Structural Engineering and Mechanics, Vol. 21, No. 2 (2005) 169-184
DOI: http://dx.doi.org/10.12989/sem.2005.21.2.169

Natural vibration characteristics of a clamped circular plate in contact with fluid

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(Received May 19, 2004, Accepted June 22, 2005)

Abstract. This study deals with the free vibration of a circular plate in contact with a fluid; submerged in fluid, beneath fluid or on fluid. An analytical method based on the finite Fourier-Bessel series expansion and Rayleigh-Ritz method is suggested. The proposed method is verified by the finite element analysis using commercial program with a good accuracy. The normalized natural frequencies are obtained in order to estimate the relative added mass effect of fluid on each vibration mode of the plate. Also, the location of plate coupled with fluid and the cases of free and bounded fluid surface are studied for the effect on the vibration characteristics.

Key words: circular plate; Fourier-Bessel series; Rayleigh-Ritz method; fluid-coupled system; normalized frequency.

1. Introduction

Recently, there have been many studies on the vibration of circular plate in contact with a fluid. The fluid-contacting structures have been used in mechanical engineering, such as in nuclear reactor internal components. Natural frequencies of the components have attracted extensive interest when exploring the structural responses to various excitations, such as seismic and pump pulsation excitations. Especially, various structures such as coaxial cylindrical shells and multiple plates coupled with a fluid, are used as a shield against irradiation damage by the neutron fluence (Jhung 1996). However, it is very difficult to identify the dynamic characteristics of these structures analytically.

It is generally known that the natural frequencies of structures in contact with a fluid, or immersed in a fluid, decrease significantly compared to the natural frequencies in air. This problem is referred to as the fluid-structure interaction problem. For this problem, many investigators have obtained some approximate solutions that have been used to predict the changes in the natural

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frequencies of a structure in a fluid (Bauer 1995). In recent literature, there has been renewed interest in the problem of plates vibrating in contact with water (Kwak 1991, Amabili 2001, Cheung and Zhou 2002). This is stimulated by new technical applications and also by the availability of powerful numerical tools based on the finite element and boundary element methods which make numerical solutions of fluid-structure interaction problems possible. However, the use of the finite element method or the boundary element method requires enormous amounts of time for modeling and computation.

This paper deals with the coupling effect of contacting fluid on the free vibration characteristics of circular plate. It is assumed that an incompressible, irrotational and frictionless fluid is bounded in the radial direction. The dynamic displacements of the plates are expanded in terms of the eigenfunctions of the plate in air with unknown weighing coefficients and then the Rayleigh-Ritz method is applied. The coefficients are determined by the boundary conditions along the edges of the plate and the compatibility requirement on the surface of the plate in contact with the fluid. Finally, the frequency equation for the coupled natural frequencies can be derived. The natural frequencies of the fluid-coupled system can be obtained by theoretical calculations and verified by three dimensional finite element analysis. The fluid–coupled natural frequency of the wet mode is normalized with respect to the natural frequency of the dry mode in order to estimate the relative hydrodynamic mass effect on each wet mode of the circular plate. Investigated also is the effect of the location of plate coupled with a fluid on the coupled natural frequency of the wet mode.

2. Theoretical development

2.1 Formulation

Considering a single circular plate submerged in a fluid-filled rigid cylinder as shown in Fig. 1, where R, h, H_1 and H_2 represent the radius and thickness of the plate, and height of upper and lower fluid respectively, the following assumptions are made for the theoretical development:

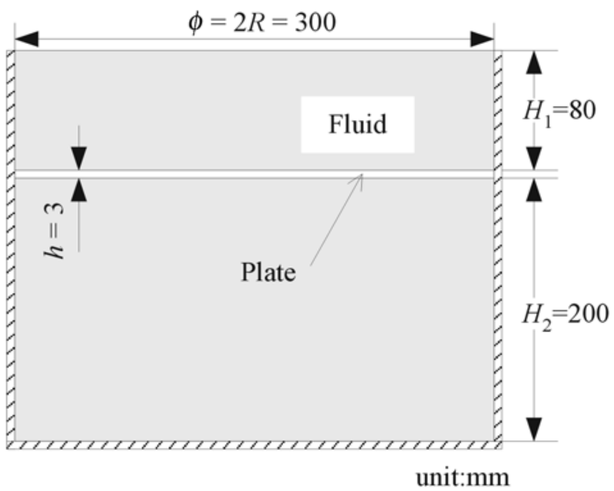


Fig. 1 Plate submerged in fluid

(a) the fluid motion is so small that it is considered to be linear,
(b) the fluid is incompressible, inviscid and irrotational,
(c) the material of plate is linearly elastic, homogeneous and isotropic.

The equation of motion for transverse displacement, w, of this plate in contact with fluid is:

$$D\nabla^4 w + \rho h w_{,tt} = \sum_{j=1}^{2} p_j, \quad j = 1, 2 \tag{1}$$

where $D = Eh^3/12(1 - \mu^2)$ is the flexural rigidity of the plate; ρ, μ, p_j and E are density, Poisson's ratio, hydrodynamic pressure on the plate and Young's modulus of the plate, respectively. The upper fluid is referred to with a subscript "1" while the lower fluid is denoted by a subscript "2". The solution of Eq. (1) takes the following form of combinations for plate deformation with respect to polar coordinates (r, θ):

$$w(r, \theta, t) = \cos(n\theta) \sum_{m=1}^{\infty} q_m W_{nm}(r) \exp(i\omega t) \tag{2}$$

where q_m is unknown coefficient and n and m are the numbers of the nodal diameters and circles of the plate, respectively. For the plate with clamped boundary condition, the displacement along the edge of the plate, $r = R$, must be zero and therefore dynamic displacement of Eq. (2) will be reduced to:

