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Earthquake response analysis of series reactor

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Abstract. A direct transfer substructure method is presented in this paper for analyzing the dynamic characteristics and the seismic random responses of a series reactor. This method combines the concept of FRF (frequency response function) and the transfer matrix algorithm with the substructure approach. The inner degrees of freedom of each substructure are eliminated in the process of reconstruction and the computation cost is reduced greatly. With the convenient solution procedure, the dynamic characteristics analysis of the structure is valid and efficient. Associated with the pseudo excitation algorithm, the direct transfer substructure method is applied to investigating the seismic random responses of the series reactor. The numerical results demonstrate that the presented method is efficient and practicable in engineering. Finally, a precise time integration method is employed in performing a time-history analysis on the series reactor under El Centro and Taft earthquake waves.

Key words: earthquake; seismic; substructure; random response; dynamic characteristics.

1. Introduction

Seismic events could destroy a lot of buildings, bridges, industrial and port facilities as well as giving rise to great economic losses. Thus, to ensure the structural safety under disastrous earthquake actions is usually of great importance in engineering practice. As a result, many analysis methods for structural seismic responses have been presented in recent years (Somerville 2000, Chandler and Lam 2001, Cofer *et al*. 2002, Lin *et al.* 2001). However, large numbers of finite elements are inevitable in dynamic analysis for large-scale and complex structures, and the numeric analysis using these known methods is difficult due to the unavoidable computational efforts. For solving this problem, many substructure synthesis methods have been developed in the past and subsequently applied to numerous different projects (Hurty 1960, Gladwell 1964, Craig and Chang 1976, Craig 1995). On the other hand, the transfer matrix algorithm is efficiently used in dynamic analysis for chain structures, especially for one-dimensional structures (Geradin and Chen 1995, Lee 2000). It is well known that the transfer matrix algorithm is not convenient for complex structures. Thus, with combination of the transfer matrix algorithm and the substructure synthesis method, an accurate and efficient method, named the direct transfer substructure method (DTSM), is presented

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in this paper and applied to calculating the dynamic characteristics and the seismic random responses of a series reactor.

The principle of DTSM is to divide a complex chain structure into a series of substructures, then construct the transfer matrix of each substructure using the concept of FRF (frequency response function). Finally, the dynamic model of the whole structure is reconstructed by combining those transfer matrices of substructures. Using the eigensolution of each substructure, the FRF matrix can be achieved conveniently and the transfer matrix of each substructure can be determined directly. Because the inner degrees of freedom of every substructure are eliminated, the numerical computational efforts required in DTSM are greatly reduced. Furthermore, the implementation of DTSM is simple and convenient. In addition, since FRF of every substructure could be obtained by experiments directly, this method could facilitate the incorporation of experimental results.

The random vibration approach has been widely recognized as a reasonable method, and therefore it has often been applied to structural seismic responses analysis. The theoretical framework of a methodology for stochastic-response analysis to random-excitation fields is already available. However, its application within the earthquake engineering community is viewed as impractical except for simple structures with a small number of degrees of freedom and supports (Ernesto and Vanmarcke 1994). When large numbers of finite elements and degrees of freedom are inevitable in random response analysis for large-scale and complex structures, some unacceptable computational efforts and a great deal of CPU time are unavoidable. A random vibration approach, called the pseudo excitation method (PEM), has been developed in Lin *et al.* (2001). The essential principle of PEM is that the random excitation applied to the structure is converted into certain harmonic excitations, and the stationary random responses are computed by means of harmonic vibration analysis for any linear structure. However, a large amount of computer time is consumed by repeated harmonic response calculations in the random response analysis of complex structures. The direct transfer substructure method, associated with the pseudo excitation algorithm, can be exploited to compute the seismic random responses of the chain structure efficiently.

The purposes of this paper are: (1) to build the proper model of the series reactor according to the experimental data, (2) to present a direct transfer substructure method for dynamic characteristic analysis of the series reactor, (3) to apply the direct transfer substructure method to calculate the seismic random response of the structure, and (4) to analyze the transient response of the structure under actual measurement seismic excitations using the precise integration method.

2. Modeling and experimental validation of a single reactor

A single reactor consists of four main components: reactor body, star frame, insulator and insulated tube. As shown in Fig. 1(a), the reactor body is sandwiched between two star frames and the lower star frame is fixed on twelve circumferential insulators with bolts. These insulators are connected with twelve insulated tubes, which are fixed in the ground base. The mass of the reactor body is 7110 kg. The inner and outer diameter of the reactor body are 1.116 m and 2.790 m, respectively. Using the finite element method (FEM) (Bathe 1982), the single reactor is discretized for dynamic analysis. The reactor body is modeled by solid brick elements and the other components, insulators and insulated tubes, are modeled by three-dimensional beam elements, as Fig. 1(b) shows. The entire structure has 168 elements, 252 nodes and 936 degrees of freedom.

