DOI: http://dx.doi.org/10.12989/sem.2005.20.6.631

Study on structural damping of aluminium using multi-layered and jointed construction

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(Received January 29, 2004, Accepted May 13, 2005)

Abstract. In this work, the mechanism of damping and its theoretical evaluation for layered aluminium cantilever structures jointed with a number of equispaced connecting bolts under an equal tightening torque have been considered. Extensive experiments have been conducted on a number of specimens for comparison with numerical results. Intensity of interface pressure, its distribution pattern, dynamic slip ratio and kinematic coefficient of friction at the interfaces, relative spacing of the connecting bolts, frequency and amplitude of excitation are found to play a major role on the damping capacity of such structures. It is established that the damping capacity of structures jointed with connecting bolts can be improved largely with an increase in number of layers maintaining uniform intensity of pressure distribution at the interfaces. Thus the above principle can be utilized in practice for construction of aircraft and aerospace structures effectively in order to improve their damping capacity which is one of the prime considerations for their design.

Key words: interface pressure; dynamic slip ratio; relative spacing; damping capacity.

1. Introduction

The study of damping and its improvement in structural members has become increasingly significant in order to control undesirable effects of vibration with simultaneously enhancing the damping capacity. This study has been taken up extensively in four major areas such as; material science, structural mechanics, vibration control and inspection methods. The damping in mechanical vibrating system has been classified into two classes depending on their energy dissipation sources as (i) material damping and (ii) system damping. Coulomb (1784) postulated that material damping arises due to interfacial friction between the grain boundaries of the material under dynamic condition. Robertson and Yorgiadis (1946) have shown that damping arises due to internal friction of engineering materials. Demer (1956) and Lazan (1968) have also established that the damping properties of materials are due to their microstructures. The system damping arises from slip and other boundary shear effects at mating surfaces, interfaces or joints between distinguishable parts. Extensive studies have been made by Belgaumkar and Murty (1968), Grootenhuis (1970), and Murty (1971) on support damping. Murty (1971) has established that the dissipated energy at the support is very small compared to material damping.

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As the available damping inherent in the structural members (material damping) is inadequate, various techniques have been adopted in practice to improve the damping capacity of structures. These techniques are; (i) use of constrained/unconstrained viscoelastic layers, (ii) fabrication of multi-layered sandwich construction, (iii) insertion of special high elastic inserts in the parent structure, (iv) application of spaced damping techniques, and (v) fabricating layered and jointed structures with welded/riveted/bolted joints. One of the most important techniques of improvement of damping capacity of structures is by use of constrained and unconstrained layers. Initial work on flexural vibration analysis of such systems was carried out in the early part of 1950. Ross *et al.* (1959), Mead (1960), Pujara *et al.* (1968), Itterbeck *et al.* (1953), Nakra (1998), Chantalakhana *et al.* (2000), Hu *et al.* (2000), Van Vuure *et al.* (2001), and Trindade *et al.* (2002) carried out more work on constrained layer damping. Grootenhuis (1970) has reported that the unconstrained layer treatment can compete with the sandwich forms of construction only when the damping material is very stiff and can be applied in a thickness several times greater than the base structure. Reddy *et al.* (1980) and Parthasarathy *et al.* (1985) have studied the damping effectiveness of unconstrained partially applied damping treatment applied to rectangular platens. Cremer *et al.* (1972) and Nashif *et al.* (1985) have established that the damping of structures improve by applying bonding surface layer (free layer damping) to the structure of interest. Appreciable structural damping has been achieved by laminated constructions of alternate layers of an elastic material such as metal and a high damping viscoelastic such as plastic. These low weight structures have good structural, fatigue and acoustic properties and this has resulted in the rapid development of sandwich structures for use in air craft and other industries. A lot of work has been reported by Plantema (1966), Jones *et al.* (1967), Mead and Markus (1970), Bert *et al.* (1967), Nakra *et al.* (1972), Bhimaraddi (1995), Wang *et al.* (2000), Patel *et al.* (2001), Trindade *et al.* (2001), Yim *et al.* (2003), and Srikantha *et al.* (2003) on damping of sandwich beams. Han (1985) has shown that the damping of sandwich type plate with metal facing and felt has been enhances by thirty times compared to the damping of an identical stiff solid plate. It has been established that the damping characteristics of a structural member can be improved considerably by using high damping elastic inserts or pins. Mallik and Ghosh (1973, 1974), and Rahmathullah and Mallik (1979) have shown that the damping capacity increases with use of a proper combination of strip and insert material. Another technique of improvement of damping capacity is by moving the damping material away from the structure. This technique was first used to damp out noise and vibration in U. S. submarines. The details of studies in this field have been done by Miller and Warnaka (1970). Joints are present in most of the structures and usually over ninety percent of the inherent damping in a fabricated structure originates in the joints. Belgaumkar *et al.* (1968), Masuko *et al.* (1973), and Nishiwaki *et al.* (1978, 1980) have reported extensive work on the technique of improvement of damping capacity of welded structures and they have established that the damping capacity of a welded machine tool structure is not different from that of a cast structure. Anno *et al.* (1970) have studied on the relationship between the forms of welded joints and the damping capacity and reported that the steel plates welded with the plug joints show a high damping capacity compared to other forms of welded joints. Pain (1957) through his investigation has established that the riveted joints are also responsible for improving the damping capacity of structures. Extensive work has been done by Fernlund (1961), Kobayashi *et al.* (1986), Shin *et al.* (1991), Masuko *et al.* (1973), Nishiwaki *et al.* (1978, 1980), Motosh (1975), Connolly *et al.* (1965), Mitsunaga (1965), Goodman and Klumpp (1956), Ito and Masuko (1971, 1975), Courtney-Pratt *et al.* (1957) and Law *et al.* (2004) on the damping of structures with bolted joints. It is generally recognized that the damping capacity of the

