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Three-dimensional structural design based on cellular automata simulation

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Abstract. This paper describes the design scheme of the three-dimensional structures based on the concept of the cellular automata simulation. The cellular automata simulation is performed according to the local rule. In this paper, the local rule is derived in the mathematical formulation from the optimization problem. The cell density is taken as the design variable. Two objective functions are defined for reducing the total weight of the structure and obtaining the fully stressed structure. The constraint condition is defined for defining the local rule. The penalty function is defined from the objective functions and the constraint condition. Minimization of the penalty function with respect to the design parameter leads to the local rule. The derived rule is applied to the design of the three-dimensional structure first. The final structure can be obtained successfully. However, the computational cost is expensive. So, in order to reduce the computational cost, the material parameters c_1 and c_2 and the value of the cell rejection criterion (CRC) are changed. The results show that the computational cost depends on the parameters and the CRC value.

Keywords: structural design; cellular automata; local rule; finite element method.

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1. Introduction

The theory of the finite automata has been presented by von Neumann (1951). After that, it is understood that, according to Ulam's suggestions, he has developed the finite automata theory to the cellular automata theory (von Neumann 1951, 1966, Waldrop 1992). In the computer simulation based on the cellular automata theory, the time interval of the simulation is divided uniformly by time steps and the simulation domain is also divided into uniform square cells. The state value of the cell at the previous time step is updated according to the local rule from the values at the cell and its neighboring cells in the adjacent time step. Since the local rule is defined as the relationship between the state values at the cell and its neighboring cells, the governing equations of the simulated phenomena are not necessary in the cellular automata simulation. This is one of the attractive features of the cellular automata simulation and therefore, it has been successfully applied to the simulation of the so-called complex systems or the complexity of which governing equations are not defined clearly. Recently, some researchers have been studying the topology and the shape optimization method of the structures based on the cellular automata model (Xie and Steven 1997, Sakamoto and Oda 1995, Kundu *et al.* 1997, Inou *et al.* 1994, 1998, Payten *et al.* 1998, Kita and Toyoda 2000).

In the field of the structural optimization, the term "Cellular automata" has been used firstly in Inou's works (Inou *et al.* 1994, 1998). In the papers, the design domain is divided into many small cells and the von Mises equivalent stress distribution on the whole domain is estimated by the finite element method. Then, the reference stress which is individually specified at each cell is updated by applying a local rule to the stress distribution. Young's modulus for each cell is taken as a design variable, which is modified so that the equivalent stress at the updated cell is to be equal to the reference stress. The cells with relative small Young's modulus are removed and therefore, the shape and topology of the structures are modified. The local rule in this study is defined as the non-linear relationship between the equivalent stress distribution and Young's modulus. Since the local rule is derived from the numerical experience, the mathematical relationship between the rule and the optimization problem is not obvious. So, for deriving the local rules, several formulations have been presented.

Oda *et al.* have presented the formulation that the local rule has been defined according to the artificial neural network model (Sakamoto and Oda 1995, Kundu *et al.* 1997). The neural model was learned according to the back propagation algorithm.

Payton *et al.* (1998) have defined the local rule from the remodeling theory of the anatomy such as bones and woods.

One of the present authors and his co-authors have presented the formulation that the local rule has been mathematically derived from the optimization problem (Kita and Toyoda 2000). After defining the optimization problem, the local rule was derived from the penalty function defined from the objective functions and the constraint conditions of the optimization problem. They applied the local rule to the shape and topology optimization of the two-dimensional structures.

Besides, we would like to note Evolutionary Structural Optimization (ESO) (Xie and Steven 1997). The CA-based optimization schemes have the similar algorithm to the ESO although they have been developed individually and concurrently. In the ESO, the design domain was divided into small square cells and the stress state was evaluated by using the finite element method. When the equivalent stress at a cell is smaller than the threshold specified by a user in advance, the cell is deleted. The process was repeated until satisfying the convergence criterion.

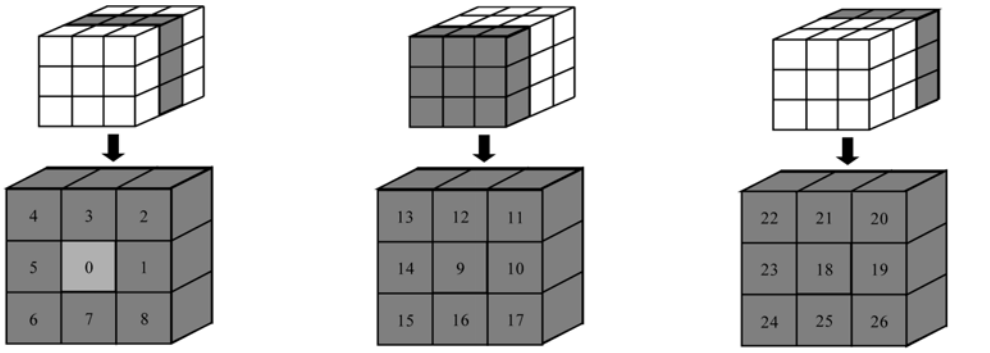
In this paper, we will develop the formulation presented in Kita and Toyoda (2000) to the design of the three-dimensional structures. For this purpose, the distribution of the material density is taken as the design variable, instead of the cell thickness in the two-dimensional structure. Two objective functions are defined to minimize the total weight of the structure and to obtain even equivalent stress distribution on the whole structure. The constraint condition is introduced to define the local rule. The local rule is derived from the penalty function which is defined from the objective functions and the constraint conditions. We also would like to discuss the algorithm to reduce the computational cost.