Based on the FEM model, the characteristic roots of the single reactor are calculated and the

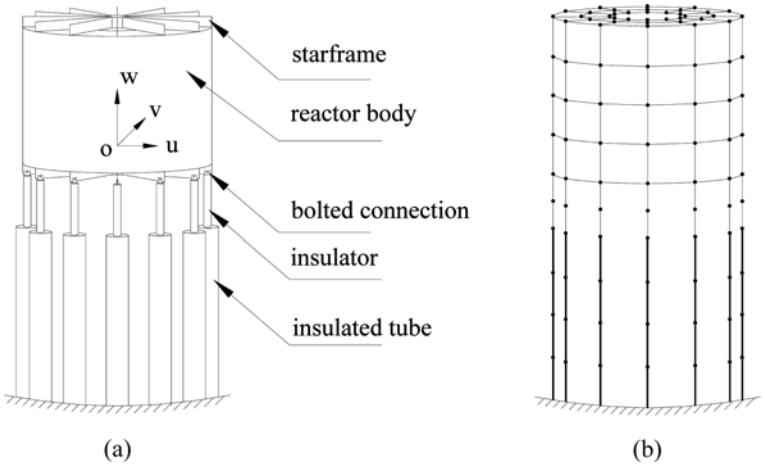

Fig. 1 (a) Single reactor configuration and (b) FEM model

Table 1 Comparison of the eigenvalues (Hz) of a single reactor obtained by FEM and experiment

Vibration mode	1st	2nd	3rd	4th	5th
FEM	17.1020	17.1020	24.1201	46.3029	52.6054
Experiment	16.75	16.75	24.75		
Error /%	2.10	2.10	2.55		

results are shown in Table 1, where the second row expresses the experimental results. The numerical results are in good agreement with the experimental results, indicating that the modeling of the structure is reasonable.

3. The eigensolutions analysis of a series reactor using the direct transfer substructure method

3.1 The model of a series reactor

As shown in Fig. 2, the structure, analyzed to obtain its earthquake response in this paper, is a series reactor, a kind of protective device of large electric appliances. As an important short-circuit protection device, a series reactor is used to ensure the security of large electric applications and plays a considerable role in the electric, chemical and metallurgical industries and so on. Hence, to ensure its safety under earthquake actions is of great significance and thus an effective seismic analysis is critical.

The series reactor is a chain structure, constituted by superimposing a single reactor above another (Fig. 3(a)). It can be divided into three substructures, as shown in Fig. 3(b). Each substructure is connected to adjacent substructures by joints of the corresponding interface. As with the single reactor, the series reactor is discretized with the aid of FEM. The reactor bodies are modeled by solid brick elements and the other components, insulators and insulated tubes, are modeled by 3D

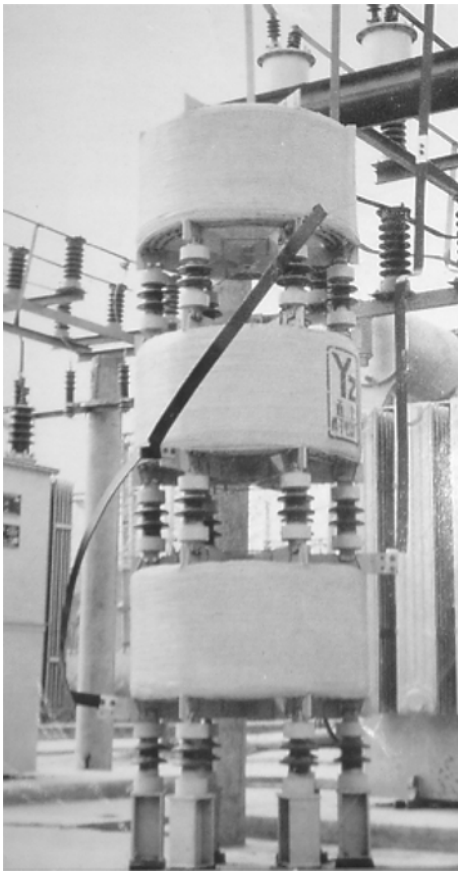

Fig. 2 A series reactor

beam elements. Then the first substructure, same as the second substructure, has 120 elements, 204 nodes and 720 degrees of freedom. The third substructure has 168 elements, 252 nodes and 936 degrees of freedom. Since we have confirmed that the model of the single reactor is correct by experiment, it is reasonable to expect that the model of the series reactor, assembled with several single reactors, is proper.

3.2 The frequency response function matrix of a substructure

It is assumed that the interface constraints on the ith (i = 1, 2, 3) substructure of the series reactor are released. The dynamic equation of the ith substructure can be expressed as

$$M_i\ddot{x}_i(t) + K_i x_i(t) = f_i(t) \tag{1}$$

where the subscript i refers to the ith substructure. M_i and K_i are the mass and stiffness matrices, respectively. $x_i(t)$ is the displacement response vector; and $f_i(t)$ is the excitation vector applied to the ith substructure.

