Structural Engineering and Mechanics, Vol. 21, No. 4 (2005) 375-392 DOI: http://dx.doi.org/10.12989/sem.2005.21.4.375

Load deformation characteristics of shallow suspension footbridge with reverse profiled pre-tensioned cables

Ming-Hui Huang[†] and David P. Thambiratnam[‡]

School of Urban Development, Queensland University of Technology, GPO Box 2434, Brisbane, Queensland 4001, Australia

Nimal J. Perera ;;

Bird & Marshall Ltd, London, England

(Received September 9, 2004, Accepted August 11, 2005)

Abstract. Cable supported structures offer an elegant and economical solution for bridging over long spans with resultant low material content and ease of construction. In this paper, a model of shallow cable supported footbridge with reverse profiled pre-tensioned cables is treated and its load deformation characteristics under different quasi-static loads are investigated. Effects of important parameters such as cable sag and pre-tension are also studied. Numerical results performed on a 3D model show that structural stiffness of this bridge (model) depends not only on the cable sag and cross sectional areas of the cables, but also on the pre-tension in the reverse profiled cables. The tension in the top supporting cables can be adjusted to a high level by the pre-tension in the reverse profiled bottom cables, with the total horizontal force in the bridge structure remaining reasonably constant. It is also evident that pre-tensioned horizontally profiled cables can greatly increase the lateral horizontal stiffness and suppress the lateral horizontal deflection induced by eccentric vertical loads.

Key words: suspension footbridge; structural stiffness; parameter study; reverse profile; pre-tension.

1. Introduction

Cable supported (pedestrian and highway) bridges are aesthetically pleasing and have gained popularity throughout the world. Due to the application of high strength materials and new technology, cable supported footbridges can be constructed to be much longer and more slender than the other types of footbridge structures and in different structural configurations. In general, long span suspension bridges achieve their load and deformation resistance and stability under vibration, oscillation and galloping effects using tension cables with significantly large sags hanging from tall towers interacting with stiffening girders or trusses. However, as footbridge structures are mainly designed for pedestrians and cyclists, the design loads are relatively smaller, compared with

[†] PhD Candidate

[‡] Professor, Corresponding author, E-mail: d.thambiratnam@qut.edu.au

^{††} Managing Director

long-span and large cable supported highway bridge structures. Due to this reason, the stiffness of girders in suspension footbridges are often weak and cross bracing (Brownjohn 1994) or wind ropes (Nakamura 2003) are added to achieve the satisfactory lateral resistance. Cable supported ribbon bridges adopt the principle of suspension bridges and develop it further by using high strength materials and modern engineering technology. They do not require tall towers and can span fairly long distances with shallow profiles. One of the main types of such bridges is the stress ribbon bridge (Morrow 1983, Strasky 1987, Cobo Del Arco et al. 2001). Stress ribbon bridges were introduced in the 1960s and have been built in various countries. The superstructures are generally composed of supporting cables with small sags and prestressed concrete decks. In a simple stress ribbon bridge, the prestressed concrete decks hang on the suspended cables and the suspenders have the same profile as the catenary cables. In order to improve the structural behaviour, other types of stressed ribbon bridges were proposed. For example, the configuration of superstructure can be designed as a stretched stress ribbon hung from supporting cables with the centres of curvature of the ribbon and of the supporting cable on opposite sides, under and over the pavements. In order to reduce the total horizontal tension forces, the stretched stress ribbon can be supported by a classical arch structure near the mid-span (Pirner and Fischer 1998, 1999). Also stress ribbon bridges can be designed as hybrid stress ribbon bridges such as stress ribbon cable-stayed suspension or stress ribbon suspension with a very light full-steel or a light concrete-steel girder (Tanaka et al. 2002). The Millennium footbridge in London (Dallard et al. 2000, 2001a, 2001b, 2001c) is another kind of tension ribbon footbridge in which the cables provides almost all the stiffness of the bridge in both vertical and horizontal directions. In the superstructure, transverse arms span between cables along the two sides and the deck structure comprises two steel edge tubes which span onto the transverse arms and the aluminium deck sheets span on the tubes.

