

# Design and optimization of steel trusses using genetic algorithms, parallel computing, and human-computer interaction

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*(Received November 29, 2004, Accepted March 14, 2006)*

**Abstract.** A hybrid structural design and optimization methodology that combines the strengths of genetic algorithms, local search techniques, and parallel computing is developed to evolve optimal truss systems in this research effort. The primary objective that is met in evolving near-optimal or optimal structural systems using this approach is the capability of satisfying user-defined design criteria while minimizing the computational time required. The application of genetic algorithms to the design and optimization of truss systems supports conceptual design by facilitating the exploration of new design alternatives. In addition, final shape optimization of the evolved designs is supported through the refinement of member sizes using local search techniques for further improvement. The use of the hybrid approach, therefore, enhances the overall process of structural design. Parallel computing is implemented to reduce the total computation time required to obtain near-optimal designs. The support of human-computer interaction during layout optimization and local optimization is also discussed since it assists in evolving optimal truss systems that better satisfy a user's design requirements and design preferences.

**Keywords:** Genetic Algorithms; Pareto Ranking; MOGA; multi-objective optimization; conceptual design; parallel computing.

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## 1. Introduction

Designs of roof truss systems have become increasingly complex due to increased span length and loading requirements. In addition, the structural designer must often incorporate novel architectural features into of the design, which increases the complexity of the design. In practice, often the optimization of a structural system is limited to only the final design stage using existing design packages or trial and error procedures. Using these approaches, the ability to design efficient and novel designs is heavily dependent on a designer's experience and the effectiveness of trial and error procedures applied. Enhancing the support available to designers during conceptual design by allowing a more thorough search and evaluation of possible designs has become a critical part of obtaining efficient designs.

Conceptual design allows designers to create efficient and innovative solutions to complex, multi-

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objective design problems. Genetic algorithms (GA) have proven to be an effective optimization technique for solving complex problems in many engineering disciplines (Goldberg 1989). In this research, GAs support the conceptual design process by providing a global exploration of the problem domain, while also supporting the final design of specific truss layouts by optimizing member sizes. It is recognized, however, that due to the increasing complexity of the structural systems designed, including large-scale roof trusses and bridges, careful consideration must be paid to the overall time required in producing the efficient designs.

This paper discusses the development of a three-stage design methodology that applies a hybrid GA/local optimization search technique to evolve near-optimal steel trusses for a specified three-dimensional design space. The main research objective is to design efficient truss topologies, geometries, and shapes that minimize volume while also satisfying stated stress and deflection limits and reducing the total computational time. A multi-objective GA (MOGA) that implements a Pareto ranking scheme (Pareto 1906) is used to perform shape optimization of the structures identified during conceptual design by optimizing member properties. Implementation of all three stages on a four-node parallel platform assists in reducing the total design time.

This research also explores the possibility of supporting human-computer interaction in order to assist in capturing the user's design requirements and design preferences. Using the defined interactive features, the user would direct the hybrid optimization process so that it supports the designer's specific design objectives and constraints.

## 2. Literature review

The concept of using the principles of biological evolution for analysis and design can be traced back to the work of Rechenberg (1965), although genetic algorithms by themselves gained prominence through the work of Holland (1975). Goldberg (1989) researched the applicability of genetic algorithms to optimization problems, which started the era of research in this area.

Rajeev and Krishnamoorthy (1992) and Jenkins (1992) were among the first researchers to successfully implement simple GAs on structural optimization problems. The former explored the application of GAs for truss sizing optimization, whereas Jenkins used GA for shape optimization of plane frames.

Other researchers have tried to provide the ability to perform limited topology and geometry optimization from different perspectives. Hajela and Lee (1995) used a two step design methodology. After achieving kinematic stability in Stage I, member sizing was carried out in the next step. Roston and Sturges (1996) advocated the combination of formal grammars with genetic programming, which they referred to as Genetic Design. Their method presented the designer with an array of viable design alternatives. Gage *et al.* (1995) developed a variable-complexity genetic algorithm procedure for topological design. The topology optimization using genetic algorithms was followed up by a gradient based optimizer to size the members. This research effort also introduced the variable length GA and the use of cut-splice crossover operation.