This paper is organized as follows. In section 2, the representation of the three-dimensional structure by using cells is described. The finite element formulation is also described briefly for convenience of the explanation of the local rule. In section 3, the local rule is defined and the optimization algorithm is described. In section 4, the algorithm is applied to the design of a three-dimensional structure. It is shown in section 5 that an improved algorithm for reducing the computational cost. The obtained results are summarized in section 6.

2. Cellular representation of structures and finite element formulation

2.1 Cellular representation of structure

The design domain is discretized with small cubic cells. In the finite element analysis, the cubic cells play a role of finite elements. In the cellular automata simulation, the density of the cell material is updated according to so-called "local rule" or "template". The local rule is defined as the nonlinear relationship of the physical states of the neighboring cells. In this study, the neighboring cells are defined by means of so-called "Moore neighborhood". Fig. 1 indicates the definition of the neighborhood cells according to the Moor neighborhood. The cell "0" means the cell of which density is updated with the local rule. In the Moor neighborhood concept, all cells neighboring the cell "0" are considered as the neighborhood of the cell "0". The neighborhood cells are numbered from 1 to 26. When the cell "0" is placed on the boundary, no cell exists outside the design domain. In this case, the cells within the design domain are taken into account.



(a) Neighborhood of center (b) Neighborhood of front (c) Neighborhood of back

Fig. 1 Neighborhoods

2.2 Finite element analysis (Zienkiewicz and Taylor 1991)

The symbols Ω, Γ_u and Γ_t denote the object domain, its displacement- and the traction-specified boundaries, respectively. The whole boundary Γ is by $\Gamma = \Gamma_u \cap \Gamma_t$.

Without the external force, the principle of the virtual work is given as

$$\int_{\Omega} \delta\varepsilon^T \sigma d\Omega = \int_{\Gamma_t} \delta u^T t d\Gamma \tag{1}$$

where u, t, ε and σ denote the displacement, traction, strain and stress vectors, respectively. The variables following the symbol δ denote the virtual values. The superscript T means the transposition of the matrix and the vector. The following relationships exist between the

Displacement-Strain relationship: $$\varepsilon = Au \tag{2}$$

Hooke's law: $$\sigma = D\varepsilon \equiv E\tilde{D}\varepsilon \tag{3}$$

Cauchy relationship: $$t_i = \sigma_{ij} n_j \tag{4}$$

where A, D and n_j denote the matrix of partial differential operators, the elastic matrix and x_j-directional component of normal vector on boundary, respectively.

Discretizing Eq. (1) with finite elements leads to

$$KU = f \tag{5}$$

where K, U and f denote stiffness matrix, the vector of nodal displacements and nodal force vector, respectively.

3. Optimization algorithm

3.1 Definition of local rule

The density distribution of the structure is taken as the design variable. The material density is considered to be constant within each cell (element).

Two kinds of objective functions are considered. The first objective function W_1 is defined to minimize the total weight and the second one W_2 is defined to make the von Mises equivalent stress at each cell to approach the reference stress;

$$W_1 = \frac{1}{2}\left(\frac{\tilde{\rho}}{\tilde{\rho}_0}\right)^2 \equiv \frac{1}{2}\rho^2 \tag{6}$$

$$W_2 = \frac{1}{2}\left(\frac{\tilde{\sigma}}{\sigma_c} - 1\right)^2 \equiv \frac{1}{2}(\sigma - 1)^2 \tag{7}$$

where $\tilde{\rho}$ and $\tilde{\rho}_0$ denote the density of cell "0" and its initial value, respectively. $\tilde{\sigma}$ and σ_c denote the von Mises equivalent stress at cell "0" and the reference stress defined in advance, respectively.

The reference stress σ_c is defined as the yield stress of the material in the actual problem. In the numerical examples of this paper, the reference stress σ_c is given be a user.

In this study, the following special constraint condition is introduced to derive the local rule. The addition of the constraint condition enable to solve the optimization problem at each element individually and as a result, the local rule can be derived.

