associated with conventional materials in the mid 1950s because of their favourable engineering properties. Effectiveness of FRPC materials has been demonstrated through extensive experimental research throughout the world in the last two decades. However there is a need to use refined analytical tools to simulate response of strengthened system. In this paper, an attempt has been made to develop a numerical model of strengthened reinforced concrete (RC) beams with FRPC laminates. Material models for RC beams strengthened with FRPC laminates are described and verified through a nonlinear finite element (FE) commercial code, with the help of available experimental data. Three dimensional (3D) FE analysis has been performed by assuming perfect bonding between concrete and FRPC laminate. A parametric study has also been performed to examine effects of various parameters like fibre type, stirrup's spacing, etc. on the strengthening system. Through numerical simulation, it has been shown that it is possible to predict accurately the flexural response of RC beams strengthened with FRPC laminates by selecting an appropriate material constitutive model. Comparisons are made between the available experimental results in literature and FE analysis results obtained by the present investigators using load-deflection and load-strain plots as well as ultimate load of the strengthened beams. Furthermore, evaluation of crack patterns from FE analysis and experimental failure modes are discussed at the end.

Keywords: RC beam; FRPC; FE analysis; strengthening; nonlinear; ultimate load.

1. Introduction

Use of FRPC laminates in civil engineering applications is gaining acceptance. Extensive experimental/analytical research across the world and recent developments have proved that FRPCs can be efficient strengthening materials because of their good engineering properties - light weight, corrosion resistance, high strength to density ratio, high fatigue endurance, etc. (ACI 440 1996). Various parameters like fibre type, its configuration, width/thickness of FRPC, wrap angle, concrete strength, level of loading, etc. have been considered in experimental research for use of FRPC as a strengthening material and also as reinforcement in new construction. However, most of the current

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research is focused on issues related to strengthening. There are various applications of FRPC for structural engineering. However, each of them must be developed and evaluated on its own merits. As effectiveness of FRPCs have been widely recognized through extensive experimental studies throughout the world, a present need of the scientific community is to have a reliable analytical tool to simulate response of strengthened or repaired RC structures with FRPC laminates by considering the actual properties of the material.

Analysis of FRPC laminate strengthened RC beams has been done in the past using closed-form solutions based on linear elastic theory and a strength of materials approach (Triantafillou and Plevris 1992). Some empirical expressions have been suggested by various researchers (ACI 440 2002). All these empirical expressions were based on certain assumptions and the experimental database varied from experiment to experiment. Such simplistic empirical models cannot take into account material and geometric nonlinearities present in a physical system. In addition, cracking of concrete cannot be included in a simplistic elastic analysis. As a result, simple models failed to simulate collapse loads of structural components. Few results are available on the mechanical behaviour of RC beams with externally bonded carbon fibre reinforced plastic (CFRP) laminates subjected to service loads. Ferrier *et al.* (2003) have tried to provide a more accurate characterization of their serviceability by conducting a full-scale field test and by utilizing an optical measurement technique. Based on their measurements, analysis of crack growth and opening in both unstrengthened and strengthened beams were presented. It was demonstrated that effects of cracking, stress-strain variation and stiffness degradation should be incorporated as the system was subjected to nonlinear changes.

Researchers have also attempted to simulate behavior of RC beams strengthened with FRPC laminate using FE analysis. Arduini and Nanni (1997) performed FE analysis with a smeared cracking approach with a view to validate experimental results. FE analysis results showed good agreement with experimental results. However, crack patterns could not be predicted and analyses were limited to monotonic loading on beams. A similar model was used by Tedesco *et al*. (1999) to study FRPC laminate strengthened concrete bridges. Another study was performed by Kachlakev *et al.* (1998), to simulate behaviour of RC beams strengthened with FRPC laminate for the Horsetail Creek Bridge using ANSYS commercial code and by assuming perfect bond between concrete and FRPC laminate. Only material nonlinearity was considered. Recently, Pesic and Pilakoutas (2005) developed a numerical model for computation of bending moment capacity and prediction of the flexural failure modes of RC beams externally bonded with FRPC laminates.

A damage plasticity model has been presented here for concrete and relevant constitute models for steel and FRPCs. Use of this particular plasticity model for concrete has many advantages. It accounts for stress history dependent behaviour, unloading-loading, stiffness degradation and recovery, and thus cyclic loading can also be modelled. Detailed 3D FE analyses of strengthened RC beams have been presented in the present work by combining geometric and material nonlinearities. The commercial code ABAQUS (v.6.4.2) is used for nonlinear analysis. Proper material constitutive models for concrete, steel and FRPC laminates are proposed and verified against available experimental data reported by Ramana *et al.* (2000) and Rahimi and Hutchinson (2001). A parametric study is also performed to study effect of laminate thickness, area of FRPC, etc. The main aim of the present study is to validate available experimental results and to provide valuable guidelines for future laboratory investigations.

2. Material models and their properties

Selection of a proper material model is a very important step in a FE analysis. Materials used in the present numerical studies are concrete, steel and FRPC laminate. Constitutive models of these materials are discussed below in brief.

2.1 Concrete

Concrete is a quasi-brittle material showing a highly non-linear behaviour due to inelastic phenomena like concrete plasticity, cracking of concrete, nonlinear multiaxial material properties, etc. Failure of concrete may be tensile or compressive and characterized as ductile or brittle. Modelling of concrete is quite difficult because of such uncertain behaviour, non-homogenity and anisotropic nature.

Development of cracks is a major source of material nonlinearity in concrete. Concrete models must be able to represent initiation and propagation of cracks. Smeared crack model (Rashid 1968) and damaged plasticity model (Lubliner *et al.* 1989) have been widely used to represent behavior of concrete. Concrete has been assumed to be homogenous in both these models and cracks are uniformly distributed over a finite area surrounding the Gauss point with the assumption of the cracked solid being continuum. Unlike smeared crack model, the damaged plasticity model can also be used for cyclic loading in addition to monotonic loading and has capability to take into account stiffness degradation associated with each failure mode as well as stiffness recovery effect during load reversal. Thus, the damaged plasticity model is used in the present study.