Though cable supported footbridges can have different types of superstructures, it is evident that the structural stiffness is mainly provided by the supporting cable system and that these bridge structures are slender and prone to vibration. However, for cable supported structures, the dynamic properties and structural behaviour depend not only on the cable profile, but also on tension force in the cables, particularly for shallow sag cable structures (Irvine 1992, Gimsing 1998) such as cable supported ribbon bridges. The study of the structural behaviour and effects of variation of tension forces in the cables under different loads are therefore important as they influence the bridge response.

A conceptual study has been undertaken to investigate the structural behaviour and dynamic characteristics of slender footbridges under human-induced dynamic loads and a cable supported footbridge model with reverse profiled pre-tensioned cables in the vertical plane and pre-tensioned side cables in the horizontal plane is proposed for this purpose. In this bridge model, the transverse bridge frames with top, bottom and side legs hang from the top suspending cables and further restrained by the reverse profiled pre-tensioned bottom cables and pre-tensioned side cables. The deck units span across beams which are simply supported on the bridge frames. In this model the cable tensions (and hence the bridge stiffness) can be easily changed to vary the natural frequencies (Huang *et al.* 2005). This feature will be useful in investigating the dynamic behaviour of such footbridges under human-induced loads. The present paper, however, investigates the static behaviour of shallow suspension footbridges, with reverse profiled pre-tensioned cables, under quasi static loads. Evaluation of the load deformation characteristics will be the main concern and effects of some important structural parameters, such as cable sag, cross sectional area and pre-tension in the reverse profiled cables will be studied. The results will be helpful to better understand the

behaviour of tensile structures with pre-tensioned reverse profiled cables, particularly shallow pretensioned cable supported footbridges and ribbon bridges.

2. Pre-tensioned cable supported footbridge model

The proposed pre-tensioned cable supported footbridge model is shown in Fig. 1. In this bridge model, the cable systems are composed of three groups of cables which may have same or different cable profiles: top supporting (or suspending) cables, bottom pre-tensioned cables (Fig. 1(a)) and side pre-tensioned cables (Fig. 1(b)). The top cables are two parallel supporting cables which have the catenary profiles and provide tension forces to support the whole structural gravity, applied loads and extra internal forces induced by the pre-tensioned bottom cables. The two parallel bottom cables are designed to have reverse profiles in the vertical plane and their function is to introduce



Fig. 1 Pre-tensioned cable supported bridge model: (a) elevation, (b) top view, (c) middle transverse bridge frame

pre-tension forces and provide extra internal vertical forces to the transverse bridge frames and the top supporting cables. The side cables are a pair of bi-concave cables which have the same cable profiles in the horizontal plane, and their main function is to provide extra internal horizontal forces and improve the horizontal stiffness. When the pre-tensioned bottom and/or side cables are slack, they could carry small tension forces only to support their own gravity load and cannot resist any external loads. In this case, they couldn't contribute stiffness and tension forces to the entire structure. However, these small tensions can provide sufficient restraining forces to prevent the transverse frames from swaying in the longitudinal direction.

When the reverse profiled cables are pre-tensioned, they provide extra internal vertical and/or horizontal forces to the supporting cables and increase the total tension forces in the whole bridge structure. Then the structural stiffness as well as the dynamic properties can be altered and the structural behaviour can be improved. Similar reverse cable systems have been proposed and used in long span suspension bridges and footbridges. For example, in a long span suspension bridge model for the Straits of Messina Bridge, Italy, stabilising reverse cables were designed to improve the torsional and lateral stiffness as well as the aerodynamic behaviour (Borri *et al.* 1993), and in the M-bridge, Japan, wind ropes with reverse profiles in nearly horizontal plane were designed to increase the lateral resistance against wind loads (Nakamura 2003). However, there is little information regarding the structural behaviour of such bridges and influence of parameters, and further information is needed to understand their performance under load.

In this conceptual study, transverse bridge frames have been designed to support the deck and hold the cables. These frames (Fig. 1(c)) comprise cross members (for the support beam and deck), top and bottom vertical legs as well as horizontal side legs and they form a set of spreaders for the cables to create required profiles. They have in-plane stiffness to protect against collapse under in plane forces and contribute very little in the way of longitudinal, lateral and rotational stiffness for the entire system. The transverse bridge frames are hung from the top cables, and further restrained by the lower reversed profile cables as well as the side cables. Two support beams of rectangular section are simply supported on cross members of the adjacent bridge frames, and the deck units are simply supported at the ends on these beams. As the main concern of this conceptual study is the static and dynamic structural behaviour and the effects of important parameters, the connection details and anchorages of cables are not important for numerical analysis, although they are very important in the design and construction of real footbridges.