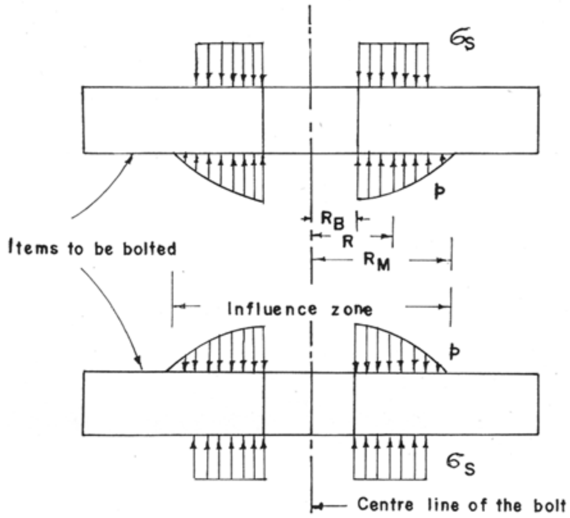

Fig. 1 Free-body diagram of a bolted joint showing the influence zone

jointed structures may be determined by the frictional loss energy caused by slip between interfaces of steel plates. Beards and Williams (1977) have shown that the interfacial slip in joints is the major contributor to the inherent damping of most fabricated structures. Goodman and Klumpp (1956) reported that the damping capacity on the bolted joint is caused by the friction between the joint surfaces, and therefore the surface topography has large effects on the damping capacity of bolted structures. Ito and Masuko (1971, 1975) conducted experiments with the bolted cantilever specimens to confirm the effects of the surface conditions such as surface roughness, machining method and machined lay orientation of surfaces and found that the optimum mean interface pressure and the logarithmic damping decrement are slightly changed by the machining method and the machined lay orientation of joint surfaces. However, the surface conditions have significant effects on the damping capacity. Courtney-Pratt *et al.* (1957) have experimentally proved that the equivalent coefficient of friction changes with the quantity of micro-slip at the interfaces of the layered and jointed cantilever beams.

Although considerable amount of work has been done on experimental study of damping in welded structures but no generalized theory has been established. Similarly, no such theory has been developed for the mechanism of damping in case of riveted structures. Hence, layered construction jointed with connecting bolts can be used more effectively with required damping capacity by controlling the influencing parameters. Therefore, attention has to be focused on such influencing parameters in order to maximize the overall damping capacity.

The logarithmic damping decrement, a measure of damping capacity of layered and jointed structures has been determined by the energy principle considering the relative dynamic slip and the pressure distribution at the interfaces of the contacting layers. These two major parameters are to be accurately assessed for correct evaluation of the damping capacity of such structures. Previous investigators, e.g. Fernlund (1961), Kobayashi and Matsubayashi (1986), and Shin *et al.* (1991) have reported on this interface pressure and its distribution characteristics without specifying the spacing of the connecting bolts between them. Masuko *et al.* (1973), Nishiwaki *et al.* (1978, 1980) and Motosh (1975) have done extensive work assuming uniform intensity of pressure distribution at the interfaces of the layered and jointed structures without considering the actual pattern but by using Rötschar's pressure cone (1973). Connolly *et al.* (1965) and Mitsunaga (1965) have reported that the pressure distribution at the jointed interfaces is not uniform but varies almost parabolically being maximum at the surface of the bolt hole. Further, Gould and Mikic (1972) and Ziada and Abd (1980) have shown that the pressure distribution at the interfaces of a bolted joint is parabolic in nature and there exists an influence zone in the form of a circle with 3.5 times the diameter of the connecting bolt which is independent of the tightening load applied on it as shown in Fig. 1. Nanda (1992) and Nanda and Behera (1999, 2000) have also done considerable amount of work on the distribution pattern of the interface pressure and established that the same becomes uniform at a distance of 2.00211 times the diameter of the consecutive connecting bolts joining the layered beams. The damping capacity of such structures can be improved substantially by varying the influencing parameters such as; intensity of interface pressure and its distribution characteristics, spacing of the connecting bolts, tightening torque applied on them, coefficient of kinetic friction at the interfaces, material used for the structure, dynamic slip ratio and the number of layers.

In the present investigation, damping capacity of such layered and jointed structures has been evaluated from analytical expressions developed in the investigation and compared experimentally for two as well as multi-layered aluminium cantilever beams under different conditions of excitation in order to establish the accuracy of the theory developed.

2. Theoretical analysis

In case of a layered structure jointed with connecting bolts, the intensity of interface pressure distribution under each bolt in a non-dimensional form has been assumed to be polynomial with even powers as:

$$p/\sigma_s = A_1 + A_2(R/R_B)^2 + A_3(R/R_B)^4 + A_4(R/R_B)^6 + A_5(R/R_B)^8 + A_6(R/R_B)^{10} \quad (1)$$

where p, σ_s, R and R_B are the interface pressure, surface stress on the jointed structure due to tightening load, any radius within the influencing zone and radius of the connecting bolt respectively and A_1, A_2, A_3, A_4, A_5 and A_6 are the constants of the polynomial. These constants are evaluated from the numerical data of Ziada and Abd (1980) by using Dunn's curve fitting software. These are; 0.68517E+00, −0.10122E+00, 0.94205E-02, −0.23895E-02, 0.29487E-03 and −0.11262E-04 respectively.

The present work is based on the loss energy due to friction at the interfaces and the strain energy of a cantilever beam as shown in Fig. 2. The energy loss per cycle of vibration (E_f) arising due to friction and relative dynamic slip (u_r) at the interfaces has been found out using the theory of Nishiwaki *et al.* (1980) as;

$$E_f = \oint F_r du_r = 2F_{rM}\, u_{rM} \quad (2)$$

where F_r, du_r, F_{rM} and u_{rM} are the frictional force at the interfaces of the beam in presence of relative dynamic slip, incremental relative dynamic slip, maximum frictional force at the interfaces of the beam during vibration and relative dynamic slip between the interfaces at the maximum amplitude of vibration respectively as shown in Fig. 3.

