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Effective length factor for columns in braced frames considering axial forces on restraining members

M. R. Mahini†

Civil Engineering Group, Persian Gulf University, Boushehr, Iran

H. Seyyedian‡

Civil Engineering Department, Shiraz University, Shiraz, Iran

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Abstract. The effective length factor is a familiar concept for practicing engineers and has long been an approach for column stability evaluations. Neglecting the effects of axial force in the restraining members, in the case of sway prevented frames, is one of the simplifying assumptions which the Alignment Charts, the conventional nomographs for K-Factor determination, are based on. A survey on the problem reveals that the K-Factor of the columns may be significantly affected when the differences in axial forces are taken into account. In this paper a new iterative approach, with high convergence rate, based on the general principles of structural mechanics is developed and the patterns for detection of the critical member are presented and discussed in details. Such facilities are not available in the previously presented methods. A constructive methodology is outlined and the usefulness of the proposed algorithm is illustrated by numerical examples.

Keywords: effective length factor; K-factor; frame stability; column buckling; braced frame.

1. Introduction

The concept of Effective Length Factor, K-Factor, which has been widely used by practicing Engineers, is an old subject which goes back to the Euler era, when column buckling was first formulated. Theoretically the K-Factor may be carried out from a stability analysis of the structure as a whole but it is preferred to use the Alignment Charts (hereafter AC) method (Hu *et al.* 1993). The method is developed in 1956 by Julian and Lawrence and is adopted by various design codes such as AISC (2005), ACI (2005), AASHTO (1998), etc.

This method is based on some basic simplifying assumptions which are hardly maintained by actual structural systems and results in nonrealistic design in some cases. The major drawback of the AC is that the stability index $u = \sqrt{PL^2/EI}$ is assumed to be the same for all columns in a typical story and the other is neglecting the axial forces in the beams (restraining members).

Bridge and Fraser (1987) considered the effects of axial forces in restraining members for the

† Instructor, Structural Engineering, Corresponding author, E-mail: mahini@pgu.ac.ir
‡ Assistant Professor, Structural Engineering, E-mail: seyyedia@shirazu.ac.ir

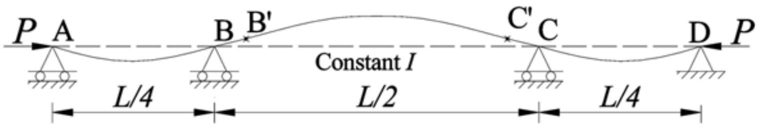


Fig. 1 Continuous 3-span column (Bridge and Fraser 1987)

braced systems and obtained K-Factor values greater than unity, contradicting the value anticipated by AC. They explained this phenomena as the effects of negative restraining and proposed an iterative method, so called "*Improved G-Factor*", to eliminate this discrepancy, which throughout a large number of curves and tabulated values and simplified functions are to be used simultaneously.

As will be shown later, the Improved G-Factor method is very sensitive to the initial guess. Also lack of an appropriate criterion on selection of the critical member, increases the error and difficulties of the before mentioned method.

Hereafter, the goal is to present a method which does not include such intricacies.

2. Theory

The effective length KL of a compression member with restrained ends is the distance between two adjacent inflection points of the member in it's buckled configuration (Duan and Chen 1997).

To examine the truth of this conclusion, consider the braced 3-span prismatic column shown in Fig. 1, which was originally investigated by Bridge and Fraser (1987).

The stability analysis of the structure yields the following numerical values for K-Factors (Bridge and Fraser 1987)

$$K_{AB} = K_{CD} = 1.4$$
$$K_{BC} = 0.7$$

It is clear from the figure that the points A and D are external inflection points and B' and C' are the internal ones which will lie on B and C, respectively, if the stability indices of all members become the same. Accepting the effective length as the length between two adjacent inflection points, the following relations should hold:

$$K_{AB} \times L_{AB} = L_{AB'}$$
$$K_{BC} \times L_{BC} = L_{B'C'}$$
$$K_{CD} \times L_{CD} = L_{C'D}$$

So, it seems that the sum of the effective lengths of all members should be equal to the total lengths of the continuous column, but:

$$K_{AB}L_{AB} + K_{BC}L_{BC} + K_{CD}L_{CD} =$$
$$2 \times 1.4(L/4) + 0.7(L/2) =$$
$$1.05L \neq L$$

In the first view it seems that the computational errors have caused the above disparity, but by computing the K-Factors up to 5 decimal digits, the obtained total length, 1.05L, only decreases to 1.0487L and even for more accuracy this will never converge to L!