In this bridge model, the entire structural stiffness is provided by the cable systems. When the structure is subjected to applied loads, all the loads can be balanced by the tension forces in the cables with deformed cable profiles since these forces can provide components in different directions.

In order to simplify the problem, all the transverse bridge frames have been assumed to have the same size, and hence the weight of frame and deck acting on the cables can be considered as equal concentrated loads.

A typical symmetric cable profile with equal concentrated loads is shown in Fig. 2. In the following description, different cables and cable profiles in the proposed bridge model are identified by the subscript j. When the subscript j equals to 1, 2 and 3, it represents the top, bottom and side cables as well as their profiles respectively.

For a cable supported bridge model with N uniform segments in the horizontal longitudinal direction, the forces from the N-1 transverse bridge frames can be modelled as N-1 equal concentrated loads acting on the cables. Assuming the horizontal distance between two adjacent



Fig. 2 Typical cable profile

transverse bridge frames (or loads) to be a, the span length will be defined as:

$$L = Na \tag{1}$$

For the *j*th symmetric cable, the sag F_j is located at the middle segment or the middle node *K*. Choosing the local x - y coordinates as shown in Fig. 2, the coordinates for the node *K* can be obtained as:

$$x_{iK} = Ka \qquad y_{iK} = F_i \qquad K = int(N/2) \tag{2}$$

where *int*() is an integer function.

For a symmetric cable subjected to equal concentrated loads, it is easy to obtain the vertical and horizontal reactions by using static equilibrium equations. Using these support reactions, equal concentrated loads and the cable sag, the cable profile and the tension forces as well as the tensile deformation in the segments can be calculated. The coordinate of the *i*th node, *j*th cable can be expressed by

$$x_{ii} = ia$$
 $y_{ii} = \alpha_i F_i$ $i = 0, 1, 2, ..., N$ (3)

Here the coefficient α_i can be calculated by the following equation.

$$\alpha_i = i(N-i)/[K(N-K)] \tag{4}$$

The tension force T_{ji} and tensile deformation ΔL_{ji} of the *i*th segment of the *j*th cable can be obtained by

$$T_{ji} = \beta_{ji}W \quad \Delta L_{ji} = \gamma_{ji}Wa/(E_{ji}A_{ji}) \quad i = 0, 1, 2, \dots, N; \quad j = 1, 2, 3$$
(5)

 E_{ji} and A_{ji} are Young's modulus and area of cross section of the *i*th cable segment of *j*th cable. *W* is the applied equal concentrated load. The coefficients β_{ji} and γ_{ji} are shown to be as follows

$$\beta_{ji} = \frac{1}{2} \sqrt{\left[K(N-K)(a/F_j)\right]^2 + \left[N-2i+1\right]^2}$$
(6)

$$\gamma_{ji} = \beta_{ji} \sqrt{1 + (\alpha_i - \alpha_{i-1})^2 (F_j/a)^2}$$
(7)



Fig. 3 Extra internal forces in cables

In the analysis of the bridge model, all the cables are stretched to keep the designed cable sags or cable profiles and then the decks can be kept in a horizontal plane before the bridge structure is subjected to the applied loads. This can be done by introducing initial distortions to the cables according to their cable sags, cross sectional areas, material properties, loads such as the weight of bridge frame and decks as well as cables, and extra internal forces produced by pre-tensioned reverse profiled cables or horizontal side cables.

