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Unified approach to predict the dynamic performance of transportation system considering wind effects

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Abstract. Natural hazards, including the wind hazard and others, threaten the integrity of the modern society. A transportation system usually consists of roadways, bridges and related vehicles. Harsh environmental conditions, caused by such as wind, exist in the real world frequently and affect the dynamic performance of the transportation system through their interactions. Long-span bridges are usually the backbones of transportation lines. In windy conditions, the information about the dynamic performance of bridges and vehicles considering full interactions of environmental factors is very essential for people to assess the overall operational conditions and safety risks of the transportation lines. Most of existent approaches target specifically at several isolated tasks considering partial interaction effects. In order to improve the understanding of these related-in-nature problems integrally as well as the consistency of different approaches, a unified approach to integrally predict the dynamic performance of long-span bridges and vehicles under wind is introduced. Such an approach can be used as a general platform to predict the dynamic responses of vehicles and bridges under various situations through adopting both commercial and in-house software. Dynamic interaction effects can be fully considered automatically for each situation. An example of a prototype bridge in US is given for the purpose of demonstration.

Keywords: bridge; vehicle; wind; vibration.

1. Introduction

Natural hazards, including the wind hazard and others, threaten the integrity of the modern society. A transportation system usually consists of roadways, bridges and transporting vehicles. Harsh environmental conditions, caused by such as wind, exist in the real world frequently and affect the dynamic performance of the transportation system through their interactions. Long-span bridges are usually the backbones of transportation lines. Wind, as one of the most devastating natural hazards in the United States, endangers the safety and operations of long-span bridges as

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well as moving vehicles on the bridges and highways. In order to predict the safety and service conditions of these bridges and vehicles considering the possible existence of wind, dynamic performance of both bridges and vehicles should be reasonably predicted considering the full interactions among vehicle/bridge/wind. Failure to consider the interaction effects among transportation infrastructures, vehicles and environmental conditions (e.g., wind) may pose a hardship to study the transportation system performance accurately.

The aerodynamic phenomenon of bridges under wind has been studied by many scholars for many years (Simiu and Scanlan 1996). However, the bridge dynamic performance in windy conditions usually does not consider the existence of vehicles. On the other hand, the vehicle performance under wind was usually predicted separately without considering the interaction with supporting structures (e.g., bridges, road surface) (Baker 1991, 1999). There have been many years since people started working on vehicle-bridge interactions. Despite the fact that wind, especially strong wind, has significant effects on bridge and vehicle dynamics, most of these interaction analyses only considered the external loading from road roughness (Timoshenko *et al.* 1974, Yang and Yau 1997). Obviously, researches on the inherently related bridge/wind/vehicle system were separated into several pieces. Accordingly, some different approaches have been developed to solve these specific problems in terms of bridge/wind aerodynamic analysis, bridge/vehicle interaction, vehicle/wind interaction, etc. These isolated simplified approaches, despite being straightforward, fail to represent the real world accurately and thus will cause error in the predicted results. Moreover, these isolated approaches targeting at several specific problems may also prevent people from understanding the mechanism behind those interrelated phenomenon as a unity. The inconsistency of these different approaches also poses some unnecessary difficulties for researchers to study these related topics together. A general platform to predict the dynamic performance of a long-span bridge and vehicles under external loadings, which can be used in many different situations, is desirable.

In the present study, a general purpose approach combining the utilization of both commercial software and in-house software is introduced. The proposed methodology can not only analyze many simpler situations (as special cases for the general model) that existent researches have covered, but also work on more complicated cases, e.g., under wind loading or some other external dynamic loading. As a result, the dynamic performance of the whole transportation systems can be assessed as a unity considering different environmental conditions seamlessly. An example of a long-span bridge-Luling Bridge near New Orleans is given to demonstrate the approach.

2. Theoretical basis

Considering the vehicle-bridge-wind system as a whole, the writers developed the vehicle-bridge-wind interaction model using virtual work theory recently (Cai and Chen 2004). In the model, the vehicle was modeled with the combinations of several rigid bodies, axle masses, springs and damping pots. Through appropriately defining parameters of the general vehicle model, different kinds of vehicles (cars, trucks, and tractor-trailers) can be modeled (Fig. 1). Wind loadings acting on both bridges and vehicles are simulated in time history. The wind loading acting on vehicles can be modeled with the quasi-static assumption or buffeting force assumptions. Non-steady wind loading acting along the bridge is simulated in time domain based on the wind velocity spectrum. The developed model is briefly revisited to help understand the unified methodology introduced hereinafter.