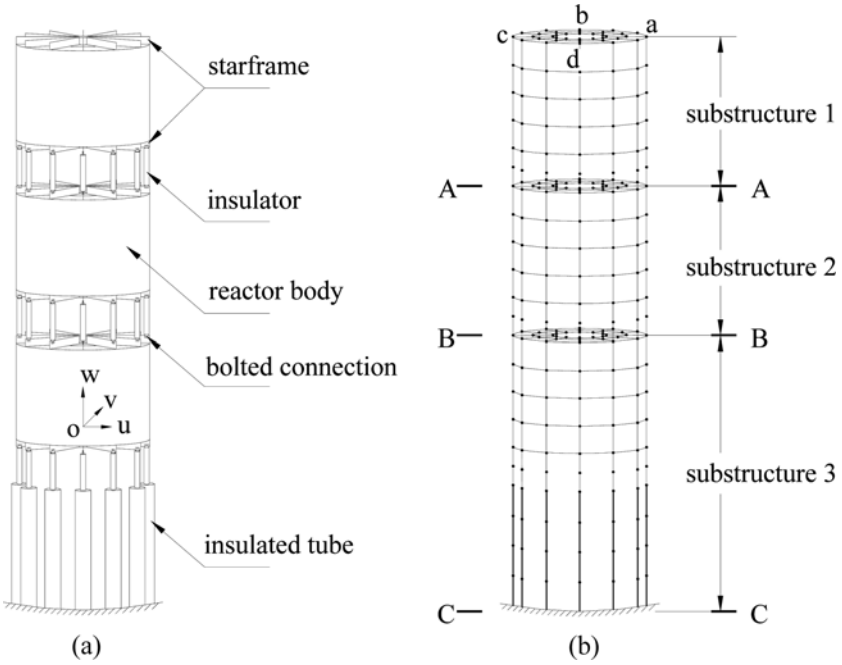

Fig. 3 (a) Series reactor configuration and (b) FEM model with three substructures

The eigenvalues and eigenvectors of substructure i can be obtained by solving the eigenproblem of Eq. (1). Let Λ_i be the spectrum matrix, a diagonal matrix of generalized eigenvalues, and Φ_i be a full matrix whose columns are the corresponding eigenvectors. The following generalized orthogonality properties can be established:

$$\Phi_i^T M_i \Phi_i = I, \quad \Phi_i^T K_i \Phi_i = \Lambda_i \tag{2}$$

Using a transformation of coordinates, the displacement response vector $x_i(t)$ can be expressed as

$$x_i(t) = \Phi_i p_i(t) \tag{3}$$

where $p_i(t)$ is the modal coordinate vector of the ith substructure. Then Eq. (1) becomes

$$\ddot{p}_i(t) + \Lambda_i p_i(t) = \Phi_i^T f_i(t) \tag{4}$$

Under harmonic excitation at a frequency ω, $f_i(t) = F_i e^{j\omega t}$ and $x_i(t) = X_i e^{j\omega t}$ are proposed. j is the imaginary unit. F_i and X_i are the amplitude vectors of the excitation and the displacement response, respectively. X_i can be expressed as

$$X_i = \Phi_i [\Lambda_i - \omega^2 I]^{-1} \Phi_i^T F_i = H_i(\omega) F_i \tag{5}$$

The frequency response function (FRF) matrix $H_i(\omega)$ can be given by

$$H_i(\omega) = \Phi_i[\Lambda_i - \omega^2 I]^{-1}\Phi_i^T \tag{6}$$

Thus, FRF can be calculated by simple multiplication when the eigensolution of the ith substructure is obtained. Note that the complete substructure modal set is not needed to obtain the FRF matrix because of the special nature of Eq. (6). The participant substructure modes can be obtained from finite element analysis or experimental testing.

3.3 Transfer matrix of a substructure

The physical characteristics matrices can be partitioned into partial matrices (vectors). The superscripts U, L and I refer to the partial matrices (vectors) corresponding to the upper joints, lower joints and the inner nodes of a substructure, respectively. Therefore, Eq. (5) can be written as follows:

$$\begin{Bmatrix} X_i^U \\ X_i^I \\ X_i^L \end{Bmatrix} = \begin{Bmatrix} \Phi_i^U \\ \Phi_i^I \\ \Phi_i^L \end{Bmatrix}[\bar{\Lambda}_i]\begin{bmatrix} \Phi_i^U \\ \Phi_i^I \\ \Phi_i^L \end{bmatrix}^T \begin{Bmatrix} F_i^U \\ F_i^I \\ F_i^L \end{Bmatrix} = \begin{bmatrix} H_i^{UU} & H_i^{UI} & H_i^{UL} \\ H_i^{IU} & H_i^{II} & H_i^{IL} \\ H_i^{LU} & H_i^{LI} & H_i^{LL} \end{bmatrix}\begin{Bmatrix} F_i^U \\ F_i^I \\ F_i^L \end{Bmatrix} \tag{7}$$

where $H_i^{ab} = \Phi_i^a \bar{\Lambda}_i \Phi_i^{b^T}$, $a, b = U, I, L$; $\bar{\Lambda}_i = [\Lambda_i - \omega^2 I]^{-1}$.