Assuming the bottom cables have a diameter D_2 , Young's modulus E_2 , and cable sag F_2 , if the internal vertical force W_{int} at each bridge frame is induced (Fig. 3), the initial distortion ΔL_{2i} introduced to the *i*th cable segment of one bottom cable can be determined to be:

$$\Delta L_{2i} = -2\gamma_{2i}W_{int}a/(\pi E_2 D_2^2)$$
(8)

The side cables are a pair of bi-concave cables in the horizontal plane which have opposite cable profile to each other. When they are pre-tensioned, internal horizontal forces can be introduced to the bridge frames. If the side cables have diameter D_3 , Young's modulus E_3 , and cable sag F_3 (in horizontal plane), and internal horizontal force Q_{int} at each bridge frame is induced by the pair of side cables, the initial distortion L_{3i} introduced to the *i*th cable segment of one side cable is determined as:

$$\Delta L_{3i} = -4\gamma_{3i}Q_{int}a/(\pi E_3 D_3^2) \tag{9}$$

When the internal vertical force W_{int} is induced at each bridge frame by pre-tensioned bottom cables, the top supporting cables are subjected to the weight (gravity) of the whole structure and the extra internal vertical forces. If the top supporting cables have diameter D_1 , Young's modulus E_1 , and cable sag F_1 , and the total weight of one bridge frame, the cables and decks between adjacent frames is G, the following initial distortion ΔL_{1i} in the *i*th cable segment of one top cable should be introduced:

$$\Delta L_{1i} = -2\gamma_{1i}(G + W_{int})a/(\pi E_1 D_1^2)$$
(10)

After the initial distortions are introduced to the cable systems, the cable profiles can have the designed cable sags and the bridge deck will be kept in the horizontal plane before it is subjected to the applied loads.

The structural analysis package software MICROSTRAN (Engineering Systems 2002) is adopted

in the numerical study. The footbridge structure is analysed as a 3D model to reveal some features that cannot be obtained by 2D models. In the 3D numerical bridge model, stainless steel (Young's modulus 2.0×10^{11} N/m² and density 7850 kg/m³) is chosen for the transverse bridge frame and support beams, and Aluminium (Young's modulus 6.5×10^{10} N/m² and density 2700 kg/m³) is chosen for the deck units to reduce the weight of the bridge structure. All members of the transverse bridge frames have a uniform rectangular cross sectional area of 250×300 mm² and the support beams have a uniform rectangular cross section of 200×250 mm². 8 deck units in size of $4000 \times 500 \times 50$ mm³ are simply supported on the support beams between the adjacent transverse bridge frames. Stainless steel cables are chosen for all the cable systems and the material properties are the same as those of bridge frames. In the numerical analysis, the span length is set to 80 m, the horizontal distance between the adjacent bridge frames is set to 4 m and the width of the deck for applied loads is set to 4 m. The cable profiles (sags), cable sectional areas (diameters) and pretension are important structural parameters and can be changed for the parameter study.

3. Numerical analysis

In the following numerical analysis, the structural behaviour of the pre-tensioned cable supported footbridge under symmetrical and eccentric vertical loads as well as lateral horizontal loads is studied. The symmetric load is modelled as uniform load acting on the deck (Fig. 4(a)) and the eccentric vertical load is modelled as uniform load distributed along the half width on bridge deck (Fig. 4(b)). The horizontal static load is modelled as uniform load acting on the deck in the transverse direction (Fig. 4(c)). The maximum load density for this bridge structure has been chosen as 8 kPa for the symmetric and eccentric vertical loads (Austroads 1992). As the lateral loads are usually quite smaller than the vertical loads, the maximum load intensity is set to one tenth of the vertical loads (0.8 kPa).

In order to compare the results and describe the structural behaviour effectively, two types of cable supported bridge models will be mentioned in the following analysis. Pre-tensioned bridge refers to a cable supported bridge model with pre-tensioned bottom and/or side cables. Un-pre-tensioned bridge, on the other hand, refers to a cable supported footbridge model with slack bottom and side cables which have no contribution to the structural stiffness but carry small tension forces to support their own gravity loads and prevent the transverse bridge frames from swaying in the longitudinal direction. To make the bottom and side cables slack, a small initial distortion (extension 0.01 m) is introduced to these cables before the loads are applied.