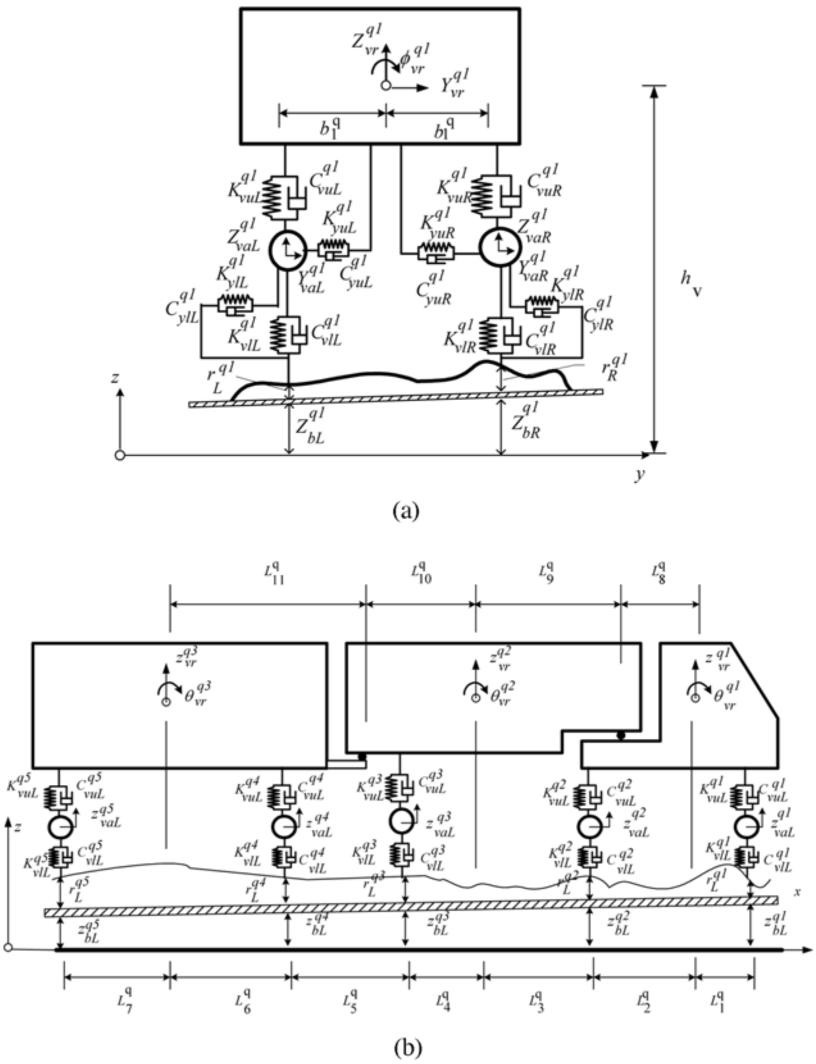

Fig. 1 General dynamic model for various vehicles: (a) cross section view and (b) elevation view

The general coupled equations of bridge-vehicle-wind system can be finally built (Fig. 2), through a lengthy derivation, as

$$\begin{bmatrix} \mathbf{M_v} & \mathbf{0} \\ \mathbf{0} & \mathbf{M_b} \end{bmatrix} \begin{Bmatrix} \ddot{\gamma}_\mathbf{v} \\ \ddot{\gamma}_\mathbf{b} \end{Bmatrix} + \begin{bmatrix} \mathbf{C_v} & \mathbf{C_{vb}} \\ \mathbf{C_{bv}} & \mathbf{C_b^s} + \mathbf{C_b^v} \end{bmatrix} \begin{Bmatrix} \dot{\gamma}_\mathbf{v} \\ \dot{\gamma}_\mathbf{b} \end{Bmatrix} + \begin{bmatrix} \mathbf{K_v} & \mathbf{K_{vb}} \\ \mathbf{K_{bv}} & \mathbf{K_b^s} + \mathbf{K_b^v} \end{bmatrix} \begin{Bmatrix} \gamma_\mathbf{v} \\ \gamma_\mathbf{b} \end{Bmatrix} = \begin{Bmatrix} \{\mathbf{F}\}_\mathbf{r}^\mathbf{v} + \{\mathbf{F}\}_\mathbf{w}^\mathbf{v} \\ \{\mathbf{F}\}_\mathbf{r}^\mathbf{b} + \{\mathbf{F}\}_\mathbf{w}^\mathbf{b} + \{\mathbf{F}\}_\mathbf{G}^\mathbf{b} \end{Bmatrix} \quad (1)$$

where subscripts "*b*" and "*v*" represent for bridge and vehicle, respectively; superscripts of "*s*" and "*v*" in the stiffness and damping terms for the bridge refer to the terms of bridge structure itself and those contributed by the vehicles, respectively; subscripts "*bv*" and "*vb*" refer to the vehicles-bridge coupled terms; "*r*", "*w*" and "*G*" represent for roughness, wind, and gravity force, respectively; n_v is