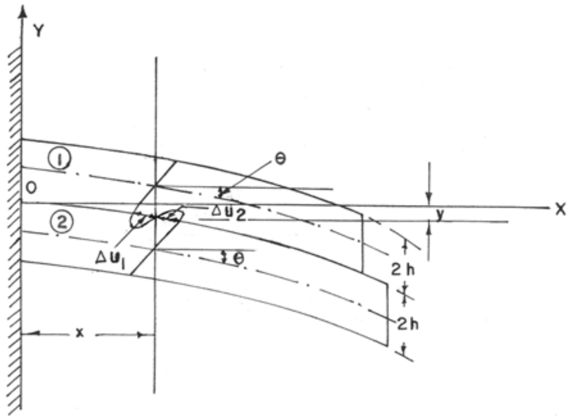

Fig. 2 Mechanism of dynamic slip at the interfaces

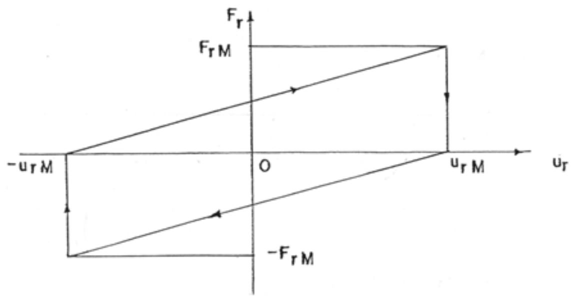

Fig. 3 Relationship between the friction force (F_r) and the relative dynamic slip (u_r) during one cycle

2.1 Determination of maximum frictional force

The maximum frictional force at the interfaces of the beam under transverse vibration is given by;

$$F_{rM} = \mu N \tag{3}$$

where μ and N are the kinematic coefficient of friction and the total normal force at the interfaces of the layers under each connecting bolt respectively.

In order to find out the normal force at the interfaces of a bolted joint, a strip ABCD as shown in Fig. 4 has been considered within the influence zone whose area is given by;

$$RdRd\theta = x'\sec^2\theta dx'd\theta \tag{4}$$

The Eq. (1) for interface pressure distribution is modified considering σ_s as a function of axial load on the connecting bolt due to tightening torque as;

$$p = [A_1 + A_2(R/R_B)^2 + A_3(R/R_B)^4 + A_4(R/R_B)^6 + A_5(R/R_B)^8 + A_6(R/R_B)^{10}]P/A' \tag{5}$$

where A' and P are the area under a connecting bolt head and axial load on the connecting bolt due to tightening torque respectively. This area A' is found out considering the Fig. 5 as;

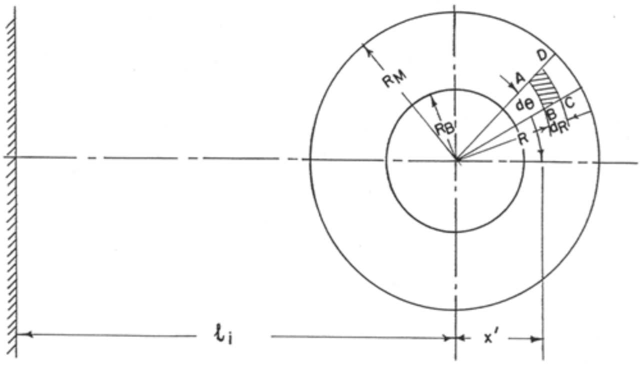

Fig 4 Influence zone of a connecting bolt

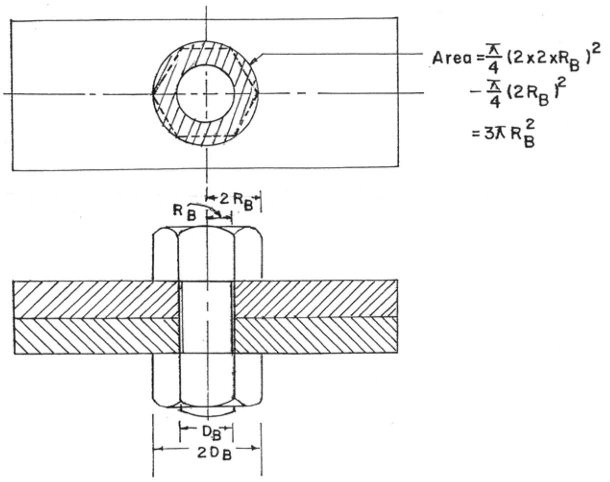

Fig. 5 Influence area under a connecting bolt head

$$A' = (\pi/4)[(4R_B)^2 - (2R_B)^2] = 3\pi R_B^2 \tag{6}$$

Hence, the Eq. (5) for interface pressure distribution becomes;

$$p = [A_1 + A_2(R/R_B)^2 + A_3(R/R_B)^4 + A_4(R/R_B)^6 + A_5(R/R_B)^8 + A_6(R/R_B)^{10}][P/3\pi R_B^2] \tag{7}$$

Combining Eqs. (4) and (7), the normal force on the above strip ABCD is given by;

$$pRdRd\theta = [A_1 + A_2(R/R_B)^2 + A_3(R/R_B)^4 + A_4(R/R_B)^6 + A_5(R/R_B)^8 + A_6(R/R_B)^{10}]$$
$$[P/3\pi R_B^2][x'\sec^2\theta dx' d\theta] \tag{8}$$

Therefore, the total normal force at the interfaces within the influencing zone under each connecting bolt is given by;

$$N = \int_{-\pi/2}^{\pi/2} \int_{R_B\cos\theta}^{R_M\cos\theta} [A_1 + A_2(R/R_B)^2 + A_3(R/R_B)^4 + A_4(R/R_B)^6 + A_5(R/R_B)^8 + A_6(R/R_B)^{10}]$$
$$[x'\sec^2\theta dx' d\theta/3\pi R_B^2]P \tag{9}$$

Integrating the above Eq. (9), the total normal force at the interfaces within the influencing zone under each connecting bolt as found out by Nanda and Behera (2000) is given by;

$$N = [A_1\{(R_M/R_B)^2 - 1\} + \{A_2/2\}\{(R_M/R_B)^4 - 1\} + \{A_3/3\}\{(R_M/R_B)^6 - 1\}$$
$$+\{A_4/4\}\{(R_M/R_B)^8 - 1\} + \{A_5/5\}\{(R_M/R_B)^{10} - 1\} + \{A_6/6\}\{(R_M/R_B)^{12} - 1\}][P/3] \quad (10)$$

where R_M is the limiting radius of the influencing zone under each connecting bolt.