The damaged plasticity model available in ABAQUS uses a concept of isotropic damaged elasticity in combination with isotropic tensile and compressive plasticity to represent the inelastic behaviour of concrete. The model adopts non-associated plastic flow to obtain proper control of the dilatancy. The Drucker-Prager hyperbolic function has been used for flow potential and evaluation

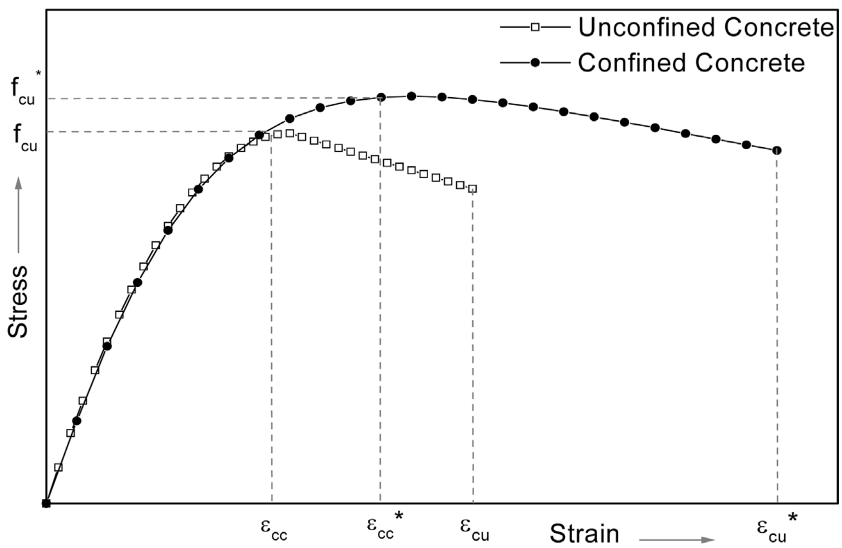


Fig. 1 Uniaxial compressive stress-strain curve for confined and unconfined concrete

Table 1 Properties of concrete and reinforcement

	Parameters	Reference			
		Ramana *et al.* (2000)	Rahimi and Hutchinson (2001)		Parametric study
Confined concrete	Compressive yield stress, f^*_{cu} (MPa)	34.0	52.5	55.0	34.0
	Young's modulus, E_c (MPa)	27397	35200	36612	27397
	Cracking strain, ε^*_{cc}	0.0022	0.0025	0.0026	0.0022
	Dilation angle ϕ (Degree)	15	15	15	15
	Flow eccentricity e	0.10	0.10	0.10	0.10
	Ultimate tensile stress, σ_t (MPa)	3.0	3.0	3.0	3.0
	Ultimate tensile strain, ε^f_t	0.00213	0.00227	0.00227	0.00213
	Poisson's ratio υ_c	0.15	0.20	0.20	0.15
Reinforce-ment	Young's modulus, E_s (MPa)	2.06E5	2.1E5	2.1E5	2.1E5
	Yield stress, f_y (MPa)	271	575	575	415
	Poisson's ratio υ_s	0.30	0.30	0.30	0.30

of the yield surface is determined through hardening variables which are linked to the degradation mechanism under tensile and compressive stress conditions. For details of this model, the reader is referred the ABAQUS manuals (Hibbitt and Sorensen 2003).

Modelling of shear reinforcement in concrete is somewhat difficult and time consuming. Shear reinforcement helps to confine concrete. As a result, the ultimate compressive stress increases with considerable increase in the ultimate compressive strain. Failure mode is also affected by shear reinforcement. To take into account contribution of closely spaced stirrups, Mander *et al.* (1988) approach is used to develop the confined concrete stress-strain curve as input data in the modelling. The uniaxial compressive stress-strain curves for unconfined and confined concrete are presented in Fig. 1. The confined compressive stress f^*_{cu}, compressive strain ε^*_{cc} at f^*_{cu} and parameters required for the damaged plasticity model including flow eccentricity, dilation angle, etc. are reported in Table 1. The ultimate compressive strain ε^*_{cu} is assumed to be 0.006 in the present study.

2.2 Steel

Steel in the FE analysis is assumed to be elastic-perfectly plastic and its behaviour is considered to be identical in tension and compression. Poisson's ratio of 0.3 is used for steel reinforcement. Details of reinforcement grade and the yield point used in the present study are presented in Table 1.

2.3 FRPC laminate

In the experimental studies considered, unidirectional laminates are used having different properties in the three mutually orthogonal planes. Thus, linear elastic-orthotropic material properties are used in the present study. Cross-sectional properties of FRPCs are adopted from Liu *et al.* (1996) and are reported in Table 2.

Table 2 Properties of FRPC laminates used in modelling

Reference	Type of fibre	FRPC laminates: Modulus of elasticity along fibre and cross-fibre direction			Poisson's ratio along fibre and cross-fibre direction			Shear modulus along fibre and cross-fibre direction		
		E_{11}	E_{22}	E_{33}	ν_{12}	ν_{13}	ν_{23}	G_{12}	G_{13}	G_{23}
Ramana *et al.* (2000)	CFRP	1.32E5	9255*	9255*	0.3340*	0.3340*	0.4862*	4795*	4795*	3113.65**
Rahimi and Hutchinson (2001)	CFRP	1.27E5	9255*	9255*	0.3000	0.3000	0.4862*	4795*	4795*	3113.65**
	GFRP	36.0E5	9367*	9367*	0.3000	0.3000	0.5071*	3414*	3414*	3107.63**
Parametric study	CFRP	1.32E5	9255*	9255*	0.3340*	0.3340*	0.4862*	4795*	4795*	3113.65**

*From Liu *et al.* (1996)

$$^{**}G_{23} = \frac{E_{22} \text{ or } E_{33}}{2(1+\nu_{13})}$$

3. Finite element modeling

Material model used in the present study is described in Section 2. Along with the material model, selection of element type is also an important consideration in FE analysis. Parametric study is performed on RC beams with linear and quadratic elements with a view to assess their efficiency and behaviour prediction. Responses of linear and quadratic elements are compared in Fig. 2 in the form of a load-deflection plot. It is observed that both element responses are identical and the ultimate load values with linear and quadratic elements are 9.01 kN and 8.98 kN, respectively which are almost same. Therefore, a linear element is adopted in the FE modeling to avoid an increase in number of degrees of freedom in the overall system. Concrete and FRPC laminate are