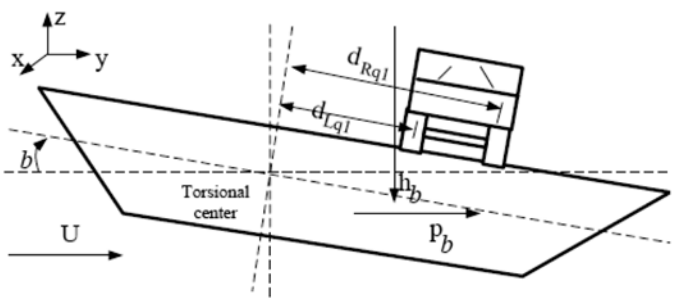

Fig. 2 Coupled model of vehicle on bridge under wind loading

the number of vehicles; and γ_v and γ_b are the displacement vectors of the vehicles and the bridge, respectively;

$$\gamma_v = \{\gamma_v^1, \ldots, \gamma_v^q, \ldots, \gamma_v^{n_v}\}^T \tag{2}$$

$$\gamma_b = \{\xi_1, \ldots, \xi_i, \ldots, \xi_n\}^T \tag{3}$$

$$\mathbf{M}_v = diag(M_v^1, M_v^2, \ldots, M_v^q, \ldots, M_v^{n_v}),\ \mathbf{K}_v = diag(\mathbf{K}_v^1, \mathbf{K}_v^2, \ldots, \mathbf{K}_v^q, \ldots, \mathbf{K}_v^{n_v});$$

$$\mathbf{C}_v = diag(\mathbf{C}_v^1, \mathbf{C}_v^2, \ldots, \mathbf{C}_v^q, \ldots, \mathbf{C}_v^{n_v}) \tag{4}$$

$$\mathbf{C}_{vb} = diag(\mathbf{C}_{vb}^1, \mathbf{C}_{vb}^2, \ldots, \mathbf{C}_{vb}^q, \ldots, \mathbf{C}_{vb}^{n_v}) \tag{5}$$

$$\mathbf{K}_{vb} = diag(\mathbf{K}_{vb}^1, \mathbf{K}_{vb}^2, \ldots, \mathbf{K}_{vb}^q, \ldots, \mathbf{K}_{vb}^{n_v}) \tag{6}$$

It is noted that Eq. (1) can be used to simulate different kinds of vehicles and different number of vehicles at different locations. Since the position of each vehicle keeps changing when it is driven, the matrices should be updated at any time step to simulate the driving process of vehicles. Such a developed model serves a general platform, which can be used in many different situations, including:

1) Vehicles driving on bridges or on roadways
   When vehicles are driven on roadways, the general platform still works after getting rid of bridge-related terms in Eq. (1).
2) Under wind loading or under other loadings
   When wind effect is weak or people just do not want to consider wind loadings, the wind-related terms in Eq. (1) can be eliminated to achieve the goal. If other external loading, e.g., seismic loading, is to be considered, only a minor revision of the right-hand loading vectors and matrices is required.
3) Bridge aerodynamic analysis or vehicle-bridge interaction analysis
   Obviously, the general platform in Eq. (1) can be used in solving traditional problems as special cases: bridge aerodynamic analyses and vehicle-bridge interactions without considering wind effects.

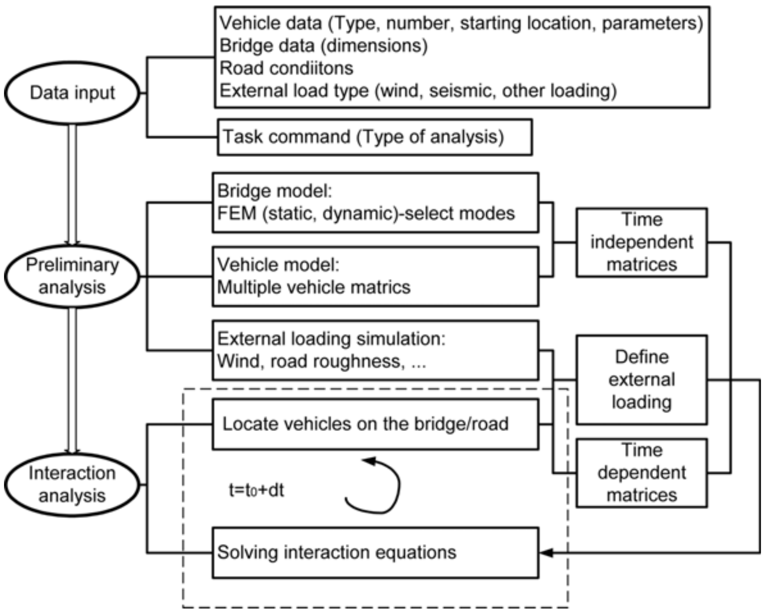

Fig. 3 Flowchart of the unified approach

3. Unified approach

The unified approach is based on the general dynamic model introduced in the previous section and is made possible for different situations through appropriately defining the computer program. The unified approach consists of three parts as shown in Fig. 3:

Part I, data input

The data input part includes four components:

Vehicle input

Vehicle number, type, starting location and other detailed dynamic data.

Bridge input

Bridge basic data (dimensions, cross-section type, material, wind tunnel measured parameters, etc.)

Other input

Road condition, external loading type considered (wind, seismic, others)

Task command

Type of analysis (A-bridge/wind, B-bridge/vehicle, C-vehicle/wind/road, D-vehicle/wind/bridge, E-vehicle/bridge/other loading, etc.)

Part II, preliminary analysis

Bridge modeling

With the bridge data, a finite element model of the bridge can be built with some commercial software, e.g., ANSYS®. Static and dynamic analyses are given to the bridge considering some necessary nonlinearity. With the help of aerodynamic key mode selection techniques (Chen *et al.* 2004), a limited number of important modes for the bridge aerodynamic analysis can be selected.