The constraint condition is defined as

$$g_j = \frac{\tilde{\sigma}_j}{\tilde{\sigma}_j^0} - 1 \equiv \sigma_j - 1 = 0 \ (j = 1, 2, \ldots, N) \tag{8}$$

where $\tilde{\sigma}_j$ and $\tilde{\sigma}_j^0$ denote the von Mises equivalent stress at the neighborhood cell j at the present time step t and its previous time step $t-1$, respectively. The value N denotes the total number of the neighborhood cells of cell "0". When cell "0" is in the domain and not on the boundary, $N = 26$. When cell "0" is on the boundary, $N < 26$ because no neighborhood cell exists outside domain.

When the constraint (8) is satisfied, it means that the stress state at the neighborhood cells dose not depend on the density of cell "0". In that case, the local objective functions W_1 and W_2 can be optimized individually at each cell.

The penalty function can be defined from Eqs. (6), (7) and (8) as follows.

$$W = \alpha W_1 + \beta W_2 + \frac{p}{2}\sum_{j=1}^{N} g_j^2 = \frac{1}{2}\alpha\rho^2 + \frac{1}{2}\beta(\sigma - 1)^2 + \frac{p}{2}\sum_{j=1}^{N}(\sigma_j - 1)^2 \tag{9}$$

where p denotes the penalty parameter and the weight parameters α and β are defined as

$$\alpha + \beta = 1, \qquad \beta = \begin{cases} \sigma & (\sigma < 1) \\ 1 & (\sigma \geq 1) \end{cases} \tag{10}$$

Expanding and linearizing σ and σ_j around $\rho + \delta\rho$ leads to

$$\sigma = \sigma + \frac{\partial\sigma}{\partial\rho}\delta\rho \equiv \sigma + \sigma' \tag{11}$$

$$\sigma_j = \sigma_j + \frac{\partial\sigma_j}{\partial\rho}\delta\rho \equiv \sigma_j + \sigma_j' \tag{12}$$

where $(\,)' = \partial/\partial\rho$. By using the above equations, Eq. (9) is expanded around $\rho + \delta\rho$ and linearized as follows.

$$W(\rho + \delta\rho) = \frac{1}{2}\alpha(\rho + \delta\rho)^2 + \frac{1}{2}\beta(\sigma + \sigma' - 1)^2 + \frac{p}{2}\sum_{j=1}^{N}(\sigma_j + \sigma_j' - 1)^2 \tag{13}$$

Stationalizing Eq. (9) with respect to $\delta\rho$ leads to

$$\frac{\partial W}{\partial\rho} = 0 \tag{14}$$

$$\delta\rho = -\frac{\alpha\rho + \beta(\sigma - 1)\sigma' + p\sum(\sigma_j - 1)\sigma_j'}{\alpha + \beta\sigma'^2 + p\sum\sigma_j'^2} \tag{15}$$

By the way, there exist so-called "stress sensitivities" σ' and σ_j' in Eq. (15). The sensitivities are calculated as follows.

According to the density formulation (Hassani and Hinton 1999), the relationship between the density ρ and the Young's modulus E is assumed as

$$E = c_1 \rho^{c_2} \tag{16}$$

where the parameters c_1 and c_2 are specified by a user. Differentiating the both sides of Eq. (3) with respect to ρ and substituting Eq. (16) leads to

$$\sigma' = E' \tilde{D} \varepsilon = \frac{c_2}{\rho} \sigma \tag{17}$$

Sensitivity σ_j' is estimated as follows. The principle of virtual work in the cell j is given as

$$\int_{\Omega_j} \delta \varepsilon^T \sigma d\Omega = \int_{\Gamma_j} \delta u^T t d\Gamma \tag{18}$$

where Ω_j and Γ_j denote the domain and the boundary of the cell j, respectively. The traction vector t in Eq. (18) denotes the effect of the stress states of the neighborhood cells. Discretizing Eq. (18) with finite elements leads to

$$K_j U_j = f_j \tag{19}$$

where K_j, U_j and f_j denote the stiffness matrix, the nodal displacement vector and the nodal traction vector of neighborhood cell j, respectively.

The material density of the cell ρ affects the vector f_j of Eq. (19). For example, the vector f_j decreases according to the increase of the density ρ and vice versa. Therefore, it is assumed that the vector f_j is inversely proportional to ρ.

$$f_j \rho = const. \tag{20}$$

Differentiating both sides of the above equation with respect to the density ρ leads to

$$f_j' \rho + f_j = 0$$

$$f_j' = -\frac{f_j}{\rho} \tag{21}$$

Since the density ρ is at cell "0" and the matrix K_j in Eq. (19) is related to the other cell k than cell "0", the matrix K_j dose not depend on ρ. Differentiating Eq. (19) with respect to ρ leads to

$$K_j U_j' = f_j'$$

$$U_j' = (K_j)^{-1} f_j' \tag{22}$$

Substituting Eq. (21) to the above equation leads to

$$U_j' = -\frac{U_j}{\rho} \tag{23}$$

The following equation is derived from Eq. (3);

$$\sigma_j' = DBU_j = DB\left(-\frac{U_j}{\rho}\right) = -\frac{\sigma_j}{\rho} \tag{24}$$

where the matrix B is the matrix composed of the differential operators in the Cartesian coordinates. Finally, substituting Eqs. (17) and (24) into Eq. (15) lead to

$$\delta\rho = -\frac{\alpha\rho^2 + \beta c_2(\sigma - 1)\sigma - p\sum(\sigma_j - 1)\sigma_j}{\alpha\rho^2 + \beta c_2^2\sigma^2 + p\sum\sigma_j^2} \cdot \rho \tag{25}$$

