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Stationary random response analysis of linear fuzzy truss

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Abstract. A new method called fuzzy factor method for the stationary stochastic response analysis of fuzzy truss with global fuzzy structural parameters is presented in this paper. Considering the fuzziness of the structural physical parameters and geometric dimensions simultaneously, the fuzzy correlation function matrix of structural displacement response in time domain is derived by using the fuzzy factor method and the optimization method, the fuzzy mean square values of the structural displacement and stress response in the frequency domain are then developed with the fuzzy factor method. The influences of the fuzziness of structural parameters on the fuzziness of mean square values of the displacement and stress response are inspected via an example and some important conclusions are obtained. Finally, the example is simulated by Monte-Carlo method and the results of the two methods are close, which verified the feasibility of the method given in this paper.

Keywords: fuzzy truss; stationary stochastic excitation; fuzzy correlation function matrix of displacement response; fuzzy factor method; fuzzy mean square values of structural dynamic response.

1. Introduction

Due to kinds of uncertain effects on the production of the structural material, its physical parameters and geometric dimensions take on uncertainty to a certain extent. Furthermore, the loads like earthquake, wind and ocean wave applying to structures often take on randomness. Therefore,

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studying the stationary stochastic response of uncertain structure has realistic engineering background and theoretic signification.

Though the random dynamic response analysis of linear stochastic structure is very complicated and difficult, some researches on compound stochastic vibration may be found in the literatures (Jensen and Iwan 1992, Zhao and Chen 2000, Lin and Yi 2001, Li and Liao 2002, Gao *et al.* 2004). Jensen and Iwan (1992) studied the response of systems with uncertain parameters to random excitation by extended the orthogonal expansion method. Gao *et al.* (2004) analyzed the dynamic response of structures with stochastic parameters under external non-stationary random excitation by means of the random factor method, the algebra synthesis method and random variable's function moment method.

With the increasing realization to the uncertainty, however, uncertainties due to inaccurate measurement, the lack of experimental data and incomplete knowledge of the structure belong to fuzziness, which should not be dealt with as randomness simply. Some researches on fuzziness have been done using fuzzy set as Valliappan and Pham (1993), Cherki *et al.* (2000), Simões (2001), Rao and Cao (2001), which was put forward by Zadeh (1978) in 1965. Valliappan and Pham (1993) proposed a model based on the theory of fuzzy sets to take account of the uncertainty in the soil behaviour, and considered the elastic modulus and Poisson's ratio as two fuzzy numbers in the elastic matrix. Rao and Cao (2001) developed a fuzzy boundary element method for the analysis of imprecisely defined systems and solved the resulting fuzzy equations using a fuzzified version of Gaussian elimination procedure coupled with truncation. Compared with the structural analysis with stochastic parameters, dynamic analysis and modeling of structures with fuzzy parameters is of significance as well, remaining to be studied further.

In this paper, the stationary stochastic dynamic response analysis of truss with fuzzy physical parameters and geometric dimensions is presented. A new method (Fuzzy Factor Method) is proposed, in which the influence of each fuzzy parameter on the structural dynamic response can be reflected expediently. Firstly, the fuzzy correlation function matrix of the displacement response in time domain is derived by means of the fuzzy factor method, interval arithmetic and the optimization method. Then fuzzy mean square values of the structural displacement and stress response in the frequency domain are developed by fuzzy factor method. Finally, the example is simulated by Monte-Carlo method to verify the feasibility of the method given in this paper.

2. Structural stationary random dynamic response analysis

Suppose that there are ne bar elements in the analyzed truss. The mass matrix $[M]$ and stiffness matrix $[K]$ in global coordinate can be expressed respectively as

$$[M] = \sum_{e=1}^{ne}[M^{(e)}] = \sum_{e=1}^{ne}\left\{\frac{1}{2}\rho^{(e)}A^{(e)}l^{(e)}[I]\right\} \tag{1}$$

$$[K] = \sum_{e=1}^{ne}[K^{(e)}] = \sum_{e=1}^{ne}\left\{[T^{(e)}]^T\frac{E^{(e)}A^{(e)}}{l^{(e)}}[G][T^{(e)}]\right\} \tag{2}$$

where $[K^{(e)}]$ and $[M^{(e)}]$ are the eth bar element's stiffness and mass matrix in global coordinate, respectively; $[I]$ is 6×6 identity matrix; $E^{(e)}$, $A^{(e)}$, $l^{(e)}$ and $\rho^{(e)}$ are the eth bar element's Young's modulus, cross-section area, length and mass density, respectively; $[G]$ is 6×6 matrix, where $g_{11} = g_{44} = 1$, $g_{14} = g_{41} = -1$, other elements are zero; $[T^{(e)}]$ is a transformation matrix that translates the local coordinates of the eth element to global coordinates, and $[T^{(e)}]^T$ is its transpose.

The dynamic equation of the structure under stationary stochastic excitation is

$$[M]\{\ddot{u}(t)\} + [C]\{\dot{u}(t)\} + [K]\{u(t)\} = \{P(t)\} \tag{3}$$

where $[M]$, $[C]$ and $[K]$ are the mass, damping and stiffness matrices, respectively. $\{u(t)\}$, $\{\dot{u}(t)\}$ and $\{\ddot{u}(t)\}$ are structural displacement vector, velocity vector and acceleration vector, respectively. $\{P(t)\}$ is stationary stochastic load force vector.

Eq. (3) is a set of differential equations coupled to each other. Its formal solution can be obtained from the decoupling transform and Duhamel integral:

$$\{u(t)\} = \int_0^t [\phi][h(\tau)][\phi]^T \{P(t-\tau)\} d\tau \tag{4}$$

where $[\phi]$ is the structural normal modal matrix, $[\phi]^T$ is its transpose; $[h(t)]$ is the structural impulse response function matrix and it can be expressed as

$$[h(t)] = diag(h_j(t)) \tag{5}$$

where

$$h_j(t) = \begin{cases} (\exp(-\xi_j \omega_j t)\sin\overline{\omega}_j t)/\overline{\omega}_j & t \geq 0 \\ 0 & t < 0 \end{cases} \qquad (j = 1, 2, \ldots, s) \tag{6}$$

where ω_j and ξ_j are the jth order structural natural frequency and mode damping respectively; $\overline{\omega}_j = \omega_j(1 - \xi_j^2)^{1/2}$.