Vehicle modeling

With vehicle input information from Part I, overall multiple-vehicle dynamic matrices can be developed.

Time independent part of coupled matrices

Through combining the bridge and vehicle dynamic matrices, the time independent terms of the coupled matrices can be built. Developments of these terms are needed only once and will not be repeated in every time step to be computational efficient.

External loading simulations

According to the nature of the task, time history for external loading (e.g., wind, seismic) may be simulated. If the wind is considered, wind loading acting on vehicles can be modeled with the quasi-static assumption or buffeting force assumptions (Cai and Chen 2004). Wind loading acting along the bridge can also be simulated in time domain considering the horizontal correlation along the bridge (Cao *et al.* 2000). The road surface roughness profile should also be simulated for both bridges and roadways according to the road surface condition (Cai and Chen 2004).

Part III, coupled interaction analysis

In time domain, at any time step, the locations of the vehicles on the road or bridge are identified first. With the locations of the vehicles, time-dependent parts of the matrices and the right-hand side external loading terms can be defined. The time-dependent parts of the matrices are combined with time-independent parts of the matrices obtained in Part II to form the final coupled matrices and the external loading at the current time step. Then we can solve the differential equations and repeat the process for next time step until the end of the simulation. The time history responses of the coupled system can be retrieved.

All these above steps except for bridge finite element analysis are made possible with the in-house software developed with Matlab® by the writers. Through appropriately defining the input commands, the dynamic response of bridges and vehicles under different situations can be obtained.

4. Numerical example: dynamic interaction analysis of Luling Bridge with vehicles in time domain

The Luling Cable-stayed Bridge spans the Mississippi River connecting the towns of Luling and Destrehan in St. Charles Parish, roughly 12 miles west of New Orleans, Louisiana. The bridge length of 2745 feet (836.7 meters) is subdivided into five spans with the center span of 1222 feet (372.5 meters) between the two piers; two anchor spans of 508 feet (154.8 meters) and 495 feet (150.9 meters), respectively, and two additional approach spans of 260 feet (72 meters) each. The cables of the bridge are arranged as a double-plane fan with 12 cables in each plane. Fig. 4 shows the elevation, plane, and cross-sectional views of the Luling Bridge (Namini 1989). The bridge is a portion of Interstate 310, formally known as Interstate 410. Luling Cable-stayed Bridge is chosen as the example for two reasons. (1) This bridge was chosen in previous studies by other researchers and the data are available in the literature (Namini 1989), and (2) this bridge is in the nearby coastal area with hurricane threats, it is desirable for the local community to know its performance.

The finite element model of the bridge is first built with commercial software, ANSYS®. It is modeled with 110 nodes and 163 elements. Of all the elements, there are 24 cable elements, 20 tower elements and 119 deck elements. Truss element type "Link 8" in ANAYS® is chosen for the