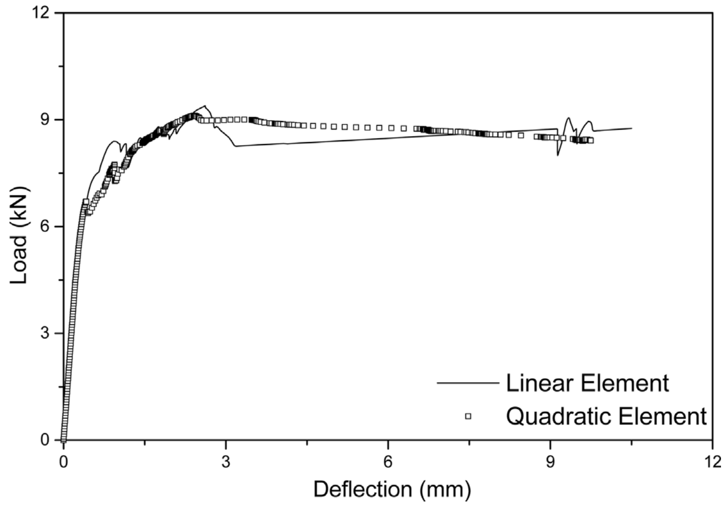
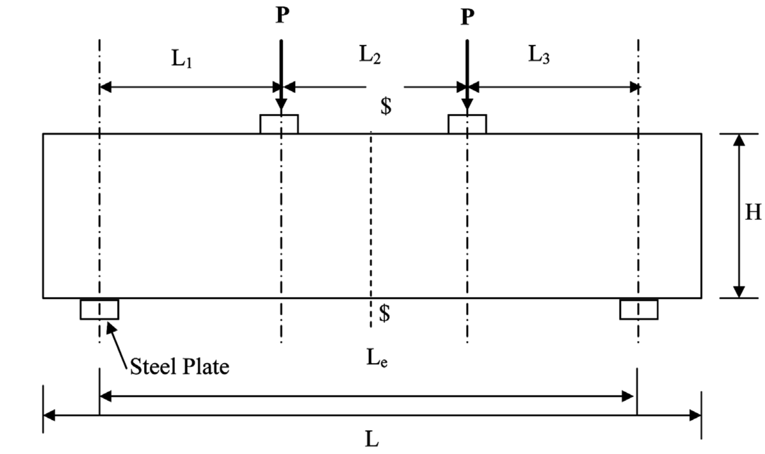
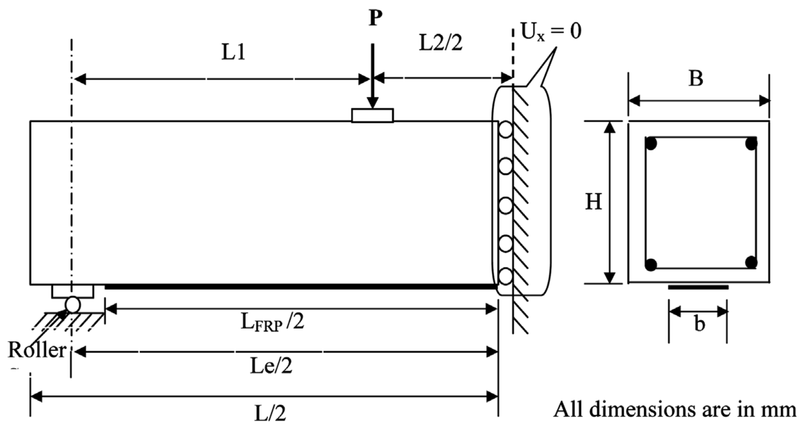


Fig. 2 Comparison between linear and quadratic elements



(a)



(b)

Fig. 3 Details of RC beam with FRPC

modelled with 8-node tri-linear brick elements having three degrees of freedom per node. 2-node truss elements having two degrees of freedom per node are used to model steel reinforcement.

The concrete model has crushing and cracking capabilities. If a load is applied at a single node, there are possibilities of localized failure. To avoid such localized failure and for a uniform distribution of load, a very small size steel plate (50-100 mm width and 2.5 mm thick) is fixed (as shown in Fig. 3) at the support and at the loading locations. This plate is modeled with a 8-node tri-linear brick element.

Due to symmetry, only half of the beam is discretized to reduce computation cost/time. A schematic diagram of the model with boundary conditions is shown in Fig. 3. Details of dimensions of the concrete beam, main reinforcement, stirrup spacing, thickness, width of laminate and distance of load points from the support are presented in Tables 3 and 4.

Table 3 Details of strengthened beams

Reference	Concrete beam				Main reinforcement			Stirrups		FRPC laminates			Designa-tion
	B mm	*H* mm	*L* mm	L_e mm	Top *r/f*	Bottom *r/f*	Cover mm	Dia. mm	Spacing mm	*b* mm	*t* mm	L_L Mm	
Ramana *et al.* (2000)	100	100	1000	900	2-6ϕ	2-6ϕ	10	6	70	10	1.50	800	RCFRP1
	100	100	1000	900	2-6ϕ	2-6ϕ	10	6	70	20	1.50	800	RCFRP2
	100	100	1000	900	2-6ϕ	2-6ϕ	10	6	70	40	1.50	800	RCFRP3
Rahimi and Hutchinson (2001)	200	150	2300	2100	2-8ϕ	2-10ϕ	30	6	150	150	0.80	2000	RCFRP4
	200	150	2300	2100	2-8ϕ	2-10ϕ	30	6	150	150	1.20	2000	RCFRP5
	200	150	2300	2100	2-8ϕ	2-10ϕ	30	6	75	150	0.4	2000	RCFRP6
	200	150	2300	2100	2-8ϕ	2-10ϕ	30	6	75	150	1.20	2000	RCFRP7
	200	150	2300	2100	2-8ϕ	2-10ϕ	30	6	75	150	1.80	2000	RCFRP8
	200	150	2300	2100	2-8ϕ	2-16ϕ	30	6	75	150	0.40	2000	RCFRP9
	200	150	2300	2100	2-8ϕ	2-16ϕ	30	6	75	150	1.20	2000	RCFRP10
	200	150	2300	2100	2-8ϕ	2-16ϕ	30	6	75	150	1.80	2000	RCFRP11
Parametric study	100	100	1000	900	2-6ϕ	2-6ϕ	10	6	70	30	1.50	800	RCFRP12
	100	100	1000	900	2-6ϕ	2-6ϕ	10	6	70	70	1.50	800	RCFRP13
	100	100	1000	900	2-6ϕ	2-6ϕ	10	6	70	80	1.50	800	RCFRP14
	100	100	1000	900	2-6ϕ	2-6ϕ	10	6	70	100	1.50	800	RCFRP15
	100	100	1000	900	2-6ϕ	2-6ϕ	10	6	70	20	3.00	800	RCFRP16
	100	100	1000	900	2-6ϕ	2-6ϕ	10	6	70	40	3.00	800	RCFRP17
	100	100	1000	900	2-6ϕ	2-6ϕ	10	6	70	60	1.00	800	RCFRP18