The axial load "P" on the connecting bolt is the force with which it clamps the member together and depends upon the torque applied in tightening. The tightening torque "T" for a connecting bolt of nominal diameter (major diameter) D_B which is to be tightened to an axial load "P" is given by Shigley (1956) as;

$$T = K_1 P D_B \quad (11)$$

where K_1 is the torque coefficient and for average and un-lubricated bolts K_1 is about 0.20.

Hence, the axial load "P" on the connecting bolt due to tightening torque is given by;

$$P = [T/0.2D_B] \quad (12)$$

2.2 Determination of relative dynamic slip at maximum amplitude of vibration

The vibration of the cantilever beam specimen, as shown in Fig. 2, can be expressed as;

$$y(x, t) = Y(x)\, f(t) \quad (13)$$

where the space function, $Y(x) = C_1\sin\lambda x + C_2\cos\lambda x + C_3\sinh\lambda x + C_4\cosh\lambda x$, and the time function, $f(t) = A\cos\omega_n t + B\sin\omega_n t$, C_1, C_2, C_3, and C_4 are constants to be evaluated from the boundary conditions with the usual notation; $\lambda^4 = \omega_n^2 A''\gamma/EIg$ and A and B are constants to be evaluated from the initial conditions. The terms ω_n is the natural circular frequency of vibration, g is the acceleration due to gravity and A'', γ, E and I are the area of cross section, weight density, modulus of elasticity and second moment of inertia of the cantilever beam respectively.

Using the initial free end displacement, $y(l, 0)$ with its boundary conditions for the cantilever beam, the equation for slope is given by;

$$[\partial y(x, t)/\partial x] = -[(\cos\lambda l + \cosh\lambda l)(\cosh\lambda x - \cos\lambda x) - (\sin\lambda l + \sinh\lambda l)(\sin\lambda x + \sinh\lambda x)]$$
$$[\lambda y(l, 0)\cos\omega_n t] \times [2(\cos\lambda l\sinh\lambda l - \sin\lambda l\cosh\lambda l)]^{-1} \quad (14)$$

The actual relative dynamic slip at the interfaces of a bolted joint, which is at a distance of "l_i" from the fixed end of a layered and jointed cantilever beam, is given by;

$$u_r(l_i, t) = \alpha u(l_i, t) \quad (15)$$

where α is the dynamic slip ratio, $u_r(l_i, t)$ and $u(l_i, t)$ are the relative dynamic slip between the interfaces at a bolted joint in the presence and absence of a friction force respectively.

If the layered and jointed beam specimen is given an initial small free end displacement, the relative dynamic slip at the interfaces of the layers due to small angle of slope of the beam as shown in Fig. 2, is given by;

$$u_r(l_i, t) = \alpha[\Delta u_1 + \Delta u_2] = 2\alpha h\tan\theta = 2\alpha h\, [\partial y(l_i, t)/\partial x] \quad (16)$$

where $2h$ is the thickness of each layer of the cantilever beam.

Modifying Eq. (14) and combining the same with Eq. (16), the maximum relative dynamic slip under a connecting bolt is found to be;

$$u_{rM} = [\alpha h][(\cos\lambda l + \cosh\lambda l)(\cosh\lambda l_i - \cos\lambda l_i) - (\sin\lambda l + \sinh\lambda l)(\sin\lambda l_i + \sinh\lambda l_i)]$$
$$[\lambda y(l,\ 0)] \times [\sin\lambda l\cosh\lambda l - \cos\lambda l\sinh\lambda l]^{-1} \tag{17}$$

In order to fabricate a layered and jointed cantilever beam, a number of connecting bolts with a definite spacing of 3.5 times their diameter are required. Influence of all these connecting bolts on overall dynamic slip ratio of the layered and jointed beam has also to be analysed for accuracy. Due to the variation of the center distance of each connecting bolt from the fixed end of the cantilever beam, the relative dynamic slip under one bolt will be different from another. Therefore, the actual overall maximum relative dynamic slip for a layered and jointed cantilever beam with "q" number of equispaced connecting bolts having a spacing of 3.5 times their diameter has been found out by Nanda (1992) from Eq. (17) and is given by;

$$u_{rM} = \alpha h X_{sum}\lambda y(l,\ 0) \tag{18}$$

where $$X_{sum} = [(\cos\lambda l + \cosh\lambda l)\sum_{i=1}^{q}(\cosh\lambda l_i - \cos\lambda l_i) - (\sin\lambda l + \sinh\lambda l)$$
$$\sum_{i=1}^{q}(\sin\lambda l_i + \sinh\lambda l_i)] \times [\sin\lambda l\cosh\lambda l - \cos\lambda l\sinh\lambda l]^{-1}$$

2.3 Determination of logarithmic damping decrement

It is assumed that the energy loss of the layered and jointed beam consists of the loss arising from interface friction under the joints (E_f) and the loss from material and support damping (E_0). Thus, the logarithmic damping decrement of a layered and jointed beam is expressed as;

$$\delta = [(E_f/E_n) + (E_0/E_n)]/2 = \delta_f + \delta_0 \tag{19}$$

where E_n is the energy stored per cycle of vibration due to the initial amplitude of excitation $[y(l,\ 0)]$ and is given by $E_n = [ky^2(l,\ 0)]/2$

The logarithmic damping decrement due to material and support damping (δ_0) being very small compared to the interface friction damping, is neglected and the equation for the logarithmic damping decrement is simplified as;

$$\delta \approx \delta_f = E_f/2E_n \tag{20}$$

The energy loss per cycle due to friction at the interfaces as given in Eq. (2), can be modified by combining Eqs. (3) and (18) and hence, the logarithmic damping decrement for such a beam is then found to be;

$$\delta = E_f/2E_n = 2\mu N\alpha h X_{sum}\lambda/ky(l,\ 0) \tag{21}$$

where k is the static bending stiffness of the layered and jointed cantilever beam.