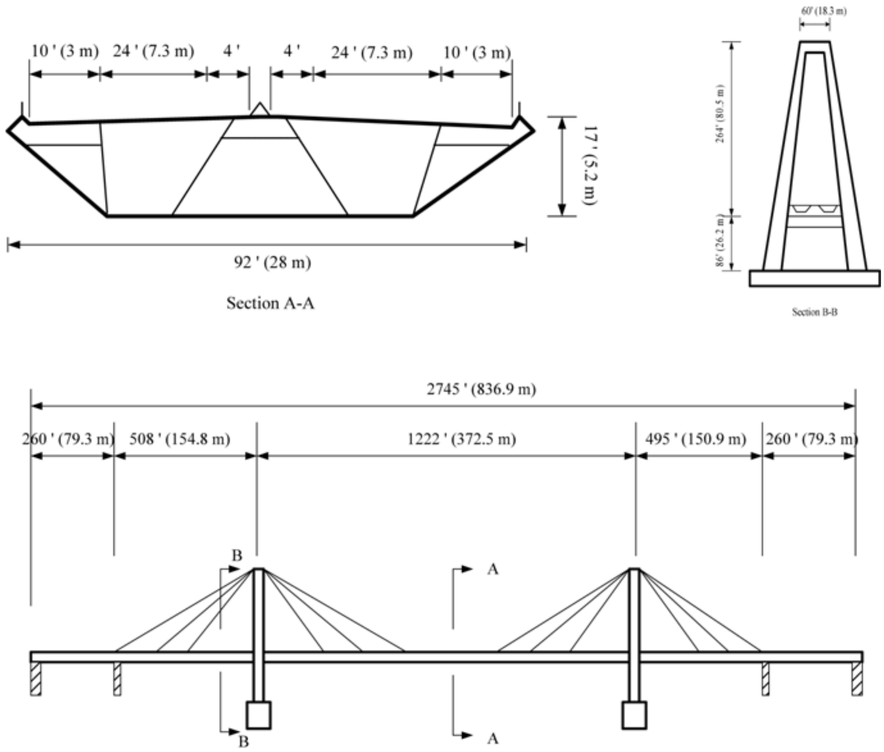

Fig. 4 Luling Bridge elevation and cross section

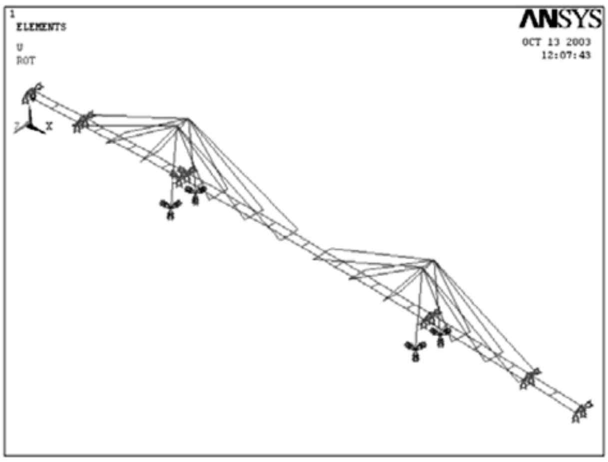

Fig. 5 Finite element analysis model of Luling Bridge with ANSYS®

cable elements and three-dimensional beam element type "Beam 4" is chosen for the tower and deck elements (Fig. 5). Since the cross section of the bridge deck is "twin-box", the deck is modeled with two longitudinal rows of elements, where each row corresponds to one box girder.

Table 1 Modal properties of Luling Bridge

No.	Natural frequency (Hz)	Mode type of deck
1	0.429	1st symmetric vertical bending
2	0.687	1st asymmetric vertical bending
3	0.704	1st symmetric lateral bending
4	1.010	2nd symmetric vertical bending
5	1.078	1st symmetric torsion
6	1.122	2nd asymmetric vertical bending
7	1.217	3rd symmetric vertical bending
8	1.673	3rd asymmetric vertical bending
9	1.780	1st asymmetric torsion
10	1.911	2nd asymmetric torsion

4.1 Nonlinear static and dynamic analysis

Modern cable-stayed bridges are usually made of steel and concrete. Steel, which is usually used for the cables, sometimes for the deck and tower, has linear deformation behavior under loading below the elastic limit. Although concrete has the nonlinear load-deformation behavior, concrete is still practically treated as linear for low service load level. In spite of the fact that the materials in the cable-stayed bridge behave in a linear way, the nonlinearity of the cable-stayed bridge still exists due to: nonlinear sag effect induced by its self-weight; axial-flexural interaction when the deck and tower are applied simultaneous large bending moments and axial forces; and large displacements due to the high flexibility (Namini 1989).

To account for the nonlinearity of the sagging cable due to its own weight, equivalent modulus of elasticity of the cable is considered. The Ernst equation for the equivalent modulus of elasticity E_{eq} is commonly adopted as (Namini 1989):

$$E_{eq} = \frac{E}{1 + \frac{(w_d l_h)^2}{12 T_c^3} E A_c} \tag{7}$$

where

w_d = uniformly distributed cable weight

A_c = cross-sectional area of the cable

T_c = tensile force of the cable

E = modulus of elasticity

In the static analysis, the initial cable forces are substituted into Eq. (7) and the equivalent modulus is then adopted in the static analysis. Other nonlinear effects such as axial-flexural interaction and large displacement are assumed to be small and not considered for demonstration purpose. Under the dead load of the bridge, the bridge will have static deformation, and correspondingly, the cable forces will also changed compared with un-deformed configuration. A dynamic analysis should be carried out based on the deformed bridge model to obtain more realistic results.

After the static analysis, the new cable forces are obtained under the dead load action. New

Table 2 Main parameters for aerodynamic analysis for Luling Bridge (Namini 1989)

Total span (m)	837	Lift coefficient at 0° attack angle	−0.14
Width of the deck (m)	20	Drag coefficient at 0° attack angle	0.3
Clearance above water (m)	40	Pitching coefficient at 0° attack angle	−1.3
Equivalent mass per length (× 10^3 kg/m)	17.69	$(\partial C_L/\partial\alpha)\vert_{\alpha=0°}$	4.2
Equivalent inertial moment of mass per length (× 10^3 kg · m)	796.05	$(\partial C_D/\partial\alpha)\vert_{\alpha=0°}$	−0.1
Structural damping ratio	0.005	$(\partial C_M/\partial\alpha)\vert_{\alpha=0°}$	1.13

equivalent modulus of elasticity is then calculated considering the static deformation and will be used in the following dynamic analysis.

The dynamic analysis here is to extract the natural modes in the vertical bending, lateral bending and torsion directions. Subspace solution option is used in the modal analysis of ANSYS®. While the natural frequencies and mode shapes for the first 20 modes are calculated, only the natural frequencies of the first 10 modes are shown in Table 1.