Fig. 4 Applied loads: (a) symmetric vertical load, (b) asymmetric vertical load, (c) lateral horizontal load

3.1 Un-pre-tensioned footbridge under symmetric vertical loads: Effect of cable sag and cross sectional area

In this section, the effects of cable sag and cross sectional area of the top supporting cables are discussed. Numerical results show that, compared with the deformation of cables, the transverse bridge frames deform slightly and they can be considered as rigid members under gravity and applied loads. The position of the maximum vertical deflection is at the mid point of the cross member of the middle frame and maximum tension force in the cables occurs at the two end segments. In order to show the deflection of the cables, the maximum deflection in the following analysis represents the deflection at the end node of the cross member of the central frame (almost at the same place as the maximum cable sag), and the maximum tension force represents the tension force in the end segment of a top, bottom or side cable, when they are mentioned in text or shown in the figures.



Fig. 5 Maximum deflection under symmetric applied vertical load with different cable sags



Fig. 6 Maximum tension force in top cables under applied vertical load with different cable sags

Load deformation characteristics of shallow suspension footbridge



Fig. 7 Maximum deflection under applied vertical load with different top cable cross sectional areas (diameters)



Fig. 8 Maximum tension force in top cables under applied vertical load with different top cable cross sectional areas

Fig. 5 and Fig. 6 show the effects of cable sag (F_1) on maximum deflection and maximum tension force in one of the top supporting cables under the applied symmetric vertical loads for the cables with a diameter of 240 mm $(D_1 = 240 \text{ mm})$ but different cable sags (F_1) . It can be seen that when the cable sag increases, the vertical structural stiffness increases and both the deflection and maximum tension force decrease.

Fig. 7 and Fig. 8 show the variations of the maximum deflection and maximum tension force in one top cable with cross sectional area (or diameter) of the top cables when the cable sag is set to be 1.8 m ($F_1 = 1.8$ m). In order to show the effect of the cable cross section area, the total weight of the whole bridge structure is kept the same by changing the diameters of the slack side cables. Results show that at the same initial cable sag, the structural stiffness increases (as expected), while the maximum tension force (which mainly depends on the sag of deformed cable) increases slightly when the cross sectional area increases.

3.2 Effect of pre-tension in the bottom cables (internal vertical forces)

When the reverse profiled bottom cables are pre-tensioned, extra internal vertical forces are induced to the bridge frames and catenary supporting cables. Here, the effects of pre-tension in the bottom cables, structural parameters of the pre-tensioned bottom cables such as cable sectional area and cable sag have been analysed. In numerical analysis, the side cables are assumed to be slack $(Q_{int} = 0)$, and the top supporting cables are stretched to keep the deck in horizontal plane before it is subjected to the applied vertical loads.

The effect of pre-tension has been investigated by changing the internal vertical force (W_{inl}), while the top and bottom cables, as well as the slack side cables, are supposed to have the same cable sag of 1.8 m and diameter of 240 mm ($F_1 = F_2 = F_3 = 1.8$ m, $D_1 = D_2 = D_3 = 240$ mm). In order to illustrate the variation of structural stiffness and the effects of cross sectional area and pre-tension, results are compared with those of an un-pre-tensioned bridge model, in this section as well as in the other sections. In this un-pre-tensioned bridge model (UPTB) ($F_1 = 1.8$ m, $D_1 = 339$ mm), it is assumed that all the cable profiles are the same as those of the pre-tensioned bridge model, but the sectional area of the top supporting cables is equal to the sum of sectional areas of the top and bottom cables in the pre-tensioned bridge model. In order to keep the same gravity loads, the diameter of the slack bottom and side cables in this model is set to be 190 mm ($F_2 = F_3 = 1.8$ m, $D_2 = D_3 = 190$ mm).

Fig. 9 shows the variation of maximum deflection under applied symmetric vertical load. Fig. 10 shows the maximum tension force in the top cables and Fig. 11 the maximum tension forces in the bottom cables. From these figures, it can be seen that for a pre-tensioned cable supported bridge, the structural behaviour depends not only on the top supporting cables, but also on the pre-tensioned bottom cables as well as the pre-tensioned forces in the bottom cables (or extra internal vertical forces – Fig. 3), and the performance can be described in two phases. In the first phase when the bottom cables are pre-tensioned and provide vertical forces to the top supporting cables, the pre-tension forces in the bottom cables decrease while the tension forces in top cables increase with the applied vertical load. The structural stiffness in this phase is almost the same as that of the un-pre-



Fig. 9 Maximum deflection under applied vertical load with different internal vertical forces



Fig. 10 Maximum tension force in top cables under applied vertical load with different internal vertical forces