$$W_{nm}(r) = \mathrm{J}_n(\lambda_{nm} r) - \mathrm{J}_n(\lambda_{nm} R)\frac{\mathrm{I}_n(\lambda_{nm} r)}{\mathrm{I}_n(\lambda_{nm} R)} \tag{3}$$

where λ_{nm} is the frequency parameter for the plate in air, which is also determined by the boundary conditions and is related to the circular frequency of the plate in air ω. J_n and I_n are the Bessel function and the modified Bessel function of the first kind, respectively. For the fixed boundary condition, the eigenvalues λ_{nm} for the plate in air can be obtained from the zero slope and zero moment boundary conditions as follows (Bauer 1995):

$$\mathrm{J}_n(\lambda_{nm} R)\mathrm{I}_{n+1}(\lambda_{nm} R) + \mathrm{J}_{n+1}(\lambda_{nm} R)\mathrm{I}_n(\lambda_R R) = 0 \tag{4}$$

2.2 Velocity potential

The fluid region contained in cylindrical rigid vessel is bisected into two parts, an upper fluid and a lower fluid by the circular plate. The three dimensional oscillatory fluid flow in the cylindrical coordinates can be described by the velocity potential. The facing side of the circular plates is contacted with inviscid and incompressible fluid. The fluid movement due to vibration of the plate is described by the spatial velocity potential that satisfies the Laplace equation:

$$\nabla^2 \Phi_j(x, r, \theta, t) = 0, \quad j = 1, 2 \tag{5}$$

It is possible to separate the function Φ with respect to r by observing that in the radial direction the vessel which supports the edges of the plate are assumed to be rigid, as in the case of the completely contact circular plate. Thus:

$$\Phi_j(x, r, \theta, t) = i\omega\phi_j(r, \theta, x)\exp(i\omega t) \tag{6}$$

Substituting Eq. (6) into Eq. (5) generates the general solution of Eq. (5) as (Jeong and Kim 2005):

$$\phi_j(r, \theta, t) = \cos(n\theta)\sum_{s=1}^{\infty} \mathrm{J}_n(\beta_{ns}r)\{E_{nsj}\sinh(\beta_{ns}x) + F_{nsj}\cosh(\beta_{ns}x)\} \tag{7}$$

For the bounded fluid, the boundary condition along the cylindrical vessel wall assures the zero fluid velocity in the radial direction given by:

$$\partial\phi_j/\partial r|_{r=R} = 0 \tag{8}$$

Insertion of Eq. (7) into Eq. (8) determines β_{ns} for every n and s by the following transcendental equation:

$$\mathrm{J}_n'(\beta_{ns}R) = 0 \tag{9}$$

When it is assumed that all the vessel walls are rigid and the plate thickness is negligible comparing with the vessel height, the velocity potential must satisfy the followings:

$$\partial\phi_1(r, \theta, H_1)/\partial x = 0 \quad \text{for the upper fluid} \tag{10}$$

$$\partial\phi_2(r, \theta, -H_2)/\partial x = 0 \quad \text{for the lower fluid} \tag{11}$$

Application of Eqs. (10) and (11) into Eq. (7) gives reduced forms of Eq. (7) for the upper and lower fluid:

$$\phi_1(r, \theta, x) = \cos(n\theta)\sum_{s=1}^{\infty} E_{ns1}\mathrm{J}_n(\beta_{ns}r) \times \{\sinh(\alpha_{ns}x) - \cosh(\alpha_{ns}x)/\tanh(\alpha_{ns}H_1)\} \tag{12a}$$

$$\phi_2(r, \theta, x) = \cos(n\theta)\sum_{s=1}^{\infty} E_{ns2}\mathrm{J}_n(\beta_{ns}r) \times \{\sinh(\alpha_{ns}x) + \cosh(\alpha_{ns}x)/\tanh(\alpha_{ns}H_2)\} \tag{12b}$$

2.3 Method of solution

In order to determine the unknown coefficients E_{ns1} and E_{ns2} of fluid motion in Eqs. (12a) and (12b), the compatibility conditions at the interface of the upper and lower fluids in contact with the plate are used. Since the plate thickness is neglected, the compatibility conditions at the fluid interface with the plate yield:

$$w = \partial\phi_1/\partial x|_{x=0} \tag{13a}$$

$$w = \partial\phi_2/\partial x|_{x=0} \tag{13b}$$

Substitution of Eqs. (2), (3), (12a) and (12b) into Eqs. (13a) and (13b) gives:

$$\sum_{m=1}^{M} q_m \left[\mathrm{J}_n(\lambda_{nm} r) - \mathrm{J}_n(\lambda_{nm} R) \frac{\mathrm{I}_n(\lambda_{nm} r)}{\mathrm{I}_n(\lambda_{nm} R)} \right] = \sum_{s=1}^{\infty} E_{ns1} \beta_{ns} \mathrm{J}_n(\beta_{ns} r) \tag{14a}$$

$$\sum_{m=1}^{M} q_m \left[\mathrm{J}_n(\lambda_{nm} r) - \mathrm{J}_n(\lambda_{nm} R) \frac{\mathrm{I}_n(\lambda_{nm} r)}{\mathrm{I}_n(\lambda_{nm} R)} \right] = \sum_{s=1}^{\infty} E_{ns2} \beta_{ns} \mathrm{J}_n(\beta_{ns} r) \tag{14b}$$