Table 4 Details of loading and support conditions (Refer Fig. 3)

Parameters	Reference		
	Ramana *et al.* (2000)	Rahimi and Hutchinson (2001)	Parametric study
Loading Point	Four Point Bending	Four Point Bending	Four Point Bending
L_1 (mm)	300	750	300
L_2 (mm)	300	600	300
L_3 (mm)	300	750	300
Support Condition	Simply Supported	Simply Supported	Simply Supported

3.1 Interaction between concrete-steel reinforcement and tension stiffening

In the present model, reinforcement is modelled with the embedded element approach. Advantages of this particular approach are that location of reinforcement and orientation can be controlled easily and there is no limitation on FE mesh generation. This is considered to be an effective and realistic way of modelling reinforcement in concrete. Effects associated with reinforcement and the concrete interface are modelled approximately by introducing some tension stiffening into the concrete model to simulate the load transfer across cracks through reinforcement.

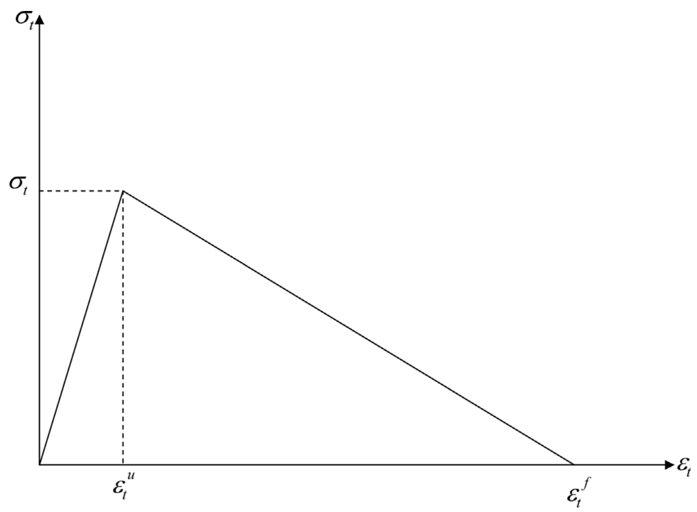


Fig. 4 Tensile stress-strain relation (Tension stiffening)

Tension stiffening also helps to model the post-failure behaviour of concrete by means of the post-failure stress-strain relation and fracture energy criterion. A linear tensile stress-strain curve is assumed as shown in Fig. 4. Bazant and Oh (1983) introduced the crack band theory as one of the simplest types of fictitious crack models. The same model is adopted here for determining the tensile strain (ε_t^f) at which tensile stress, Eq. (1) is equal to zero. On the other hand, the fracture energy required to open a unit area of crack is determined with the help of Eq. (2).

$$\varepsilon_t^f = \frac{2G_f}{b\sigma_t} \tag{1}$$

$$G_f = (2.72 + 0.0214\sigma_t)\frac{\sigma_t^2 D_a}{E_c} \tag{2}$$

Here in the above empirical equations,

G_f : Fracture energy of concrete/area of crack (pound per inch)
σ_t : Uniaxial tensile stress (psi)
b : Element width (inch)
D_a : Maximum diameter of aggregates (inch)

3.2 Interaction between concrete-FRPC laminate

Perfect bond is assumed between concrete and the FRPC laminate. Layer of glue is not modelled in the present model. A surface contact element is used to model contact between concrete and FRPC. Hard contact and rough friction are assigned for contact element to ensure perfect bond. Assumption of hard contact (Fig. 5) ensures transmission of full pressure between the contact pair. On the other hand rough friction keeps the surface closed during analysis.

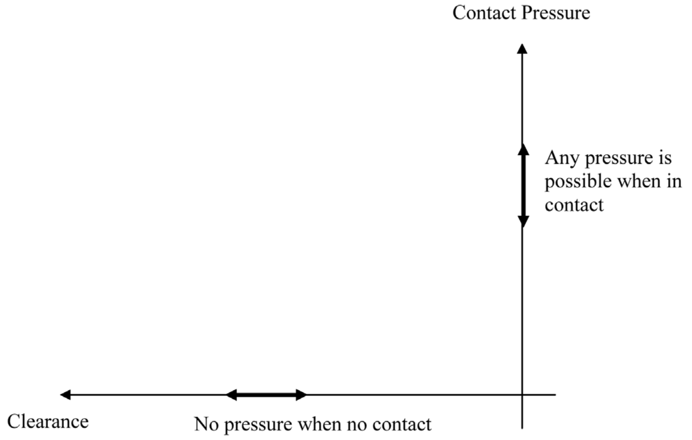


Fig. 5 Hard-contact relationships (Hibbitt and Sorensen 2003)

3.3 Finite element mesh discretization

A very important step in FE analysis is selection of optimum mesh density. A convergence of result is obtained when an adequate number of elements are used in the model. This is practically achieved when an increase in mesh density has a negligible effect on results. Hence, it is necessary to perform a convergence study to fix the mesh density.

It is well documented in literature that FE analysis of concrete is highly mesh sensitive. Crack band theory developed by Bazant and Oh (1983) can be used advantageously to reduce mesh sensitivity of results. In concrete structures, very fine mesh creates numerical convergence problems when concrete starts to crack and the problem becomes unstable because of narrower crack band width. On the other hand, a coarse mesh used in analysis results in overestimation of the ultimate load and also crack patterns are not correctly predicted.