From Eq. (4), the correlation function matrix of the structural displacement response $[R_u(\varepsilon)]$ is

$$\begin{aligned} [R_u(\varepsilon)] &= E(\{u(t)\}\{u(t+\varepsilon)\}^T) \\ &= \int_0^t \int_0^t [\phi][h(\tau)][\phi]^T [R_P(\tau - \tau_1 + \varepsilon)][\phi][h(\tau_1)]^T [\phi]^T d\tau d\tau_1 \end{aligned} \tag{7}$$

where $[R_P(\tau - \tau_1 + \varepsilon)]$ is the correlation function matrix of the load $\{P(t)\}$.

By performing a Fourier transformation to $[R_u(\varepsilon)]$, the power spectral density matrix of the displacement response $[S_u(\omega)]$ can be obtained as follows:

$$[S_u(\omega)] = [\phi][H(\omega)][\phi]^T [S_P(\omega)][\phi][H^*(\omega)][\phi]^T \tag{8}$$

where $[S_P(\omega)]$ is the power spectral density matrix of load $\{P(t)\}$, $[H^*(\omega)]$ is the conjugate matrix of $[H(\omega)]$, $[H(\omega)]$ is the structural frequency response function matrix and can be expressed as

$$[H(\omega)] = diag[H_j(\omega)] \tag{9}$$

where

$$H_j(\omega) = 1/(\omega_j^2 - \omega^2 + i \cdot 2\xi_j\omega_j\omega) \quad (i = \sqrt{-1}, j = 1, 2, \ldots, s) \tag{10}$$

Integrating $[S_u(\omega)]$ within the frequency domain, the mean square value matrix of the structural displacement response, that is, $[\psi_u^2]$ can be obtained

$$[\psi_u^2] = \int_0^\infty [S_u(\omega)]d\omega = \int_0^\infty [\phi][H(\omega)][\phi]^T[S_P(\omega)][\phi][H^*(\omega)][\phi]^T d\omega \tag{11}$$

Then the mean square value of the kth degree of freedom of structural displacement response is

$$\psi_{uk}^2 = \vec{\phi}_k \cdot \int_0^\infty [H(\omega)][\phi]^T[S_P(\omega)][\phi][H^*(\omega)]d\omega \cdot \vec{\phi}_k^T \qquad (k = 1, 2, \ldots, n) \tag{12}$$

where $\vec{\phi}_k$ is the kth line vector of the matrix $[\phi]$.

From the relationship between node displacement and element stress, the stress response of the eth bar element in the truss is

$$\{\sigma(t)^{(e)}\} = E^{(e)} \cdot [B] \cdot \{u(t)^{(e)}\} \qquad (e = 1, 2, \ldots, n_e) \tag{13}$$

where $\{u(t)^{(e)}\}$ is the displacement response of the eth bar element's nodes, $\{\sigma(t)^{(e)}\}$ is stress response of the eth element, $[B]$ is geometric matrix of the eth element, $E^{(e)}$ is the Young's module of the eth element.

From Eq. (13), the correlation function matrix of the eth bar element's stress response $[R_\sigma^{(e)}(\tau)]$ is

$$[R_\sigma^{(e)}(\tau)] = E(\{\sigma(t)^{(e)}\}\{\sigma(t+\tau)^{(e)}\}^T) = E^{(e)}[B][R_u^{(e)}(\tau)][B]^T E^{(e)} \tag{14}$$

Thus, the power spectral density matrix of the stress response of the eth element $[S_\sigma^{(e)}(\omega)]$ is

$$[S_\sigma^{(e)}(\omega)] = E^{(e)}[B][S_u^{(e)}(\omega)][B]^T E^{(e)} \tag{15}$$

Then, the mean square value matrix of the eth element's stress response $[\psi_{\sigma^{(e)}}^2]$ is

$$[\psi_{\sigma^{(e)}}^2] = E^{(e)}[B][\psi_{u^{(e)}}^2][B]^T E^{(e)} \tag{16}$$

3. Fuzzy factor method and interval arithmetic

3.1 L – R representation of fuzzy numbers (Dubois and Prade 1980) and fuzzy factor method

The definition of fuzzy set A is as follows:

Let X be a classical set of objects, called the *universe*, whose generic elements are denoted x. Membership in a classical subset A of X is often viewed as a characteristic function μ_A from X to $\{0, 1\}$ such that

$$\mu_A(x) = \begin{cases} 1 & \text{iff } x \in A \\ 0 & \text{iff } x \notin A \end{cases}$$

(N.B.: "iff" is short for "if and only if".); $\{0, 1\}$ is called a valuation set.

If the valuation set is allowed to be the real interval [0, 1], A is called a fuzzy set. $\mu_A(x)$ is the grade of membership of x in A. The closer the value of $\mu_A(x)$ is to 1, the more x belongs to A.

The definition of fuzzy number is: for *universe X*, a fuzzy set A is called a fuzzy number.

The definition of $L-R$ type fuzzy number is: A function, usually denoted L or R, is a reference function of fuzzy numbers iff: (1) $L(x) = L(-x)$; (2) $L(0) = 1$; (3) L is nonincreasing on $[0, +\infty)$.

A fuzzy number M is said to be an $L-R$ type fuzzy number iff

$$\mu_M(x) = \begin{cases} L((m-x)/\alpha) & \text{for} \quad (x \le m, \alpha > 0) \\ R((x-m)/\beta) & \text{for} \quad (x \ge m, \beta > 0) \end{cases}$$

L is for left and R for right reference. m is the main value of M. α and β are called *left* and *right* spreads respectively. When the spreads are zero, M is a nonfuzzy number by convention. As the spreads increase, M becomes fuzzier and fuzzier. Symbolically, an $L-R$ type fuzzy number is written as $M = (m, \alpha, \beta)_{LR}$.

The fuzzy factor method below is aimed at $L-R$ type fuzzy number.

(1) Let $M = (M_M, \alpha, \beta)_{LR}$ be an $L-R$ type fuzzy number, M_M is its main value, α and β are its left and right spreads. Taking the grade of membership λ as small as we can to do λ-level cut, the approximate maximum M_R and minimum M_L in the value range of M can be obtained, their deviations from M_M represent the fuzzy dispersion degree of M too. Then M can be described as a fuzzy number $M = (M_M, M_L, M_R)$. Introduce a fuzzy number without dimension $\gamma = M/M_M$ as the fuzzy factor of M, main value of γ is 1, its value range representing the fuzzy dispersion degree of M too is $[\gamma_L = M_L/M_M, \gamma_R = M_R/M_M]$. Then $M = \gamma \cdot M_M$.

