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A fast construction sequential analysis strategy for tall buildings

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Abstract. In structural analysis of tall buildings the traditional primary loading analysis approach that assumes all the loads are simultaneously applied to the fully built structure has been shown to be unsuitable by many researches. The construction sequential analysis that reflects the fact of the level-by-level construction of tall buildings can provide more reliable results and has been used more and more. However, too much computational cost has prevented the construction sequential analysis from its application in CAD/CAE software for building structures, since such an approach needs to deal with systematic changing of resultant stiffness matrices following level-by-level construction. This paper firstly analyzes the characteristics of assembling and triangular factorization of the stiffness matrix in the finite element model of the construction sequential analysis, then presents a fast construction sequential analysis strategy and a corresponding step-by-step active column solver by means of improving the existing skyline solver. The new strategy avoids considerably repeated calculation by only working on the latest appended and modified part of resultant stiffness matrices in each construction level. Without any simplification, the strategy guarantees accuracy while efficiency is greatly enhanced. The numerical tests show that the proposed strategy can be implemented with high efficiency in practical engineering design.

Keywords: high performance computing; FEA; construction sequential analysis; building structures; structural analysis.

1. Introduction

The traditional structural analysis approach named primary loading analysis is based on the hypothesis that all the loads are simultaneously applied to the fully built structure. Researches show that there are usually significant differences between reality and computational results in internal forces of tall buildings under vertical loads by the primary loading analysis (Choi and Kim 1985, Choi *et al.* 1992, Prado *et al.* 2003). If the primary loading analysis is performed to a fully built reinforced concrete tall building structure, the relative vertical displacements caused by heavy vertical loads at a higher level are often quite large. This phenomenon causes sometimes tension for

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columns as well as large positive moment for beams at higher levels. However, in the construction process vertical loads are applied in the manner of level-by-level and the deformation of lower levels does not transferred to higher levels, since in the level-by-level construction process, lower levels are physically a flat foundation for an appended higher level. Therefore, tension for columns as well as large positive moment for beams do not appear in construction reality. On the other hand, the construction sequential analysis is an approach that reflects the level-by-level construction. It can remedy the demerits of primary loading analysis model and provide more reliable results, especially in tall building analysis. Hence this approach recently is becoming more and more popular in CAD/CAE software packages (ETABS 2002, GT STRUDL 2005). In the Chinese technical specification for concrete structures of tall building (JBJ3-2002 2002), such an approach is recommended. In addition, ACI 318 also recommends to consider construction loads (2005).

However since such an approach needs to solve a number of linear systems coresponding to the construction process, a great deal of computational cost is involved. If installing and removal of shores and reshores in construction stage need to be considered, the numerical efforts are huge and such a task can be hardly fulfilled for large scale tall buildings. Therefore despite its accuracy the efficiency of the construction sequential analysis by availiable solution procedures is not completely adequate. Some researches discussed the improvement in efficiency of the construction sequential analysis, but few of them have made much progress. In order to put the construction sequential analysis for large scale tall buildings into design practice with an acceptable computational effort, several simplified models or approaches based on empirical studies were proposed, for example, in (Choi *et al.* 1992, JBJ3-2002 2002).

In order to implement the construction sequential analysis accurately for large scale buildings within acceptable time cost, this paper firstly carefully re-examines the procedure of the active column solution in FEA and reports a speed-up strategy with loop-unrolling. After features of assembling and triangular factorization of resultant stiffness matrices in the construction sequential analysis are given, it is found that a sequential feature of matrix operations coresponding to the construction process can be utilized to raise the efficiency of the solution process for the construction sequential analysis. Based on such a feature this paper presents a fast strategy by means of improving the existing active column solver. The new strategy avoids tremendous repeated calculation by a deliberate arrangement of assembling and triangular factorization for the resultant step stiffness matrices. The proposed strategy does not require any simplified approximation or empirical assumption, so that accuracy is guaranteed while the solution efficiency is greatly raised. The numerical tests show that the proposed strategy can be implemented with high efficiency in practical engineering design.

This paper is organized as follows. The computational cost of FEM model for the construction sequential analysis is first discussed in section 2. Section 3 reviews the existing active column solver and gives its enhancement with loop-unrolling. Based on the features of assembling and triangular factorization of the stiffness matrices in corresponding finite element model of the construction sequential analysis, a fast construction sequential analysis strategy is proposed in section 4. Section 5 presents the corresponding step-by-step active column solver. Some testing results and discussions are given in section 6. The paper ends with a brief conclusion in section 7.