As Eq. (21) for logarithmic damping decrement is valid for a two-layered and jointed cantilever beam, a generalized equation has been developed for a multi-layered and jointed cantilever beam and is given by;

$$\delta = 2(m - 1)\mu N\alpha h X_{sum}\lambda / ky(l, 0) \tag{22}$$

where m is the number of layers.

Since direct evaluation of the dynamic slip ratio, α, and kinematic coefficient of friction, μ, were not possible, the product of these two parameters, i.e., $\alpha \times \mu$ has been found out from the experimental results for logarithmic decrement for two-layered aluminium specimens with 10 mm diameter connecting bolts. For this purpose, Eq. (21) has been modified as;

$$\alpha \times \mu = [ky(l, 0)\ \delta] / [2NhX_{sum}\lambda] \tag{23}$$

2.4 Determination of logarithmic damping decrement under uniform intensity of pressure distribution at the interfaces

In order to obtain a uniform intensity of pressure distribution at the interfaces, the consecutive influencing zones are to be superimposed by decreasing the spacing of the consecutive bolts on the structure. This spacing between the consecutive bolts for uniform pressure distribution at the interfaces has been evaluated with the help of a suitable software package and is found to be 2.00211 times the diameter of the connecting bolts, as reported by Nanda and Behera (1999) which is independent of the tightening torque on the connecting bolts. The magnitude of the uniform intensity of pressure distribution with the above spacing has been determined as shown in Fig. 5 and found to be;

$$p = 0.671P/3\pi R_B^2 \tag{24}$$

For a layered and jointed beam, the damping ratio, Ψ, is expressed as the ratio between the loss energy dissipated due to the relative dynamic slip between the interfaces and the total energy introduced into the system and is expressed as;

$$\Psi = [E_{loss}/(E_{loss} + E_{net})] \tag{25}$$

where E_{loss} and E_{net} are the energy loss due to interface friction and energy introduced during unloading process in to the system.

In the present analysis for determining the logarithmic damping decrement under uniform intensity of pressure distribution at the interfaces, the frictional energy loss as well as energy introduced in to the layered and jointed specimen per half cycle during unloading process has been considered for mathematical simplicity. Accordingly frictional loss energy per half cycle during unloading process has been found to be;

$$E_{loss} = \int_0^{\pi/\omega_n} \int_0^{l} [\mu p b [\{\partial u_r(x, t)/\partial t\} dx dt] \tag{26}$$

However, the energy introduced in to the specimen during unloading process per half cycle during vibration soon after its initial displacement is given by;

$$E_{net} = (3EI/l^3)\, y^2(l, 0) \tag{27}$$

From the above Eqs. (26) and (27) we get;

$$E_{loss}/E_{net} = \int_0^{\pi/\omega_n} \int_0^{l} [[\mu p b\{\partial u_r(x, t)/\partial t\} dx dt]/[(3EI/l^3) y^2(l, 0)] \tag{28}$$

Considering uniform pressure distribution throughout the contact area of the interfaces and assuming dynamic slip ratio, α, to be independent of the distance from the fixed end of the cantilever beam and time, the above Eq. (28) can be modified as;

$$E_{loss}/E_{net} = [2\mu bhp\alpha/\{(3EI/l^3)y^2(l, 0)\}]\int_0^{\pi/\omega_n}\int_0^{l}[\partial\{\tan\partial y(x, t)/\partial x\}dxdt]/\partial t \tag{29}$$

Moreover, the slope of the cantilever beams $\partial y(x, t)/\partial x$ being quite small, $[\tan\partial y(x, t)/\partial x] \approx \partial y(x, t)/\partial x$.

Therefore, Eq. (29) is modified as;

$$E_{loss}/E_{net} = [2\mu bhp\alpha/\{(3EI/l^3)y^2(l, 0)\}]\int_0^{\pi/\omega_n}\int_0^{l}[\{\partial^2 y(x, t)/\partial x\partial t\}dxdt] \tag{30}$$

Considering the boundary and the initial conditions of the cantilever beam as $y(l, 0) = y_0$ (positive downward deflection) and $\partial y(l, 0)/\partial t = 0$ (no initial velocity) respectively, the bending deflection of the beam under vibration can be expressed as;

$$y(x, t) = Y(x)\{y_0/Y(l)\}\cos\omega_n t \tag{31}$$

where $Y(x)$ is the space function and the rest is the time function.