The value of $\delta\rho$ is estimated from Eq. (25) and density ρ is updated by

$$\rho^{k+1} = \rho^k + \delta\rho \tag{26}$$

where the superscript k denotes the iterative time step.

3.2 Cell rejection criterion (CRC)

The cell rejection criterion (CRC) is introduced in order to improve the convergence speed of the solutions. The CRC value is defined as the threshold related to the initial cell density ρ^0. If the cell density is smaller than the CRC value specified in advance, the cell is deleted.

3.3 Convergence criterion

A convergence criterion should be introduced to confirm whether the final structures can be obtained or not. The criterion is defined as the convergence rate of the maximum or mean stress and the total weight of the structure. However, in the numerical examples, the criterion is not adopted.

3.4 Computational algorithm

The computational algorithm of the present method is as follows

1. Specify the initial data such as the object domain, the boundary conditions, the material parameters, the design parameters c_1, c_2, CRC value and so on.
2. Perform a stress analysis.
3. Check the convergence criterion. If the criterion is satisfied, the solution is found.
4. Estimate the value of $\delta\rho$ from Eq. (25).
5. Update the design variable according to Eq. (26).
6. Delete the unnecessary cells according to the cell rejection criterion (CRC).
7. Return to the step 2.

Finite element analysis is performed with FElt version 3.06 (The FElt home page http://www.sourceforge.net 2000) and PC/AT compatible computer.

3.5 Comparison of local rules

The algorithm of the present method is similar to the Evolutionary Structural Optimization (ESO) method by Xie and Steven (1997) and the optimality criterion method (Hassani and Hinton 1999). So, the present method is compared with these two methods.

Firstly, the relationship between the present method and the ESO method is discussed. In the ESO method, the design domain is uniformly divided by small square cells (in 2D) or cubes (in 3D). After the stress analysis with the finite element method, one deletes the cells at which the equivalent stress is smaller than the threshold named as "the cell rejection criterion (CRC)" value. We can say that, in the ESO method, the CRC value just acts as the local rule and therefore, the final profile depends on the value. In the present method, the shape and topology modification is performed according to the local rule, and the CRC value is just used for the acceleration of the convergence speed of the shape and topology optimization.

Next, the present method is compared with the optimality criterion for the design of the fully stressed structure (Hassani and Hinton 1999). In the optimality criterion method, the expression of the optimality criterion is firstly defined and the optimal structure is determined by the iterative scheme so that the criterion is satisfied. The optimality criterion for the fully stressed design is given as

$$z_j^{k+1} = z_j^k \cdot \frac{\tilde{\sigma}}{\sigma_c} \tag{27}$$

where the value z_j^k is the design variable at the iterative step k, the values $\tilde{\sigma}$ and σ_c denote the equivalent stress and the yield stress of the material, respectively. The most important difference between the local rule (25) and Eq. (27) is that there exist the terms related to the stress values at the neighborhood cells in the local rule (25).

4. Numerical example

The three-dimensional continuum structure shown in Fig. 2 is considered. The plane ABCD is fixed in all directions and the load P is applied at the center of the plane EFGH in y-direction. The

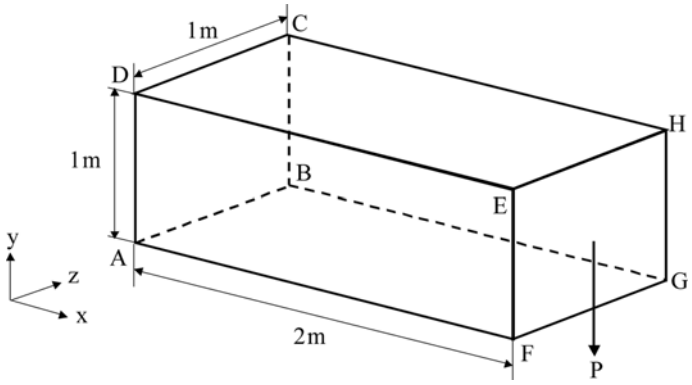


Fig. 2 Object under consideration

Table 1 Design parameters

Number of cells	$20 \times 10 \times 10$
Initial Young's modulus	$E = 10^{10}$ Pa
Poisson's ratio	$\nu = 0.3$
Initial cell density	$\rho^0 = 1.0$ kg/m^3
Force	$P = 1000$ N
Penalty coefficient	$p = 1$
Initial maximum stress	$\sigma^0_{\max} = 6.0 \times 10^2$N/m^2
Reference stress	$\sigma_c = 0.8 \times \sigma^0_{\max}$
Material parameters	$c_1 = 1.0 \times 10^{10}$, $c_2 = 1.0$
Cell Rejection Criterion	CRC $= 0.1 \times \rho^0$