2. Cost analysis of the finite element model

Fig. 1 outlines the scheme of the sequential analysis approach. In such an approach the structural

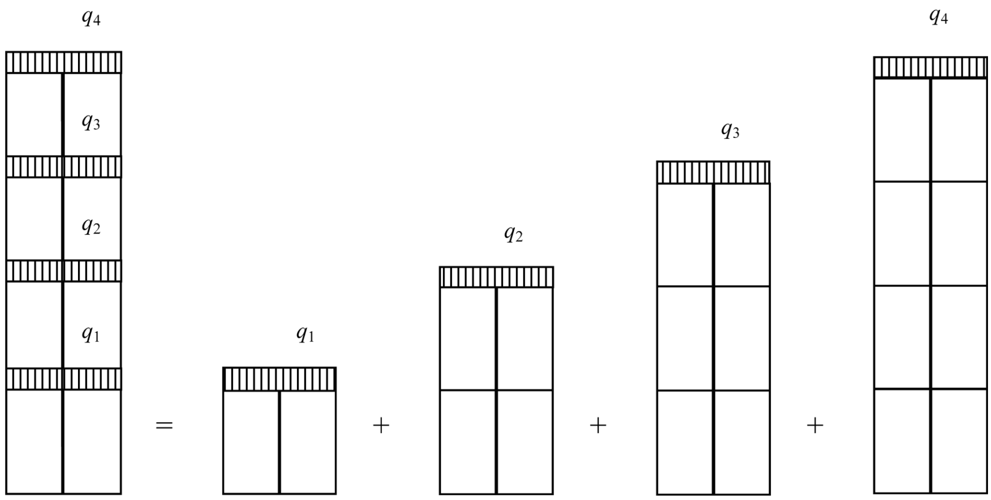

Fig. 1 Scheme of sequential analysis approach

stiffness matrices and load vectors of the partially constructed parts will be formed in the sequence of level-by-level construction. After the rth level is completely constructed, the system of linear equations

$$\mathbf{K}_r u_r = p_r \tag{1}$$

should be solved. Here $\mathbf{K}_r$ is the resultant stiffness matrix of the structure from the 1st to the rth level. The right-hand-side vector p_r is the corresponding load vector of the rth level including the self-weight and construction auxiliary loads. The unknown u_r is the nodal displacement vector from the 1st to the rth level accordingly. During the calculation the structural members below the rth level are regarded as weightless and unloaded base. With the development of construction, resultant stiffness matrices and load vectors vary step-by-step. After the structure is fully constructed the stress and displacement vector of the completed structure will be the superposition of corresponding items in each calculating step.

Following the scheme described above it is easy to find the complexity of the time cost. Suppose that the time consumed in the primary loading analysis of an m-story building is T. When the construction sequential analysis of this building is performed, there are m steps corresponding to the construction of m levels. Denote the average time cost for one level analysis as t_0 and the construction sequential analysis as T_m. Let t_k be the time required for the kth step in this process, and $T = t_m$, the time cost for the whole building structure analysis. Approximately $t_k = k\, t_0$. For each step we rebuild the resultant stiffness matrix for all existing levels and then solve linear system (1) through $\mathbf{LDL}^T$ triangular factorization. Therefore

$$T_m = \sum_{k=1}^{m} t_k \approx \sum_{k=1}^{m} k t_0 \approx T(m+1)/2 \tag{2}$$

As a matter of fact the construction sequential analysis is very time consuming, especially for the tall buildings with a large number of levels m.

If installing and removal of shores and reshores in construction stage are considered, the number of steps to be calculated for the construction sequential analysis is much larger than m (typically more than $2m$) for an m-story building (Prado *et al.* 2003). Thus the numerical effort increases substantially and the estimation of time cost is far beyond the expression T_m in Eq. (2).

This fact just blocked the practical application of the construction sequential analysis. In the past years researchers had proposed several simplified models (Choi *et al.* 1992, Prado *et al.* 2003, JBJ3-2002 2002) for the construction sequential analysis. However these simplified models definitely reduced the reliability and rationality of the results and the requirement of analysis accuracy cannot be satisfied.

3. The existing active column solver and its enhancement

3.1 Basic considerations

A fast and precise construction sequential analysis strategy is of great value in civil engineering design. Major design software packages for building structures, such as ETABS† (2002) and GT STRUD‡ (2005), can perform this analysis in different ways. It is obvious that the efficiency of the construction sequential analysis can be enhanced effectively by increasing the solution speed of Eq. (1) in each step. Nowadays, computer hardware has been well developed, so do the high performance solvers for the linear system with symmetric positive definite coefficient matrices. One of them is the Cell Sparse Fast Direct Solver presented by Chen *et al.* (2003, 2005). It is about ten to fifty times faster than traditional active column solvers (Bathe 1996, Fellipa 1975) and makes the construction sequential analysis feasible in engineering applications (Nie *et al.* 2006). Nevertheless if the improvement of speed purely depends on enhancements of linear system solvers, the analysis of very tall buildings, say m = 60, is still quite time consuming because that the huge amount of computational effort is required by 60 steps. Hence improvement of the solution strategy for the construction sequential analysis is still necessary.

Nowadays, the direct solution technique is the dominant approach in solution of linear system of equations in FEA. It is well known that the process of direct linear equation solving includes three phases, i.e., $\mathbf{LDL}^T$ triangular factorization, forward reduction and back substitution. Among them $\mathbf{LDL}^T$ triangular factorization costs the most of CPU time. In the primary loading analysis, a sparse solver runs much faster than a traditional active column solver (Bathe 1996, Fellipa 1975). However it is a different situation in the construction sequential analysis with m steps. Since any sparse solver requires an optimization to minimize fill-in's, a complete $\mathbf{LDL}^T$ triangular factorization of the resultant stiffness matrix $\mathbf{K}_r$ ($r = 1, 2,\ldots, m$) for each step is unavoidable. In contrast by means of arranging calculation strategy in the active column solver elaborately, what we need to do is just a partial triangular factorization in each construction step and most of the repeated operations for the triangular factorization associated with the previous steps can be avoided. This concept leads to the fast algorithm proposed in this paper for the construction sequential analysis.