4.2 Validation of the aerodynamic analysis model

As introduced earlier in Parts II and III, a general interaction model can be built with Matlab based on the finite element analysis results. Before the interaction model is used, some validations should be made. Bridge/vehicle interaction techniques (without wind) are pretty well reported. Since the complete data of some bridges can not be obtained from the literature, the comparison can not be made using the exact same bridge data between the present model and other results from peer researchers. The program, however, has already been tested and validated for vehicle dynamics through comparing with peer publications (Guo and Xu 2001). Bridge aerodynamic analysis is more complicated and has not been tested for the current complicated interaction analysis model before. Hence, validations of the bridge aerodynamic analysis function of the program are given as follows. In order to compare with the existent results without considering vehicles, the task command of the unified approach should be set as "bridge aerodynamics", namely the vehicle/bridge coupled terms of the unified approach are temporarily removed. Table 2 gives the parameters for aerodynamic analysis of Luling Bridge. With the simulated wind velocity time history, the time history buffeting responses of the bridge in vertical and torsional directions can be obtained.

Bridge aerodynamic buffeting analysis usually can be conducted in two domains, one is in the traditional frequency domain and the other one is in the time domain. In order to validate the bridge aerodynamic analysis function in the time history as proposed, the traditional bridge buffeting analysis in frequency domain was also conducted for comparison purpose. Fig. 6 shows the response spectrum converted from the time history of the response and the response spectrum obtained in the frequency domain directly, for comparison purposes. The results suggest that these two approaches can lead to pretty similar results with some small difference. It is found that the proposed unified approach in time domain can give pretty reasonable (theoretically more accurate than in frequency domain) buffeting response prediction.

The flutter stability of Luling Bridge is also studied using a complex eigenvalue approach (Chen and Cai 2003). With the increase of wind speed, the modal frequencies and modal damping ratios

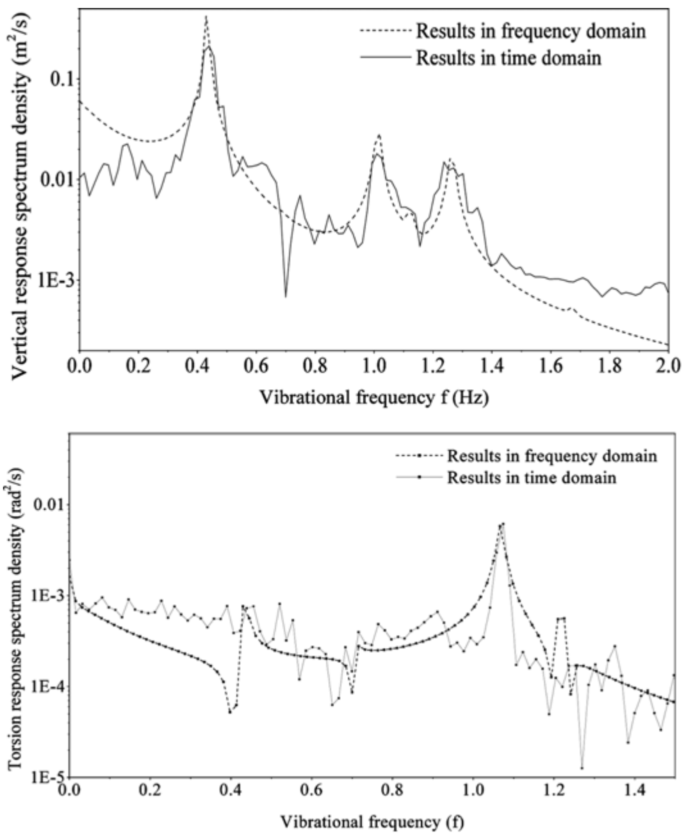

Fig. 6 Response spectrum comparison when $U = 30$ m/s

are obtained as shown in Fig. 7. The flutter critical wind speed is identified as 89 m/s (322 km/hr), which is close to the results obtained in Namini (1989) (342 Km/hr). It can be found that the proposed unified approach can also lead to reasonable prediction of flutter stability performance for the bridge.

4.3 Vehicle-bridge-wind interaction analysis

The dynamic performance of the coupled system is numerically analyzed during the whole process when vehicles are driven though the bridge. The study simulates the period from vehicles moving on the road, then on the bridge and finally off the bridge. During the whole driving period, wind action always remains. The same number and types of vehicles as in Cai and Chen (2004) are used in the present study. It is assumed that the distance before entering the bridge is 200 meter. Since the bridge's total length is 837 m, the vehicle will have moved 1037 meter when it gets off the bridge. In this example, the driving speed is 20 m/s and the vehicle response of a total time period of 1 minute is predicted. Consequently, the distance after the vehicle gets off the bridge will be (20*60−1037 = 163 m).

The relative response of the vehicle should be studied to investigate the dynamic interaction effects. The bridge responses at the contact point with the vehicle tires are identified at any time