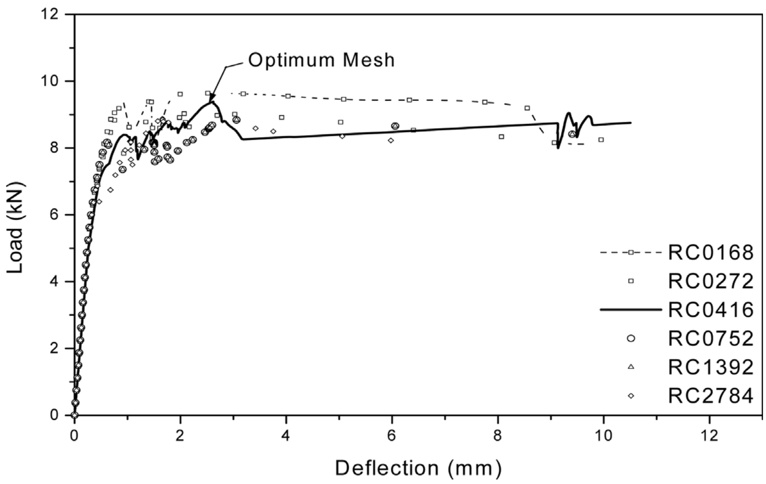


Fig. 6 Load-deflection plot for mesh convergence study (Ramana *et al.* 2000)

Table 5 Convergence study of virgin beam (Ramana *et al.* 2000)

Designation	Number of elements	Ultimate load (kN)
RC0168	168	9.64
RC0272	272	9.06
[1]RC0416	416	9.01
RC0752	752	8.85
RC1392	1392	8.84
RC2784	2784	8.82

[1]Mesh used in present study

In crack band theory, the ultimate failure tensile strain is based on element size and fracture energy, which depends on aggregate size Eqs. (1) and (2). In the present study, the same theory is used to determine the ultimate tensile failure strain. A convergence study is performed on the reinforced concrete beam to fix the element size for all examples. For brevity, results of convergence study only for one case are reported in the paper. Load-deflection plots and the predicted ultimate loads with number of elements are presented in Fig. 6 and Table 5, respectively. Based on the above, all numerical computations with the experimental results of Ramana *et al.* (2000) have been performed with a mesh density of 416 in this study.

4. Simulation of structural response for strengthened beams

The non-linear FE analysis results are compared with experimental results in the form of a load-deflection plot at mid-span; load-strain plots for concrete and laminate at mid-span; ultimate load; deflection at the ultimate load; and cracking pattern at failure. Effects of different types of fibre, cross-sectional area of FRPC by changing the width and thickness of laminate, quantity of main reinforcement and spacing of stirrups are all investigated. The main focus has been to predict increase in strength and stiffness of RC beams provided by bonded FRPC on the tension side.

4.1 Ultimate load and deflection

It can be observed from the experimental results (Ramana *et al.* 2000, Rahimi and Hutchinson 2001) as well as the FE analysis performed here that all strengthened beams performed significantly better in terms of strength and stiffness than the virgin beam. The ultimate load and deflection at the ultimate load observed in FE analysis and experiments are presented in Table 6. It is seen from Table 6 and Figs. 7-8 that nonlinear FE analysis results show good agreement with experimental results. After reaching the ultimate load value, numerical as well as experimental load-deflection plots drop down suddenly, indicating failure of the strengthened beams. In the elastic range, the FE model showed higher stiffness than experiment. This is perhaps due to the assumption of perfect bond between concrete-steel and concrete-FRPC laminates. Also, the presence of micro cracks from shrinkage and creep are not incorporated in the FE analysis. However, the predicted ultimate loads from FE analysis are within ±10-20% of experimental results. Load-deflection trends are also similar to experimental plots. In general, increase in the ultimate loads in experiments as well as in

Table 6 Comparison of ultimate load and deflection at ultimate load

Reference	Beam designation	Ultimate load (kN)		Deflection at ultimate load (mm)		Failure mode**
		Expt.	FE analysis	Expt.	FE analysis	
Ramana *et al.* (2000)	RCFRP1	21.20	17.50	8.40	11.83	S/P
	RCFRP2	23.20	21.64	7.60	8.17	S/P
	RCFRP3	28.00	26.10	6.25	5.04	S/P
Rahimi and Hutchinson (2001)	RCFRP4	62.55	55.70	31.65	23.72	S/P
	RCFRP5	65.00	70.10	25.50	34.34	S/P
	RCFRP6	53.85	51.93	38.40	39.70	C/P
	RCFRP7	69.65	72.70	29.30	30.84	C/P
	RCFRP8	60.35	54.30	33.75	32.10	C/P
	RCFRP9	76.05	83.86	28.15	38.90	CC/P
	RCFRP10	102.25	94.70	31.90	30.03	CC/P
	RCFRP11	86.90	83.55	32.85	28.67	CC/P
Parametric study	RCFRP12	---[a]	24.93	---[a]	6.04	---[a]
	RCFRP13	---[a]	30.70	---[a]	4.10	---[a]
	RCFRP14	---[a]	32.40	---[a]	3.90	---[a]
	RCFRP15	---[a]	32.60	---[a]	3.20	---[a]
	RCFRP16	---[a]	28.82	---[a]	4.15	---[a]
	RCFRP17	---[a]	29.94	---[a]	3.06	---[a]
	RCFRP18	---[a]	27.55	---[a]	5.61	---[a]

---[a] Experimental results not available
**Experimental failure mode
Note: - S/P = concrete shear failure followed by plate separation
C/P = cover failure followed by plate separation
CC/P = concrete crushing and plate separation

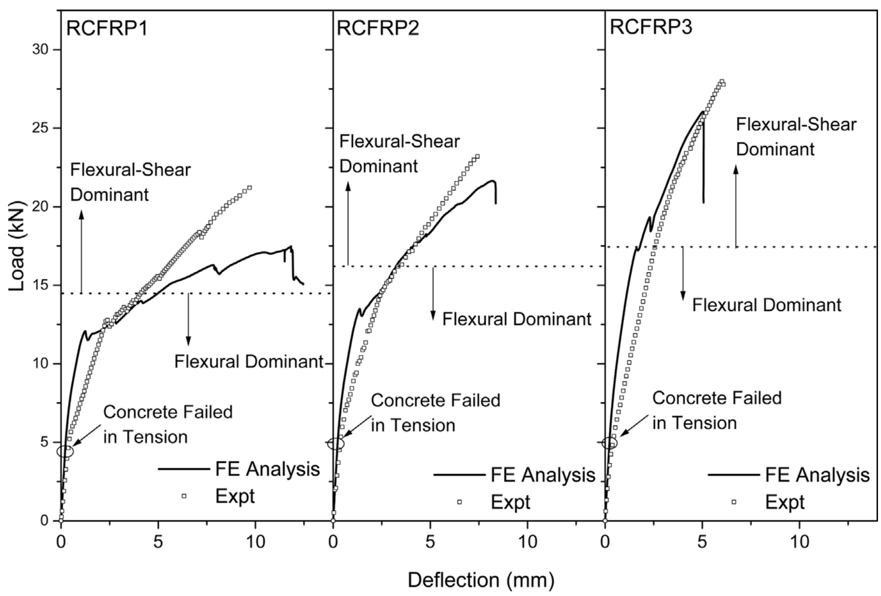


Fig 7 Load versus deflection comparison of FE analysis and experimental results (Ramana *et al.* 2000)