†ETABS is a trademark of Computers and Structures, Inc.
‡GT STRUDL is a registered service mark of the Georgia Tech Research Corporation, Atlanta, Georgia, USA

3.2 Review of active column solver

In order to keep the completeness of the solution method that we proposed we firstly review the well-known active column solver as described in (Bathe 1996, Fellipa 1975, Wilson and Dovey 1978) and its enhancement with loop-unrolling in finite element analysis (Nguyen *et al.* 1997, Zheng and Chang 1995).

It is well-known that the resultant stiffness matrix of the element assemblage is not only symmetric positive definite but also banded in traditional FEA if the equations are properly reordered through certain algorithms, such as Reversed Cuthill-Mckee (RCM), Minimum Front, or Gipspoole-Stockmyer (Cuthill and McKee 1969, Gibbs and Stockmeyer 1976, Hoit and Wilson 1983). The fact that all nonzero entries are clustered around the diagonal of the resultant system matrices greatly reduces the total number of operations and the core storage requirement in the solution process. However an equation numbering for tall building analysis is physically plausible without activating any reorder algorithm, i.e., equation numbering from bottom to top. Basic matrix algebra indicates that for any symmetric positive definite matrix, there is a unique triangular factorization $\mathbf{LDL}^T$, in which $\mathbf{L}$ is a unit lower triangular matrix and $\mathbf{D}$ is a diagonal matrix. Mathematically, there are several approaches to calculate $\mathbf{L}$ and $\mathbf{D}$. In FEA the active column solution procedure based on the skyline storage scheme is one of the most favorite approaches.

Assume that a specific nodal point numbering for a given finite element assemblage has been determined and the corresponding height of each column in the stiffness $\mathbf{K}$ has been calculated. Inductively, we can write the $\mathbf{LDL}^T$ triangular factorization of a $n \times n$ matrix $\mathbf{K}$ as

$$\mathbf{K}_s = \begin{pmatrix} \mathbf{K}_{s-1} & \alpha \\ \alpha^T & k_{ss} \end{pmatrix} = \begin{pmatrix} \mathbf{L}_{s-1}\mathbf{D}_{s-1}\mathbf{L}_{s-1}^T & \alpha \\ \alpha^T & k_{ss} \end{pmatrix} = \begin{pmatrix} \mathbf{L}_{s-1} & \\ \lambda^T & 1 \end{pmatrix} \begin{pmatrix} \mathbf{D}_{s-1} & \\ & d_s \end{pmatrix} \begin{pmatrix} \mathbf{L}_{s-1}^T & \lambda \\ & 1 \end{pmatrix} \quad (3)$$

for $s = 1, 2, \ldots, n$, where $\mathbf{K}_s$ is the leading principal sub-matrix of $\mathbf{K}$ and

$$\begin{aligned} \mathbf{L}_{s-1}\mathbf{D}_{s-1}\lambda &= \alpha \\ \lambda^T\mathbf{D}_{s-1}\lambda + d_s &= k_{ss} \end{aligned} \quad (4)$$

Since the banded feature of the element assemblage $\mathbf{K}$, the factor $\mathbf{L}$ is also banded. Thus, the calculation of nonzero entries of the vector λ starts at the first nonzero of column s. Related to the fast construction sequential analysis, it is to emphasize here that change of α does not affect the factorization of the leading principal submatrix $\mathbf{K}_{s-1}$. In addition, since $\mathbf{K}$ is generally banded, the solution of $\mathbf{L}_{s-1}(\mathbf{D}_{s-1}\lambda) = \alpha$ in (4) is only necessary to consider a few of latest equations from the first nonzero of α to the diagonal in implementation.

Conventionally, one dimensional storage scheme has been used to store the resultant stiffness of the skyline format, in which the upper parts of $\mathbf{K}$ as well as $\mathbf{D}/\mathbf{L}^T$ are treated as a combination of ordinal column sets, each column consists of entries from the skyline down to its diagonal. The capacity of the in-core memory cannot be evaluated adequately in priori for the global stiffness matrix. Appropriate partition can be employed to extend the capacity, i.e., splitting the matrix and its factor into blocks of a similar size (Fellipa 1975, Wilson and Dovey 1978), which must be stored on backup storage and loaded into core memory in an effective manner. In skyline storage scheme the original matrix $\mathbf{K}$ and its factor $\mathbf{L}^T$ require the same partition, since nonzero entries of the factor do not exceed the skyline. In this paper, the out-of-core treatment proposed by (Wilson and Dovey 1978) is used.