(2) If y is a dissymmetrical normal fuzzy number near m, its membership function about fuzzy set A (Luo 1989) is

$$\mu_L(y) = \exp\{-(y-m)^2/\alpha^2\} \quad (y \le m) \tag{17}$$

$$\mu_R(y) = \exp\{-(y-m)^2/\beta^2\} \quad (y \ge m) \tag{18}$$

where m is the main value of y, α and β are left and right spreads. y can then be described as an $L-R$ type fuzzy number $(m, \alpha, \beta)_{LR}$. When $\alpha = \beta$, y is a symmetrical normal fuzzy number and it can be described as an $L-R$ type fuzzy number $(m, \alpha, \alpha)_{LR}$, and the uniform representation of Eqs. (17) and (18) (Luo 1989) is $\mu(y) = \exp\{-(y-m)^2/\alpha^2\} \ (y \in R)$.

Because the normal fuzzy number is a special case of the $L-R$ type fuzzy number, and it is impossible to reach zero and get a bounded fuzzy number in this case, here taking grade of membership λ small enough to do λ-level cut, the approximate minimum y_L and the approximate maximum y_R in the value range of y can be obtained

$$y_L = m - \sqrt{(-\ln\lambda) \cdot \alpha^2} = m - \alpha\sqrt{(-\ln\lambda)} \tag{19}$$

$$y_R = m + \sqrt{(-\ln\lambda)\cdot\beta^2} = m + \beta\sqrt{(-\ln\lambda)} \quad (20)$$

Then y can be described as a fuzzy number $y = (m, y_L, y_R)$. Now introducing fuzzy number $\gamma = y/m$ as the fuzzy factor of y. $\gamma = (\gamma_M = 1, \gamma_L = y_L/m, \gamma_R = y_R/m)$, and γ represents the fuzziness of y. Then $y = \gamma\cdot m$, main value of γ is 1, and its value range is $[\gamma_L = y_L/m, \gamma_R = y_R/m]$.

Then other $L-R$ type fuzzy number $(m, \alpha, \beta)_{LR}$ like triangle fuzzy number can be denoted $M = (M_M, M_L, M_R)$ or $M = \gamma\cdot M_M$ as well, where γ is the fuzzy factor of M obtained from the method above, M_M (that is, m) is its main value.

3.2 Interval arithmetic (Alefeld and Claudio 1998)

Interval analysis is a conventional method to deal with uncertainty.

From the addition, multiplication and division of interval, for two closed intervals: $[a] = [\underline{a};\overline{a}] = \{x\in R|(\underline{a}\le x\le\overline{a})\}$ and $[b] = [\underline{b};\overline{b}] = \{x\in R|(\underline{b}\le x\le\overline{b})\}$, then $[a]+[b] = [\underline{a}+\underline{b}, \overline{a}+\overline{b}]$, $[a]\times[b] = [\min\{\underline{a}\,\underline{b}, \underline{a}\,\overline{b}, \overline{a}\,\underline{b}, \overline{a}\,\overline{b}\};\max\{\underline{a}\,\underline{b}, \underline{a}\,\overline{b}, \overline{a}\,\underline{b}, \overline{a}\,\overline{b}\}], [a]/[b] = [\underline{a};\overline{a}]\times[1/\overline{b};1/\underline{b}]$.

4. Dynamic response of fuzzy truss under stationary stochastic excitation

Considering the fuzziness of structural parameters $\rho^{(e)}, E^{(e)}, A^{(e)}$ and $l^{(e)}$, and the randomness of applied load $\{P(t)\}$ simultaneously. The engineering background of this case is that the stationary stochastic applied loads act on the truss structure with fuzzy parameters.

When these fuzzy structural parameters are all $L-R$ type fuzzy number, suppose that all bars' material is homology, from fuzzy factor method, they can be described as $\rho^{(e)} = \tilde{\rho}\cdot\rho_M$, $E^{(e)} = \tilde{E}\cdot E_M$. where ρ_M and E_M are main values of $\rho^{(e)}$ and $E^{(e)}$ respectively; $\tilde{\rho}$ and $\tilde{E}$ are their fuzzy factors respectively. From fuzzy factor method, $\tilde{\rho} = (\tilde{\rho}_M = 1, \tilde{\rho}_L = \rho_L/\rho_M, \tilde{\rho}_R = \rho_R/\rho_M)$ and $\tilde{E} = (\tilde{E}_M = 1, \tilde{E}_L = E_L/E_M, \tilde{E}_R = E_R/E_M)$. where ρ_R and ρ_L are the approximate maximum and minimum of ρ_e respectively; E_R and E_L are the approximate maximum and minimum of E_e respectively.

Suppose that the fuzzy dispersion degree of all bars' length is equivalent, and the dispersion degree of all bars' cross section area is equivalent as well, from fuzzy factor method, $A^{(e)} = \tilde{A}\cdot A_M^{(e)}$ and $l^{(e)} = \tilde{l}\cdot l_M^{(e)}$. where $A_M^{(e)}$ and $l_M^{(e)}$ are fuzzy main values of $A^{(e)}$ and $l^{(e)}$ respectively; $\tilde{l}$ is the fuzzy factor of the shortest bar (from fuzzy factor method, value ranges of $\tilde{l}$ including those of other bars' fuzzy factors), $\tilde{l} = (\tilde{l}_M = 1, \tilde{l}_L = l_L/l_M, \tilde{l}_R = l_R/l_M)$, where l_M, l_R and l_L are main value, the approximate maximum and minimum of the shortest bar's length, respectively; in the same way, $\tilde{A}$ is the fuzzy factor of the bar with minimum cross-section area, $\tilde{A} = (\tilde{A}_M = 1, \tilde{A}_L = A_L/A_M, \tilde{A}_R = A_R/A_M)$, where A_M, A_R and A_L are main value, the approximate maximum and minimum of bar's smallest cross section area, respectively.