During the last decade, multi-objective optimization has also been discussed by a number of researchers who used different approaches to obtain feasible solutions. Cheng and Li (1997) used MOGA to successfully address the trade-off between weight and strain energy for a 72-bar space truss. Along with selection, crossover and mutation, a niche and a pareto-set filter were used to improve their results.

Coello and Christiansen (1999) used a specialized min-max optimization approach to transform the multi-objective problem into several single objective optimization problems that were easier and faster to solve. Ruy *et al.* (2001) proposed a hybrid MOGA that generated promising topologies with consideration to the multi-objective environment. Following the selection of topologies, sizing and geometry optimization was carried out. Reynolds and Azarm (2002) used a hybrid MOGA that combined a standard MOGA with heuristics specifically tailored to address the deficiencies in multi-objective design for trusses. An 'advance pareto' technique was able to generate more-optimal solutions with respect to all objectives. The 'search between' and the 'extend pareto' techniques were able to increase the number of unique solutions while improving the distribution along the pareto frontier. Deb (2001) provides a detailed description of implementing different multi-objective optimization methods using evolutionary algorithms.

### 3. Generation of conceptual design alternatives

The first stage of the design module focuses on the generation of alternative truss topologies and geometries for a specific design space defined by the user. The truss generation model uses triangles as the basic building unit. Angles defining each triangle are randomly picked from a set of pre-defined angles. The construction of each truss topology and geometry is based on several geometric constraints. The production of consequent triangles is based on the areas of the previously defined triangles, while boundary constraint limits the truss geometry to within the user-defined design space. A symmetric structure is created by overlapping similar triangles on previously existent triangles on each end. An additional constraint forces truss configurations that are more optimal towards converging the structure from both ends. Finally, a closing mechanism is defined that assists in correctly, and efficiently, closing the truss based on the current truss layout generated.

The results obtained from the first stage of the design module are a set of conceptual design alternatives for the problem domain specified by the user. The number of design alternatives generated in the first stage is controlled by the user. Consideration of a larger range of design alternatives allows for a more thorough global exploration of the design space, but may also require more computational effort. The information required to define each individual includes the nodal coordinates, member connectivity matrix, and the loading information for each node. Collecting this information for each design alternative allows the user the view the set of generated truss topologies using an OpenGL graphical user interface.

The total roof load is specified by the user since it depends on the actual dead, live and wind loads applied to the structure. The loads applied to each generated truss alternative, however, are assigned based on tributary area to each top chord nodal point defined in that truss. Therefore, different designs may have different loading configurations and magnitude of loads at the truss nodal locations.

### 4. Multi-objective genetic algorithm design optimization

Each truss topology created in the first stage undergoes multi-objective shape optimization in the next stage of the hybrid design method. For each individual structure, member areas are randomly generated and encoded as binary numbers in a GA individual. A population size of hundred

individuals is considered for each truss topology configuration. During each generation, standard GA crossover and mutation operations are applied to the population resulting in improved designs in subsequent generations.

The simultaneous requirement of minimizing the total volume, stresses, and deflections defines the design of steel trusses as a multi-objective design problem. An overview of MOGA techniques is provided in the literature (Deb 2001). In this research, a pareto-front ranking procedure is implemented to determine the Pareto-optimal set of individuals (truss designs) that satisfy the conflicting design objectives. This set of Pareto-optimal solutions captures the tradeoffs that occur between the objectives. The entire population undergoes Pareto ranking, which identifies the non-dominated individuals in the population and assigns them the first rank that corresponds to the highest fitness. A similar procedure is used to define the non-dominated individuals in the other ranks, except that previously ranked individuals are excluded from consideration (Eschenauer *et al.* 1990).

A modified approach is adopted for performing crossover. Each individual member encoded in one parent is individually crossed with the corresponding member of the other parent. The probabilistic nature of this operation is retained by randomly selecting the crossover location for each member. Elitism is incorporated to allow a sub-set of the current Pareto-optimal set of individuals found to date to remain in the population for the next generation. A maximum of ten elite individuals are passed into the next generation. If the number of individuals in the Pareto-optimal set exceeds ten, then an elite subset of individuals is randomly selected from among all of the Pareto-optimal solutions.