For natural vibration, the external forces acting on the ith substructure are zero, i.e., $F_i^I = 0$. By expanding Eq. (7), an equation is deduced to show the relation between the upper and the lower joint response vectors.

$$\begin{Bmatrix} X_i^U \\ F_i^U \end{Bmatrix} = \begin{bmatrix} H_i^{UU}(H_i^{LU})^{-1} & H_i^{UU}(H_i^{LU})^{-1}H_i^{LL} - H_i^{UL} \\ (H_i^{LU})^{-1} & (H_i^{LU})^{-1}H_i^{LL} \end{bmatrix}\begin{Bmatrix} X_i^L \\ -F_i^L \end{Bmatrix} = T_i\begin{Bmatrix} X_i^L \\ -F_i^L \end{Bmatrix} \tag{8}$$

where T_i is the transfer matrix of the ith substructure. H_i^{ab} ($a, b = U, I, L$) is only correlated with the eigensolution corresponding to the upper and the lower joints without regard to the inner degrees of freedom of the ith substructure. Therefore, the degrees of freedom at the inner nodes of the ith substructure are eliminated completely in the transfer matrix and the computation cost has a dramatic decrease.

3.4 The natural frequencies of the series reactor

The conditions for geometric compatibility and force equilibrium of the ith substructure can be given as

$$\begin{Bmatrix} X_i^L \\ -F_i^L \end{Bmatrix} = \begin{Bmatrix} X_{i+1}^U \\ F_{i+1}^U \end{Bmatrix} \tag{9}$$

Table 2 Comparison of the eigenvalues (Hz) of a series reactor obtained by DTSM and FEM

Method	Vibration mode								
	1,2	3	4	5,6	7	8,9	10	20	30
DTSM	4.4262	9.7960	14.9483	16.1072	31.6035	39.1171	46.8752	66.6209	86.2192
FEM	4.4110	9.7725	14.9361	16.0200	31.5750	38.6434	46.7231	66.2226	85.9725

According to the boundary conditions expressed in Eq. (9), the transfer equations of 1~3 substructures of the series reactor can be obtained as follows:

$$\begin{Bmatrix} X_1^U \\ F_1^U \end{Bmatrix} = T \begin{Bmatrix} X_3^L \\ -F_3^L \end{Bmatrix} = \begin{bmatrix} T_{11} & T_{13} \\ T_{31} & T_{33} \end{bmatrix} \begin{Bmatrix} X_3^L \\ -F_3^L \end{Bmatrix} \tag{10}$$

where $T = T_1 T_2 T_3$ is the transfer matrix of the entire structure.

In order to calculate the structural natural frequencies, the terminal conditions of the whole structure should be introduced to Eq. (10). For the series reactor, its top is a free boundary and its base is a fixed boundary; the total boundary conditions can be expressed as

$$F_1^U = 0, \quad X_3^L = 0 \tag{11}$$

According to the terminal conditions of the whole structure, the second line of Eq. (10) is

$$T_{33}(\omega) F_3^L = 0 \tag{12}$$

To guarantee that Eq. (12) has nonzero roots, the determinant of $T_{33}(\omega)$ must be zero,

$$\det \| T_{33}(\omega) \| = 0 \tag{13}$$

where $\det \| T_{33}(\omega) \|$ is a function of frequency ω. The natural frequencies of the series reactor can be determined from Eq. (13). Through scanning the particular desired frequency range, the results of Eq. (13) can be determined. The structural natural frequencies are shown in Table 2 where DTSM denotes the direct transfer substructure method. Since the order of $T_{33}(\omega)$ is just the number of physical connections at one end of a substructure, the numerical computation time required for resolving Eq. (13) could be reduced markedly. A Pentium-IV personal computer (main frequency 1.2 GHz) is used for analysis. Under the identical computational condition, the computation time consumed using DTSM is 35.42 s, as compared with 112.46 s consumed using FEM. Moreover, the results obtained by both methods have a uniform precision. It indicates that DTSM is an efficient method for dynamic characteristics analysis.

3.5 The modal shapes of the structure

Let a natural frequency of the structure solved from Eq. (13) be introduced into Eq. (12). The force vector F_3^L for this order natural frequency can be calculated. Substituting F_3^L for the corresponding vector in Eq. (8), the corresponding modal shape at the joints can be determined in