Expanding $\mathrm{J}_n(\lambda_{nm} r)$ and $\mathrm{I}_n(\lambda_{nm} r)$ of Eqs. (14a) and (14b) into Bessel-Fourier series of the form (Hagedorn 1994, Sneddon 1951) will give:

$$\mathrm{J}_n(\lambda_{nm} r) = \sum_{s=1}^{\infty} a_{nms} \mathrm{J}_n(\beta_{ns} r) \tag{15a}$$

$$\mathrm{I}_n(\lambda_{nm} r) = \sum_{s=1}^{\infty} b_{nms} \mathrm{J}_n(\alpha_{ns} r) \tag{15b}$$

where the Bessel-Fourier coefficients a_{nms} and b_{nms} are

$$a_{nms} = \frac{2(\beta_{ns}R)^2 (\lambda_{nm}R) \mathrm{J}_n'(\lambda_{nm}R)}{[(\beta_{ns}R)^2 - n^2][(\beta_{ns}R)^2 - (\lambda_{nm}R)^2] \mathrm{J}_n(\beta_{ns}R)} \tag{16a}$$

$$b_{nms} = \frac{2(\beta_{ns}R)^2 (\lambda_{nm}R) \mathrm{I}_n'(\lambda_{nm}R)}{[(\beta_{ns}R)^2 - n^2][(\beta_{ns}R)^2 + (\lambda_{nm}R)^2] \mathrm{J}_n(\beta_{ns}R)} \tag{16b}$$

Therefore, the velocity potential of the fluid can be written in terms of unknown constants q_m instead of the unknown coefficient E_{nsj}.

$$\phi_1(r, \theta, x) = \cos(n\theta) \sum_{m=1}^{M} q_m \sum_{s=1}^{\infty} \Lambda_{nms} \mathrm{J}_n(\beta_{ns} r) \times \{\sinh(\beta_{ns} x) - \cosh(\beta_{ns} x)/\tanh(\beta_{ns} H_1)\} \tag{17a}$$

$$\phi_2(r, \theta, x) = \cos(n\theta) \sum_{m=1}^{M} q_m \sum_{s=1}^{\infty} \Lambda_{nms} \mathrm{J}_n(\beta_{ns} r) \times \{\sinh(\beta_{ns} x) + \cosh(\beta_{ns} x)/\tanh(\beta_{ns} H_2)\} \tag{17b}$$

where Λ_{nms} is a derived coefficient:

$$\Lambda_{nms} = \frac{4R(\beta_{ns}R)(\lambda_{nm}R)^3 \mathrm{J}_n'(\lambda_{nm}R)}{[(\beta_{ns}R)^2 - n^2][(\beta_{ns}R)^4 - (\lambda_{nm}R)^4] \mathrm{J}_n(\beta_{ns}R)} \tag{18}$$

When the gravity is neglected, it is useful to introduce the Rayleigh quotient in order to calculate the coupled natural frequencies of the circular plate submerged in the ideal fluid.

$$\omega^2 = \frac{V_d}{T_d + T_F} \tag{19}$$

where V_d is the potential energies of the plate and T_d and T_F are the reference kinetic energies of the plate and the fluid, respectively. In order to perform numerical calculations for each fixed n value, a sufficiently large finite M number of terms must be considered in all the previous sums of the expanding term, m. For this purpose, a vector **q** of the unknown parameters is introduced as:

$$\mathbf{q} = \{q_1 \; q_2 \; q_3 \; \ldots \; q_M\}^T \tag{20}$$

Now, it is necessary to know the reference kinetic energies of the plate and containing fluids in order to calculate the coupled natural frequencies of the circular plate in contact with fluids. Using the hypothesis of irrotational movement of the fluid, the reference kinetic energy of the fluids can be evaluated from its boundary motion.

$$T_F = \frac{1}{2}\rho_o \int_0^{2\pi}\int_0^R \frac{\partial\phi_1(r,0)}{\partial x}\phi_1(r,0)\cos^2(n\theta)r\,dr\,d\theta + \frac{1}{2}\rho_o \int_0^{2\pi}\int_0^R \frac{\partial\phi_2(r,0)}{\partial x}\phi_2(r,0)\cos^2(n\theta)r\,dr\,d\theta \tag{21}$$

Now, where ρ_o is the mass density of the fluid. Application of Eqs. (13a) and (13b) into Eq. (21) reduces to Eq. (22):

$$T_F = -\frac{1}{2}\rho_o\kappa_\theta\left[\int_0^R w\phi_1(r,0)r\,dr + \int_0^R w\phi_2(r,0)r\,dr\right] \tag{22}$$

where $\kappa_\theta = 2\pi$ for $n = 0$ and $\kappa_\theta = \pi$ for $n > 0$. Insertion of Eqs. (2), (3) and (17) into Eq. (22) gives the reference kinetic energy of the fluid :

$$T_F = \rho_o\kappa_\theta\mathbf{q}^T\mathbf{G}\mathbf{q} \tag{23}$$

where the $M \times M$ symmetric matrix $\mathbf{G}$ for the fixed n is given by Eqs. (15a,b), (16a,d) and (22) as follows and it is called added virtual mass incremental (AVMI) matrix (Kwak and Kim 1991, Chiba 1994).

$$G_{ik} = -\sum_{s=1}^{\infty}\frac{8R^3(\beta_{ns}R)\Xi_{is}\Xi_{ks}B_{ns}}{[(\beta_{ns}R)^2 - n^2]}, \quad i, k = 1, 2, \ldots, M \tag{24}$$

with

$$\Xi_{is} = \frac{(\lambda_{ni}R)^3 J_n'(\lambda_{ni}R)}{[(\beta_{ns}R)^4 - (\lambda_{ni}R)^4]} \tag{25a}$$

$$\Xi_{ks} = \frac{(\lambda_{nk}R)^3 J_n'(\lambda_{nk}R)}{[(\beta_{ns}R)^4 - (\lambda_{nk}R)^4]} \tag{25b}$$

$$B_{ns} = \tanh(\beta_{ns}H_1), \quad \text{for the upper fluid} \tag{25c}$$

$$B_{ns} = \tanh(\beta_{ns}H_2), \quad \text{for the lower fluid} \tag{25d}$$

The reference kinetic energy of the circular plate is presented:

$$T_d = \frac{\rho h\kappa_\theta}{2}\int_0^R w^2 r\,dr \tag{26}$$

Insertion of Eq. (2) into Eq. (26) gives the kinetic energy of the plate as:

$$T_d = \rho h \kappa_\theta \mathbf{q}^T \mathbf{Z} \mathbf{q} \tag{27}$$

where $\mathbf{Z}$ is the $M \times M$ matrix given as

$$Z_{ik} = \delta_{ik} \int_0^R r W_{ni1} W_{nk1} dr \tag{28}$$

with δ_{ik} of Kronecker delta. When Eq. (3) is inserted into Eq. (28) and the integration is carried out, matrix $\mathbf{Z}$ is simply represented as:

$$Z_{ik} = R^2 \{\mathrm{J}_n(\lambda_{ni} R)\}^2 \delta_{ik} \tag{29}$$

The maximum potential energy of the plate can be computed as:

$$V_d = \kappa_\theta D \int_0^R \left([\nabla^2 w]^2 - 2(1-\mu) \left\{ \frac{\partial^2 w}{\partial r^2} \left(\frac{1}{r} \frac{\partial w}{\partial r} + \frac{1}{r^2} \frac{\partial^2 w}{\partial \theta^2} \right) - \left(\frac{\partial}{\partial r} \left[\frac{1}{r} \frac{\partial w}{\partial \theta} \right] \right)^2 \right\} \right) r dr \tag{30}$$

As the first term $[\nabla^2 w]^2$ of Eq. (30) is identical to $\lambda_{ni}^4 w^2$ and the other terms of Eq. (30) are negligible comparing with the first term, the maximum potential energy may be approximated by

$$V_d \approx \kappa_\theta D \mathbf{q}^T \mathbf{P} \mathbf{q} \tag{31}$$

where $\mathbf{P}$ is the $M \times M$ diagonal matrix given by

$$P_{ik} = \frac{(\lambda_{ni} R)^4}{R^2} \{\mathrm{J}_n(\lambda_{ni} R)\}^2 \delta_{ik} \tag{32}$$

The correspondence between the reference total kinetic energy of each mode multiplied by its square circular frequency and the maximum potential energy of the same node are used. In order to find natural frequencies and mode shpaes of the plate in contact with fluid, the Rayleigh quotient for the plate vibration coupled with ideal fluid is used. Minimizing Rayleigh quotient $V_d/(T_d + T_F)$ with respect to the unknown parameters q_m, the non-dimensional Galerkin equation can be obtained:

$$\{D\mathbf{P} - \omega^2 (\rho h \mathbf{Z} + \rho_o \mathbf{G})\} \mathbf{q} = \{0\} \tag{33}$$

Eq. (33) gives an eigenvalue problem and the natural frequencies ω can be calculated.

3. Analysis

3.1 Theoretical analysis

On the basis of the preceding analysis, the determinant of the left side in Eq. (33) is numerically solved using MathCAD in order to find the natural frequencies of circular plate coupled with fluid. In order to check the validity and accuracy of the results from the theoretical study, finite element analyses are also performed and frequency comparisons between them are carried out for the fluid-coupled system.

Table 1 Dimensions and material properties

	Unit	Plate	Container	Fluid
Young's modulus	Pa	69E9	172E9	
Poisson's ratio		0.3	0.3	
Density	kg/m^3	2700	7800	1000
Sound speed	m/sec			1483
Bulk modulus of elasticity	Pa			2.2E9
Thickness	mm	3		
Diameter	mm	300		

The circular plate is connected to the fixed closed-type container which is made of carbon steel. The plate is made of aluminum having a radius of 150 mm and a thickness of 3 mm (Fig. 1). The physical properties of the material are as follows: Young's modulus = 69.0 GPa, Poisson's ratio = 0.3, and mass density = 2700 kg/m^3. Water is used as the contained fluid, having a density of 1000 kg/m^3. The sound speed in water is 1483 m/s, which is equivalent to the bulk modulus of elasticity of 2.2 GPa (Table 1).

The frequency equations derived in the preceding sections involve an infinite series of algebraic terms. Before exploring the analytical method to obtain the natural frequencies of the fluid-coupled plate, it is necessary to conduct convergence studies and establish the number of terms required in the series expansions involved. In the numerical calculation, the Bessel-Fourier expansion term s is set to 200 and the expanding term m for the admissible function is set to 40, which gives an exact enough solution by convergence. In general, the solution approaches the exact frequency from above as the number of terms included in the series Eqs. (23), (27) and (31) increases, which may increase the calculation time significantly.

3.2 Finite element analysis

Finite element analyses using a commercial computer code ANSYS 6.1 (2001) are performed to verify the analytical results for the theoretical study. The results from finite element method are used as the baseline data. Three-dimensional model is constructed for the plate submerged in fluid as shown in Fig. 2. Also, three different models are developed for plates in air, on fluid and beneath fluid by eliminating upper fluid and/or lower fluid as shown in Fig. 3. The fluid region is divided into a number of 3-dimensional contained fluid elements (FLUID80) with eight nodes having three degrees of freedom at each node. The fluid element FLUID80 is particularly well suited for calculating hydrostatic pressures and fluid/solid interactions. The circular plate is modeled as elastic shell elements (SHELL63) with four nodes.

The perimeter nodes of the plate are coupled with the nodes of the container which are fixed in all six degrees of freedom. The fluid movement at top and bottom of the container is considered to be constrained in the vertical direction for the bounded surface fluid case. The vertical velocities of the fluid element nodes adjacent to each surface of the wetted circular plate coincide to those of plate so that the model can simulate Eqs. (13a) and (13b).

The Block Lanczos method is used for the eigenvalue and eigenvector extractions to calculate 1000 frequencies including fluid modes.