Each individual truss layout undergoes MOGA optimization until there is no further improvement in the design objective for minimizing volume after 10 generations, while still meeting the stress and deflection limits, or for a maximum of 30 generations. During the multi-objective optimization stage, three different truss spacings were investigated. Trial results for spacings of 6.096 m (20 feet), 9.144 m (30 feet), and 12.192 m (40 feet) are presented, but other spacings could be defined by the user based on design requirements. Varying the spacing of the trusses changes the loading applied to the truss since the actual load depends on the truss geometry and topology, which varies between design alternatives. Although the overall area load is constant, the loads that each truss carries will vary in location and magnitude due to different tributary areas at the defined nodes of the truss.

The MOGA optimization results obtained from the second stage define the characteristics of the tradeoffs that occur between the conflicting design objectives of minimizing volume, deflection, and member stresses. The tradeoffs can be visually defined by plotting the objective fitness values for

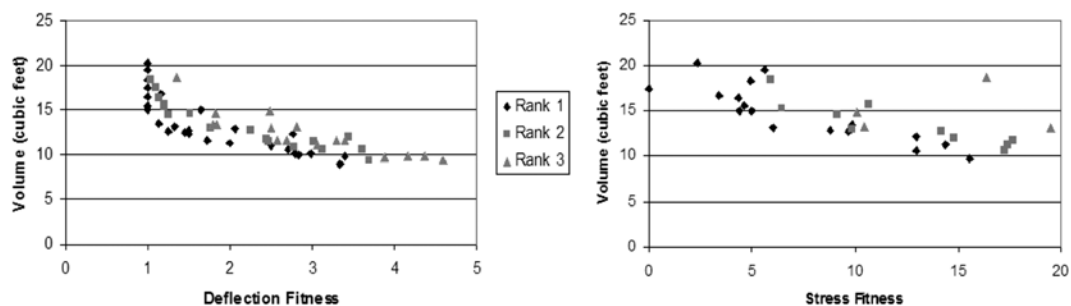


Fig. 1 Pareto-surface ranking used for MOGA

each individual in the Pareto-optimal set of design alternatives evolved. Fig. 1 shows two views of the defined Pareto front. The first graph shows the tradeoff between minimizing volume and deflection, while the second graph highlights the tradeoff between minimizing volume and member stress. The individuals defined in the Pareto-optimal set, which consist of individuals with rank 1, undergo local optimization in the third stage.

## 5. Local optimization of near-optimal truss designs

The third stage in the design methodology developed applies a local search method to further optimize the Pareto-optimal truss designs evolved in the previous stage. The defined truss topology, geometry, and member sizes for each individual serve as the initial solution for the local search trials. Local search is performed by perturbing (mutating) each truss geometry to locally explore for more optimal truss systems. The mutations implemented include moving the nodal locations within a specific range from the current position. Geometric constraints are applied to prevent overlapping and crossing of truss members during local search. The mutation procedure of changing the node locations is carried out iteratively until no further improvement is found in reducing volume, while still meeting the imposed stress and deflection limits.

The local search method is applied to the fittest truss structure evolved during the MOGA shape optimization stage. This requires that the user select between the fittest truss structures evolved for the 6.096 m (20 feet), 9.144 m (30 feet), and 12.192 m (40 feet) truss spacing. As a default, the truss with the overall minimum volume is selected. The user can view the final near-optimal truss design obtained during the third stage in the OpenGL GUI. In addition, the information about the final design that defines the node locations, connectivity matrix, loaded nodes, and member properties can be exported to a structural engineering package to allow further analysis.

## 6. Parallel platform implementation

The three-stage design methodology is implemented on a four-node parallel platform, in which MPI programming is used to communicate between the different nodes. As a direct result of the parallel implementation, a dramatic improvement in computational time is achieved. The total time is approximately reduced to one fourth of the time taken for carrying out the entire process on a single computer.