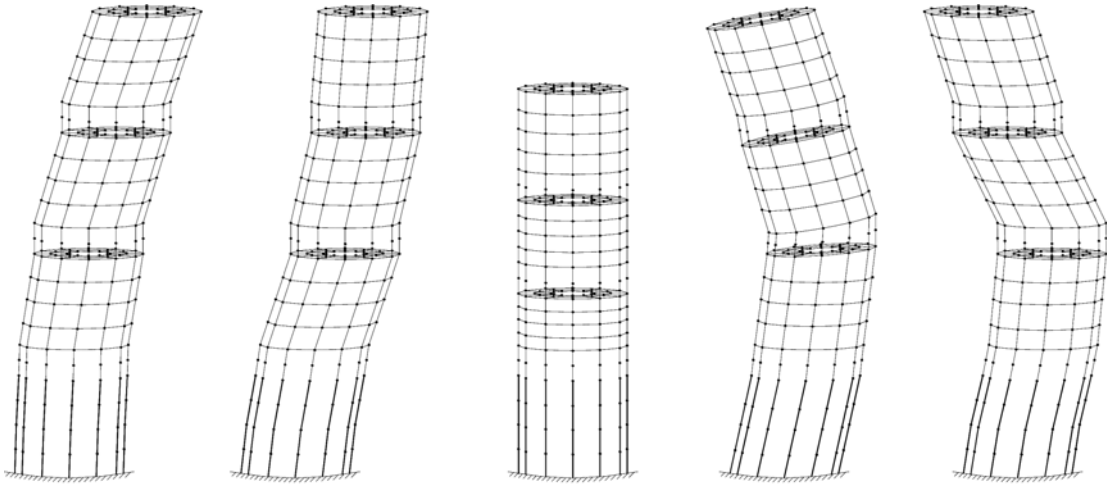

Fig. 4 Modal shapes of the series reactor

turn and the modal shape at the inner nodes can be calculated directly from Eq. (7). In the same way, other modal shapes can be obtained. The first seven modal shapes are plotted in Fig. 4.

4. Stochastic-response analysis for the series reactor

Lin *et al.* (2001) have suggested the pseudo excitation method (PEM) for random seismic response analysis. According to PEM, the random excitations applied to a structure are converted into certain harmonic excitations. The stationary random responses can be represented by means of harmonic vibration results. However, using this method, a large amount of computation time is consumed by repeated harmonic response analysis in the numerical integral procedure for calculating the displacement (stress or strain) variance. On the other hand, the harmonic response of a chain structure can be calculated effectively using DTSM. Thus, DTSM, combining with PEM, is exploited to calculate the seismic random response of the series reactor. First, the seismic random excitations are transformed to sinusoidal ones. Then, DTSM is employed to compute the response under these certain sinusoidal excitations and the seismic random response of the series reactor is analyzed accurately and conveniently.

4.1 The harmonic response of the structure

It is assumed that the ith substructure is subjected to a harmonic excitation; $\tilde{\omega}_i$ is the frequency and $\tilde{F}_i$ is the amplitude vector of the harmonic excitation. Corresponding to the upper joints, the lower joints and the inner nodes, $\tilde{F}_i$ can also be partitioned into partial vectors and the superscripts U, L and I respectively refer to the partial vectors, i.e., $\tilde{F}_i = [\tilde{F}_i^U \ \tilde{F}_i^I \ \tilde{F}_i^L]^T$. The steady-state response of substructure i can be expressed as

$$\begin{Bmatrix} X_i^U \\ X_i^I \\ X_i^L \end{Bmatrix} = \begin{bmatrix} H_i^{UU} & H_i^{UI} & H_i^{UL} \\ H_i^{IU} & H_i^{II} & H_i^{IL} \\ H_i^{LU} & H_i^{LI} & H_i^{LL} \end{bmatrix} \left(\begin{Bmatrix} F_i^U \\ 0 \\ F_i^L \end{Bmatrix} + \begin{Bmatrix} \tilde{F}_i^U \\ \tilde{F}_i^I \\ \tilde{F}_i^L \end{Bmatrix} \right) \tag{14}$$

By substituting $\tilde{\omega}_i$ for ω in Eq. (5), the transfer equations indicate the relation of the state vectors between the upper and the lower joints.

$$\begin{Bmatrix} X_i^U \\ F_i^U \\ 1 \end{Bmatrix} = \begin{bmatrix} \tilde{T}_{XX}^i & \tilde{T}_{XF}^i & \tilde{T}_{X1}^i \\ \tilde{T}_{FX}^i & \tilde{T}_{FF}^i & \tilde{T}_{F1}^i \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} X_i^L \\ -F_i^L \\ 1 \end{Bmatrix} = \tilde{T}_i \begin{Bmatrix} X_i^L \\ -F_i^L \\ 1 \end{Bmatrix} \tag{15}$$

where $\tilde{T}_i$ is the transfer matrix for calculating the steady-state response of the *i*th substructure under harmonic excitations. Each block in the transfer matrix is stated as follows:

$$\tilde{T}_{XX}^i = H_i^{UU}(H_i^{LU})^{-1} \tag{16}$$

$$\tilde{T}_{XF}^i = H_i^{UU}(H_i^{LU})^{-1}H_i^{LL} - H_i^{UL} \tag{17}$$

$$\tilde{T}_{X1}^i = (H_i^{UL} - H_i^{UU}(H_i^{LU})^{-1}H_i^{LL})\tilde{F}_i^L + (H_i^{UI} - H_i^{UU}(H_i^{LU})^{-1}H_i^{LI})\tilde{F}_i^I \tag{18}$$

$$\tilde{T}_{FX}^i = (H_i^{LU})^{-1} \tag{19}$$

$$\tilde{T}_{FF}^i = (H_i^{LU})^{-1}H_i^{LL} \tag{20}$$

$$\tilde{T}_{F1}^i = -(H_i^{LU})^{-1}H_i^{LL}\tilde{F}_i^L - (H_i^{LU})^{-1}H_i^{LI}\tilde{F}_i^I - \tilde{F}_i^U \tag{21}$$

The block matrix of the frequency response function matrix can be simply acquired with the aid of the corresponding eigensolution of the *i*th substructure. Furthermore, DTSM does not need to transform the excitations applied to the inner nodes into equivalent excitations acting on the joint nodes and the solution process for the harmonic response becomes very convenient.