Therefore, Eq. (1) and Eq. (2) can be expressed respectively as

$$[M] = \sum_{e=1}^{ne}[M^{(e)}] = \sum_{e=1}^{ne}\left\{\frac{1}{2}A^{(e)}\cdot\rho^{(e)}\cdot l^{(e)}\cdot[I]\right\} = \tilde{A}\tilde{\rho}\tilde{l}\sum_{e=1}^{ne}\left\{\frac{1}{2}A_M^{(e)}\cdot\rho_M\cdot l_M^{(e)}\cdot[I]\right\} = \tilde{A}\tilde{\rho}\tilde{l}\cdot[M]^{\#} \quad (21)$$

$$[K] = \sum_{e=1}^{ne}[K^{(e)}] = \sum_{e=1}^{ne}[T^{(e)}]^T\frac{E^{(e)}\cdot A^{(e)}}{l^{(e)}}[G][T^{(e)}] = \frac{\tilde{E}\tilde{A}}{\tilde{l}}\cdot\left(\sum_{e=1}^{ne}[T^{(e)}]^T\frac{E_M\cdot A_M^{(e)}}{l_M^{(e)}}[G][T^{(e)}]\right)$$

$$= \left(\frac{\tilde{E}\tilde{A}}{\tilde{l}}\right)[K]^{\#} \tag{22}$$

where $[K]^{\#}$ is the deterministic part of $[K]$ when $E_e = E_M$, $l_e = l_M^{(e)}$ and $A_e = A_M^{(e)}$; $[M]^{\#}$ is the deterministic part of $[M]$ when $\rho_e = \rho_M$, $l_e = l_M^{(e)}$ and $A_e = A_M^{(e)}$.

The fuzziness of structural parameters leads to fuzziness of dynamic characteristic ω_i and $\{\phi\}_i$. The fuzziness of the dynamic characteristic and the randomness of loads will lead to fuzziness of the mean square values of dynamic displacement and stress response in the end.

Substituting Eqs. (21) and (22) into Raleigh quotient of structural vibration $\omega_i^2 = (\{\phi\}_i^T[K]\{\phi\}_i/\{\phi\}_i^T[M]\{\phi\}_i)$, then

$$\omega_i = \sqrt{\tilde{E}/(\tilde{\rho}\tilde{l}^2)}\cdot\sqrt{(\tilde{\phi}^2\cdot\{\phi\}_i^{\#T}[K]^{\#}\{\phi\}_i^{\#})/(\tilde{\phi}^2\cdot\{\phi\}_i^{\#T}[M]^{\#}\{\phi\}_i^{\#})} = \tilde{\omega}\omega_{iM} \tag{23}$$

where $\tilde{\omega} = \sqrt{\tilde{E}/(\tilde{\rho}\tilde{l}^2)}$ is the fuzzy factor of ω_i; from the interval arithmetic, the value range of $\tilde{\omega}$ is $[\tilde{\omega}_L = \sqrt{\tilde{E}_L/(\tilde{\rho}_R\cdot\tilde{l}_R^2)}, \tilde{\omega}_R = \sqrt{\tilde{E}_R/(\tilde{\rho}_L\cdot\tilde{l}_L^2)}]$ and its main value is 1; ω_{iM} is the main value of ω_i. The fuzzy value range of ω_i is $[\omega_{iL} = \tilde{\omega}_L\cdot\omega_{iM}, \omega_{iR} = \tilde{\omega}_R\cdot\omega_{iM}]$.

$\tilde{\phi}$ in Eq. (23) is fuzzy factor of $\{\phi\}_i$ and $\tilde{\phi}$ in Eq. (23) is counteracted, and it is independent to number of order the same as fuzzy factor $\tilde{\omega}$; $\{\phi\}_i^{\#}$ is the ith order deterministic natural mode shape after fuzzy factor $\tilde{\phi}$ abstracted from $\{\phi\}_i$; $\{\phi\}_i^{\#T}$ is the transpose of $\{\phi\}_i^{\#}$.

The solution to the fuzzy factor $\tilde{\phi}$ is as follows: considering the fuzziness of amplitude of the fuzzy normal modal matrix $[\phi]$, where $[\phi] = [\{\phi\}_1, \{\phi\}_2, \ldots, \{\phi\}_n]^T = [\phi_1, \phi_2, \ldots, \phi_n] = \tilde{\phi}\cdot[\phi_1^{\#}, \phi_2^{\#}, \ldots, \phi_n^{\#}] = \tilde{\phi}\cdot[\phi]^{\#}$, $[\phi]^{\#}$ is the deterministic normal modal matrix after $\tilde{\phi}$ is abstracted from $[\phi]$. From mode theory of structural vibration, the equation below holds:

$$[\phi]^T[K][\phi] = [\Omega] = diag(\omega_i^2) \tag{24}$$

where $[\phi]^T$ is the transpose of $[\phi]$; $[K]$ is fuzzy stiffness matrix; $[\Omega]$ is a diagonal matrix, its elements are square of every order fuzzy natural frequency ω_i^2. Further

$$\tilde{\phi}^2\cdot((\tilde{E}\tilde{A})/\tilde{l})\cdot([\phi]^{\#T}[K]^{\#}[\phi]^{\#}) = (\tilde{E}/(\tilde{\rho}\tilde{l}^2))\cdot diag(\omega_{iM}^2) \tag{25}$$

where $[\phi]^{\#T}$ is the transpose of $[\phi]^{\#}$; $[K]^{\#}$ is the deterministic stiffness matrix after fuzzy factor $(\tilde{E}\tilde{A})/\tilde{l}$ is abstracted from $[K]$ (in (22)); ω_{iM}^2 is the deterministic part after fuzzy factor $\tilde{E}/(\tilde{\rho}\tilde{l}^2)$ is abstracted from ω_i^2 (in (23)), hence $\tilde{\phi} = \sqrt{1/(\tilde{\rho}\tilde{A}\tilde{l})}$. From interval operation, main value of $\tilde{\phi}$ is 1 and its value range is $[\tilde{\phi}_L = \sqrt{1/(\tilde{\rho}_R\tilde{A}_R\tilde{l}_R)}, \tilde{\phi}_R = \sqrt{1/(\tilde{\rho}_L\tilde{A}_L\tilde{l}_L)}]$.

In the following, fuzzy correlation function matrix of the structural displacement response in time domain will be derived by means of the fuzzy factor method and optimization method.