The parallel implementation details are shown in Fig. 2. The master node is responsible for carrying out the truss topology layout performed in the first stage and the local optimization of the fittest, or user-selected, truss design in the third stage. After the complete execution of the first stage on the master node, all of the relevant information required for each of the truss layouts is passed to three nodes. These nodes work in parallel to carry out the MOGA optimization performed in the second stage. In addition, each of the three parallel nodes considers a different truss spacing. This allows the problem domain to be defined as a three-dimensional space. Each node is provided with the same set of defined truss layouts generated during stage I, but will consider one of three truss tributary widths (6.096 m (20 feet), 9.144 m (30 feet) and 12.192 (40 feet)). In the presented trials, the total width of the structural design space is defined as 73.152 m (240 feet), which results in twelve trusses at 6.096 m (20 feet), eight trusses at 9.144 m (30 feet), and six trusses at 12.192 m

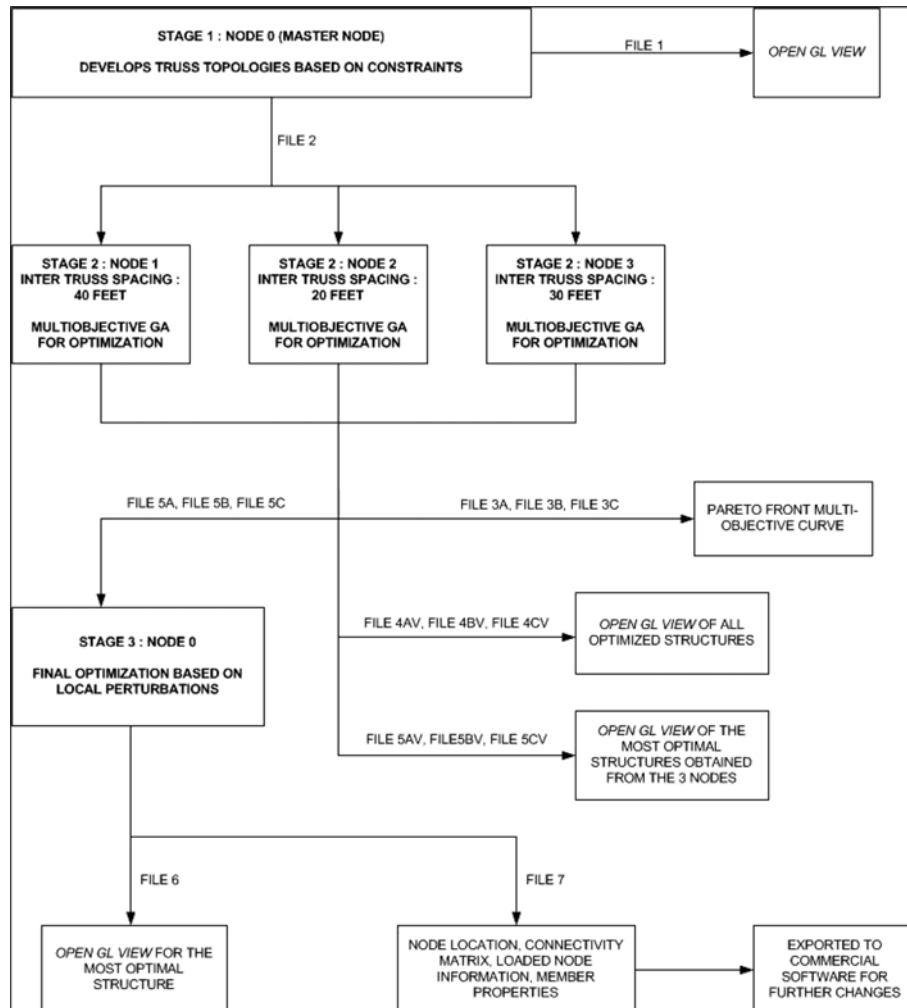


Fig. 2 Schematic representation of the parallel implementation

(40 feet). Varying the truss spacing affects the magnitude of the loads carried by the trusses being optimized as discussed previously. As Fig. 2 shows, after completion of the MOGA optimization performed in the second stage, the execution is passed to the master node to proceed with the third stage.

## 7. Discussion of research results

### 7.1 Generation of conceptual design alternatives

The following results were obtained for a random trial performed using the three-stage hybrid design methodology implemented on a four-node parallel platform. A Unix-based compiler