According to the boundary conditions for geometric compatibility and force equilibrium of the *i*th substructure, as expressed in Eq. (9), the transfer equation of the 1~3 substructure of the series reactor can be obtained as follows:

$$\begin{Bmatrix} X_1^U \\ F_1^U \\ 1 \end{Bmatrix} = \tilde{T} \begin{Bmatrix} X_3^L \\ -F_3^L \\ 1 \end{Bmatrix} = \begin{bmatrix} \tilde{T}_{XX} & \tilde{T}_{XF} & \tilde{T}_{X1} \\ \tilde{T}_{FX} & \tilde{T}_{FF} & \tilde{T}_{F1} \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} X_3^L \\ -F_3^L \\ 1 \end{Bmatrix} \tag{22}$$

where $\tilde{T} = \tilde{T}_1\tilde{T}_2\tilde{T}_3$ is the transfer matrix of entire structure.

The terminal conditions of the entire chain structure should be introduced into Eq. (22) for solving this equation. Using the terminal boundary conditions proposed in Eq. (11), the second line of Eq. (22) can be shown as

$$\tilde{T}_{FF}F_3^L = \tilde{T}_{F1} \tag{23}$$

The force vector F_3^L can simply be obtained from above equation. After introducing F_3^L into Eq. (22), X_1^U can be obtained directly from the following equation

$$X_1^U = \tilde{T}_{X1} - \tilde{T}_{XF}F_3^L \tag{24}$$

Using these known vectors, the interface force and displacement vectors of each substructure can be calculated from Eq. (15). The procedure of computing the interface force and displacement vectors of each substructure is convenient and simple because the transfer matrix of each substructure has been determined. The force and displacement vectors of the inner nodes of each substructure can be obtained from Eq. (14).

4.2 The seismic random response of the series reactor

If the ith (i = 1, 2, 3) substructure of the series reactor is subjected to a horizontal seismic random excitation, the equation of motion can be expressed as

$$M_i\ddot{x}_i + C_i\dot{x} + K_i x_i = -M_i E\ddot{x}_s \tag{25}$$

where E is the indicating vector for locating the loads. $\ddot{x}_s$ is a stationary random process of seismic acceleration and its power spectral density (PSD) is known. C_i is a uncoupled damping matrix of this substructure and the following equation should be satisfied.

$$\Phi_i^T C_i \Phi_i = D_i \tag{26}$$

where D_i is a diagonal matrix and its kth diagonal element is $2\zeta_{ik}\omega_{ik}$, where ζ_{ik} is damping ratio of kth mode and ω_{ik} is the kth natural frequency of substructure i. Its frequency response function matrix can be given as

$$H_i(\omega) = \Phi_i[\Lambda_i + j\omega D_i - \omega^2 I]^{-1}\Phi_i^T \tag{27}$$

where j is the imaginary unit.

The PSD of seismic acceleration can be determined from the model of filtered white noise (Kanai-Tajimi model) and expressed as Tajimi (1960)

$$S_{\ddot{x}_s}(\omega) = \frac{1 + 4\zeta_s^2\dfrac{\omega^2}{\omega_s^2}}{\left(1 - \dfrac{\omega^2}{\omega_s^2}\right)^2 + 4\zeta_s^2\dfrac{\omega^2}{\omega_s^2}} S_0 \tag{28}$$

where S_0 is the spectral density of zero-mean-valued stationary white noise; ζ_s and ω_s are the damping ratio and natural frequency of a soil stratum, respectively. In this study the seismic excitation is prescribed using the model of filtered white noise. When the earthquake intensity is

8 degree, S_0 is 73.776 $cm^2 \cdot s^{-3}$. $\zeta_s = 0.623$ and $\omega_s = 15.708$ rad/s are taken.

According to the PEM (Lin *et al.* 2001), the random excitation can be converted into the pseudo harmonic excitation and Eq. (25) becomes the following harmonic equation

$$M_i\ddot{x}_i + C_i\dot{x} + K_i x_i = -M_i E\sqrt{S_{\ddot{x}_s}(\omega)}e^{j\omega t} \tag{29}$$

Therefore, using the algorithm of DTSM introduced in section 4.1, the harmonic response can be calculated efficiently and stated in a form as

$$y = Y(\omega)e^{j\omega t} \tag{30}$$

where y could be the structural displacement or force response and Y is the corresponding amplitude on the interfaces of the substructures. Note that Y is a function of ω. The PSD matrix of y is calculated from the following equation:

$$S_{yy}(\omega) = y^* \cdot y^T = Y^* \cdot Y^T \tag{31}$$

The variance of y can be obtained in the form of

$$\sigma_{yy}^2 = 2\int_0^\infty S_{yy}(\omega)d\omega \tag{32}$$

In order to compute variances, the above integral equation is solved in the region $\omega = [0,18]Hz$ and the integration step is 0.1 Hz. The displacement variances along the X direction of the a,b,c,d nodes on top of the series reactor (Fig. 3(b)) are listed in Table 3. In this table, MSM is defined as the mode-superposition method. The first twenty modes of the entire structure are employed in the mode-superposition analysis. The critical damping is 0.01 for analyzing the damped structure. From Table 3, it can be seen that the results obtained by both methods have a uniform precision, but both the mode extraction time and computation time consumed by DTSM are greatly reduced compared with those of MSM.