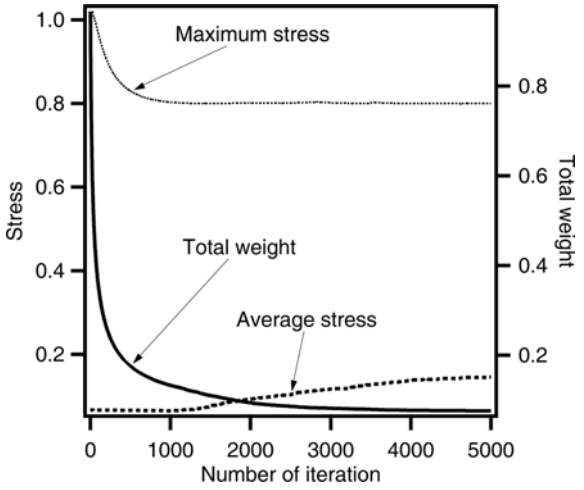


Fig. 3 Convergence histories of stress and total weight

simulation parameters are shown in Table 1. The reference stress is specified as the 80% of the initial maximum stress $\sigma^0_{\max}$, which is the maximum stress yields in the finite element analysis of the initial assumed structure. The material parameters c_1 and c_2 can be selected by a user. Now, they are taken as $c_1 = 1.0 \times 10^{10}$ and $c_2 = 1.0$. The cell rejection criterion (CRC) value is taken as CRC = $0.1 \times \rho_0$, where the value ρ_0 is the initial density of the cell. In this case, the cells of which density is smaller than 10% of the initial density are deleted.

Fig. 3 illustrates the convergence histories of the maximum and the mean stresses and the total weight of the structure. The abscissa and the left- and the right-ordinates denote the number of the iteration and the stress divided by the value $\sigma^0_{\max}$ and the total weight divided by the initial total weight, respectively. The total weight decreases drastically to the 300th iteration step and converges to about 8% of the initial total weight. The maximum stress $\sigma_{\max}$ slightly increases to the 10th iterative step and then, gradually decreases to the reference stress. The convergence speed of the mean stress is slower than those of the maximum stress and the total weight.

Figs. 4 and 5 illustrate the side and the elevation views of the final structure at 5000th iterative step, respectively.

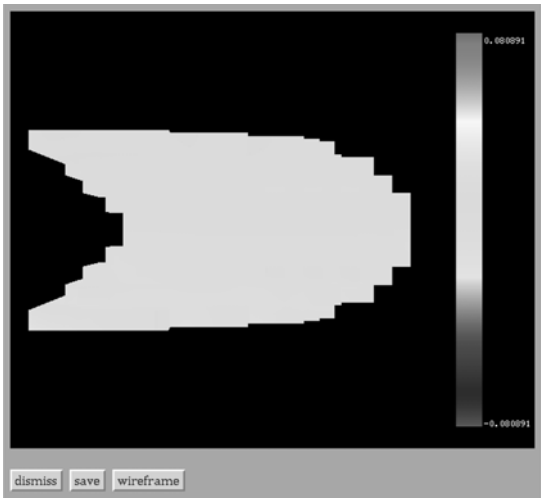


Fig. 4 Edge view of a final structure

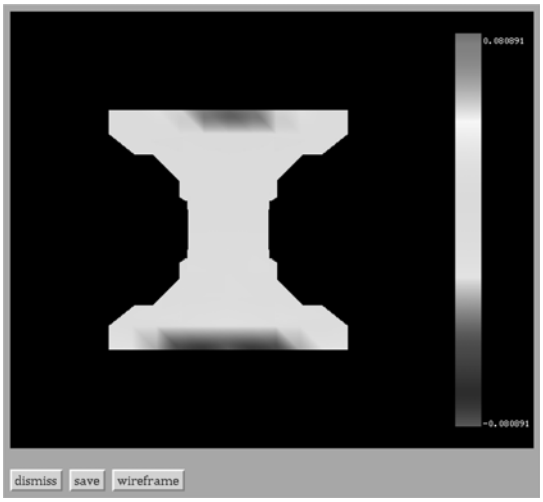


Fig. 5 Elevation view of a final structure

5. Improvement of convergence speed

The results in the section 4 show that the convergence speed of the present method is very slow. So, in this section, we would like to discuss the improvement of the convergence speed by changing the material parameter c_2 and the CRC value.