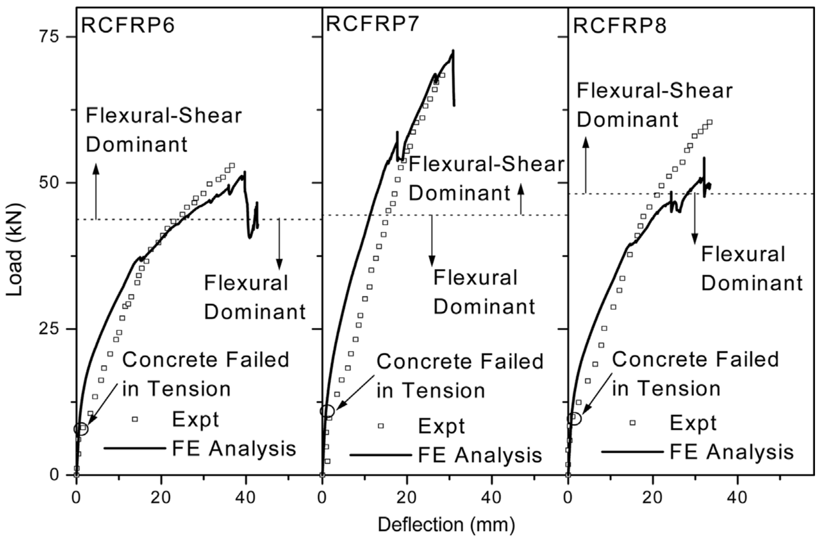


Fig 8 Load versus deflection comparison of FE analysis and experimental results (Rahimi and Hutchinson 2001)

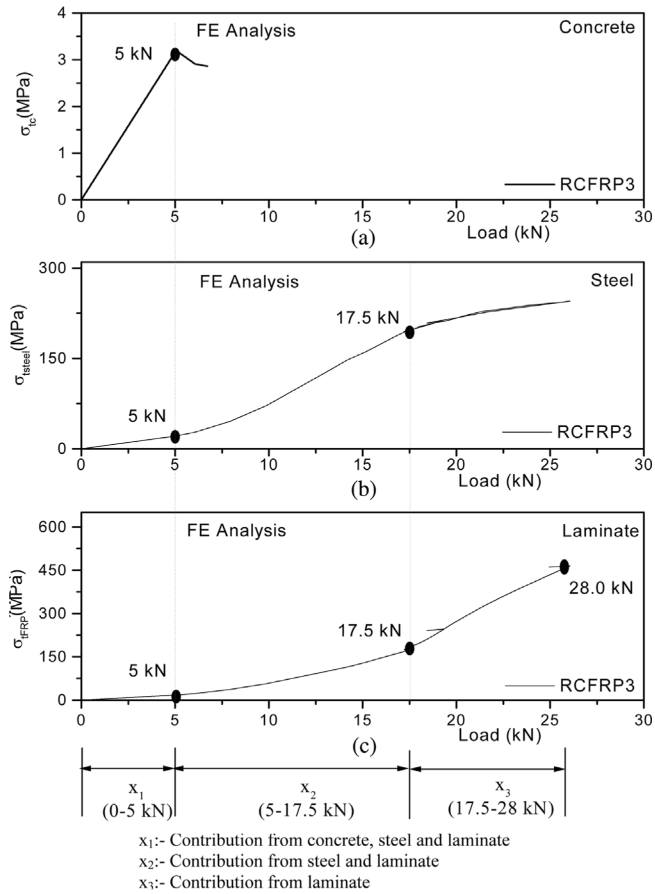


Fig. 9 Longitudinal tensile stresses in concrete (σ_{tc}), laminate (σ_{tFRP}) and steel reinforcement (σ_{tsteel}) at different load levels (RCFRP3)

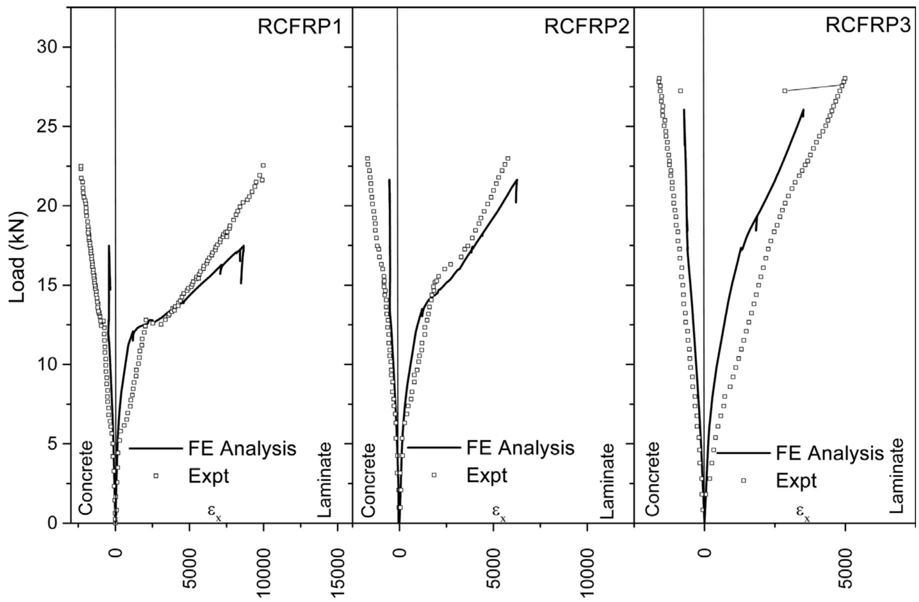


Fig 10 Load versus strain in concrete and laminate comparison of FE analysis and experimental results (Ramana *et al.* 2000)

the FE analysis can be attributed to increase in the tensile strength due to the laminate restraining effect. Contribution of concrete, steel reinforcement and FRPC laminate to load carrying capacity of strengthened beam RCFRP3 are shown in Fig. 9. At a 5 kN load, the tensile stress in concrete reaches 3 MPa (ultimate tensile stress of concrete, σ_t) and concrete starts to crack on the tension side. Further, the load sharing from steel and FRPC laminate is marginal as seen in Fig. 10(a). During the load range of 5-17.5 kN, both steel and FRPC laminate share load significantly with no contribution coming from concrete as seen in Fig. 10(b). Beyond 17.5 kN load, it is only the FRPC laminate which contributes to the load sharing since steel also begins to yield and this behaviour is clearly seen in Fig. 10(c).