From the property of matrix integral and after fuzzy factor $\tilde{\phi}$ abstracted from $[\phi]$, Eq. (7) is described as:

$$[R_u(\varepsilon)] = \int_0^t\int_0^t [\phi][h(\tau)][\phi]^T[R_P(\tau-\tau_1+\varepsilon)][\phi][h(\tau_1)]^T[\phi]^T d\tau d\tau_1$$
$$= \tilde{\phi}^4 \cdot [\phi]^{\#} \cdot \left(\int_0^t\int_0^t [h(\tau)][\phi]^{\#T}[R_P(\tau-\tau_1+\varepsilon)][\phi]^{\#}[h(\tau_1)]^T d\tau d\tau_1\right)[\phi]^{\#T} \quad (26)$$

Given the known correlation function matrix of load $[R_P(\tau-\tau_1+\varepsilon)]$, the load action time t and time interval ε, let $[r_{ij}]_{s\times s} = \int_0^t\int_0^t [h(\tau)][\phi]^{\#T}[R_P(\tau-\tau_1+\varepsilon)][\phi]^{\#}[h(\tau_1)]^T d\tau d\tau_1$, and the matrix $[r_{ij}]_{s\times s}$ can be obtained by using engineering analytical software MATLAB to integrate. Its element is

$$r_{ij} = a_1 \cdot \Bigg\{\sum_{k=1}^{p} b_{1k}\cdot\cos(c_{1k}\omega_i + c_{2k}\omega_j)\cdot\exp(d_{1k}\omega_i + d_{2k}\omega_j) + \sum_{m=1}^{l} b_{1m}\cdot\sin(c_{1m}\omega_i + c_{2m}\omega_j)$$
$$\exp(d_{1m}\cdot\omega_i + d_{2m}\cdot\omega_j) + b_3\cdot(\cos(c_2\cdot\omega_i))\cdot\exp(d_2\cdot\omega_i) + b_4\cdot(\sin(c_3\cdot\omega_i))\cdot\exp(d_3\cdot\omega_i)$$
$$+ b_5\cdot(\cos(c_4\cdot\omega_j))\cdot\exp(d_4\cdot\omega_j) + b_6\cdot(\sin(c_5\cdot\omega_j))\cdot\exp(d_5\cdot\omega_j) - a_3\Bigg\}/(\omega_i^2\cdot\omega_j^2)$$
$$(i,j = 1, 2, \ldots, s;\ k = 1, 2, \ldots, p;\ m = 1, 2, \ldots, l) \quad (27)$$

where r_{ij} is the ith line and the jth column element of matrix $[r_{ij}]_{s\times s}$, and it is a fuzzy variable; a_1, $a_3, b_{1k}, b_{1m}, b_3, b_4, b_5, b_6, c_{1k}, c_{2k}, d_{1m}, d_{2m}, c_2, c_3, c_4, c_5, d_2, d_{1k}, d_{2k}, c_{1m}, c_{2m}, d_3, d_4, d_5$ are constants; ω_i and ω_j are the ith and jth structural fuzzy natural frequency respectively.

From the optimization theory, r_{ij} is a function with two variables ω_i and ω_j. Value ranges of ω_i and ω_j are $[\omega_{iL}, \omega_{iR}]$ and $[\omega_{jL}, \omega_{jR}]$ respectively, and they construct a protruding set (when ω_i is taken as X-axis and ω_j is taken as Y-axis) which is the constraints of the function r_{ij}. From the theory of the non-linear programming and by using indirect optimization method like sequential unconstrained minimization technique (that is, SUMT), the maximum r_{ijR} and the minimum r_{ijL} of r_{ij} within the constraints can be obtained. When ω_i and ω_j in Eq. (27) are taken as ω_{iM} and ω_{jM} simultaneously, the main value r_{ijM} of r_{ij} can be obtained.

From fuzzy factor method, the fuzzy factor $\tilde{r}_{ij} = (1, \tilde{r}_{ijL}, \tilde{r}_{ijR})$ of r_{ij} can be obtained. The main value of $\tilde{r}_{ij}$ is 1, and its value range is $[\tilde{r}_{ijL} = r_{ijL}/r_{ijM}, \tilde{r}_{ijR} = r_{ijR}/r_{ijM}]$. The fuzzy factor $\tilde{r}_{ij}$ with largest value range among all $\tilde{r}_{ij}(i,j = 1, 2, \ldots, s)$ is taken as the uniform fuzzy factor $\tilde{r}$ of matrix $[r_{ij}]_{s\times s}$.

Given the known loads' self-power spectral density and mutual-power spectral density (that is, the correlation function matrix $[R_P(\tau-\tau_1+\varepsilon)]$ of applied loads vector can be obtained by using Fourier transformation) and the structural fuzzy parameters, for the same load action time t, fuzzy factor $\tilde{r}$ of the matrix $[r_{ij}]_{s\times s}$ is independent to the time interval ε. As an evidence, the fuzzy factor $\tilde{r}$ of a 10-bar fuzzy truss is computed and the results are listed in Table 1, where the load's self-power spectral density $S_{pp} = 20\ \mathrm{N^2/s}$ which is a white noise stationary stochastic process acting

Table 1 $\tilde{r}$ of 10-bar fuzzy truss when different time intervals ε are taken

Time interval $\varepsilon(s)$	Time of applied load action $t(s)$	Self-power spectral density $S_{pp}(\mathrm{N^2/s})$	Self-correlation function of applied load R_{pp}	Fuzzy factor $\tilde{r}$ of $[r_{ij}]_{s\times s}$
1e-4	10	20	1799	(1, 0.7935, 1.2764)
1e-2	10	20	−187.098	(1, 0.7935, 1.2764)
1	10	20	−3.0705e-5	(1, 0.7935, 1.2764)

on one of the nodes, the time of applied load action $t = 10$s and the structural fuzzy parameters are the same as those of the example 5 below.