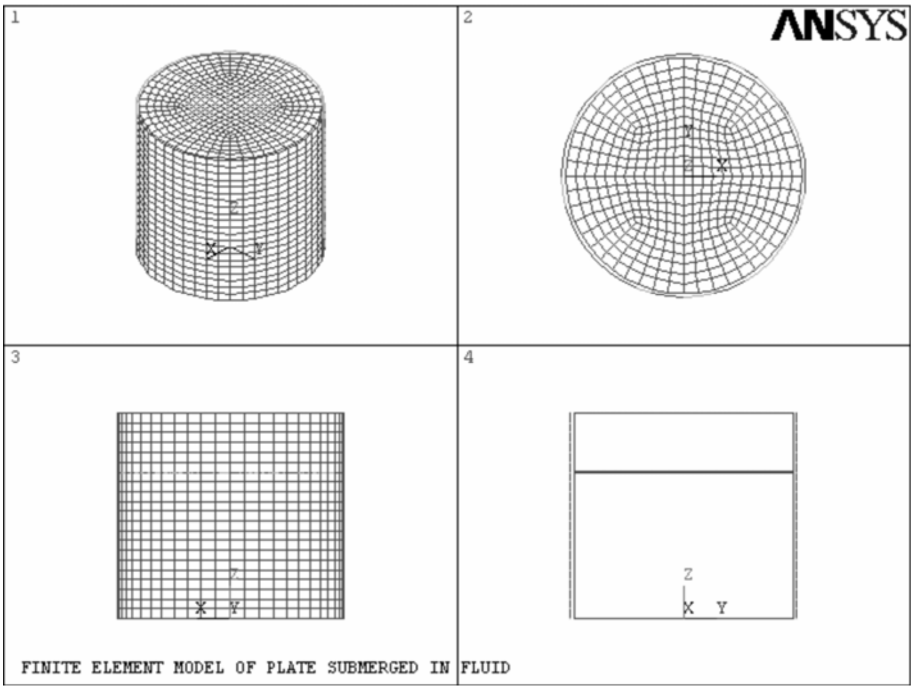
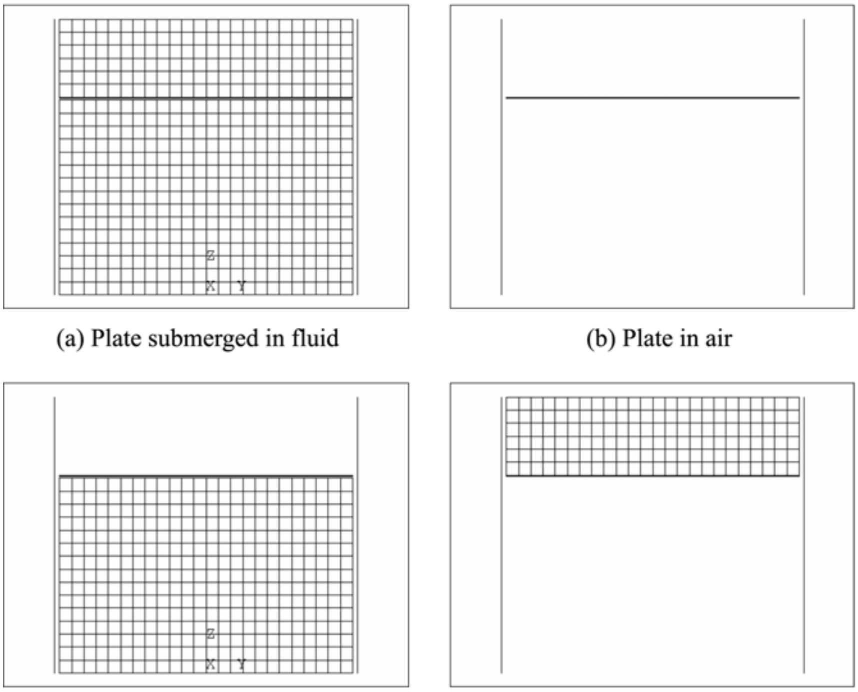


Fig. 2 Finite element model of plate submerged in fluid



(a) Plate submerged in fluid (b) Plate in air

(c) Plate with lower fluid (d) Plate with upper fluid

Fig. 3 Four different types of finite element models

Table 2 Comparison of frequencies between FEM and theory for plate in air

Mode		Natural frequency (Hz)		Error (%)
n	*m'*	FEM	Theory	
1	1	690	690	0.0
	2	1971	1975	0.2
	3	3885	3898	0.3
	4	6427	6462	0.5
2	1	1131	1132	0.1
	2	2739	2746	0.3
	3	4971	4993	0.4
	4	7828	7879	0.6
3	1	1653	1657	0.2
	2	3591	3604	0.4
	3	6142	6178	0.6
	4	9315	9388	0.8
4	1	2255	2262	0.3
	2	4527	4548	0.5
	3	7399	7451	0.7
	4	10886	10986	0.9

Table 3 Comparison of frequencies between FEM and theory for plate submerged in fluid with bounded surface

Mode		Natural frequency (Hz)		Error (%)
n	*m'*	FEM	Theory	
1	1	172	173	0.6
	2	697	707	1.4
	3	1639	1685	2.7
	4	3006	3154	4.7
2	1	360	362	0.6
	2	1085	1105	1.8
	3	2245	2318	3.1
	4	3847	4041	4.8
3	1	597	604	1.2
	2	1534	1572	2.4
	3	2917	3030	3.7
	4	4746	5013	5.3
4	1	889	902	1.4
	2	2046	2107	2.9
	3	3656	3820	4.3
	4	5712	6071	5.9

4. Results and discussion

The frequency comparisons between analytical solution developed here and finite element method are shown in Tables 2 and 3 for plates in air and submerged in fluid with bounded surface, respectively. The symbol *m*' in the tables represents the number of nodal circles of the wet mode and the symbol *n* means the number of nodal diameter. The frequency differences of plate in air are almost negligible as shown in Table 2. But the largest discrepancy of 5.9 % in *m*' = 4, *n* = 4, as shown in Table 3, is obtained for the plate submergerd in fluid with bounded surface. As the mode number increases, the discrepancy becomes large, which can be reduced by using the sufficient number of node in the radial direction in the finite element modelling. Also, the compressibility of the fluid was found to reduce the natural frequency of the lower wet modes in the case of a fluid-filled cylindrical shell (Jeong and Kim 1998). Therefore, discrepancies in Table 3 may, also, be caused by the assumption that the water is incompressible in the theory. But the frequency comparisons between theoretical and finite element analysis results are generally found to be in good agreement within 6%.