Fig. 11 Maximum tension force in bottom cables under applied vertical load with different internal vertical forces

tensioned bridge model (UPTB). This feature demonstrates that in a pre-tensioned bridge model, the structural stiffness depends on the total cross sectional areas of the top and bottom cables, irrespective of their profiles, i.e., catenary or reverse profile. In the second phase, the pre-tension forces have been released, the bottom cables gradually become slack, and they have no ability to provide extra internal vertical forces to the top supporting cables and can only carry the tension forces to support their own gravity. In this case, the bottom cables do not contribute to the structural stiffness and the bridge structure behaves as an un-pre-tensioned one, since the structural stiffness depends only on the top cables.

Fig. 12 shows the total horizontal tension force in a bridge section. It can be seen that in a pretensioned cable supported bridge, the total horizontal force remains almost constant with increase in applied load except when the pre-tensioned bottom cables slack, for which case the total horizontal force increases with applied load. These are interesting features of load transfer and balance in this



Fig. 12 Total horizontal tension forces under applied vertical load with different internal vertical forces

type of structure. The reason is that the top cables and pre-tensioned bottom cables form a selfbalancing system when extra internal vertical forces exist. When the internal vertical forces have been released, the self-balancing system disappears and the applied loads are resisted only by the top supporting cables.

3.3 The effects of cable sag and cross sectional area of bottom cables

The sectional area and cable sag of the pre-tensioned bottom cables can affect the structural performance to some extent. These effects have been shown in Fig. 13 to Fig. 16. In these figures, the bottom cables are pre-tensioned to provide 30 kN extra internal vertical force ($W_{ml} = 30$ kN, $Q_{iml} = 0$) to the top cables at each bridge frame before the symmetric vertical load is applied and the



Fig. 13 Maximum deflection under applied load with different bottom cable sags

Load deformation characteristics of shallow suspension footbridge



Fig. 14 Tension forces in bottom cables under applied load with different bottom cable sags



Fig. 15 Maximum deflection under applied load with different bottom cable sections



Fig. 16 Tension forces in bottom cables under applied load with different bottom cable sections

cable sag and diameter of the top cables are assumed to be 1.8 m and 240 mm respectively ($F_1 = 1.8 \text{ mm}$, $D_1 = 240 \text{ mm}$).

Fig. 13 and Fig. 14 show the maximum deflection and the maximum tension force in the bottom cables when the bottom cables have diameter of 240 mm with different cable sags. It can be seen that when the cable sag of the pre-tensioned bottom cables is greater than that of the top cables, the structural stiffness can be greater than that of an un-pre-tensioned bridge (UPTB, $D_1 = 339$ mm, $D_2 = D_3 = 190$ mm). The bottom cables are easier to slack when they have greater cable sag. Fig. 15 and Fig. 16 show the maximum deflection and maximum tension force in the bottom cables respectively when the bottom cables have different diameters. Here the total weight of the bridge structures is kept the same. When the diameter of the pre-tensioned bottom cables is larger, the structural stiffness is larger and the bottom cables are easier to slack.

From these figures, it can be concluded that when the pre-tensioned bottom cables have small sectional area or small cable sag, they are slender and not easy to slack, and the extra internal forces induced by the pre-tensioned bottom cables are released very slowly.

3.4 Effect of pre-tension in the side cables (internal lateral forces)

When horizontal side cables are introduced, internal horizontal forces can be provided by pretensioning the side cables and the horizontal stiffness can be improved significantly. However, the vertical structural stiffness increases only slightly, since the horizontal cables are flexible in the vertical direction and provide small vertical force when they deform. Fig. 17 shows the maximum vertical deflection with pre-tensioned bottom and side cables when the symmetrical vertical load is applied. Here it is assumed that all the top, bottom and side cables have the same cable sag of 1.8 m and diameter of 240 mm ($F_1 = F_2 = F_3 = 1.8$ m, $D_1 = D_2 = D_3 = 240$ mm). It can be seen that the vertical stiffness mainly depends on the top and bottom cables and the effect of pretensioned side cables on the vertical structural stiffness is much smaller than that of the pretensioned bottom ones.