Then Eq. (26) can be described as

$$[R_u(\varepsilon)] = (\tilde{\phi}^4 \cdot \tilde{r}) \cdot ([\phi]^{\#} \int_0^t \int_0^t [h(\tau)]^{\#} [\phi]^{\#T} [R_P(\tau - \tau_1 + \varepsilon)][\phi]^{\#} [h(\tau_1)]^{\#T} d\tau d\tau_1 [\phi]^{\#T})$$
$$= (\tilde{\phi}^4 \cdot \tilde{r})[R_u(\varepsilon)]^{\#} = \tilde{R}_u \cdot [R_u(\varepsilon)]^{\#} \quad (28)$$

where $\tilde{R}_u$ is fuzzy factor of $[R_u(\varepsilon)]$, from interval arithmetic, $\tilde{R}_u = (\tilde{R}_{uM} = 1, \tilde{R}_{uL} = \tilde{\phi}_L^4 \cdot \tilde{r}_L \ \ \tilde{R}_{uR} = \tilde{\phi}_R^4 \cdot \tilde{r}_R)$; $[R_u(\varepsilon)]^{\#}$ is the deterministic part of $[R_u(\varepsilon)]$ when $\tilde{R}_u = 1$.

From Eq. (8), the power spectral density matrix of the fuzzy displacement response $[S_u(\omega)]$ is

$$[S_u(\omega)] = \tilde{R}_u \cdot ([\phi]^{\#} [H(\omega)]^{\#} [\phi]^{\#T} [S_P(\omega)][\phi]^{\#} [H^*(\omega)]^{\#} [\phi]^{\#T}) = \tilde{R}_u \cdot [S_u(\omega)]^{\#} \quad (29)$$

where $[S_u(\omega)]^{\#}$ is the deterministic part of $[S_u(\omega)]$.

From Eq. (11), in frequency domain $[0, \omega_c]$, the mean square value matrix of displacement response is

$$[\psi_u^2] = \tilde{R}_u \cdot \int_0^{\omega_c} [\phi]^{\#} [H(\omega)]^{\#} [\phi]^{\#T} [S_P(\omega)][\phi]^{\#} [H^*(\omega)]^{\#} [\phi]^{\#T} d\omega = \tilde{R}_u \cdot [\psi_u^2]^{\#} \quad (30)$$

where $[\psi_u^2]^{\#}$ is the deterministic part of $[\psi_u^2]$ when $\tilde{R}_u = 1$; when fuzzy factor $\tilde{R}_u = \tilde{R}_{uL}$ and $\tilde{R}_u = \tilde{R}_{uR}$, the minimum and maximum of $[\psi_u^2]$ are obtained respectively.

From Eq. (12), the fuzzy mean square value of displacement response of the *k*th degree of freedom is

$$\psi_{uk}^2 = \tilde{R}_u \cdot \left(\vec{\phi}_k^{\#} \int_0^{\omega_c} [H(\omega)]^{\#} [\phi]^{\#T} [S_P(\omega)][\phi]^{\#} [H^*(\omega)]^{\#} d\omega \cdot \vec{\phi}_k^{\#T} \right) = \tilde{R}_u \psi_{uk}^{2\#} \quad (k = 1, 2, \ldots, n) \quad (31)$$

where $\psi_{uk}^{2\#}$ is the deterministic part of ψ_{uk}^2; $\vec{\phi}_k^{\#}$ is the *k*th line vector of deterministic normal modal matrix $[\phi]^{\#}$, and $\vec{\phi}_k^{\#T}$ is its transpose.

From Eq. (14), the fuzzy correlation function matrix of stress response of the *e*th bar element $[R_\sigma^{(e)}(\tau)]$ is

$$[R_\sigma^{(e)}(\tau)] = E^{(e)}[B][R_u^{(e)}(\tau)][B]^T E^{(e)} = (\tilde{E}^2 \tilde{R}_u) \cdot E_M^{(e)}[B][\phi]^{\#} (\int_0^t \int_0^t [h(\tau)]^{\#} [\phi]^{\#T} [R_P(\tau - \tau_1 + \varepsilon)]$$
$$\cdot [\phi]^{\#} [h(\tau_1)]^{\#T} d\tau d\tau_1)[\phi]^{\#T} [B]^T E_M^{(e)} = (\tilde{E}^2 \cdot \tilde{R}_u)[R_\sigma^{(e)}(\tau)]^{\#} = \tilde{R}_\sigma \cdot [R_\sigma^{(e)}(\tau)]^{\#} \quad (32)$$

where $E_M^{(e)}$ is the main value of $E^{(e)}$; $\tilde{R}_\sigma$ is the fuzzy factor of $[R_\sigma^{(e)}(\tau)]$, and $\tilde{R}_\sigma = (\tilde{R}_{\sigma M} = 1, \tilde{R}_{\sigma L} = \tilde{E}_L^2 \cdot \tilde{R}_{uL}, \tilde{R}_{\sigma R} = \tilde{E}_R^2 \cdot \tilde{R}_{uR})$; $[R_\sigma^{(e)}(\tau)]^{\#}$ is the deterministic part of $[R_\sigma^{(e)}(\tau)]$ when $\tilde{R}_\sigma = 1$.

From Eq. (15), the fuzzy power spectral density matrix of the *e*th element's stress response $[S_\sigma^{(e)}(\omega)]$ is

$$[S_\sigma^{(e)}(\omega)] = E^{(e)}[B][S_u^{(e)}(\omega)][B]^T E^{(e)}$$
$$= \tilde{R}_\sigma \cdot (E_M^{(e)}[B][\phi]^{\#} [H(\omega)]^{\#} [\phi]^{\#T} [S_P(\omega)][\phi]^{\#} [H^*(\omega)]^{\#} [\phi]^{\#T} [B]^T E_M^{(e)}) = \tilde{R}_\sigma [S_\sigma^{(e)}(\omega)]^{\#} \quad (33)$$

where $[S_\sigma^{(e)}(\omega)]^\#$ is the deterministic part (that is, fuzzy main value) of $[S_\sigma^{(e)}(\omega)]$ when $\tilde{R}_\sigma = 1$.

From Eq. (16), the fuzzy mean square value of the eth element's stress response is

$$[\psi^2_{\sigma^{(e)}}] = E^{(e)}[B][\psi^2_{u^{(e)}}][B]^T E^{(e)} = \tilde{R}_\sigma(E_M^{(e)}[B] \cdot \int_0^{\omega_c}[\phi]^\#[H(\omega)]^\#[\phi]^{\#T}[S_P(\omega)]$$
$$\cdot [\phi]^\#[H^*(\omega)]^\#[\phi]^{\#T}d\omega[B]^T E_M^{(e)} = \tilde{R}_\sigma \cdot [\psi^2_{\sigma^{(e)}}]^\# \quad (34)$$

where $[\psi^2_{\sigma^{(e)}}]^\#$ is the deterministic part (that is, the fuzzy main value) of $[\psi^2_{\sigma^{(e)}}]$ when $\tilde{R}_\sigma = 1$; when $\tilde{R}_\sigma = \tilde{R}_{\sigma L}$, the minimum of $[\psi^2_{\sigma^{(e)}}]$ is obtained; when $\tilde{R}_\sigma = \tilde{R}_{\sigma R}$, the maximum of $[\psi^2_{\sigma^{(e)}}]$.