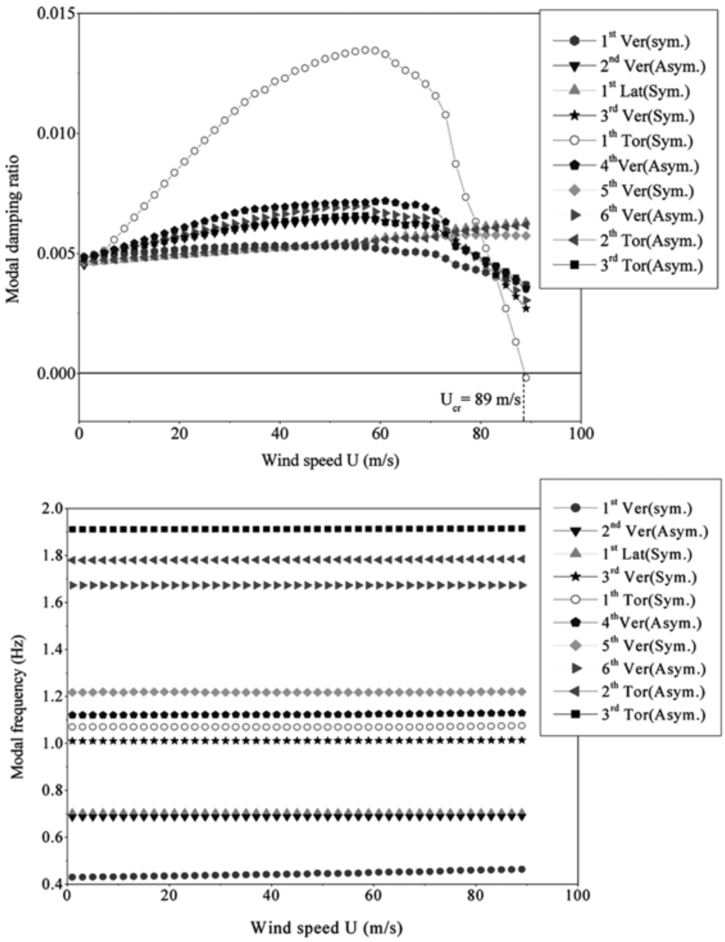

Fig. 7 Modal damping ratio and frequency of Luling Bridge versus wind speeds

with the moving of the vehicle. The absolute response of the vehicle at any time should deduct the corresponding bridge contact point response to get the vehicle relative response. The relative response of the vehicle on Luling Bridge is shown in Figs. 8-10. Labels of “On bridge” and “On road” as shown in Figs. 8-10 refer to the different periods when vehicles are driven through the bridge. Through the comparison of the relative responses of the vehicles at different time periods, it can be found from these figures that the stronger interactions between the vehicles and the bridge in vertical and torsion directions exist when vehicles arrive at the middle of the main span of the bridge. Such interaction effects may differ from bridge to bridge due to their different dynamic properties of the bidges.

The dynamic response of the Luling Bridge with vehicles running on it is also compared with the situation when there is no vehicle on the bridge. Here, we also choose the location of the middle point of the main span as the point of interests. Figs. 11-12 show the time histories of the bridge *w/* and *w/o* the vehicle. It is found that obvious difference of the response is found especially when the vehicle get to the middle range of the bridge’s main span. The bridge response has slightly suppressed with the existence of the vehicle running near the middle range of the main span in the

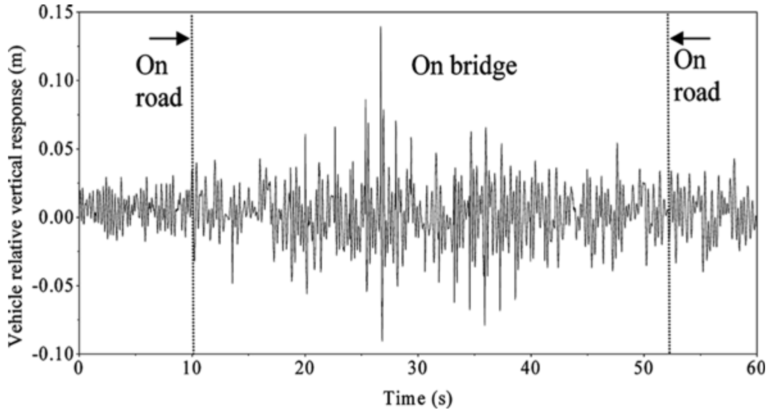

Fig. 8 Vehicle relative vertical response on the bridge when U = 30 m/s and V = 20 m/s

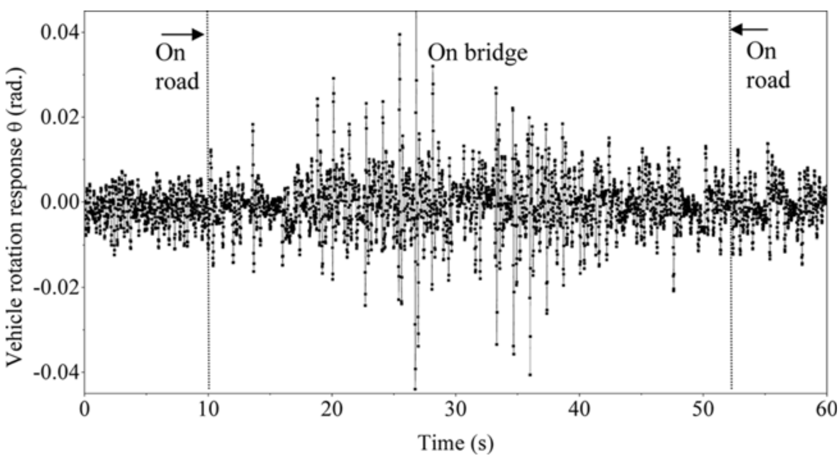

Fig. 9 Vehicle relative pitching response on the bridge when U = 30 m/s and V = 20 m/s