Originally, the values of the K-Factors which satisfy the above relation would be obtained without any major problem and requirement to stability analysis. It is just necessary to let $K_{AB} = K_{CD} = 2K_{BC} = 2K$, which is clear from buckling behavior of a continuous system. Afterwards, by applying the concept mentioned before about the effective length, we get:

$$2K(L/4) + K(L/2) + 2K(L/4) = L$$

Which results in:

$$K = \frac{2}{3} \approx 0.67 \neq 0.70$$

How can this difference be explained?

This implies that the inflection points should be selected from the pure buckled shape of the single member, not those appearing on the whole structure buckled shape. But what is important here, is the possibility of K-Factor determination for a braced frame without any requirement of stability analysis (at least for such a non-practical structure, the K-Factors have been determined with great simplicity and an error of less than 5 percent). So, this concept can set a new approach in K-Factor evaluation for columns in braced frames.

It becomes clear that the effective length of a column slightly differs from the length between two adjacent inflection points when the latter is selected on the buckled shape of the structure. So, in the process of forming the total length of the system, the effective length KL of each column enters some error into the total length expression. On the other hand, it rarely occurs that we are encountered with a problem which consists of just three columns with hinged end supports.

Accordingly, it would be better to select a piece of structure containing the main column, which is hereafter called "*critical member*" and its adjacent beams and columns instead of handling the whole structure.

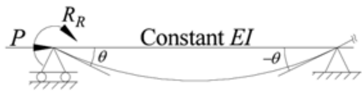
3. Structural modeling

To construct the structural model there are two basic steps. First, finding an equivalent hinged far-end member for a member with fixed or rigid connection at far end and next, condensing a series of concurrent hinged far-end members to a single one.

Since the adjacent members affect the main column through their stiffness, in above substitutions, it is reasonable to equalize the stiffness in each case.

Considering hinged, rigidly connected and fixed far end conditions in sequence for a member of length L, axial force P, moment of inertia I, and modulus of elasticity E, and using the second order analysis by following the slope-deflection method (Chen and Lui 1991), the rotational stiffness can be easily derived as tabulated in Table 1, where C and S are stability functions defined in almost all stability books, (Chen and Lui 1991) and $u = \sqrt{PL^2/EI}$ is known as stability index and may be expressed in terms of the K-Factor of the member, K, using $u = \pi/K$.

Table 1 Rotational stiffness of members with several far end condition

Far end con.	Rotational stiffness function	Buckled shape
Hinged	$R_H(EI/L, u) = \frac{EI}{L}\frac{C^2(u) - S^2(u)}{C(u)}$	
Rigid	$R_R(EI/L, u) = \frac{EI}{L}(C(u) - S(u))$	
Fixed	$R_F(EI/L, u) = \frac{EI}{L}C(u)$	

As shown in Table 1, the flexural bending of the member with rigid far end condition is assumed in symmetric single curvature, which is a simplifying assumption.

To equalize the rotational stiffness of a rigidly connected far end member with that of a hinged far end member, some equivalent maker constants γ_R and η_R are to be determined such that:

$$R_H(\eta_R EI/L, \gamma_R u) = R_R(EI/L, u) \tag{1}$$

From a numerical curve-fitting process, the best values of the before mentioned constants will be obtained as $\gamma_R = 0.974$ and $\eta_R = 0.600$.

In the case of a member with fixed far end, we are concerned with the best values for γ_F and η_F throughout an equation of the form:

$$R_H(\eta_F EI/L, \gamma_F u) = R_F(EI/L, u) \tag{2}$$

Using the same curve fitting process as before, yields $\gamma_F = 0.700$ and $\eta_R = 1.331$. These equivalent maker coefficients are arranged in Table 2.

Accordingly, each member with non-hinged far end condition, of length L, elasticity modulus E, moment of inertia I, and axial force P, could be replaced by a hinged far end member with equivalent length $L_{eq} = \gamma^2 \eta L$, equivalent moment of inertia $I_{eq} = \gamma^2 \eta^2 I$ and the same axial force and elasticity modulus, which was our goal in the first step. The equivalent length and moment of inertia L_{eq} and I_{eq}, are obtained through simultaneous solution of the equations $\eta EI/L = EI_{eq}/L_{eq}$ and $\gamma\sqrt{PL^2/EI} = \sqrt{PL_{eq}^2/EI_{eq}}$.

To derive a K-Factor formula for the case of a continuous column, as stated before, the critical member and its adjacent members shall be selected to construct a model consisting of the same critical member and only two substituted hinged far end restraining members.