5.1 Discussion about material parameter c_2

In the numerical examples in section 4, the material parameter c_2 is specified as $c_2 = 1.0$. In this section, the parameter c_2 is taken as $c_2 = 2.0$ or 3.0. As a result, one defines the non-linear

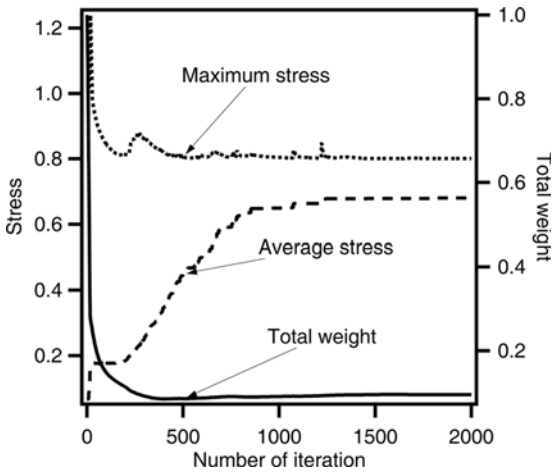


Fig. 6 Convergence histories of stress and total weight ($c_2 = 2.0$)

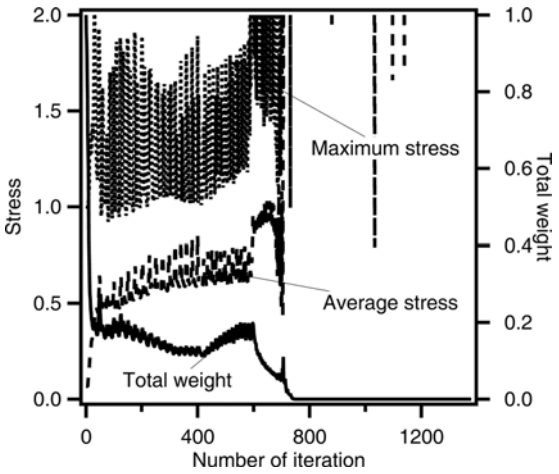


Fig. 7 Convergence histories of stress and total weight ($c_2 = 3.0$)

relationship between the density and the Young's modulus. The object domain and the boundary conditions are specified as shown in Fig. 2. The design parameters are taken as shown in Table 1 except for c_2. In the cases of $c_2 = 2.0$ and 3.0, the convergence histories of the stresses and the total weight are shown in Figs. 6 and 7, respectively.

In the case of $c_2 = 2.0$, as shown in Fig. 6, the maximum stress converges to the reference stress at the 500th iterative step and the mean stress converges to about 68% of the initial maximum stress at the 1200th iterative step. We can say that the result in the case of $c_2 = 2.0$ is better than that in the case of $c_2 = 1.0$.

As shown in Fig. 7, all quantities diverge in the case of $c_2 = 3.0$. We can say that the parameter c_2 is very important for the accurate and the stable convergence of the present method.

As the results, we will take the parameter $c_2 = 2.0$ in the following numerical examples.

5.2 Discussion about cell rejection criterion (CRC)

In section 4, the cell rejection criterion (CRC) is taken as CRC = $\rho_0 \times 0.1$. In this section, it is taken as CRC = $\rho_0 \times 0.5$ and CRC = $\rho_0 \times 0.8$ for the improvement of the convergence speed. The convergence properties in the case of CRC = $\rho_0 \times 0.5$ and CRC = $\rho_0 \times 0.8$ are illustrated in Figs. 8 and 9, respectively.

In the case of CRC = $\rho_0 \times 0.5$, as shown in Fig. 8, the maximum stress converges to the reference stress at the 700th iterative step. The mean stress rapidly increases to the 400th iterative step. After that, although the convergence speed descends slightly, the mean stress converges to the reference stress. The total weight of the structure rapidly decreases to the 100th iterative step, and after that, converges to 9.7% of the initial total weight.

In the case of CRC = $\rho_0 \times 0.8$, as shown in Fig. 9, the maximum stress converges to the reference stress to the 1100th iterative step. The total weight converges to 10% of the initial total weight to the 600th iterative step. We notice that the convergence speeds in the case of CRC = $\rho_0 \times 0.8$ are slower than those in the case of CRC = $\rho_0 \times 0.5$.

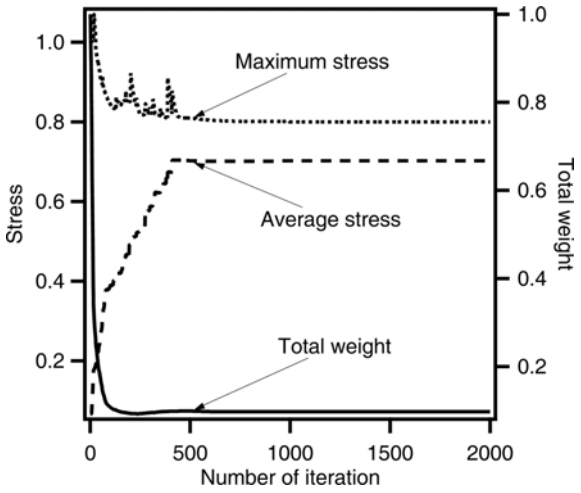


Fig. 8 Convergence histories of stress and total weight (CRC = $\rho_0 \times 0.5$)