Using the above Eq. (31) in Eq. (30) and changing the limits of the time interval from 0 and π/ω_n to 0 and $\pi/2\omega_n$ and multiplying the expression by two for yielding definite solution we get;

$$E_{loss}/E_{net} = [4\mu bhp\alpha/\{(3EI/l^3)y^2(l, 0)\}]\int_0^{\pi/2\omega_n}\int_0^{l}\partial^2[Y(x)\{y_0/Y(l)\}\cos\omega_n t]dxdt/[\partial x\partial t] \tag{32}$$

Differentiating Eq. (31) with respect to "x" we get;

$$[\partial y(x, t)/\partial x] = [\{\partial Y(x)/\partial x\}\{y_0/Y(l)\}\cos\omega_n t] \tag{33}$$

Again differentiating the above Eq. (33) with respect to time we get;

$$[\partial^2 y(x, t)/\partial x\partial t] = [(-\omega_n)\{\partial Y(x)/\partial x\}\{y_0/Y(l)\}\sin\omega_n t] \tag{34}$$

Integrating for time and putting the limits 0 to $\pi/2\omega_n$, the above Eq. (34) is modified as;

$$\int_0^{\pi/2\omega_n}[\{\partial^2 y(x, t)/\partial x\partial t\}dt] = [\partial y(x, t)/\partial x]_0^{\pi/2\omega_n} = [\{\partial Y(x)/\partial x\}\{y_0/Y(l)\}] \tag{35}$$

Again integrating for space and putting the limits from 0 to l, the above Eq. (35) becomes;

$$\int_0^{\pi/\omega_n}\int_0^{l}[\{\partial^2 y(x, t)/\partial x\partial t\}dt] = \int_0^{\pi/2\omega_n}\int_0^{l}[\partial^2[Y(x)\{y_0/Y(l)\}\cos\omega_n t]dxdt/\partial x\partial t = y_0 \tag{36}$$

Using the Eq. (36) in Eq. (32), we get;

$$E_{loss}/E_{net} = [4\mu bhp\alpha y(l, 0)]/[(3EI/l^3)y^2(l, 0)] \tag{37}$$

Replacing $3EI/l^3 = k$, i.e., the equivalent spring constant (static bending stiffness) of the layered and jointed beam, the above Eq. (37) reduces to

$$E_{loss}/E_{net} = [4\mu bhp\alpha]/[ky(l, 0)] \tag{38}$$

Eq. (25) is modified as;

$$\Psi = [E_{loss}/(E_{loss} + E_{net})] = 1/[1 + E_{net}/E_{loss}] \tag{39}$$

Putting the values of E_{loss}/E_{net} from Eq. (38) in Eq. (39) we get;

$$\Psi = 1/[1 + \{ky(l, 0)\}/\{4\mu bhp\alpha\}] \tag{40}$$

The logarithmic damping decrement, δ, is usually expressed as, $\delta = \ln(a_n/a_{n+1})$. Assuming that the energy stored in the system is proportional to the square of the corresponding amplitude, the relationship between logarithmic damping decrement and damping ratio can be written as;

$$\delta = \ln(E_n/E_{n+1})^{1/2} = [\ln\{1/(1 - \Psi)\}]/2 \tag{41}$$

where E_n and E_{n+1} are the energy stored in the system with amplitudes of vibration a_1 and a_{n+1} respectively.

In case of $\Psi \leq 1$, the Maclaurin expansion of the Eq. (41) will yield;

$$\delta = [\Psi + (\Psi^2/2)]/2 \tag{42}$$

Similarly, in order to find out the logarithmic damping decrement for multi-layered cantilever beams, the numbers of interfacial layers are to be taken into consideration. If "m" number of layers are jointed together with connecting bolts to construct the multi-layered cantilever beams so as to have uniform interface pressure, the damping ratio for such beams is given by;

$$\Psi = 1/[1 + \{ky(l, 0)\}/\{4(m - 1)\mu bhp\alpha\}] \tag{43}$$

3. Experimental techniques and experiments

In order to find out the logarithmic damping decrement of layered and jointed beams and to compare it with the numerical results evaluated from analytical expressions, an experimental set-up with a number of specimens has been fabricated. The experimental set-up with detailed instrumentation is shown in Fig. 6. The specimens are prepared from commercial aluminium flats of the sizes as presented in Table 1 by joining two as well as more number in layers with the help of equispaced connecting bolts of same tightening torque on them. The distance between the consecutive connecting bolts have been kept as 3.5 and 2.00211 times their diameter depending on non-uniform and uniform intensity of pressure distribution at the interfaces respectively. The cantilever lengths of the specimens have been varied accordingly in order to accommodate the corresponding number of connecting bolts as presented in Table 1.

The specimens are rigidly fixed to the support to obtain perfect cantilever condition and experiments are conducted initially to determine the bending modulus of elasticity (E) of the specimen materials. Solid cantilever specimens out of the same stock of commercial flats are held rigidly at the fixed end and its free end deflection (Δ) is measured by applying static loads (W). From these static loads and corresponding deflections, average static bending stiffness (W/Δ) is determined. The bending modulus for the specimen material is then evaluated from the expression $E = [(W/\Delta)(l^3/3I)]$. The average value of "E" for the aluminium specimens used in the experiments is found to be 63.30 GN/m^2.

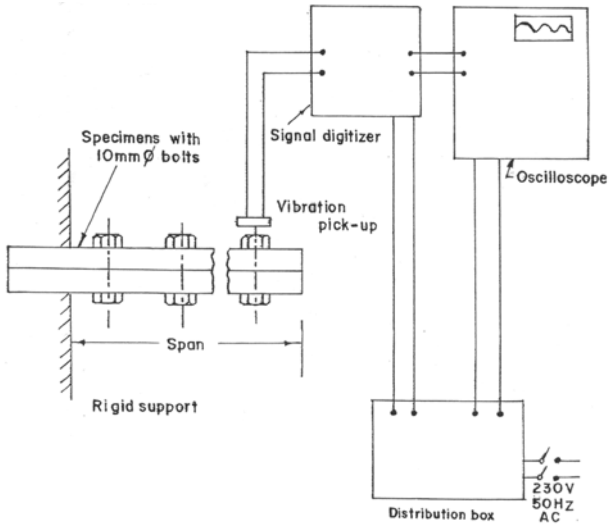

Fig 6 Schematic diagram of experimental set-up with detailed instrumentation

Table 1 Details of the specimens used in the experiment

Dimensions of the specimen (thickness × width), (mm × mm)	Diameter of the connecting bolt, (mm)	Number of layers used	Condition of interface pressure	Number of bolts used	Cantilever length, (mm)
3.20 × 35.00				11	385.00
5.60 × 35.00	10	2	non-uniform	10	350.00
12.00 × 37.20				9	315.00
4.80 × 35.00				11	385.00
	10	3	non-uniform	10	350.00
8.40 × 35.00				9	315.00
6.40 × 35.00				11	385.00
	10	4	non-uniform	10	350.00
11.20 × 35.00				9	315.00
3.20 × 40.04				18	360.38
5.60 × 40.04	10	2	uniform	17	340.36
12.00 × 37.20				16	320.34
4.80 × 40.04				18	360.38
	10	3	uniform	17	340.36
8.40 × 40.04				16	320.34
6.40 × 40.04				18	360.38
	10	4	uniform	17	340.36
11.20 × 40.04				16	320.34
6.40 × 40.04	Solid beam				360.38