Table 2 Equivalent maker coefficients

Coefficient\Far End	Hinged	Rigid	Fixed
γ	1	0.974	0.700
η	1	0.600	1.331

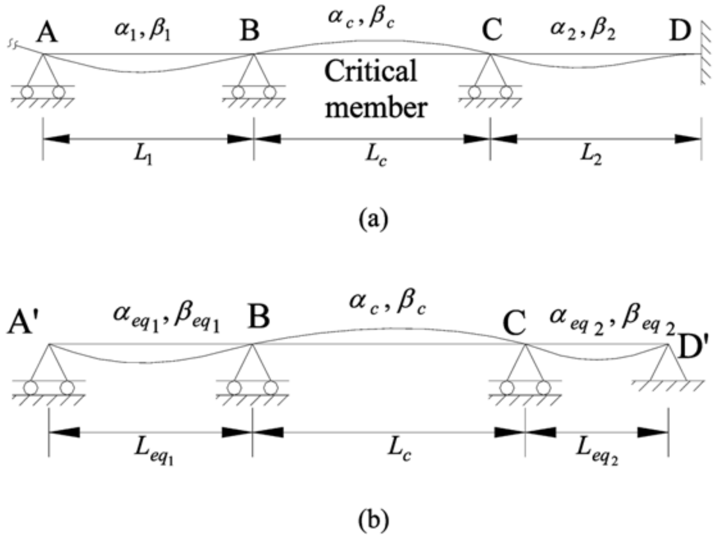


Fig. 2 (a) Isolated part of the structure, (b) Structural model

Shown in Fig. 2(a) is the selected part of the structure and the presented α_i and β_i parameters are the relative stability index and relative flexural stiffness of each member, respectively, which are defined using the relative length, axial force, moment of inertia, and the elasticity modulus as follows:

$$\alpha_i = \frac{\sqrt{P_i L_i^2 / E_i I_i}}{\sqrt{P L^2 / E I}} \tag{3a}$$

$$\beta_i = \frac{E_i I_i / L_i}{E I / L} \tag{3b}$$

In the above formulas the index i adopts t and b for the top and bottom joints, respectively. Also P, L, I, and E are the conventional axial force, length, moment of inertia, and elasticity modulus of the problem if any.

In the Fig. 2(b), restraining members with non-hinged far end are replaced by some hinged far end members $A'B$ and CD', which, with the same axial force P_c as the critical member and modified length $L_{eq_i} = \gamma_i^2 \eta_i L_i P_i / P_c$ and moments of inertia $I_{eq_i} = \gamma_i^2 \eta_i^2 I_i P_i / P_c$, produce the same rotational stiffness as that of the original members AB and CD. Consequently, the α and β parameters of the primary members may be modified as $(\alpha_{eq})_i = \gamma_i \alpha_i$ and $(\beta_{eq})_i = \eta_i \beta_i$.

Letting K_c for the effective length factor of the critical member, and defining normalized properties of the restraining members in the model as

$$\alpha_i' = \frac{\alpha_{eq}}{\alpha_c} \tag{4a}$$

$$\beta_i' = \frac{\beta_{eq}}{\beta_c} \tag{4b}$$

the K-Factor of the other members in the model, K_i', may be expressed in terms of K_c as follows:

$$K_i' = \frac{K_c}{\alpha_i'} \tag{5}$$

It must be kept in mind that the K-Factor of the equivalent hinged far end members in the model is not necessarily the same as those of the related members in the original structure.

According to the accepted concept, an approximation to the effective length factor of the critical member may be achieved by solving the below equation for K_c:

$$K_c L_c + \sum_{i=t,b} \frac{K_c}{\alpha_i'} L_{eq_i} = L_c + \sum_{i=t,b} L_{eq_i} \tag{6}$$

which lead us to:

$$K_c = \frac{1 + \sum_{i=t,b} L_{eq_i}/L_c}{1 + \sum_{i=t,b} \frac{L_{eq_i}/L_c}{\alpha_i'}} \tag{7}$$

Using the defined parameters, it can be easily shown that:

$$\frac{L_{eq_i}}{L_c} = \alpha_i'^2 \beta_i' \tag{8}$$

which simplifies the K-Factor formula into the form:

$$K_c = \frac{1 + \sum_{i=t,b} \alpha_i'^2 \beta_i'}{1 + \sum_{i=t,b} \alpha_i' \beta_i'} \tag{9}$$