4.2 Strains in concrete and laminate

Strains in concrete and laminates are measured at mid-span of beam. Load versus longitudinal strains plots for the strengthened beams, RCFRP1, RCFRP2 and RCFRP3 are presented in Fig. 10 for the experimental studies of Ramana *et al.* (2000). Stresses at the ultimate load in concrete, steel reinforcement and FRPC laminates from FE analysis are presented in Table 7.

FE analysis shows somewhat lower strains than experimental results for model marked RCFRP1. However, the values are close to experimental results for model RCFRP2 and RCFRP3.

4.3 Evaluation of failure mode

The major failure mode observed for the strengthened beams is delamination or separation of FRPC laminate from concrete surface because of failure of glue. In addition to delamination of the FRPC laminates, some other failure modes including shear or compressive failure of concrete,

Table 7 Longitudinal stresses in concrete, steel and laminate at mid-span from FEM analysis

Reference	Beam designation	Concrete top fibre stress at mid-span MPa	Steel stress at mid-span MPa	Laminate stress at mid-span MPa
Ramana *et al.* (2000)	RCFRP1	10.67	222.542	1147.220
	RCFRP2	13.32	257.417	828.087
	RCFRP3	16.57	245.211	464.444
Rahimi and Hutchinson (2001)	RCFRP4	22.50	567.292	639.780
	RCFRP5	28.37	575.032	714.228
	RCFRP6	25.72	569.048	1205.000
	RCFRP7	27.02	556.958	720.713
	RCFRP8	24.90	569.662	253.335
	RCFRP9	36.36	575.000	1108.160
	RCFRP10	37.38	542.181	493.238
	RCFRP11	35.98	575.000	226.773
Parametric study	RCFRP12	15.11	253.168	583.840
	RCFRP13	19.60	238.751	528.067
	RCFRP14	20.71	239.263	310.751
	RCFRP15	20.80	228.420	249.625
	RCFRP16	16.35	231.051	441.638
	RCFRP17	19.10	223.104	262.285
	RCFRP18	17.16	254.302	431.702

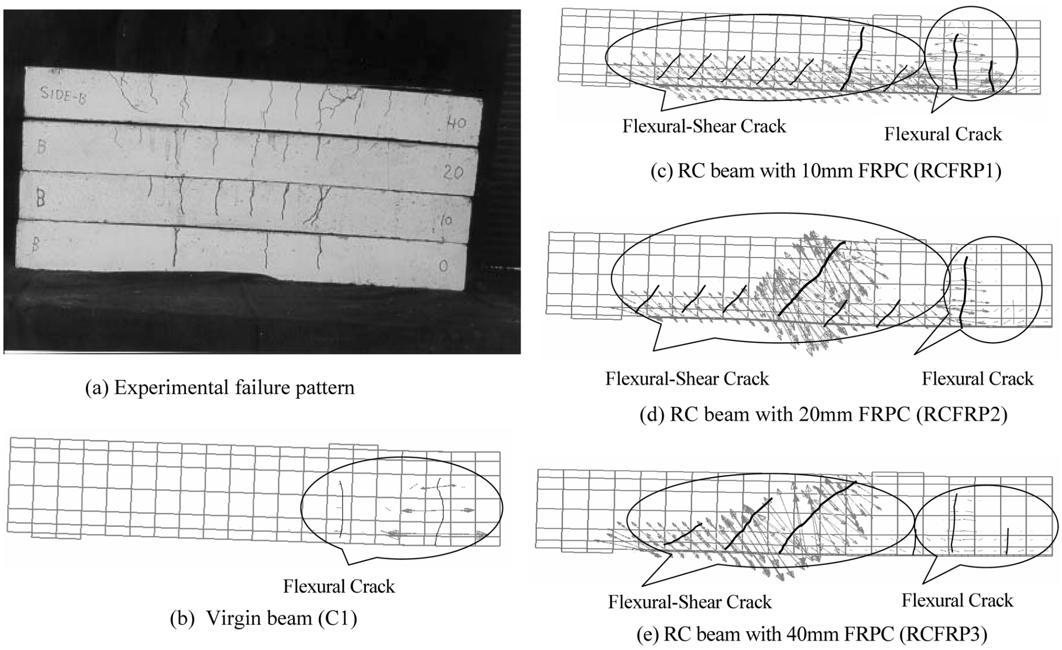


Fig. 11 Crack pattern evaluation (Ramana *et al.* 2000)

yielding of steel, FRPC rupture, debonding of concrete layer along the concrete-reinforcement interface are also observed in experiments. Crack patterns at collapse are compared in Fig. 11 from the experimental data of Ramana *et al.* (2000) and the present FE analysis. Direction of the maximum principal strains from the FE analysis and cracks perpendicular to these strain directions are clearly seen in Fig. 11. Failure mode for virgin beam is primarily in flexure due to yielding of steel reinforcement. Cracks are observed near the center of the beam (Fig. 11(a)) and the same is predicted by FE analysis (Fig. 11(b)). Composite action shifted the failure mode from flexure to peeling of the laminate due to flexure-shear cracks near the support zone. The present FE model is not capable of capturing delamination failure of the FRPC laminate. However it is observed from Fig. 11 that flexure-shear cracks developed near the end zone at the ultimate load and the concrete failed in shear leading to delamination of the FRPC laminate from the concrete surface. It is observed in all examples that cracks normally start in the vertical direction and move in an inclined direction as load increases due to combined effects of flexure and shear. Experimental failure modes for all examples are reported in Table 6. FE analysis has predicted that all strengthened beams failed due to flexure-shear failure of concrete.

4.4 General discussion based on parametric study

A parametric study is performed to examine effect of FRPC laminate widths by considering 30 mm, 70 mm, 80 mm and 100 mm widths with 1.5 mm thickness. The RC beam details were kept the same as that of Ramana *et al*. (2000) for consistent comparison. Results are presented in Fig. 12 in form of the ultimate load versus area of FRPC laminate. It is observed that the ultimate load increased with reduced deflection as cross-sectional area of the FRPC laminate increases by increasing width of laminate. However, no appreciable increase in the ultimate load is observed (Fig. 12) for laminates with width beyond 60 mm (cross-sectional area is 90 mm^2). This indicates that the tensile side of RC beam need not be strengthened over the full width. Since the concrete itself failed in shear, a further increase in flexural strengthening does not help to improve the ultimate load, although it helps to reduce deflection. In such a case, shear strengthening is also required in addition to flexural strengthening.