Frequencies of plate coupled with various fluid conditions are represented in Figs. 4 and 5. In all cases, as the number of nodal circles increases, frequencies increase, which were not shown in cylindrical shells (Jhung *et al.* 2002, 2003). Typical mode shapes of radial modes are shown in Fig. 6 and mode shapes which are too hard to predict and to be categorized in the frequency table are shown in Fig. 7. Also, very different types of mode shapes with small participation factor are appeared as shown in Fig. 8, which are expected to appear due to fluid.

The effect of fluid on the frequencies of circular plate wetted with fluid can be assessed using the normalized frequency defined as the natural frequency of a structure in contact with a fluid divided by the corresponding natural frequency in air. The normalized natural frequencies have values between one and zero due to the added mass effect of fluid. Figs. 9 and 10 show the normalized

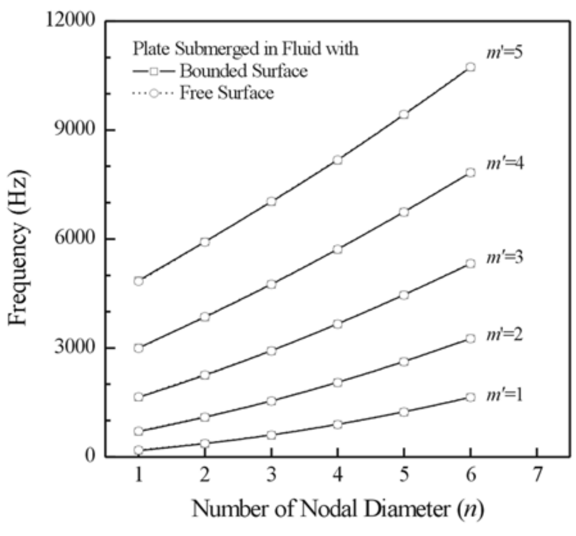


Fig. 4 Frequencies of plate submerged in fluid with bounded and free surface by FEM