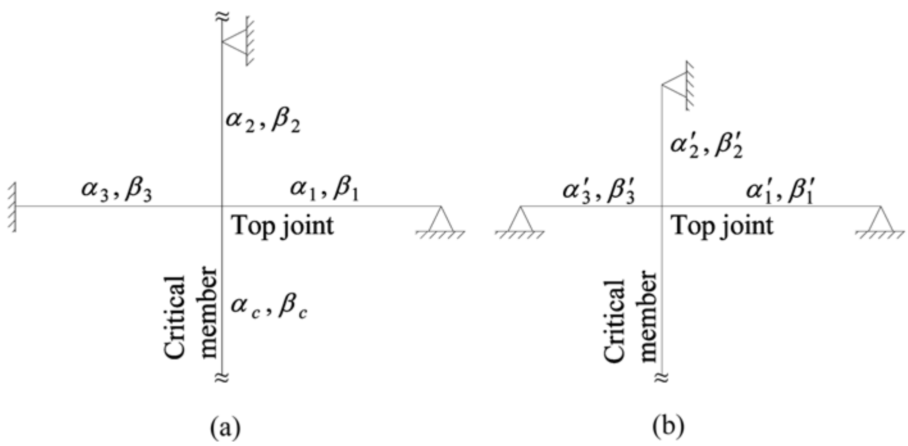


Fig. 3 (a) Isolated part of structure, (b) Equivalent restraining members (model)

For the other members in the main structure, the K-Factor may be determined through the relation:

$$K_i = \frac{\alpha_c}{\alpha_i} K_c \tag{10}$$

which is based on the frame stability principles.

It should be noted that in the presented K-Factor formula, the critical member is not to be an end one and in such cases the approach will be presented later.

It is time to consider a critical column restrained by more than one member, with several end conditions, Fig. 3(a).

Using γ and η equivalent maker coefficients determined earlier, all restraining members, for example in the top joint, could be replaced by equivalent hinged far end ones, Fig. 3(b). The mathematical expression for rotational stiffness of a hinged far-end restraining member with flexural stiffness EI/L, and stability index u, as used in the previous step, is of the form

$$R_H(EI/L, u) = \frac{EI}{L}\frac{C^2(u) - S^2(u)}{C(u)} \tag{11}$$

which, by substituting the trigonometric expressions for $C(u)$ and $S(u)$, could be simplified in a more conventional form as:

$$R_H(EI/L, u) = \frac{EI}{L}\frac{u^2 \tan u}{\tan u - u} \tag{12}$$

The behavior of such a mathematical function may be approximated by a more simple rational form:

$$R_H(EI/L, u) = \frac{EI}{L}\left(a + \frac{b}{2K^2 - 1}\right) \tag{13}$$

where a and b are curve fitting constants and u is replaced in terms of effective length factor K, using the relation $u = \pi/K$.

The best values of a and b could be easily determined through a curve fitting process but their magnitudes would not be required.

The condensation of several hinged far end members into a single one may be achieved by equating the sum of their stiffness to the stiffness of a single equivalent member. The properties of such a substituted member, i.e., relative stability index $\bar{\alpha}_t$ and flexural stiffness $\bar{\beta}_t$, are those satisfying the relation

$$\sum_{j=1}^{n_t} (\beta_j')_t \left(a + \frac{b}{2(K_c/(\alpha_j')_t)^2 - 1}\right) = \bar{\beta}_t\left(a + \frac{b}{2(K_c/\bar{\alpha}_t)^2 - 1}\right) \tag{14}$$

where n_t is the number of restraining members at the top joint t of the model and the K-Factor for all of them being replaced in terms of the effective length factor K_c of the critical member.

Unifying the coefficients a and b in both sides of Eq. (14), the required properties for the condensed member will be obtained as follows:

$$\bar{\beta}_t = \sum_{j=1}^{n_t} (\beta_j')_t \tag{15a}$$

$$\overline{\alpha}_t = \sqrt{\frac{\sum_{j=1}^{n_t} \frac{(\alpha_j')_t^2(\beta_j')_t}{2K_c^2 - (\alpha_j')_t^2}}{\sum_{j=1}^{n_t} \frac{(\beta_j')_t}{2K_c^2 - (\alpha_j')_t^2}}} \tag{15b}$$

which are applicable to the bottom joint by replacing t with b.

To estimate the effective length of the critical member, a more general form of the K-Factor formula may be presented as:

$$K_c = \frac{1 + \sum_{i=t,b} \overline{\alpha}_i^2 \overline{\beta}_i}{1 + \sum_{i=t,b} \overline{\alpha}_i \overline{\beta}_i} \tag{16}$$

It is clear that when no condensation is required, $\overline{\alpha}_i$ and $\overline{\beta}_i$ may be replaced by α_i' and β_i', respectively. Also, regarding the obtained formula for the $\overline{\alpha}_t$, it is concluded that in such cases the solution will be carried out through an iterative process, considering an initial guess for the effective length factor of the critical member.