Fig. 17 Maximum vertical deflection with pre-tensioned bottom and side cables

Load deformation characteristics of shallow suspension footbridge



Fig. 18 Maximum horizontal deflection under lateral horizontal applied load

3.5 Performance under lateral horizontal loads and eccentric vertical loads

Bridge structures are always subject to lateral horizontal loads (such as wind) and eccentric vertical loads. The behaviours of the pre-tensioned cable supported bridges under such quasi static loads have been investigated and described in this section. In the numerical analysis, it is supposed that all the cables of the bridge model have the same cable sag of 1.8 m and diameter of 240 mm $(F_1 = F_2 = F_3 = 1.8 \text{ m}, D_1 = D_2 = D_3 = 240 \text{ mm})$. The lateral horizontal load is modelled here as a distributed uniform load with density of 800 N/m² acting on the bridge deck (Fig. 4(c)), and eccentric vertical load is modelled as distributed uniform load (with density of 8 kN/m²) acting on the half width of the deck (Fig. 4(b)).

Fig. 18 shows the maximum horizontal deflection at the end of the cross member in the middle transverse bridge frame under horizontally applied load. Results show that the horizontal stiffness is much smaller than the vertical stiffness for bridge structures without pre-tensioned bottom and side cables (UPTB), even if the sectional areas of the top supporting cables are increased. The reason is that the top cables are in the vertical plane, and their tension forces have only small components in the lateral horizontal direction to resist the lateral loads, after they deformed in the lateral direction. When the bottom cables are pre-tensioned, the lateral horizontal stiffness can be improved since the tension forces in the deformed top and bottom cables can provide more components in the lateral horizontal direction. However, the most effective measure to improve the lateral stiffness is to introduce the pre-tensioned side cables. It can be seen from the Fig. 18 that, after the pre-tensioned side cables have been introduced, the cables in the vertical plane have only slight effect on the lateral stiffness.

Under eccentric loads, lateral horizontal deflection is produced accompanying the vertical deflection, since the structural stiffness is mainly provided by the cable systems which are always weak in the lateral direction, the torsion may change the direction of the vertical loads and produce small lateral horizontal component, and large lateral deflection may be induced. Fig. 19 and Fig. 20 show the lateral horizontal and vertical deflections of the ends of cross members at the side of applied eccentric loads along the bridge length. It also can be seen that although the pre-tensioned



Fig. 19 Lateral horizontal deflection along bridge under eccentric vertical load



Fig. 20 Vertical deflection along bridge under eccentric vertical load

bottom cables can improve the vertical structural stiffness, the best measure to suppress the lateral horizontal deflection is to introduce the pre-tensioned side cables.

4. Conclusions

Cable supported footbridges are very efficient structures due to low cost, material consumption and ease of construction. However, the structures always tend to be slender and flexible as the structural stiffness mainly depends on the supporting cables. A cable supported footbridge with pretensioned reverse profiled bottom and side cables is proposed for a conceptual study to conduct extensive investigation into the dynamic behaviour of slender footbridges under human-induced dynamic loads. This paper is part of the research and it investigates the load performance of the proposed footbridge model under different applied quasi static loads as well as the effect of

parameters such as cable sag and pre-tension.

Numerical results performed on the 3D footbridge model show that structural stiffness of un-pretensioned cable supported bridges depends mainly on the cable sag and cross sectional area of the top supporting cables. For pre-tensioned cable supported bridges with pre-tensioned bottom cables, the structural stiffness depends not only on the cable sag and cross sectional area of both the (top) supporting cables and (bottom) pre-tensioned cables, but also on the pre-tension in the bottom cables. Considering a pre-tensioned cable supported bridge with pre-tensioned bottom cables and an un-pre-tensioned one, if they have the same cable profile for the supporting cables and the same total cable cross sectional area, the pre-tensioned bridge can have different structural stiffness, less or greater than that of the un-pre-tensioned one, before the pre-tensioned cables slack. The tension force in the supporting cables of un-pre-tensioned bridge is determined by the bridge gravity loads, applied loads and deformed cable profile, but the tension force in the supporting cables of a pretensioned bridge with pre-stressed bottom cables can be adjusted to a high level by the pretensioned bottom cables and the total horizontal force in a structural section remains almost at the same level until the pre-tensioned cables slack. Numerical results also show that pre-tensioned side cables in horizontal plane can greatly increase the lateral horizontal stiffness and suppress the lateral horizontal deflection induced by eccentric vertical loads.