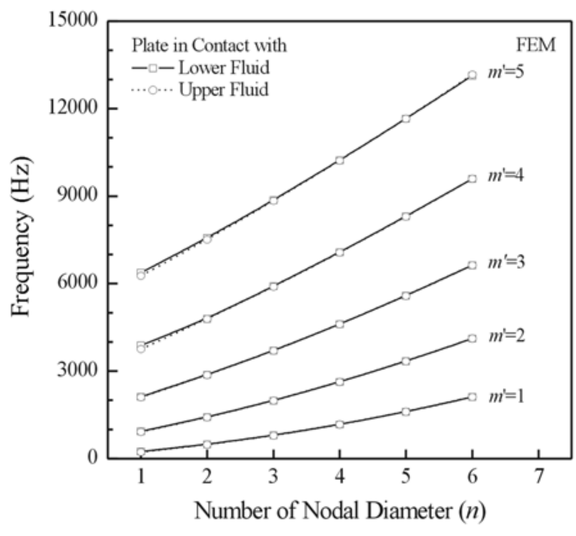


Fig. 5 Frequencies of plate with upper and lower fluid by FEM

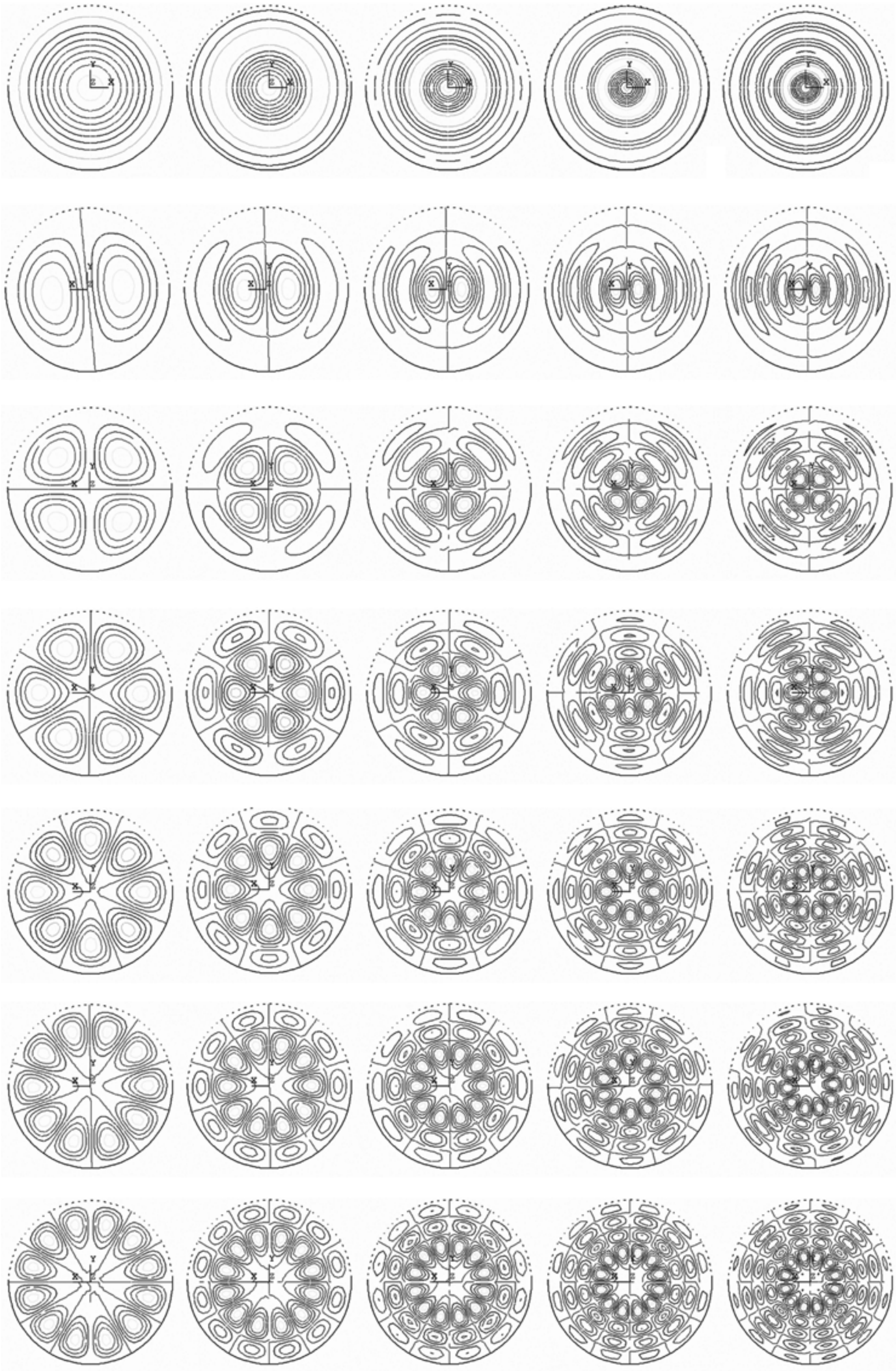


Fig. 6 Typical mode shapes of plate for radial mode

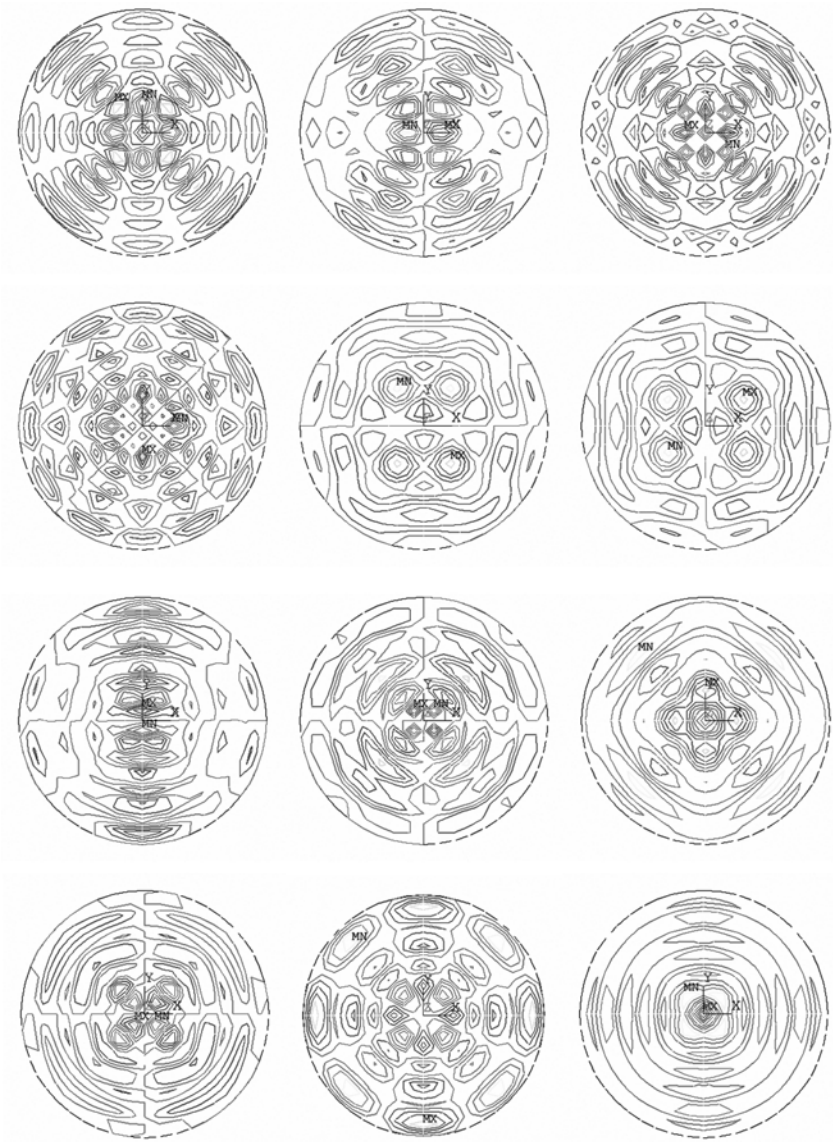


Fig. 7 Typical mode shapes of plate uncategorized

natural frequencies for the plate modes submerged in fluid and with lower and upper fluid, respectively. The fluid affects the plate modes submerged in fluid more significantly than those with only one side of fluid by comparing Fig. 9 with Fig. 10. Especially the plate with fluid of only one side has almost the same effect irrespective of the location of fluid; on or beneath. But it is noted that the modes of plate with upper fluid have a little more effect than those of plate with lower fluid. As the number of nodal circles or diameters of the plates increases, the normalized natural frequencies increase by the gradual reduction of the relative added mass effect. Therefore, an increase of nodal lines or nodal circles causes an increase in the normalized natural frequencies for all cases of modes.