3.3 Enhancement of active column solver by loop-unrolling

The active column solver can be enhanced by so-called loop-unrolling (Zhang 1998). Conventionally, we can perform the matrix operation C = C + AB as

Algorithm 1: Simple IJK multiplication

```
FOR I = 1:N
    FOR J = 1:N
        FOR K = 1:N
            C(I,J) = C(I,J) + A(I,K)*B(K,J)
        END
    END
END
```

In order to make use of caches, the I- and J-loops can be unrolled with, for example, depth 2 as

Algorithm 2: IJK multiplication with two-way loop-unrolling

```
FOR I = 1:N:2
    FOR J = 1:N:2
        FOR K = 1:N
            C(I,J) = C(I,J) + A(I,K)*B(K,J)
            C(I+1,J) = C(I+1,J) + A(I+1,K)*B(K,J)
            C(I,J+1) = C(I,J+1) + A(I,K)*B(K,J+1)
            C(I+1,J+1) = C(I+1,J+1) + A(I+1,K)*B(K,J+1)
        END
    END
END
```

The concept of loop-unrolling is to increase operations in each loop and thus to decrease count of loops. The loop-unrolling brought a speedup of 200% to 2700% on different type of machines (Zhang 1998). The unrolling depth, which is 2 in Algorithm 2, can be optimized on each individual machine. In fact many features of modern computer architectures such as memory caching and instruction-level parallelism are widely used to improve the computer's performance. Loop-unrolling in this study is well suited to take advantage of such features. Loop-unrolling enables compilers to reduce the overhead of variable indexing and thus to improve the performance of a code.

In the triangular factorization with the skyline storage scheme, loop-unrolling requires the unrolled columns have the same skyline heights. This is however not rare in engineering finite element analysis since the degrees of freedom belonging to a node are always in successive numbering and thereby the corresponding columns in the global stiffness matrix share the same skyline height. Loop-unrolling can greatly enhance the efficiency of $\mathbf{LDL}^T$ factorization with skyline storage schemes (Nguyen *et al.* 1997, Zheng and Chang 1995, Nguyen 2001). Our own test showed a speed-up of about 200% on Pentium based machines. More details about loop-unrolling enhancement can be found in (Chen and Sun 2005).

4. The fast strategy for the construction sequential analysis

From the solution procedure of the corresponding finite element model for the construction sequential analysis one can find tremendous repeated calculations existed. If these repeated calculations cannot be reduced as much as possible, the substantial enhancement in efficiency of triangular factorization for each step cannot be achieved. In the construction the members of the latest constructed rth level in the building only intersect with the $(r-1)$th level. In other word, the element assemblage regarding to levels from 1 to $r-2$ does not change when the rth level is appended to the structure. Unfortunately, this very important feature was neglected in most of conventional construction sequential analysis strategies. Making use of this feature we propose a fast strategy and a 'step-by-step active column solver' for construction sequential analysis in this paper.

4.1 Characteristics of assembly and triangulization for stiffness matrix

As mentioned before an equation numbering from bottom to top for tall building analysis is physically plausible without activating any reordering. It has been proven that the equation numbering in the ascending sequence of vertical coordinate (z coordinate) is efficient in terms of solution time and storage requirement for finite element analysis of tall buildings. This pivoting is compliant with the construction process. In the construction sequential analysis, it is assumed that the equations are sorted in the ascending sequence of the construction process, i.e., from the bottom to the top. This sort can be done through vertical coordinates related to nodes and thus to equations. Suppose that there are m levels and n_m degrees of freedom in the building. Then there are n_m equations as a total. If n_r denotes the maximal equation number of the rth level, then

$$0 < n_1 < n_2 < ...< n_m = neq \tag{5}$$

where neq denotes the total degrees of freedom of the fully built structure.

On the other hand, the structure is constructed by m 'construction steps'. Each step adds a level to the structure. Correspondingly, the resultant stiffness matrix of the whole building can be regarded as a combination of m 'level stiffness matrices' named as $\mathbf{K}^{(r)}$ ($r = 1, 2, ..., m$). The construction sequence makes it ture that the entries in the full resultant stiffness matrix contributed by the members of the rth level are only related to equations between $n_{r-2}+1$ and n_r. Denote the resultant stiffness matrix of the structure from the 1st to the rth levels as $n_r \times n_r$ matrix $\mathbf{K}_r$ ($r = 1, 2, ..., m$), which is located at the position of a leading principal submatrix of the global stiffness matrix $\mathbf{K}$ of the fully built structure, if the numbering of Eq. (5) is satisfied. Recall the symbols(See Fig. 2):

$\mathbf{K}_r$, ($r = 1, 2, \dots, m$) — the resultant stiffness matrix for the rth step
$\mathbf{K}^{(r)}$, ($r = 1, 2, \dots, m$) — the level stiffness matrix for the rth level