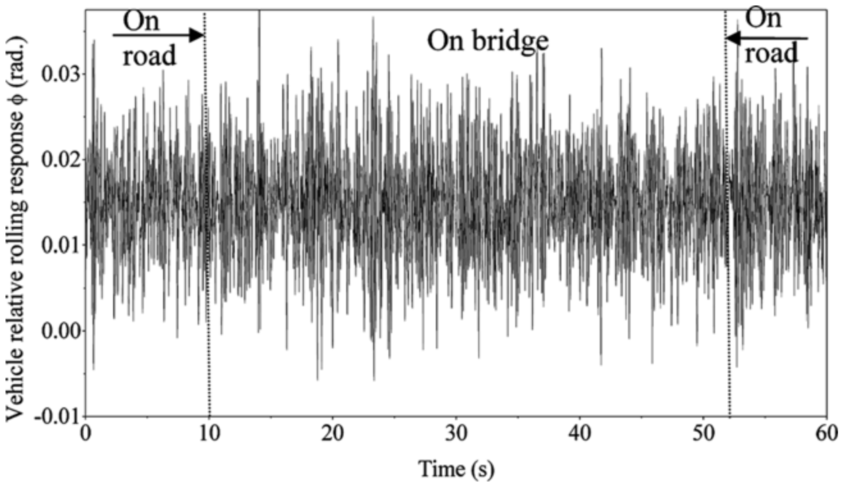

Fig. 10 Vehicle relative rolling response on the bridge when U = 30 m/s and V = 20 m/s

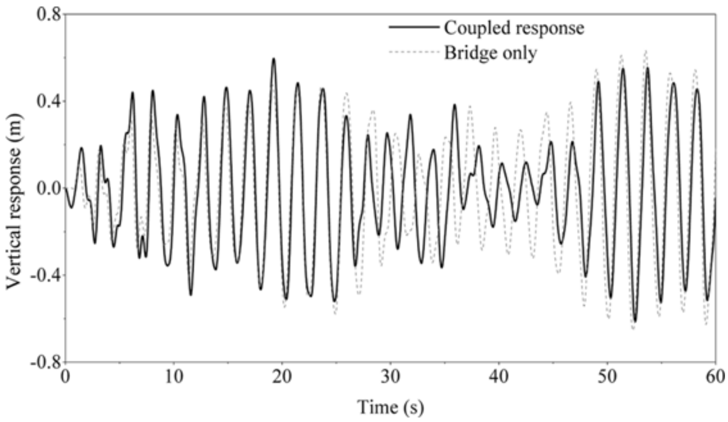

Fig. 11 Bridge vertical response *w/* and *w/o* vehicles when U = 30 m/s and V = 20 m/s

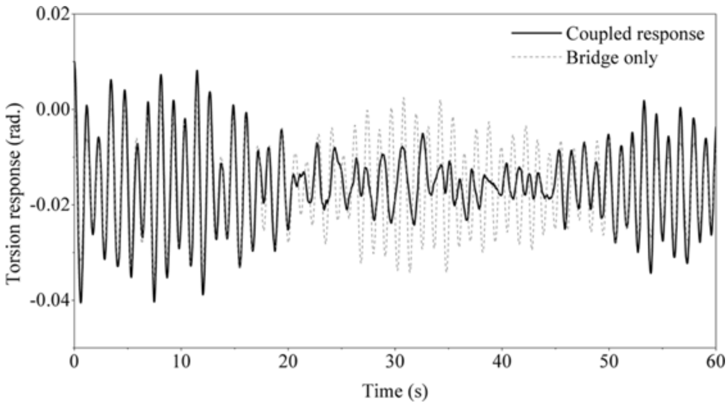

Fig. 12 Bridge torsion response *w/* and *w/o* vehicles when U = 30 m/s and V = 20 m/s

vertical direction (Fig. 11) and more significant suppression can be observed for the torsion response (Fig. 12) when the vehicle arrives at the middle of the span (around t = 31 seconds). It can be concluded that the moving vehicle can affect the bridge dynamic response when the vehicle drives to some specific local locations on the bridge. In this example, the suppression effects happen at the middle point of main span.

5. Conclusions

The paper presented a unified dynamic analysis model of bridge-vehicle system under external loading. Such an approach can serve as a general platform to predict the dynamic responses of bridges and vehicles in various situations as a unity. Different from existent isolated approaches targeting at traditional topics, e.g., vehicle/bridge, bridge/wind, vehicle/road and vehicle/wind interaction analysis separately, the proposed unified approach can be utilized for the complicated vehicle/bridge/wind interaction, as well as all above mentioned traditional dynamic analysis, as a unity. Moreover, the proposed general platform can be used for situations with different types of vehicles, different number of vehicles, different vehicle locations on the bridge/road and different external loadings. The unified approach relies on both commercial software and in-house software. Through appropriately setting the function of the in-house program according to the research needs, various analyses can be easily carried out. The analysis of Luling Bridge was given for the purpose of demonstration of the whole process.

Acknowledgements

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