Using the DTSM, the force variances on interfaces of each substructure can be efficiently calculated using the same process as that for solving the displacement variances. The pressure force variances along the axial direction of the series reactor are depicted in Fig. 5, where the A, B and C interfaces are used to denote the bottom cross-sections of substructures, and their specific positions are marked in Fig. 3(b).

Table 3 Peak displacement variance and computation time

Structure	Method	Displacement variance /μm				Mode extraction time /s	Computation time /s
		a	b	c	d		
Damped	MSM	15.9704	15.9059	15.9704	15.9059	73.54	126.40
	DTSM	16.7096	16.6234	16.7133	16.6411	22.12	45.14
Undamped	MSM	68.3934	68.1149	68.3934	68.1149	73.54	125.32
	DTSM	66.3516	66.0602	66.3654	66.1142	22.12	32.19

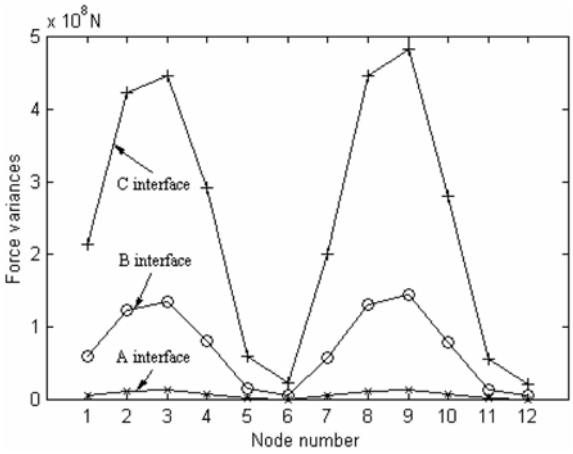

Fig. 5 Force variances of circumference nodes

5. Precise time integration of the series reactor under earthquake

For time domain response analysis, we employ the precise time integration algorithm, which has been presented in Zhong *et al.* (1994, 1996). The algorithm is explicit and the time-step sizes are not constrained by any of the natural periods of the discretized structure.

The equation of motion of the series reactor can be expressed in the form of Zhong *et al.* (1994)

$$\dot{v} = Hv + f(t), \quad v(0) = v_0 \tag{33}$$

where v is the state vector, defined as $v = [x \ \dot{x}]^T$ and v_0 is the initial state vector. Before solving Eq. (33), we should first get the solution of the corresponding homogeneous equation.

$$\dot{v} = Hv \tag{34}$$

If it is assumed that H is a matrix whose elements are independent of t, the general solution of Eq. (34) can be written as

$$v = \exp(H \cdot t)v_0 \tag{35}$$

Let the length of a time step be Δt and $T = \exp(H \cdot \Delta t)$. The solutions can be given by repeated matrix-vector multiplications.

$$T = \exp(H \cdot \Delta t) = [\exp(H \cdot \Delta t/m)]^m = [\exp(H \cdot \tau)]^m \tag{36}$$

where m can be selected as an integer power of 2, $m = 2^N$. When m is large enough, $\tau = \Delta t/m$ could be too small to cause any significant truncation error. Therefore, $\exp(H \cdot \tau)$ can be approximated using the truncated Taylor expansion.

$$\exp(H \cdot \tau) \approx I + H \cdot \tau + (H \cdot \tau)^2/2 + (H \cdot \tau)^3/3! + (H \cdot \tau)^4/4! = I + T_a \tag{37}$$

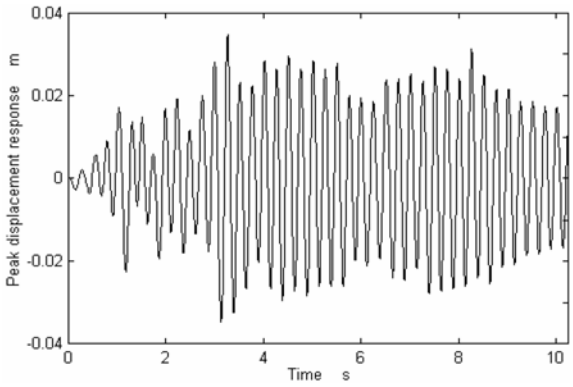

Fig. 6 Peak displacement response of the series reactor subject to El Centro earthquake

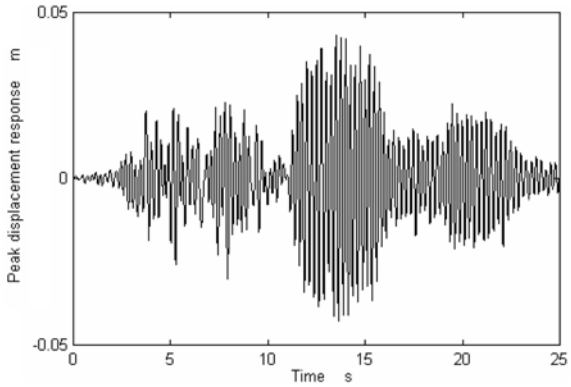

Fig. 7 Peak displacement response of the series reactor subject to Taft earthquake

With Eq. (37), matrix T can be computed as

$$T = (I + T_a)^{2^N} = (I + T_a)^{2^{N-1}} \times (I + T_a)^{2^{N-1}} \tag{38}$$

Using only N matrix multiplications, a precise approximation to T can be obtained.