When the critical member is an end one, the rotational stiffness provided by all members in the junction node may be estimated through the relation (14), wherein all members become condensed into a single hinged far end member with the properties $\overline{\alpha}$ and $\overline{\beta}$. Referring to (12), the rotational stiffness of the substituted member could be written as:

$$R_H(\overline{\beta}EI/L, \pi\overline{\alpha}/K_c) = \overline{\beta}\frac{EI}{L}\frac{(\pi\overline{\alpha}/K_c)^2\tan(\pi\overline{\alpha}/K_c)}{\tan(\pi\overline{\alpha}/K_c) - (\pi\overline{\alpha}/K_c)} \tag{17}$$

According to the stability concepts, buckling occurs as soon as the above stiffness vanishes, which happens for $\pi\overline{\alpha}/K_c = \pi$ or, in other words, buckling takes place when $\overline{\alpha}$ and K_c become the same in magnitude. This indicates that in such cases the effective length factor may be determined through the iteration:

$$K_c = \sqrt{\frac{\sum_{j=1}^{n} \frac{\alpha_j'^2 \beta_j'}{2K_c^2 - \alpha_j'^2}}{\sum_{j=1}^{n} \frac{\beta_j'}{2K_c^2 - \alpha_j'^2}}} \tag{18}$$

where j counts the critical member as well as the restraining ones.

4. Selection of critical member

Selection of the critical member, which has been left unsaid, is the key to the solution of such problems. In the Improved G-Factor method, presented by Bridge and Fraser (1987), the lack of such a criterion increases the difficulties of the method. They recommend to carry out the calculations several times by taking each member as critical in sequence and then accept the largest set of the resulted K-Factors as the most accurate one.

Herein, according to the basic concepts of the proposed method, the selection of the critical member shall be performed considering two criteria demonstrated below:

1- The critical member must be chosen such that the isolated part of the frame shall behave like the model as closely as possible. This could be controlled by considering the far end condition of the restraining members in the isolated part of the frame parallel to the contribution of the members in the model.

It is clear that the conditions for such a selection may not be available in all structures, but selecting the critical member through this criterion leads us to more accurate solutions.

2- Selection of the member with the largest stability index as the critical one will decrease the amount of the errors. This can be proved through an error analysis and will be declared with examples.

5. Proposed algorithm

1- Calculate the relative stability indexes using the formula (3a).

2- Select the critical member following these two patterns:

- Member which enters no error due to the modification of far end condition of its adjacent members.
- Member with the largest value of stability index when the previous criterion doesn't work.

3- For the members which contribute to the model (adjacent to the critical member), calculate the relative flexural stiffness using the formula (3b).

a- *If the critical member is not an end one*:

a-1- Determine the equivalent maker coefficients γ_i and η_i for the restraining members considering their far end condition from Table 2, and then evaluate the parameters α_i' and β_i' using the formulas (4a) and (4b).

a-2- If there exist just two restraining members, a direct use of the K-Factor formula (16) yields the effective length of the critical member. Else, assume an initial value for the effective length factor of the critical member and calculate $\overline{\alpha}_i$ and $\overline{\beta}_i$ in the joints with more than one restraining members and then use Eq. (16) to get a better approximation for K-Factor and repeat it until a desirable convergence is achieved.

b- *If the critical member is an end one*:

b-1- Determine the coefficients γ_i and η_i for the critical member as well as the restraining ones, considering their far end condition and using Table 2. Then, evaluate the parameters α_i' and β_i' according to the formulas (4a) and (4b).

b-2- Assume an initial value for the K-Factor of the critical member and explore for its more accurate value, using iteration (18) and considering a desirable convergence.

4- Determine the K-Factor for the other members in the structure, through the formula (10).

6. Numerical examples

Example 1- The continuous braced column is a common occurrence in buildings where the beams

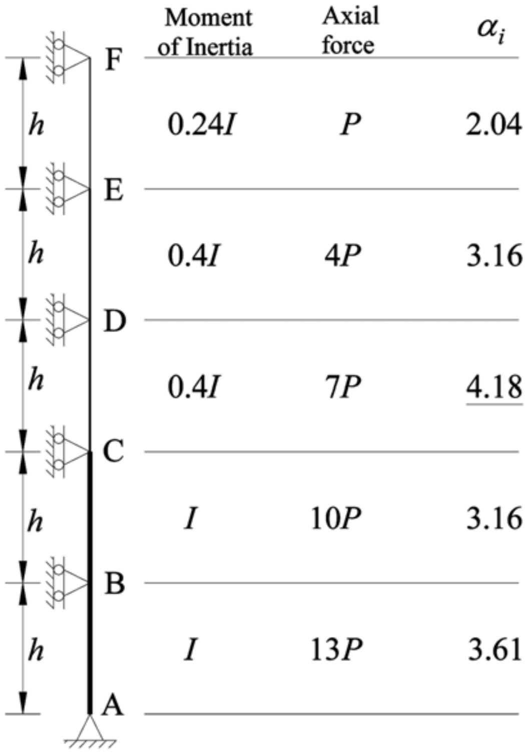


Fig. 4 Tied continuous column (Bridge and Fraser 1987)

are simply connected to the columns and the structure is erected in a manner known as simple framing. What is shown in Fig. 4 is a five span continuous column studied by Bridge and Fraser (1987). Examination of the proposed method and comparing it with what has been offered by them, is considered herein.