In the computational course above, taking the main values of all structural parameters as the deterministic structural parameters, here ω_{iM} and $\{\phi\}_i^\#$ can be obtained by engineering software ANSYS, and $[R_u(\varepsilon)]^\#$, $[S_u(\omega)]^\#$, $[\psi_u^2]^\#$, $\psi_{uk}^{2\#}$, $[R_\sigma^{(e)}(\tau)]^\#$, $[S_\sigma^{(e)}(\omega)]^\#$, $[\psi^2_{\sigma^{(e)}}]^\#$ can be obtained by using engineering software MATLAB to program expediently. Additionally, the method presented in this paper is only applicable to the truss structures with the global fuzzy structural parameters, that is, the fuzziness of the structural parameters of all elements is respectively equal to each other.

5. An example

A 20-bar planar truss is taken as the example. All structural parameters are symmetric normal fuzzy numbers $M = (M^\#, \alpha, \alpha)_{LR}$: Young's module $E = (2.1,\ 0.04,\ 0.04) \times 10^{11}$ pa, mass density $\rho = (7.8,\ 0.02,\ 0.02) \times 10^3$ kg/m^3, the bars' fuzzy dispersion degree is equivalent: $\{L\} = (\{l\}, 0.001\{l\}, 0.001\{l\})$, bar's cross-sectional area $A = (1.0,\ 0.003,\ 0.003) \times 10^{-4}$ m^2; taking membership

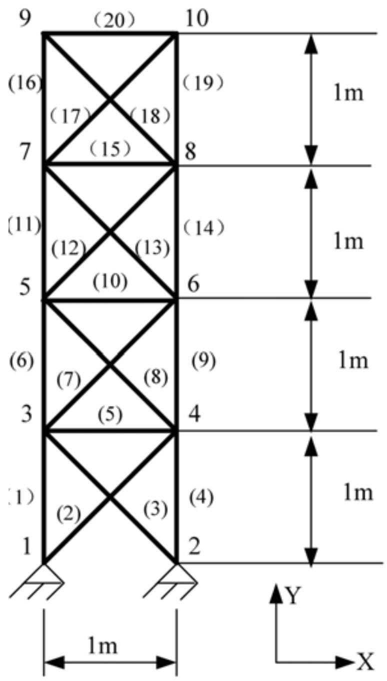

Fig. 1 20 bar planar truss system

grade $\lambda = 1.365 \times 10^{-11}$, from fuzzy factor method, they are transferred into fuzzy numbers $M = (M_M, M_L, M_R)$: $E = (2.1, 1.9, 2.3) \times 10^{11}$ pa, $\rho = (7.8, 7.7, 7.9) \times 10^3$ kg/m^3, $\{L\} = (\{l\}, \{l - 0.005l\}, \{l + 0.005l\})$, $A = (1.0, 0.985, 1.015) \times 10^{-4}$ m^2. The structural mode damping $\xi_j = \xi = 0.01$. A stationary stochastic (white noise stationary stochastic process) applied load $P(t)$ acts on node 9 along the direction of X-axis. Its self-power spectral density is $S_0 = 20$ N^2/s and its center frequency is $\omega_0 = 45$ Hz, and its band width is $B = 90$ Hz. In the course of computation, the end frequency is $\omega_c = \omega_0 + B/2 = 90$ Hz. The first ten order modes are chosen to compute. The mean square value of the 9th-node displacement response in the direction of X-axis ψ^2_{X9} and mean square value of the 1st element's stress response $\psi^2_{\sigma 1}$ are listed in Table 2, and the fuzzy factors of structural parameters, natural frequencies and normal mode shapes are given simultaneously.

To verify the feasibility of the modeling and method given in this paper, the example above is simulated by Monte-Carlo simulation method in which all structural fuzzy parameters are simulated by similar distribution. The simulation number is 1000. The results of these two methods are listed in Table 3 and they are very close.

Table 2 Dynamic response of 20-bar planar truss

Fuzzy variable	$\tilde{l}$	$\tilde{E}$	$\tilde{A}$	$\tilde{\rho}$	$\tilde{\phi}$	$\tilde{\omega}$	$\psi^2_{X9}(/10^{-4}\text{m}^2)$	$\psi^2_{\sigma 1}(/\text{MPa}^2)$
Main value	1	1	1	1	1	1	5.4066	4.7733×10^4
Minimum	0.995	0.905	0.985	0.987	0.984	0.942	4.0409	2.9217×10^4
Maximum	1.005	1.095	1.015	1.013	1.017	1.058	7.3254	7.7545×10^4

Table 3 Results of 20-bar planar truss by using different methods

Results of method in this paper		Results of Monte-Carlo simulation method	
	The mean square value of maximal displacement response $\psi^2_{X9}(/10^{-4}\text{m}^2)$		The mean square value of maximal displacement response $\psi^2_{X9}(/10^{-4}\text{m}^2)$
Fuzzy main value	5.4066	Mean value μ	5.3890
Minimum	4.0409	$\mu - 3\sigma$	4.0780
Maximum	7.3254	$\mu + 3\sigma$	6.7170

Table 4 The main value ψ^2_{X9M}, minimum ψ^2_{X9L} and maximum ψ^2_{X9R} of the mean square value of the 9th-node displacement response in the direction of X-axis ψ^2_{X9}