For the nonhomogeneous system (33), assume that the nonhomogeneous term $f(t)$ is linear in a time step (t_k, t_{k+1}), then

$$\dot{v} = Hv + r_0 + r_1(t - t_k) \tag{39}$$

Thus, the precise time integration formula for the nonhomogeneous Eq. (33) can be written as

$$v_{k+1} = T[v_k + H^{-1}(r_0 + H^{-1}r_1)] - H^{-1}[r_0 + H^{-1}r_1 + r_1\tau] \tag{40}$$

The precise time integration method is applied to performing a time history analysis on the series reactor. El Centro and Taft earthquake waves are used as ground acceleration excitations. Both earthquake waves are scaled to simulate the ground excitations at earthquake intensity 8 degree and the peak values of acceleration are adjusted to 400 cm/s^2. The top displacement response time histories in the series reactor subject to corresponding earthquake waves are plotted in Fig. 6 and Fig. 7 respectively. The peak top displacement is 0.03464 m under the El Centro earthquake wave, and 0.04306 m under the Taft earthquake wave.

6. Conclusions

An analysis procedure for obtaining the seismic responses of a series reactor is presented in this paper.

(1) A single reactor is modeled using the finite element method. The results obtained from the model are in agreement with experimental data. It is indicated that the model of the series reactor, assembled with several single reactors, is reasonable.

(2) A direct transfer substructure method (DTSM) is presented for analyzing the dynamic characteristics and the harmonic responses of the series reactor. Since the transfer matrix of each substructure is calculated from the corresponding FRF matrix directly, not only the solution procedure of DTSM is convenient, but also the computation cost is markedly reduced. Associated with the pseudo excitation algorithm, DTSM is applied to calculate the seismic random response of the series reactor. The numerical results demonstrate that DTSM is exact and efficient.
(3) For time domain response analysis, the precise integration method is employed to solve the transient response of the series reactor under EI Centro and Taft earthquake waves.

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References

Bathe, K.J. (1982), *Finite Element Procedures in Engineering Analysis*. New Jersey Prentice-Hall Inc.
Chandler, Adrian M. and Lam, Nelson T.K. (2001), "Performance-based design in earthquake engineering: A multi disciplinary review", *Eng. Struct.*, **23**(12), 1525-1543.
Cofer, William F., Zhang, Yi, McLean, David I. (2002), "A comparison of current computer analysis methods for seismic performance of reinforced concrete members", *Finite Elements in Analysis and Design*, **38**(9), 835-861.
Craig, R.R. Jr and Chang, C.J. (1976), "A review of substructure coupling methods for dynamic analysis", *Advances in Engineering Science*, **2**, 393-408.
Craig, R.R. Jr. (1995), "Substructure methods in vibration", *J. Mech. Design*, ASME, **117B**, 207-213.
Ernesto, H.Z. and Vanmarcke, E.H. (1994), "Seismic random vibration analysis of multi-support structural systems", *J. Eng. Mech.*, ASCE, **120**, 1107-1128.
Geradin, M. and Chen, S.L. (1995), "An exact model reduction technique for beam structures: Combination of transfer and dynamic stiffness matrices", *J. Sound Vib.*, **185**(3), 431-440.
Gladwell, G.M.L. (1964), "Branch mode analysis of vibration systems", *J. Sound Vib.*, **1**, 41-59.
Hurty, W.C. (1960), "Vibration of structure systems by component mode synthesis", *J. Eng. Mech.*, ASCE, **86**, 51-59.
Lee, Usik (2000), "Vibration analysis of one-dimensional structures using the spectral transfer matrix method", *Eng. Struct.*, **22**(6), 681-690.
Lin, Jiahao, Zhao, Yan and Zhang, Yahui (2001), "Accurate and highly efficient algorithms for structural stationary/non-stationary random responses", *Computer Meth. Appl. Mech. Eng.*, **191**, 103-111.
Somerville, P. (2000), "Seismic hazard evaluation", Bulletin of the New Zealand National Society for Earthquake Engineering, **33**(3), 371-386.
Tajimi, H. (1960), "A statistical method of determining the maximum response of a building structure during an earthquake", *Proc. of the 2nd World Conf. on Earthquake Engineering*, Tokyo, Japan, July.
Zhong, Wanxie, Zhu, J.P. and Zhong, X.X. (1994), "A precise time integration algorithm for non-linear systems", *Proc. 3rd World Congress on Computational Mechanics*, Chiba, Japan, 4.
Zhong, Wanxie, Zhu, Jianing and Zhong, Xiang-Xiang (1996), "On a new time integration method for solving time dependent partial differential equations", *Comput. Meth. Appl. Mech. Eng.*, **130**, 163-178.