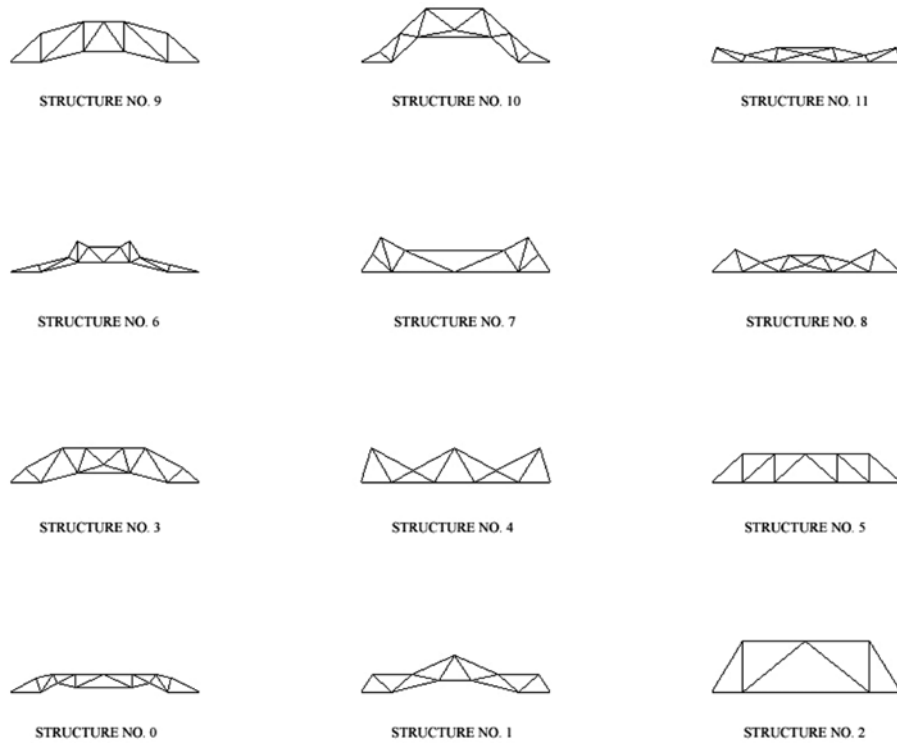


Fig. 3 Set of design alternatives generated during Stage I

supporting the parallel implementation using MPI programming was used.

Fig. 3 shows the set of design alternatives generated during the first stage. The design domain considered is a truss that spans 18.288 m (60 feet). Twelve random structures were generated that met the specified design objectives and constraints. As shown in Fig. 3, the first stage is capable of generating a diverse set of truss topologies and geometries. These structures are then passed to the second stage to undergo MOGA optimization that considers varying the truss tributary width from 6.096 m (20 feet), 9.144 m (30 feet), and 12.192 m (40 feet).

## 7.2 Multi-objective genetic algorithm design optimization

The second stage of the design method performs multi-objective shape optimization using MOGA to evolve optimal or near-optimal truss designs. This stage, as discussed previously, runs in parallel

Table 1 Trial results obtained in Stage II

Parallel node number	Truss spacing	Number of trusses optimized	Truss number
1	6.096 m (20 feet)	7	0,1,3,5,7,9,10
2	12.192 m (40 feet)	5	0,3,5,9,10
3	9.144 m (30 feet)	6	0,1,3,5,9,10

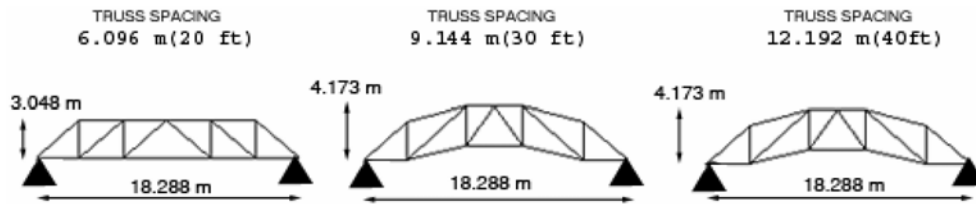


Fig. 4 Most optimal truss layouts for different truss spacing (tributary width)

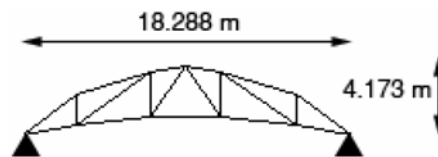


Fig. 5 Final truss design for 12.192 m (40 feet) spacing obtained after Stage III

on three nodes to optimize the member properties of each individual structure. Table 1 identifies the trusses optimized in the second stage. In this specific trial, several of the trusses generated in the first stage (Trusses 2, 4, 6 & 8) did not undergo the MOGA shape optimization. The reason for excluding these trusses was that they did not meet appropriate fitness criterion. These structures when analyzed exceeded the allowable limits for either stresses or deflections. Therefore, no optimal member property combination could be found using the MOGA approach.