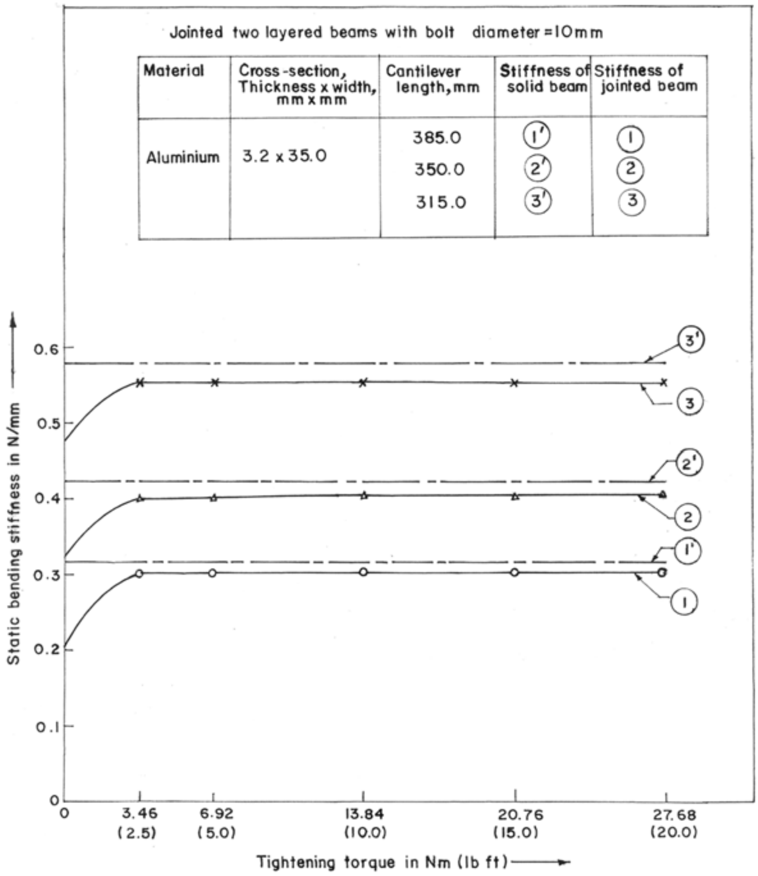

Fig. 7 Variation of static bending stiffness with applied tightening torque on the connecting bolts

The static bending stiffness (k) of the specimens are determined and is found that the same for layered and jointed beam is always less than that of an equivalent solid one (k') and increases with increase in tightening torque on the connecting bolts and remains almost constant after a limiting value, i.e., 10.370 N m (7.5 lb ft) as shown in Fig. 7 for a particular case. The ratio of this bending stiffness at the limiting tightening torque condition with the equivalent bending stiffness of a solid one (α') is found out for all specimens. The average value of α' for each group of specimens has been utilized in the numerical analysis.

The logarithmic damping decrement and natural frequency of vibration of all the specimens at their first mode of free vibration are found out experimentally. The tightening torques on all the connecting bolts of the specimens are maintained equal for each set of observations and varied in steps as 3.46, 6.92, 13.84, 20.76, and 27.68 N m (i.e., 2.50, 5.00, 10.00, 15.00 and 20.00 lb ft respectively). The lengths of these specimens during experimentation are also varied. In order to excite the specimens at their free ends, a spring loaded exciter was used. The amplitude of excitation was varied in steps and maintained as 0.1, 0.2, 0.3, 0.4, and 0.5 mm for all the specimens tested under the different conditions of the tightening torque on the connecting bolts. The free vibration was sensed with a non-contacting type of vibration pick-up and the corresponding signal was fed to a cathode ray oscilloscope through a digitizer to obtain a steady signal. The logarithmic damping decrement was then evaluated from the measured values of the amplitudes of the first cycle (a_1), last cycle (a_{n+1}) and the number of cycles(n) of the steady signal by using the equation $\delta = \ln(a_1/a_{n+1})/n$. The corresponding natural frequency was also determined from the time period (T_1) of the signal by using the relationship $f = 1/T_1$. It is found that the natural frequency of vibration of specimens is always less than that of their equivalent solid ones. It increases with increase in tightening torque on the connecting bolts and remains constant after a limiting value of the torque, i.e., 10.370 N m (7.5 lb ft). This increase is due to higher static bending stiffness of the layered and jointed specimens.