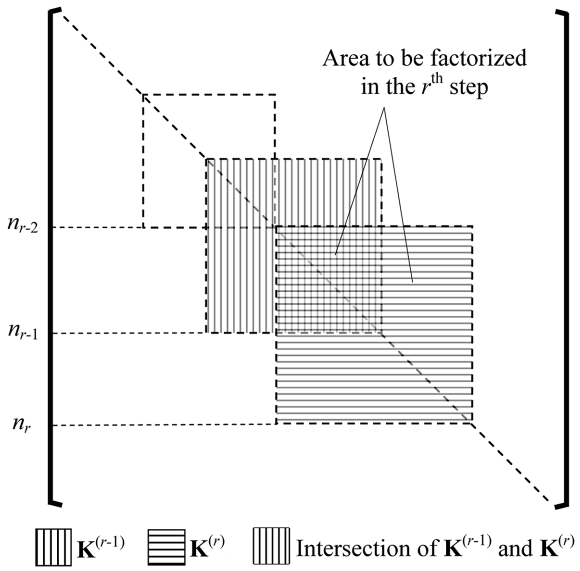

Fig. 2 Map of $\mathbf{K}_r$

we obtained symbolically

$$\mathbf{K}_r = \mathbf{K}_{r-1} + \mathbf{K}^{(r)} = \mathbf{K}_{r-2} + \mathbf{K}^{(r-1)} + \mathbf{K}^{(r)} = \ldots = \sum_{k=1}^{r} \mathbf{K}^{(k)} \tag{6}$$

In the rth step, $\mathbf{K}_r$ is formed by appending $\mathbf{K}^{(r)}$ to $\mathbf{K}_{r-1}$. The $(n_r - n_{r-2}) \times (n_r - n_{r-2})$ 'level stiffness matrix' $\mathbf{K}^{(r)}$ is located in the right-bottom corner of $\mathbf{K}_r$. As illustrated in Fig. 2, this process will change the previous $n_{r-1} \times n_{r-1}$ resultant stiffness matrix $\mathbf{K}_{r-1}$ in two ways. Firstly it will increase the order of the matrix from n_{r-1} to n_r, with additional entries contributed by the appended members of the rth level. Secondly some entries in $\mathbf{K}_{r-1}$ which are the intersected components between completed $(r-1)$th level and newly constructed rth level will be changed.

Because of the construction sequence the rth step will only change the components of the rth level in the structure, and these components do not intersect with $(r-2)$th level. So $\mathbf{K}^{(r)}$ is only concerned with equations in interval $(n_{r-2}, n_r]$ $(n_{-1} = n_0 = 0)$ in the global stiffness matrix (Fig. 2), namely the $n_{r-2} \times n_{r-2}$ leading principal submatrix in the resultant stiffness matrix does not change in the step r. Therefore its factorization remains unchanged in the rth step. This is the key feature used in the fast analysis strategy proposed in this paper.

When the resultant stiffness matrix $\mathbf{K}_r$ is factorized, what we only need to factorize is the principal submatrix in the equation interval $(n_{r-2}, n_r]$ related to $\mathbf{K}^{(r)}$ as illustrated in Fig. 2. The lowest shadow part is the new contribution of the rth level and the adjacent shadow part is the intersection area between $\mathbf{K}^{(r)}$ and $\mathbf{K}^{(r-1)}$ of the previous step, in which the values of the stiffness have been changed during the rth step. The number of equations in these two shadow parts is generally less than $n_r - n_{r-2}$, and much less than n_r. Obviously this strategy dramatically reduced the calculation of triangular factorization (see Section 5).

4.2 Scheme for forming and storing the step stiffness matrices

The feature that the element assemblage regarding to the $(r-2)$th level does not intersect the one regarding to the rth level also leads to a more economic scheme for forming and storing the resultant stiffness matrix $\mathbf{K}_r$ for each step. According to Eq. (3) we form and store the 'level stiffness matrices' $\mathbf{K}^{(r)}$ ($r = 1, 2, \ldots, m$) in the first stage. Since $\mathbf{K}^{(r)}$ and $\mathbf{K}^{(r-2)}$ do not share any degrees of freedom we can form $\mathbf{K}^{(r)}$ in odd levels and even levels respectively and store them as two global stiffness matrices $\mathbf{K}^{odd}$ and $\mathbf{K}^{even}$ in any sparse format. When the construction sequential analysis is in implementation the resultant stiffness matrix is assembled by the entries, which are extracted from these two matrices according to the construction sequence. Notice that the stiffness entries in the cross-shadowed triangle in Fig. 2 are extracted and assembled twice, one for the $(r-1)$th level and the another one for the rth level, respectively. This assembly scheme requires generally only two sweeps over element stiffness matrices since the $\mathbf{K}^{odd}$ and $\mathbf{K}^{even}$ in sparse format can be loaded in core-memory without any trouble in modern personal computers. Thus, I/O is reduced.

5. Step-by-step active column solver

The 'step-by-step active column solver' is based on the analysis model in Section 4 and well-known existing active column solver in finite element analysis. According to the analysis in Section 4, the elements which belong to the levels lower than $r-1$ will not be changed in the rth step. So it is possible to confine the triangular factorization only among the entries of levels r and $r-1$ and keep the results of the triangular factorization obtained before the $(r-1)$th step. Then a tremendous repeated calculation can be avoided and thus the efficiency of the triangular factorization for each step will be significantly raised.