Fig. 4 defines the near-optimal truss topologies and geometries evolved for each truss tributary width during the second design stage. Fig. 5 shows the reduction in total volume obtained after the MOGA method is performed on each of the truss layouts after 12 generations. A truss tributary width of 12.192 m (40 feet) was selected in this trial. During this stage, the topology and geometry of each truss remains fixed while the MOGA performs shape optimization.

The MOGA process is capable of efficiently reducing the overall volume of a diverse range of truss designs. Starting from a random set of section layouts for these trusses, the MOGA is able to produce light weight trusses that satisfy the strength and stiffness criteria. The reduction in volume over subsequent generations leads to the near-optimal solutions.

Information defining the layout and member properties of the three truss configurations is provided to the third stage of the design module, which selects one of these trusses as the “best” (as identified by the user or as the fittest design) and performs local optimization on the truss geometry to further optimize the selected.

### 7.3 Local optimization of near-optimal truss designs

After performing local optimization by investigating local perturbations to the geometry of the most optimal truss evaluated in Stage II, the truss design shown in Fig. 6 is obtained. This truss design provides a near-optimal truss design that meets the stated design criteria and constraints.

A significant reduction in the final volume produced using the search method in Stage III was obtained. The total volume before implementing Stage III was 0.2712 cubic metre (9.6 cubic feet), while after its implementation it was reduced to 0.255 cubic metre (8.99 cubic feet). Therefore, an additional 11% reduction in total volume is obtained using this procedure. The design details of the



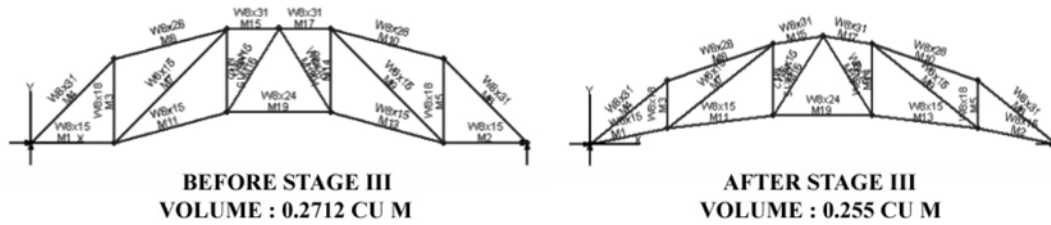


Fig. 6 Design details of the final truss design before and after Stage III

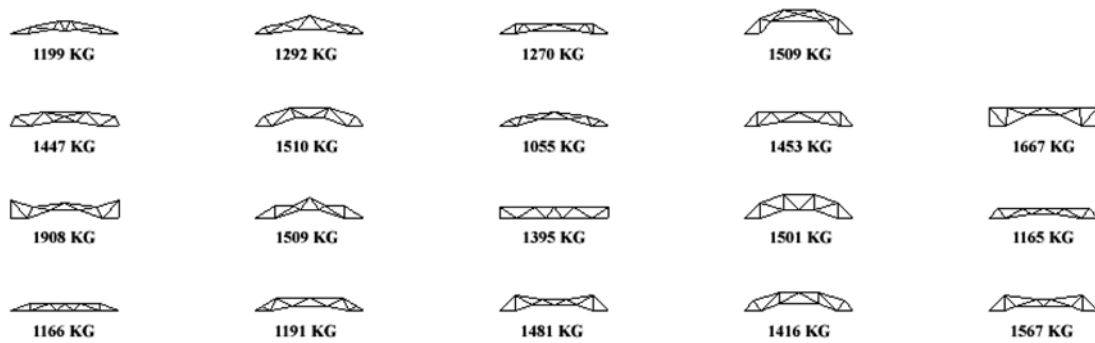


Fig. 7 Optimal solutions at the end of Stage II

final truss obtained before and after the implementation on Stage III are provided in Fig. 7. In general, after a large number of trial runs, it is observed, that the truss tries to achieve an arched geometry. As is evident by Fig. 7, the nodes on the bottom chord try to move up, where as those on the upper chord move down. The final structure obtained has a more parabolic geometry than the initial truss.