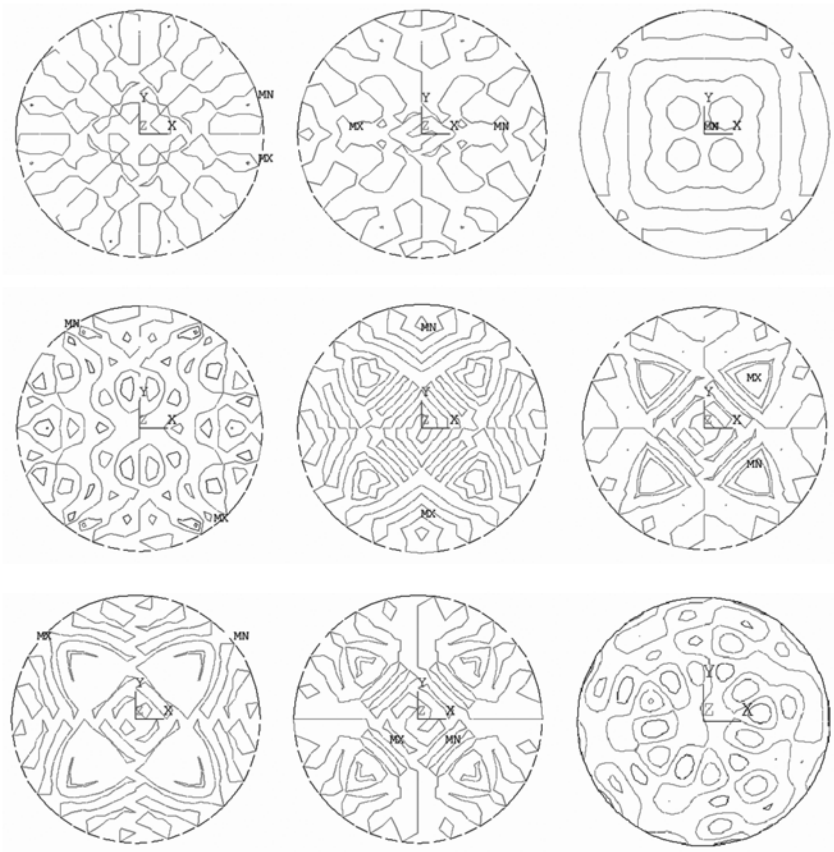


Fig. 8 Typical mode shapes with small participation factor

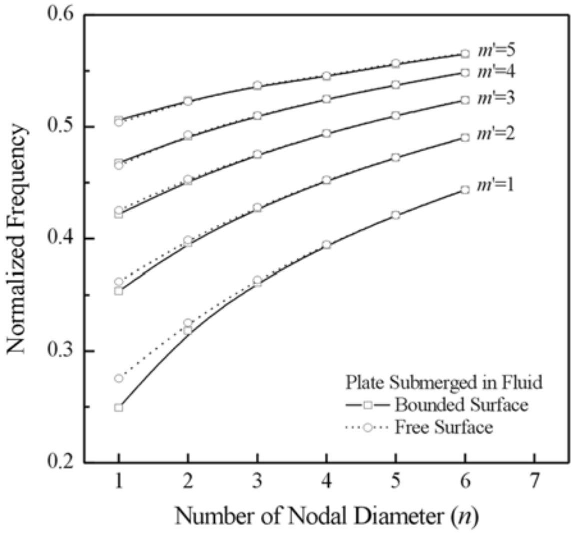


Fig. 9 Normalized frequencies of plate submerged in fluid with respect to in-air condition

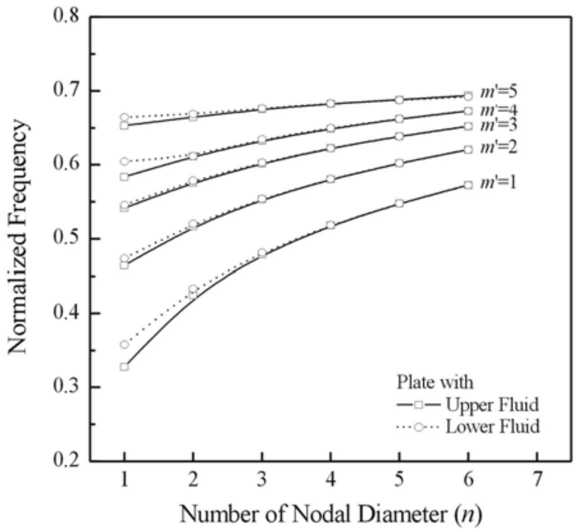


Fig. 10 Normalized frequencies of plate with lower and upper fluid with respect to in-air condition

The fluid surface bounding effects are shown in Fig. 4. Generally the natural frequencies of the free surface case are higher than those of the bounded surface case because the fluid is free to move vertically and the added mass of the mass is reduced and eventually increase the natural frequency of the wet modes, but the difference is very small. Especially it is to be noted that as the number of diametrical mode increases the difference between them becomes small and eventually the frequencies of the bounded surface began to be the same or higher than those of free surface beyond $m' = 6$. This can be predicted from the normalized frequency comparisons between bounded and free fluid surface cases shown in Fig. 9, which show that the bounded surface case has more fluid effect than the free surface case in the lower modes but as for the higher modes they have almost the same fluid effect irrespective of the bounded or free fluid surface cases.

5. Conclusions

An analytical method to estimate the coupled frequencies of the circular plate in contact with fluid is developed using the finite Fourier-Bessel series expansion and Rayleigh-Ritz method. To verify the validity of the analytical method developed, finite element method is used and the frequency comparisons between them are found to be in good agreement. The effect of the fluid on the plate frequencies is found to be more severe in the plate submerged in fluid. Especially as number of diametrical and circular modes decrease, the effect is more significant. Also, if there is a fluid only one side of the plate the fluid effect is almost the same irrespective of the location of the fluid. Finally, the frequencies of free surface case are almost the same as those of bounded surface case.

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