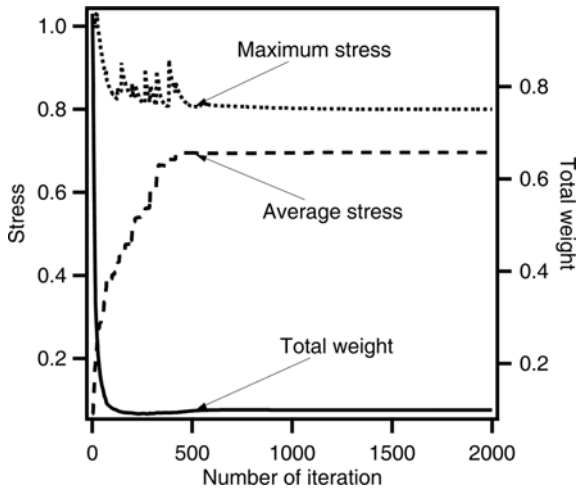


Fig. 9 Convergence histories of stress and total weight (CRC = $\rho_0 \times 0.8$)

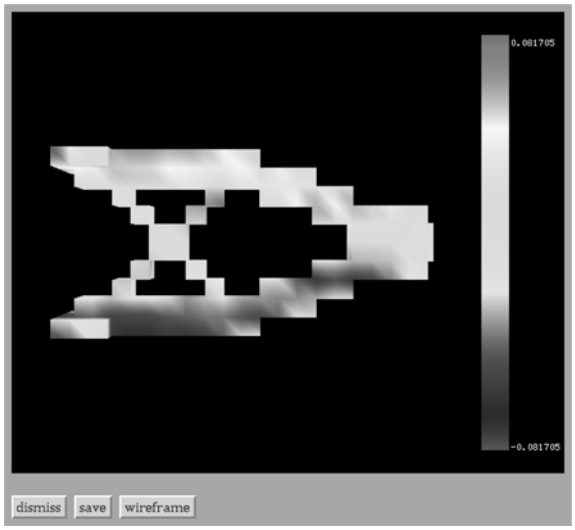


Fig. 10 Edge view of a final design (CRC = $\rho_0 \times 0.8$)

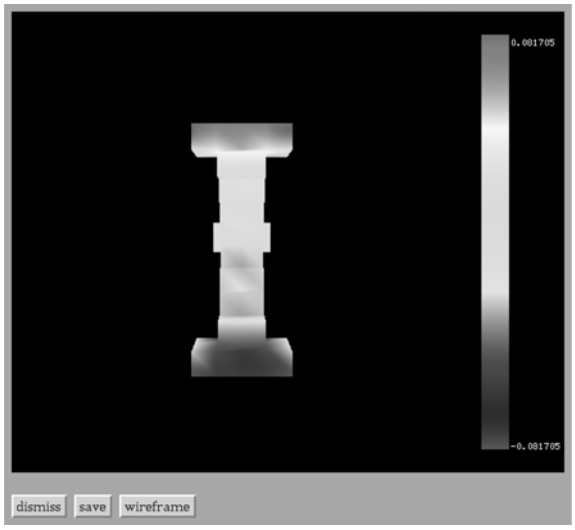


Fig. 11 Elevation view of final design (CRC = $\rho_0 \times 0.5$)

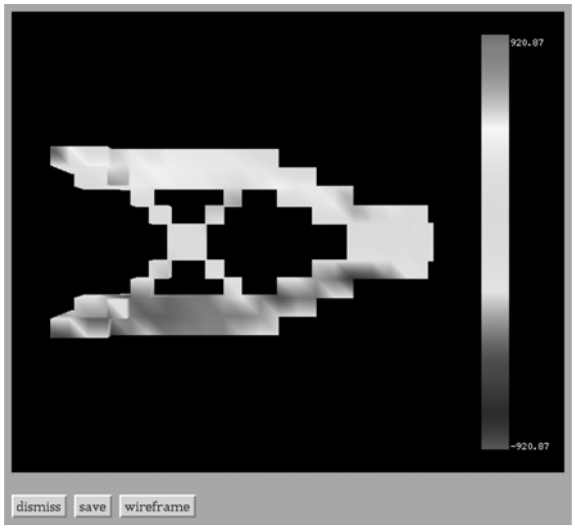


Fig. 12 Edge view of a final design (CRC = $\rho_0 \times 0.8$)