The total computational time required to obtain the final truss design shown in Fig. 7 was less than 20 seconds. Overall, the three-stage design and optimization methodology defined in this research is able to efficiently satisfy all of the objectives of minimizing volume, member stresses, and deflections, while also reducing the required computational design time.

## 8. Discussion of roof benchmark problem results

The design domain for the benchmark roof problem is defined for a 15.24 m (50 feet) and a maximum height of 6.096 m (20 feet). The center to center truss spacing along the Z-axis was considered to be 6.096 m (20 feet), 9.144 m (30 feet) & 12.192 (40 feet). The total area load on the roof is 4.788 MPa (100 psf).

The material properties are standard for steel (201 GPa ( $E = 29000$  ksi),  $f_y = 344.736$  MPa (50 ksi), and  $\rho = 7851.03$  kg/m<sup>3</sup> (0.49 k/ft<sup>3</sup>)). The AISC LRFD design specifications are followed : the allowable tension force is  $0.9f_yA_g$  ; the allowable slenderness ratio is 300 for tension members and 200 for compression members; the allowable joint displacement is limited to 25.4 mm (1 inch); and the allowable compression force for each member is determined from buckling considerations,

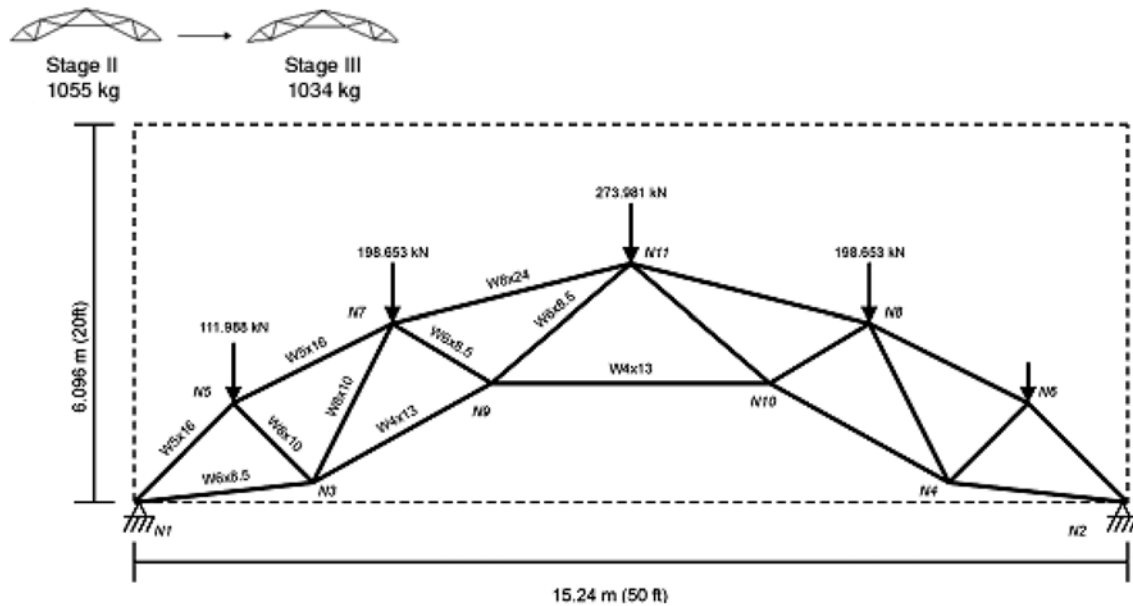


Fig. 8 Geometry and loading details of the final truss design

Table 2 Joint co-ordinates for the final truss design

Node	X(m)	Y(m)	Node	X(m)	Y(m)	Node	X(m)	Y(m)
N1	0	0	N5	1.22	1.22	N9	5.32	1.67
N2	15.24	0	N6	14.02	1.22	N10	9.92	1.67
N3	2.44	0.3	N7	3.05	2.28	N11	7.62	3.34
N4	12.8	0.3	N8	12.19	2.28			

using the relevant parts of the AISC specification (AISC 2001). The members sections were selected from a set of 16 standard sections between W4x13 to W8x24 from the AISC LRFD Manual (2001).