Another parametric study was performed to study effect of ratio of width to thickness of laminate by keeping the same cross-sectional area of FRPC laminates (Table 6 and Fig. 13). Both the

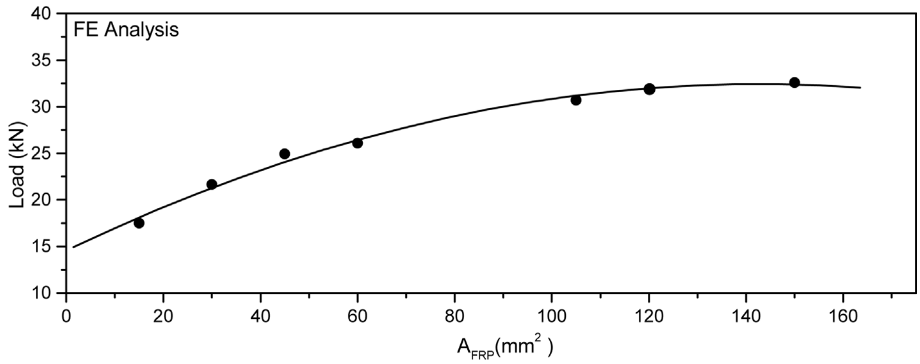


Fig. 12 Ultimate load versus area of FRPC (A_{FRP}) laminate

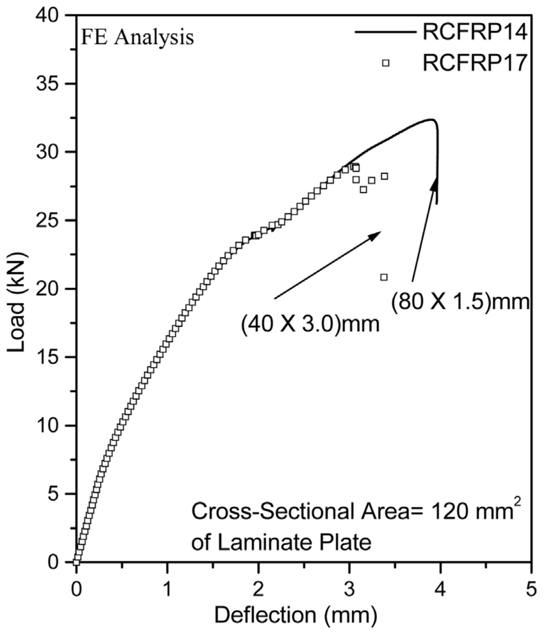


Fig. 13 Effect of ratio of width to thickness of laminate

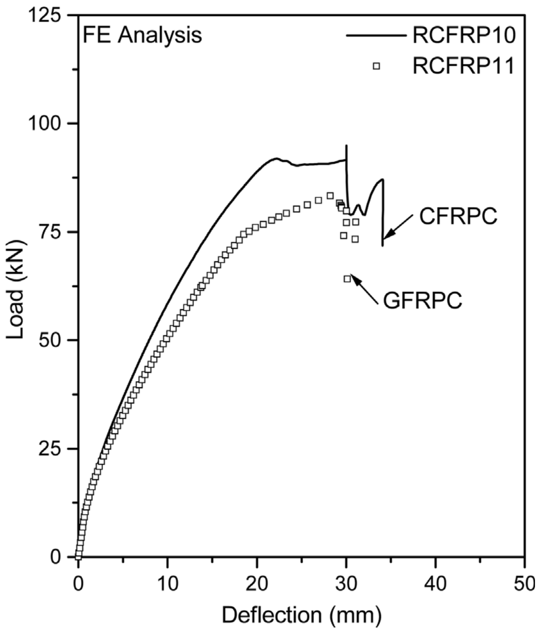


Fig. 14 Effect of different types of fibre

strengthened beams, RCFRP14 and RCFRP17, have a cross-sectional area of FRPC laminates as 120 mm^2 but dimensions are different (Table 3). It is seen that the ultimate loads and deflection at the ultimate loads for RCFRP14 and RCFRP17 are 34.40 kN, 3.90 mm and 29.94 kN, 3.06 mm respectively. Both load and ductility parameters have improved for laminates having larger width and lesser thickness. This is a very significant behaviour prediction that has come out of this numerical parametric study. It is also seen both from experimental and FE analysis that various parameters affected the performance of the strengthened system, e.g., fibre type (Fig. 14), amount of main reinforcement, area of FRPC, concrete grade, stirrup spacing, etc. Both experimental and numerical studies (Figs. 7 and 8) have confirmed that an increase in cross-sectional area of FRPC laminate delays initiation of shear cracks. The load at which the initiation of shear cracks occurs is marked by a dotted line, dividing the load-deflection plots for flexural dominant and flexural-shear dominant behaviours. In the flexural dominant region, cracks are mostly vertical, which subsequent becomes inclined in the flexural-shear dominant region, indicating a shift from a flexural mode to a flexural-shear mode of failure.

5. Conclusions

Past experimental research has shown that FRPC materials are an effective alternative to conventional methods of retrofitting. There is clearly a growing need for a numerical model to simulate response of strengthened system. A 3D nonlinear FE analysis is performed, assuming perfect bond between concrete and laminate to understand the flexural behaviour of strengthened RC beams and to validate the available experimental results.

Behaviour of the concrete from initiation of cracking to failure can be predicted by incorporating both geometric and material nonlinearities. It is important to note that, very fine mesh creates numerical convergence problems in analysis of reinforced concrete members as concrete starts cracking due to less crack band width. Therefore, a convergence study based on crack band theory and fracture energy of concrete (Bazant and Oh 1983) is required to obtain a reasonably good mesh density.

Overall behaviour from FE analysis results shows good agreement with available experimental results and the numerically simulated failure loads are within ±10-20% of experimental values. Stiffness and strength of the strengthened beams are seen to increase substantially, depending on the width and thickness of the laminates, area of FRPC laminates, spacing of stirrups, area of main reinforcement, fibre type, etc. There is a limiting point beyond which no further increase of load carrying capacity can be achieved just by increasing the area of strengthening material.

The FE model is not capable of capturing delamination of the FRPC laminates from the concrete surface. Further study on simulation of delamination failure of the strengthened system is required for accurate predictions of collapse loads.

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