Solution- Using the problem data, the relative stability indexes α_i for each segment are evaluated and displayed on the figure.

As it is impossible to select the critical member using the first criteria, the member *CD*, with the largest stability index is selected as the critical one following the second pattern. Through this selection it'll become obvious that just two restraining members *BC* and *DE* contribute to the model. So, using the properties and considering the far end condition of the before mentioned members, relative flexural stiffness, equivalent maker coefficients, normalized relative stability index, and normalized flexural stiffness are determined for them. The results are tabulated in Table 3.

Table 3 The required parameters for example 1

Member	α_i	β_i	γ_i	η_i	α_i'	β_i'
BC	3.16	1.00	0.974	0.600	0.736	1.500
DE	3.16	0.40	0.974	0.600	0.736	0.600

Table 4 Comparison between the resulting K-Factors in example 1

Column\Method	AC	Improved G-Factors	Stability analysis	Proposed
AB	1.00	1.02	0.97	0.97
BC	1.00	1.17	1.11	1.11
CD	1.00	0.88	0.84	0.84
DE	1.00	1.17	1.11	1.11
EF	1.00	1.73	1.72	1.72

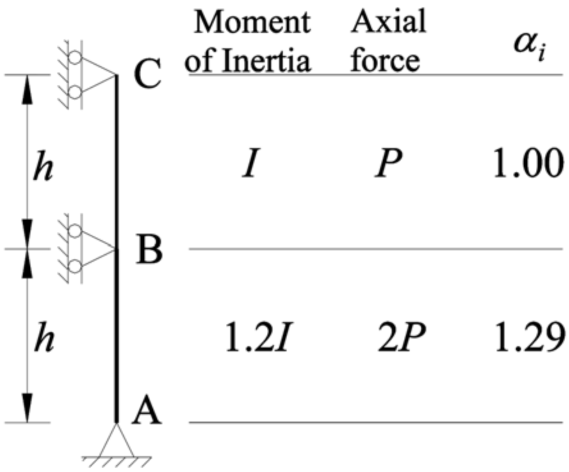


Fig. 5 Braced columns (Duan and Chen 1997)

Because there are just two restraining members, a direct use of the K-Factor formula (9) yields the effective length factor of the member *CD*, without any requirement for iteration as follows:

$$K_{CD} = \frac{1 + 0.736^2 \times 1.500 + 0.736^2 \times 0.600}{1 + 0.736 \times 1.500 + 0.736 \times 0.600} = 0.84$$

The results are tabulated besides the K-Factors obtained from AC, Improved G-Factor Method, and those obtained through a stability analysis in Table 4.

A glance at the results, indicates the excellent agreement between the K-Factors resulting from the proposed method and the accurate ones. Furthermore, the results of the proposed method, which could be called more accurate than those of the Improved G-Factor method, have been obtained without any iteration and requirements to reduplication of the process for other members.

Example 2- Shown in Fig. 5 is also a continuous column, but with two segments. This column has been selected from Duan and Chen (1997) where it has been solved by the method of Improved G-Factors which resulted in $K_{AB} = 0.93$. Here, we try to solve it using the proposed method.

Solution- The relative stability index α_i for each segment is evaluated and displayed in the figure. As shown in Fig. 5, the system contains only two members, so either can be selected as the critical member and throughout all members will contribute to the model; no error will arise due to

Table 5 The required parameters for example 2

Member	α_i	β_i	γ_i	η_i	α_i'	β_i'
AB	1.29	1.20	1	1	1	1
BC	1.00	1.00	1	1	0.775	0.833

predicting the behavior of the far end of the restraining member, and anyhow, the critical member will be an end one. So, the member *AB* is selected as the critical. Since the critical member is an end one, the relative flexural stiffness, equivalent maker coefficients, normalized relative stability index, and normalized relative flexural stiffness are to be determined for both critical and restraining members, *AB* and *BC* respectively. The results are summarized in Table 5.