To explain this clearly, the stiffness matrix symbolically can be rewritten as follows:

$$
\begin{aligned}
\mathbf{K}_r &= \mathbf{K}_{r-2} + \mathbf{K}^{(r-1)} + \mathbf{K}^{(r)} \\
&= \mathbf{L}_{r-2}\mathbf{D}_{r-2}\mathbf{L}_{r-2}^T + \mathbf{K}^{(r-1)} + \mathbf{K}^{(r)} \\
&= \mathbf{L}_{r-2}\mathbf{D}_{r-2}\mathbf{L}_{r-2}^T + \mathbf{K}^{(r-1)} \cap (n_{r-3}, n_{r-2}] + \mathbf{K}^{r-1} \cap (n_{r-2}, n_{r-1}] + \mathbf{K}^{(r)} \\
&= \tilde{\mathbf{L}}_{r-2}\tilde{\mathbf{D}}_{r-2}\tilde{\mathbf{L}}_{r-2}^T + \mathbf{K}^{(r-1)} \cap (n_{r-2}, n_{r-1}] + \mathbf{K}^{(r)}
\end{aligned}
$$

Here $\tilde{\mathbf{L}}_{r-2}\tilde{\mathbf{D}}_{r-2}\tilde{\mathbf{L}}_{r-2}^T$ is the triangular factorization of the leading principal $n_{r-2} \times n_{r-2}$ sub-matrix of the resultant stiffness matrix $\mathbf{K}_{r-1}$. This factorization, which is obtained at the $(r-1)$th step, differs from $\mathbf{L}_{r-2}\mathbf{D}_{r-2}\mathbf{L}_{r-2}^T$, the triangular factorization of $\mathbf{K}_{r-2}$, because it has included the contribution of the $(r-1)$th level. Since $\mathbf{K}^{(r)}$ is related to equations in the equation interval $(n_{r-2}, n_r]$ $(n_{-1} = n_0 = 0)$ in the global stiffness matrix, and $\mathbf{K}^{(r-1)} \cap (n_{r-2}, n_{r-1}]$ is in the equation interval $(n_{r-2}, n_{r-1}]$ $(n_{-1} = n_0 = 0)$, therefore $\tilde{\mathbf{L}}_{r-2}\tilde{\mathbf{D}}_{r-2}\tilde{\mathbf{L}}_{r-2}^T$ and $\mathbf{K}^{(r-1)} \cap (n_{r-2}, n_{r-1}] + \mathbf{K}^{(r)}$ do not share any common nonzeros at all. We design thus an algorithm in which only triangular factorization of the level stiffness $\mathbf{K}^{(r-1)} \cap (n_{r-2}, n_{r-1}] + \mathbf{K}^{(r)}$ is necessary for each step. We do not need to re-factorize the equations in interval $[1, n_{r-2}]$ in the resultant stiffness matrix $\mathbf{K}_r$.

The amount of floating point operations needed by the triangular factorization of $\mathbf{K}^{(r-1)} \cap (n_{r-2}, n_{r-1}] + \mathbf{K}^{(r)}$ in each step r ($r = 1, 2, \ldots, m$) is evaluated theoretically about twice of that needed for the stiffness matrix of one level structure in primary load analysis if the structure has a similar construction at each level. Hence running time is greatly reduced. Notice that $\mathbf{LDL}^T$ is the most time consuming step in the whole solving procedure. Denote the total time cost for the fast strategy as $T_m = aT$, theoretically we evaluated that $a \approx 2$, i.e., the computing time needed by the fast strategy is about two times more than corresponding primary loading analysis. Practically, a is more than 2 since overhead cost exists. For example in the numerical examples of this paper $a \approx 3\sim5$. However it is much less than $(m + 1)/2$ and is acceptable in engineering. Obviously the

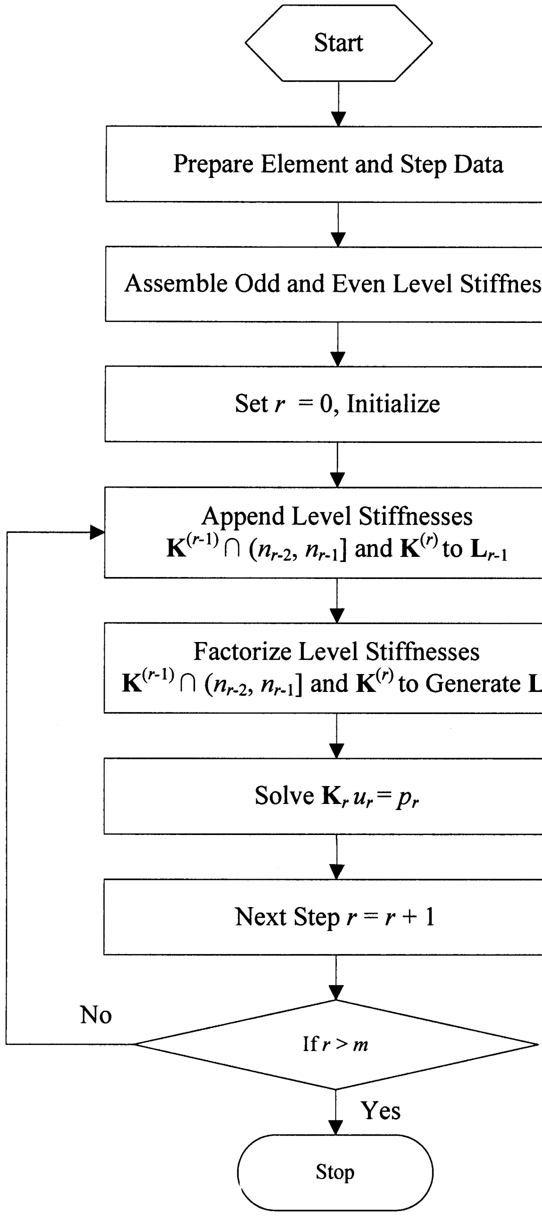

Fig. 3 Flowchart of the fast step-by-step strategy

more levels there are in the structure, the more significant enhancement of the efficiency will be achieved.