The information presented in this paper will be helpful to better understand the behaviour of tensile structures with pre-tensioned reverse profiled cables, particularly slender shallow pre-tensioned cable supported footbridges and ribbon bridges. Since the tensions in reverse profiled pre-tensioned cables can be adjusted, they provide more design options and opportunities for bridge engineers to improve the structural behaviour of slender and light footbridges under static as well as dynamic loads.

References

Austroads (1992), Australian Bridge Design Code - Section 2: Design Loads, Austroads, NSW, Australia.

- Borri, C., Majowiecki, M. and Spinelli, P. (1993), "The aerodynamic advantages of a double-effect large span suspension bridge under wind loading", J. Wind Eng. Ind. Aerod., 48, 317-328.
- Brownjohn, J.M.W., Dumanoglu, A.A. and Taylor, C.A. (1994), "Dynamic investigation of a suspension footbridge", *Eng. Struct.*, 16(6), 395-406.
- Cobo Del Arco, D., Aparicio, A.C. and Mari, A.R. (2001), "Preliminary design of prestressed concrete stress ribbon bridge", J. Bridge Eng., 6(4), 234-242.
- Dallard, P., Fitzpatrick, T., Flint, A., Low, A., Ridsdill Smith, R. and Willford, M. (2000), "Pedestrian-induced vibration of footbridges", *The Struct. Eng.*, **78**(23/24), 13-15.
- Dallard, P., Fitzpatrick, T., Flint, A., Low, A. and Ridsdill Smith, R. (2001a), "The Millennium Bridge, London: Problems and solutions", *The Struct. Eng.*, **79**(8), 15-17.
- Dallard, P., Fitzpatrick, A.J., Flint, A., Le Bourva, S., Low, A., Ridsdill Smith, R.M. and Willford, M. (2001b), "The London Millennium Footbridge", *The Struct. Eng.*, **79**(22), 17-32.
- Dallard, P., Fitzpatrick, T., Flint, A., Low, A., Ridsdill Smith, R., Willford, M. and Roche, M. (2001c), "London Millennium Bridge: Pedestrian-induced lateral vibration", J. Bridge Eng., 6(6), 412-416.
- Engineering Systems Pty Ltd. (2002), Microstran V8, User's Manual, Engineering Systems Pty Limited, Turramurra, NSW, Australia.
- Gimsing, N.J. (1998), Cable Supported Bridges: Concept & Design, 2nd ed., Chichester: John Wiley & Sons Ltd.
- Huang, M.-H., Thambiratnam, D.P. and Perera, N.J. (2005), "Vibration characteristics of shallow suspension bridge with pre-tensioned cables", *Eng. Struct.*, **27**, 1220-1233.

Irvine, M. (1992), Cable Structures, Dover Publications, Inc., New York.

- Morrow, P.J., Howes, G., Bridge, R.Q. and Wheen, R.J. (1983), "Stress-ribbon bridge A viable concept", *Civil Engineering Transactions*, Institution of Engineers, Australia, **25**(2), 83-88.
- Nakamura, S.-I. (2003), "Field measurements of lateral vibration on a pedestrian suspension bridge", *The Struct. Eng.*, **18**(November), 22-26.
- Pirner, M. and Fischer, O. (1998), "Wind-induced vibrations of concrete stress-ribbon footbridges", J. Wind Eng. Ind. Aerod., 74-76, 871-881.
- Pirner, M. and Fischer, O. (1999), "Experimental analysis of aerodynamic stability of stress-ribbon footbridges", *Wind Struct.*, **2**(2), 95-104.
- Strasky, J. (1987), "Precast stress ribbon pedestrian bridge in Czechoslovakia", PCI Journal (Prestressed Concrete Institute), 32(3), 52-73.
- Tanaka, T., Yoshimura, T., Gimsing, N.J., Mizuta, Y. Kang, W.-H., Sudo, M. Shinohara, T. and Harada, T. (2002), "A study on improving the design of hybrid stress-ribbon bridges and their aerodynamic stability", J. Wind Eng. Ind. Aerod., 90(12-15), 1995-2006.