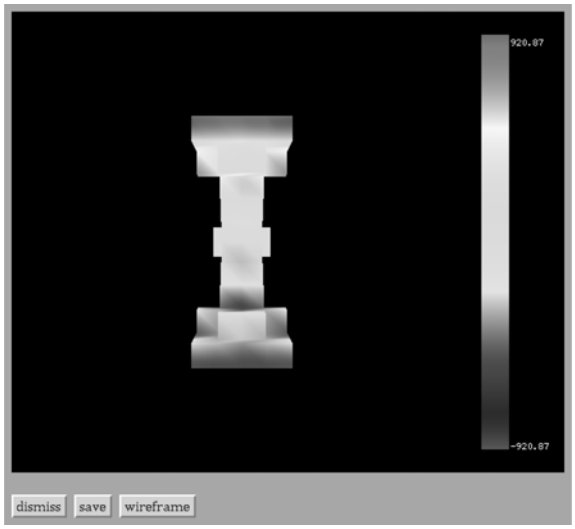


Fig. 13 Elevation view of final design (CRC = $\rho_0 \times 0.8$)

The final structures at the 2000th iterative step are shown in Figs. 10, 11, 12, and 13, respectively. We notice that there is a X-shaped reinforcement sub-structure in the final structures and that they are very similar. The obtained final structures are also similar to those by the other researchers (Xie and Steven 1997, Inou *et al.* 1994).

The results shown in Figs. 11, and 13 seem to be different from that in Fig. 5. Since the CRC value at the latter case is smaller than the former ones, the convergence speed in the latter case is slower than the former ones. If the optimization procedure is performed sufficiently in the latter case, the similar final structure can be obtained.

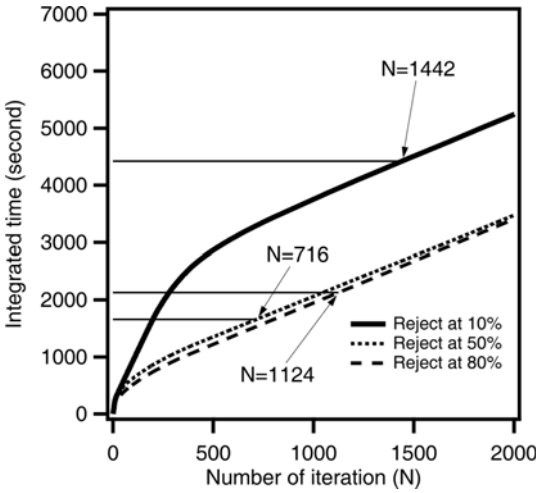


Fig. 14 Comparison of calculating time

5.3 Discussion on computational cost

One discusses the relationship between the computational cost and the CRC value. The material parameter c_2 is taken as $c_2 = 2.0$. Fig. 14 shows the CPU times at CRC = $\rho_0 \times 0.1$, 0.5 and 0.8. The abscissa and the ordinate indicate the number of the iterations and the CPU time, respectively. The outgoing lines indicate the number of the computations when the maximum stress reaches to the reference stress.

Fig. 14 shows that the convergence speed of CRC = $\rho_0 \times 0.5$ is the fastest among them and its CPU time is 1658 seconds. The CPU times of CRC = $\rho_0 \times 0.1$ and 0.8 are 2127 seconds and 4423 seconds, respectively.

6. Conclusions

The shape and topology optimization of the three-dimensional structure based on the concept of the cellular automata has been described in this paper. The object domain is divided into cubic cells. The cell density is taken as the design variable. The objective functions are defined for minimizing the total weight and for obtaining the even distribution of the equivalent stress on the whole structure. The constraint condition is defined so that the equivalent stress is almost unchanged for the variation of the equivalent stress at the neighborhood cells. The penalty function is defined from the objective functions and the constraint condition and it is stationalized with respect to derive the local rules. The optimization algorithm using the local rule was applied to the design of the three-dimensional structure. The final structure satisfying the design objectives was obtained successfully.

One has discussed the improvement of the convergence speed by changing the material parameter c_2 and the cell rejection criterion (CRC) value. The effect of the material parameter c_2 to the convergence property has been discussed firstly. The convergence properties of the stress and the total weight in $c_2 = 1.0$, 2.0 and 3.0 are compared. As a result, it is concluded that the parameter $c_2 = 2.0$ is the most adequate. Secondly, the effect of the CRC value has been discussed. The

convergence properties in CRC = $\rho_0 \times 0.1$, $\rho_0 \times 0.5$ and $\rho_0 \times 0.8$ have been compared. As a result, the CPU time in CRC = $\rho_0 \times 0.5$ is the shortest among them.

Finally, the reservations of the present method should be discussed. The first is the effect of the design variables to the optimum solution. The present method needs the design parameters such as the material parameter c_1 and c_2, the CRC value, and the penalty parameter p. The penalty parameter p effects not the optimum solution but the convergence property (speed) alone. The effect of the other parameters such as the design parameter c_1 and c_2 and the CRC value should be discussed a bit more. This paper presented the basic formulation and the algorithm improved for improving the computational cost. However, the computational cost is still expensive. We are going to study the algorithm improved for reducing the computational cost.

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