The proposed strategy and the step-by-step active column solver are applicable to the case that the members of the latest constructed rth level in the building only intersect with the $(r-1)$th level. If installing and removal of shores and reshores need to be considered, stiffness change cannot be confined within adjoining two levels in each construction stage and may be related to several levels. After proper modification, the proposed strategy can deal with such a situation. As the end of this section we illustrate in Fig. 3 the flowchart of the fast algorithm.

6. Engineering examples

This section gives the results of various numerical tests. The performance of the proposed construction sequential analysis is tested on the platform Pentium II 550 using Compaq Visual FORTRAN 6.5, with default release compiler options, 128 MB RAM. We compared three implementations of the construction sequential analysis:

A. The step-by-step active column solver proposed in this paper;
B. A full step solver based on the Cell Sparse Solver (Nie *et al.* 2006);
C. ETABS[†] 8.0 construction sequential analysis (ETABS 2002).

Three examples are selected to demonstrate the accuracy of the fast strategy proposed in this paper as well as to compare the running time of this strategy with the relevant ones in (Nie *et al.* 2006) and ETABS. All the results are realized on SAP84[‡†] (Yuan 2003) and ETABS (2002).

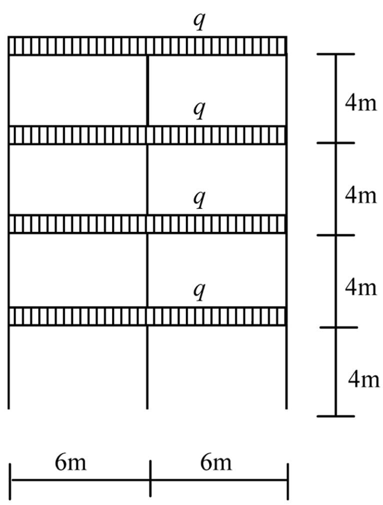

Fig. 4 A frame structure

[‡†] SAP84 is a trademark of the software package developed by the authors.

Table 1 Comparison of moments of the frame structure (kN · m)

	Numerical model	1st Floor	2nd Floor	3rd Floor	4th Floor
Moment at the bottom end of the outer column	Primary loading	4.53	12.94	11.77	13.59
	A/B	6.04	4.54	4.48	5.00
	C	6.02	4.63	4.54	5.17
Moment at the upper end of the outer column	Primary loading	−9.18	−12.39	−11.52	−16.29
	A/B	−12.35	−11.90	−11.73	−13.33
	C	−12.32	−11.95	−11.72	−13.47
Moment at the outer edge of the beam	Primary loading	−22.12	−24.15	−25.11	−16.29
	A/B	−16.89	−16.38	−16.72	−13.33
	C	−16.94	−16.49	−16.89	−13.47
Moment at the inner edge of the beam	Primary loading	−44.49	−43.40	−42.85	−47.10
	A/B	−46.94	−47.07	−46.90	−48.69
	C	−46.88	−46.98	−46.79	−48.58

6.1 Example 1: A 4-story frame

Fig. 4 shows a 4-story reinforced concrete frame. The beam section is 300 mm × 600 mm, the section of outer columns is 400 mm × 400 mm, the section of inner columns is 600 mm × 600 mm, Young's modulus of concrete is 2.8×10^7 kN/m^2, uniform loading for each floor is q = 8 kN/m, self-weight is considered. We performed primary loading and the construction sequential analyses using the 'step-by step active column solver' proposed in this paper and compared the numerical results with ETABS. From Table 1 the necessity of the construction sequential analysis in structural engineering is unavoidable because the numerical results of this model are quite different with primary loading analysis. The results by the proposed solver are very close to ETABS, which testified the precision of the strategy.

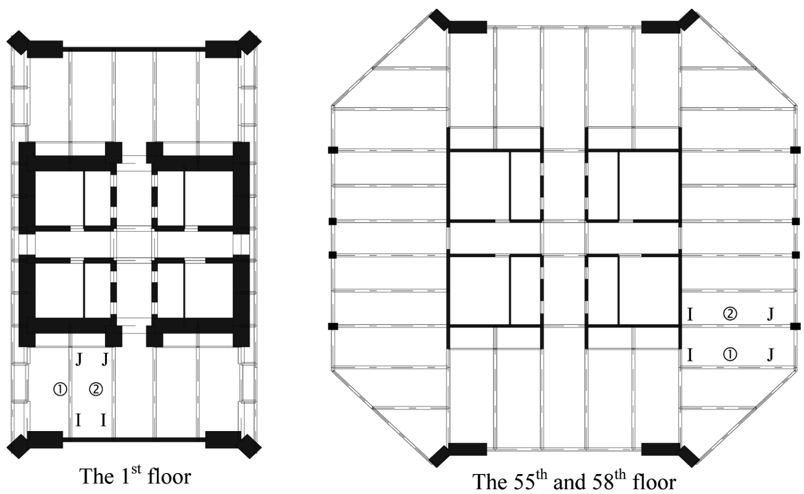

Fig. 5 Plan of a mansion

Table 2 Comparison of moments (kN·m)