Fifty truss designs were evolved for the benchmark problem domain using Stage I. The MOGA optimized nineteen of the truss designs using Stage II for a 12.192 m (40 feet) spacing. Fig. 8 shows the set of near optimal trusses obtained at the end of Stage II.

The most-optimal truss design was selected from the designs evolved and was further optimized using the local search method in Stage III. An additional reduction of 2% in volume was obtained. Fig. 8 and Table 2 present the design details of the truss obtained at the end of Stage III.

## 9. Human-computer interaction during design

Providing effective user interaction with the computational design methodology developed is critical in meeting the human designer's objectives and constraints, since often these design criteria cannot be directly modeled. One of the future objectives of the research effort discussed is to more

Table 3 Analogy between creative human design and MOGA design and optimization

Creative human design		Moga design & optimization	
		Input	Outcome
<b>STAGE I INTERACTIONS</b>			
I	Set up Design Space	Design space	Restricts alternatives within specified design space
II	Determine Truss Complexity	Max no of members allowed	Controls overall truss complexity and average length of individual members
III	Shape of Truss : Arch or Flat base, etc.	First node next to the boundary node	Determines overall shape and arching of truss
<b>STAGE II INTERACTIONS</b>			
IV	Fabrication/Construction Feasibility	Selection of Alternatives from numerous generated	Identification of feasible/possible Trusses
V	Aesthetics	Selection of Alternatives from numerous generated	Identification of aesthetically pleasing Trusses
VI	Architectural Features : Height of Apex, etc.	Selection of Alternatives from numerous generated	Identification of architecturally agreeable Trusses
VII	Inter-truss Spacing for roof trusses	Three different Inter-Truss Spacing	Identification of total load on each Truss
<b>STAGE II INTERNAL INTERACTIONS</b>			
VIII	Shape Availability	The Shape set to be used in MOGA	Restricts the Selectable Shapes to be used in design and optimization
IX	Design Requirements : Stability, Strength, etc.	Design Rules for Stress and Deflection Checks	Customization of design process by specifying Codal provisions
<b>STAGE III INTERACTIONS</b>			
X	Total Weight	Final Selection from numerous alternatives optimized	Selection of Truss with Optimal Weight
XI	Deviation from Standard Design Alternatives	Final Selection from numerous alternatives optimized	Perturbations to Local Geometry in search for innovative alternatives

efficiently support human-computer interaction during each of the three stages of the design methodology developed.

The current research effort involves the designer in defining the three-dimensional design space. The designer defines the maximum number of members desired in the truss layout, the support locations, and the location of the first node closest to the support. Several of the planned human-computer interactions include allowing the user to select a preferred set of truss topologies and geometries from the large set of generated truss layouts after stage one; allowing the user to select the desired range of member shapes, truss spacing, and total design space before stage two begins; and allowing the user to select the trusses that will undergo further refinement through local optimization in stage three.

Table 3 has been prepared by working in close consonance with experienced design engineers. It emulates the analogy between the engineer's perspective towards creative design and the optimization method used in this research.

Adding these human-computer interactions would assist in obtaining final truss designs that match the user's design objectives and constraints. Default parameters for all the above interactions could also be specified in case the user decides to by-pass these options.

## 10. Conclusions

The three-stage design and optimization methodology developed in this research is able to evolve efficient truss designs. The diverse truss layouts generated in stage one support the global exploration of the problem domain search space, which allows the user's design objectives and constraints to be satisfied more efficiently. The MOGA approach implemented in the second stage greatly reduced the total volume of a diverse set of truss designs, while also satisfying stress and deflection constraints. The innovative approach of applying local perturbations in the final stage was shown to be effective in further reducing the total volume of the truss design. Overall, a significant reduction in the total computational time was obtained by implementing the design method on a four-node parallel platform.

The final truss design evolved using the developed design methodology meets all of the primary design objectives explicitly defined. Another salient feature of this research effort is the wide range of design alternatives being produced after Stage II. In comparison to many other research studies where only a single solution is finally obtained, this research produces a range of optimal trusses. This allows the designer to select the truss that best meets their design requirements from among the set of near-optimal trusses.

The inclusion of human-computer interactions in the future may further assist in matching specific user-defined design preferences. Based on the preliminary results obtained, the design method developed is capable of assisting designers in the design, analysis, and optimization of truss systems by allowing different truss spacings and configurations to be investigated, while significantly reducing the required design time.

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