Model	ψ^2_{X9M} $(/10^{-4}\text{m}^2)$	ψ^2_{X9L} $(/10^{-4}\text{m}^2)$	ψ^2_{X9R} $(/10^{-4}\text{m}^2)$	Fuzzy factor $\tilde{R}_u$
$\tilde{E} = (1, 0.95, 1.05), \tilde{\rho} = \tilde{A} = \tilde{l} = 1$	5.4066	4.7994	6.0700	(1, 0.8877, 1.1227)
$\tilde{\rho} = (1, 0.95, 1.05), \tilde{E} = \tilde{A} = \tilde{l} = 1$	5.4066	3.7741	6.6441	(1, 0.6981, 1.2289)
$\tilde{E} = \tilde{\rho} = (1, 0.95, 1.05), \tilde{A} = \tilde{l} = 1$	5.4066	3.9277	7.2677	(1, 0.7265, 1.3442)
$\tilde{A} = (1, 0.95, 1.05), \tilde{E} = \tilde{\rho} = \tilde{l} = 1$	5.4066	4.9059	5.9905	(1, 0.9074, 1.1080)
$\tilde{l} = (1, 0.95, 1.05), \tilde{E} = \tilde{\rho} = \tilde{A} = 1$	5.4066	3.9959	7.2809	(1, 0.7391, 1.3467)
$\tilde{A} = \tilde{l} = (1, 0.95, 1.05), \tilde{E} = \tilde{\rho} = 1$	5.4066	3.6233	8.0668	(1, 0.6702, 1.4920)
$\tilde{E} = \tilde{\rho} = \tilde{A} = \tilde{l} = (1, 0.95, 1.05)$	5.4066	2.9201	9.8493	(1, 0.5401, 1.8217)
$\tilde{E} = \tilde{\rho} = \tilde{A} = \tilde{l} = (1, 0.995, 1.005)$	5.4066	4.9082	5.8514	(1, 0.9078, 1.0823)

Table 5 The main value $\psi^2_{\sigma 1M}$, maximum $\psi^2_{\sigma 1R}$ and minimum $\psi^2_{\sigma 1L}$ of mean square value of the 1th element's stress response $\psi^2_{\sigma 1}$

Model	$\psi^2_{\sigma 1M}$ (/Mpa^2)	$\psi^2_{\sigma 1L}$ (/Mpa^2)	$\psi^2_{\sigma 1R}$ (/Mpa^2)	Fuzzy factor $\tilde{R}_\sigma$
$\tilde{E}$ = (1, 0.95, 1.05), $\tilde{\rho} = \tilde{A} = \tilde{l} = 1$	4.7733 × 10^4	3.8239 × 10^4	5.9084 × 10^4	(1, 0.8011, 1.2378)
$\tilde{\rho}$ = (1, 0.95, 1.05), $\tilde{E} = \tilde{A} = \tilde{l} = 1$	4.7733 × 10^4	2.8406 × 10^4	5.9027 × 10^4	(1, 0.6981, 1.2289)
$\tilde{A}$ = (1, 0.95, 1.05), $\tilde{E} = \tilde{\rho} = \tilde{l} = 1$	4.7733 × 10^4	4.3313 × 10^4	5.2888 × 10^4	(1, 0.9074, 1.1080)
$\tilde{l}$ = (1, 0.95, 1.05), $\tilde{E} = \tilde{\rho} = \tilde{A} = 1$	4.7733 × 10^4	3.5278 × 10^4	6.4282 × 10^4	(1, 0.7391, 1.3467)
$\tilde{E} = \tilde{\rho}$ = (1, 0.95, 1.05), $\tilde{A} = \tilde{l} = 1$	4.7733 × 10^4	3.1294 × 10^4	7.0740 × 10^4	(1, 0.6557, 1.4820)
$\tilde{A} = \tilde{l}$ = (1, 0.95, 1.05), $\tilde{E} = \tilde{\rho} = 1$	4.7733 × 10^4	3.1991 × 10^4	7.1218 × 10^4	(1, 0.6702, 1.4920)
$\tilde{E} = \tilde{\rho} = \tilde{A} = \tilde{l}$ = (1, 0.95, 1.05)	4.7733 × 10^4	2.3265 × 10^4	9.5848 × 10^4	(1, 0.4874, 2.0080)
$\tilde{E} = \tilde{\rho} = \tilde{A} = \tilde{l}$ = (1, 0.995, 1.005)	4.7733 × 10^4	4.2902 × 10^4	5.2177 × 10^4	(1, 0.8988, 1.0932)

To investigate the effects of the fuzzy dispersal degree of E, ρ, l and A on the structural dynamic response, the different models are selected and the parameter E, ρ, l or A is respectively taken as fuzzy variable in different groups. The main value ψ^2_{X9M}, the maximum ψ^2_{X9R} and minimum ψ^2_{X9L} of the mean square value of the 9th-node displacement response in the direction of X-axis ψ^2_{X9} are listed in Table 4 respectively. The main value $\psi^2_{\sigma 1M}$, the maximum $\psi^2_{\sigma 1R}$ and minimum $\psi^2_{\sigma 1L}$ of the mean square value of the 1st element's stress response $\psi^2_{\sigma 1}$ are listed in Table 5 respectively.

The results in Tables 4 and 5 show that

1) When the fuzzy dispersion degree of E, ρ, l and A equals, the effects of their fuzziness on the fuzziness of mean square value of the structural displacement and stress response are different. The fuzziness of bar's length l produces the greatest effect on the fuzziness of the mean square value of the structural displacement and stress response, mass density ρ takes second place.

2) When fuzzy dispersion degree of physical parameters are equal to that of geometric dimensions, the fuzziness of the later affects more on the fuzziness of the mean square value of structural displacement response, while the fuzziness of former has a greater effect on that of the mean square value of stress response.

3) Comparing with the conditions that only one kind of fuzziness of E, ρ, l and A is considered, under the condition that their fuzziness are all considered, the fuzziness of the mean square value of structural displacement and stress response is greater.

4) With the increase of the fuzziness of E, ρ, l and A, the fuzzy dispersal degree of the mean square value of structural dynamic response will increase as well.

6. Conclusions

The fuzzy factor method is presented for the dynamic response analysis of the fuzzy truss structures under the stationary stochastic excitation, in which the influences of fuzziness of structural parameters on the fuzziness of the mean square values of structural displacement and stress response can be reflected expediently and objectively. Specially, we analyze the stationary stochastic dynamic response of the conventional deterministic structure only one time so that to obtain the deterministic parts (main values) of the mean square values of the dynamic response by means of engineering analytical software ANASYS or MATLAB, then the fuzzy mean square values of displacement and stress response can be obtained by fuzzy factor method with a small computational amount. The example shows that the modeling and method of the stationary stochastic dynamic response analysis of fuzzy truss structure presented in this paper are rational and feasible.

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