Component	Location	Primary loading analysis	A/B
No.1 of 55th floor	I-end	−121.6718	−106.2360
	Center of Span	179.3379	181.9735
	J-end	−89.1547	−99.3192
No.2 of 55th floor	I-end	−305.0667	−219.9512
	Center of Span	95.3555	96.9042
	J-end	−70.1460	−152.1641
No.1 of 58th floor	I-end	−392.1189	−194.9439
	Center of Span	20.1266	45.7382
	J-end	217.2820	71.3302
No.2 of 58th floor	I-end	−307.6285	−181.7499
	Center of Span	−22.9958	26.7779
	J-end	41.9520	15.6207

6.2 Example 2: A mansion

The mansion is 218.1 meters high with 60 floors. Fig. 5 displays its plane diagraph. There are 6712 beams and columns, 3590 shear walls, 7168 nodes and 21075 degrees of freedom. Shear walls are modeled by shell elements and rigid floor assumption is adopted in the analysis. We also used SAP84 to perform the primary loading and the construction sequential analysis. The numerical results are listed in Table 2.

To illustrate the calculating efficiency we performed primary loading analysis using an active column solver with loop-unrolling and the construction sequential analysis using the step-by-step active column solver proposed in this Paper, cell sparse fast direct solver (Nie *et al.* 2006) and ETABS V. 8.0. The running time is compared in Table 3. Here the running time statistics includes formation and solving of global stiffness matrices, but excludes the formation of element stiffness matrices and load vectors and internal forces (stresses) calculation. Loop-unrolling speeding technique is adopted in active column solver and doubles the solving speed.

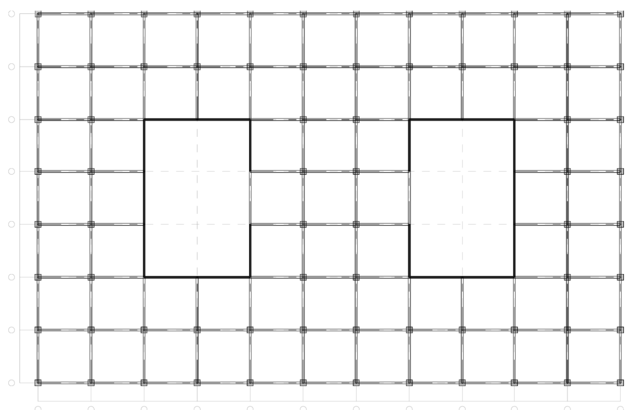

Fig. 6 Plan of a 40 story frame-wall structure

Table 3 Statistics of running time (seconds)

Engineering projects	Primary loading analysis by active column solver	Construction sequential analysis		
		A	B	C
Example 2	25	124	275	N/A
Example 3	63	186	694	2893

6.3 Example 3: A 40-story frame wall

A 40 story frame-wall structure is shown in Fig. 6. There are 8480 frame elements, 720 shear wall elements, 22,080 degrees of freedom in this model. Like Example 2, corresponding running time statistics is listed in Table 3.

Compared with the construction sequential analysis in (Nie *et al.* 2006) and ETABS, the strategy proposed in this paper significantly enhanced the solving speed. In our test platform, under the case of a big number of levels or over 30,000 degrees of freedom, ETABS 8.0 failed to deliver results of the construction analysis.

7. Conclusions

The construction sequential analysis is necessary in the vertical load analysis of structures. In this paper, we take advantage of the fact that each level in a building only intersects with adjoining levels in the level-by-level construction. Based on building construction process, we proposed a fast construction sequential analysis strategy by means of amending existing skyline solver. The new strategy avoided tremendous repeated calculation and thus significantly reduced the total operations in the construction sequential analysis. In addition, loop-unrolling is employed in the $\mathbf{LDL}^T$ procedure to enhance the efficiency about 200% of triangular factorization for each step. The strategy proposed in this paper does not require any simplified assumption or approximate empirical formula, so that accuracy is guaranteed while the solving speed is greatly raised. In the test it runs several times faster than the current ETABS construction sequential analysis program. The fast strategy proposed in this paper provides an applicable technique for the construction sequential analysis in large-scale engineering structures.

In addition to the easy aspect we discussed in this paper construction loads in a tall reinforced concrete building are largely influenced by the construction schedule. These loads may exceed the designed strength of the building's structure or produce early cracking of the floor, with adverse consequences to its service conditions (Prado *et al.* 2003). The proposed approach can be easily used to model floors supported by previously cast floors in different periods, since only the last floors are required to be considered in our computation. Finally it is to emphasis that the greater self-weight of a concrete building structure (especially for a core-frame structure), the more significant differences may be produced between a sequential analysis and a traditional primary loading analysis. Normally, a sequential analysis is not necessary for a steel building structure.

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