Using the formula 18, a fixed point iteration of the form

$$K_{AB}^{k+1} = \sqrt{\frac{3.00(K_{AB}^{k})^2 - 1.10}{3.67(K_{AB}^{k})^2 - 1.43}}$$

may be created. Assuming $K_{AB}^{0} = 1.00$, the value of K-Factor will be improved as follows:

$$K_{AB}^{1} = 0.921 \Rightarrow K_{AB}^{2} = 0.926 \Rightarrow K_{AB}^{3} = 0.926$$

Using the obtained K-Factor, the effective length factor of the other member may be determined as $K_{BC} = 1.20$.

From a stability analysis the correct value of the effective length factor for member *AB* is determined to be 0.922, which indicates an error less than 0.5 percent on the conservative side.

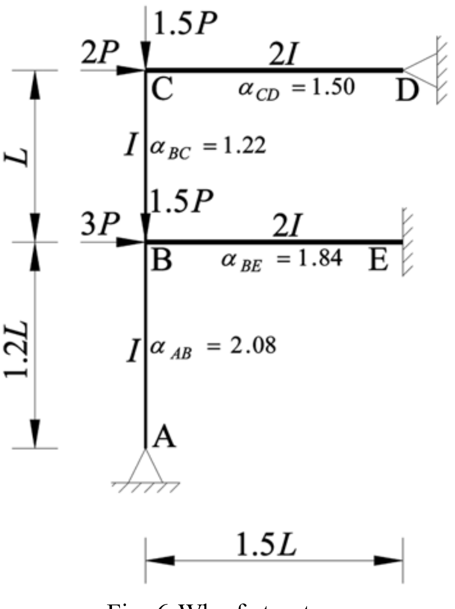


Fig. 6 Wharf structure

Table 6 The required parameters for example 3 (*AB* is selected as critical)

Member	α_i	β_i	γ_i	η_i	α_i'	β_i'
AB	2.08	0.83	0.700	1.331	0.700	1.331
BC	1.22	1.00	0.974	0.600	0.574	0.720
BE	1.84	1.33	0.700	1.331	0.619	2.130

Example 3- The final example is a sway-prevented wharf structure shown in Fig. 6. This problem has been studied by Bridge and Fraser (1987), and they enumerate it as a structure wherein that it is not obvious which member is the most critical. Now, let us verify if the proposed method is capable of handling this problem without inconvenience.

Solution- The relative stability indexes, are calculated and displayed in figure. As shown, member *AB* has the largest stability index among the others, but if member *BC* is selected as the critical one, all members will contribute to our calculations and there wouldn't be any error due to predicting the buckled shape of any restraining member. However, the above-mentioned error is not significant.

To demonstrate the preference of the critical member selecting criteria, we solve this problem in two cases.

a- *AB* is the critical member:

In this case the critical member is an end one and all required parameters are determined and arranged in Table 6.

Substituting the values of α_i' and β_i' into (18) and starting with $K_{AB}^0 = 1$, the iteration will converges after three tries, as follows:

$$K_{AB}^1 = 0.640 \Rightarrow K_{AB}^2 = 0.646 \Rightarrow K_{AB}^3 = 0.646$$

For the design purposes, convergence up to two decimal digits is sufficient, which is almost achieved after two iterations, but more digits are considered to indicate the rate and type of convergence for the proposed method. Generally, unlike the Improved G-Factor method, high convergence rate and least sensitivity to the initial guess are among the features of the proposed method.

For example, by using a far-off initial guess $K_{AB}^0 = 10$, the iteration converges after just as many tries as before:

$$K_{AB}^1 = 0.639 \Rightarrow K_{AB}^2 = 0.646 \Rightarrow K_{AB}^3 = 0.646$$

while the Improved G-Factor method for the same critical member and $K_{AB}^0 = 1$ results in:

$$K_{AB}^1 = 0.57 \Rightarrow K_{AB}^2 = 0.95 \Rightarrow K_{AB}^3 = 0.58 \Rightarrow$$

$$K_{AB}^4 = 0.83 \Rightarrow K_{AB}^5 = 0.59 \ \dots \ K_{AB}^? = 0.65$$

which shows the lower convergence rate in the Improved G-Factor method as well as its high sensitivity to the initial guess.

b- *BC* is the critical member

In this case, the critical member is restrained in its both ends and the parameters are to be determined only for the restraining members. The results are listed in Table 7.

Table 7 The required parameters for example 3 (*BC* is selected as critical)

Joint	Mem.	α_i	β_i	γ_i	η_i	α_i'	β_i'
Top	*CD*	1.50	1.33	1	1	1.225	1.333
Bot	*BE*	1.84	1.33	0.70	1.331	1.050	1.775
	AB	2.08	0.83	0.70	1.331	1.188	1.109

Since there is just one restraining member at the top joint, there would not be any requirement for condensation. So, $\overline{\alpha}_t = 1.225$ and $\overline{\beta}_t = 1.333$. But at the bottom joint, where there exist more than one restraining member, appropriate values of $\overline{\alpha}_b$ and $\overline{\beta}_b$ should be determined using the relations (15a) and (15b) which result in:

$$\overline{\beta}_b = 1.775 + 1.109 = 2.884$$

and

$$\overline{\alpha}_b = \sqrt{\frac{7.044K_{BC}^2 - 4.487}{5.768K_{BC}^2 - 3.728}}$$

It is clear that $\overline{\alpha}_b$ could not be evaluated without any K_{BC} in hand. Assuming $K_{BC}^0 = 1$ results in $\overline{\alpha}_b = 1.120$ and then a better estimation of K_{BC} could be obtained using the K-Factor formula (16), which would be $K_{BC}^1 = 1.129$. For more accurate effective length factor, above process should be repeated. More iteration steps could be found in Table 8.

To give another reason for the high convergence rate of the proposed method, let us start with $K_{BC}^0 = 10$ which throughout the iteration converges after just one try more than before:

$$K_{BC}^1 = 1.121 \Rightarrow K_{BC}^2 = 1.125 \Rightarrow K_{BC}^3 = 1.125$$

Table 8 Iteration process in example 3 (*BC* is selected as critical)

Iteration k	K_{BC}^k	$\overline{\alpha}_b^k$	K_{BC}^{k+1}
0	1	1.120	1.129
1	1.129	1.113	1.125
2	1.125	1.113	1.125

Table 9 Comparison between the resulting K-Factors in example 3

Mem.	AC	Improved G-Factor	Proposed method		Stability analysis
			AB as critical	*BC* as critical	
AB	0.61	0.66	0.65	0.66	0.67
BC	0.71	1.12	1.10	1.13	1.13
BE	0.59	0.75	0.73	0.75	0.76
CD	0.88	0.92	0.90	0.92	0.93

The K-Factor of the other members may be determined using the obtained values for the critical ones in the cases “a” and “b”. The results are tabulated in Table 9 to be compared with those obtained from Alignment Charts (AC), Improved G-Factor method, and exact stability analysis.

The results obtained in this example also show the efficiency of the proposed method. It is seen that selecting *BC* as the critical member, which is a selection based on the first criterion, results in a more accurate solution, and this reveals the advantage of the first criterion for the selection of the critical member. However, the resulting K-Factors by following the second pattern will even contain a small and satisfactory amount of error.

7. Conclusions

The value of K-Factor which gives the magnitude of the critical buckling load of columns obtained from Alignment Charts, may be significantly improved by taking into account the axial force of restraining members and the variation of stability index of adjacent columns.

The main published research concerning the sources of these inaccuracies, is that followed by a method called Improved G-Factor, an iterative method based on linearized stability functions which necessitate the use of a table containing several coefficients and functions and a family of curves. In addition to the difficulties of using a variety of coefficients and functions, other problems such as low rate of convergence, high sensitivity to the initial guess and lack of a criterion for selection of the critical member, increase the difficulties involved in this method.

Herein, by phrasing the effective length of a compressive member as the length between two adjacent inflection points, an iterative procedure has been presented. Using the curve-fitting principles, the explicit form of the stability functions are replaced by more simple rational forms and also throughout the same process, members with non-hinged far end condition are converted to equivalent hinged far end ones, considering the rotational stiffness in each case.

Numerical examples have been presented to illustrate the proposed method and show its high convergence rate, effectiveness and straightforwardness which are all problems faced when using the Improved G-Factor method.

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Notation

The following symbols are used in this paper:

a, b : Curve-fitting constants;
C : Stiffness stability function;
E : Young's modulus of material;
e : Lengths ratio;
E_r : Relative error;
I : Second moment of area;
K : Effective length factor;
L : length of members;
P : axial load;
S : Stability function;
u : Stability index;
x : length variable;
$y(x)$: Buckled shape;
α : Relative stability index;
β : Relative flexural stiffness;
α' : Normalized relative stability index w.r.t. the relative stability index of the critical member;
β' : Normalized relative flexural stiffness w.r.t. the relative flexural stiffness of the critical member;
$\overline{\alpha}$: Condensed relative stability index;
$\overline{\beta}$: Condensed relative flexural stiffness;
Δ : total difference between the location of the single member's inflection point and that of the system in the model;
δ : difference between the location of the single member's inflection point and that of the system;
γ : stability index equivalent maker coefficient;
η : flexural stiffness equivalent maker coefficient.