4. Determination of the product of dynamic slip ratio and kinematic coefficient of friction ($\alpha \cdot \mu$)

The experimental logarithmic damping decrement values for two layered and jointed beams with 10 mm diameter connecting bolts under different conditions of excitation have been used to

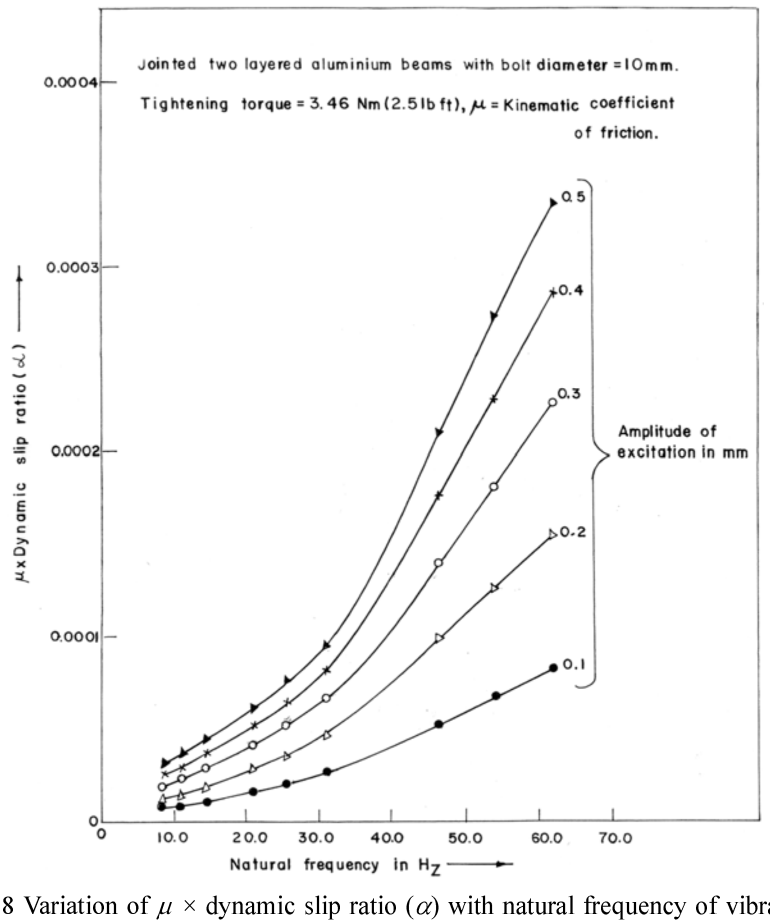

Fig. 8 Variation of μ × dynamic slip ratio (α) with natural frequency of vibration

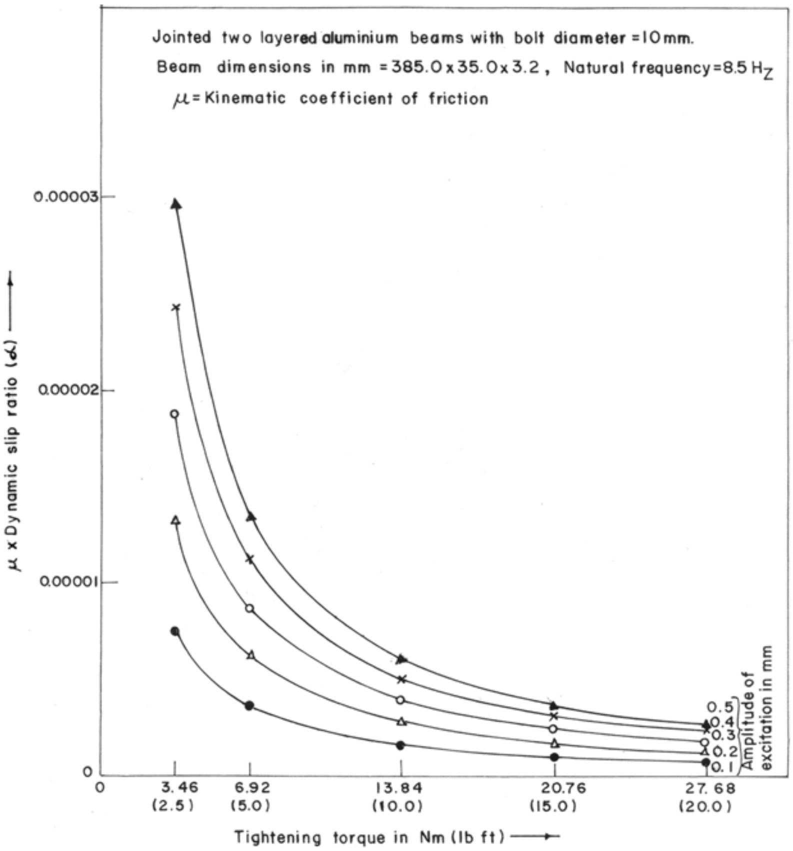

Fig. 9 Variation of $\mu \times$ dynamic slip ratio (α) with applied tightening torque on the connecting bolts

evaluate the corresponding values of the product of dynamic slip ratio and kinematic coefficient of friction using Eq. (23). The variation in the dynamic slip ratios and natural frequency of the first mode transverse vibration for a particular tightening torque on connecting bolts was determined under different initial amplitudes of excitation. The results have been plotted and one such sample is presented in Fig. 8. Moreover, the variation of these dynamic slip ratios with applied tightening torque on the connecting bolts for different specimens have also been plotted and one such sample has been shown in Fig. 9. All of these plots have been further used in the evaluation of the numerical results for the logarithmic damping decrement of multi-layered jointed beams using the Eqs. (22) and (42).

5. Comparison of experimental and numerical results

The logarithmic damping decrements of three and four layered cantilever specimens with 10 mm diameter connecting bolts have been found out from Eq. (22) using the values of the product of dynamic slip ratios and kinematic coefficient of friction from the respective plots as already discussed. These numerical results have been determined along with the corresponding experimental ones for comparison and one such result from each has been shown in Figs. 10 and 11. It is observed that both the curves are very close to each other with a maximum variation of 1.18% which authenticates the accuracy of the values of the product of dynamic slip ratios and kinematic coefficient of friction determined numerically from the experimental results for the logarithmic damping decrement.

Further, numerical results for two, three and four layered and jointed cantilever beams with uniform intensity of interface pressure distribution at the interfaces and 10 mm diameter connecting bolts have been found out using Eq. (42) in order to verify the accuracy of the numerical analysis. These numerical results for logarithmic damping decrement have also been plotted along with the corresponding experimental ones and one such plot for each case has been shown in Figs. 12, 13 and 14, showing that both the plots are very close to each other with a maximum variation of 1.17%.

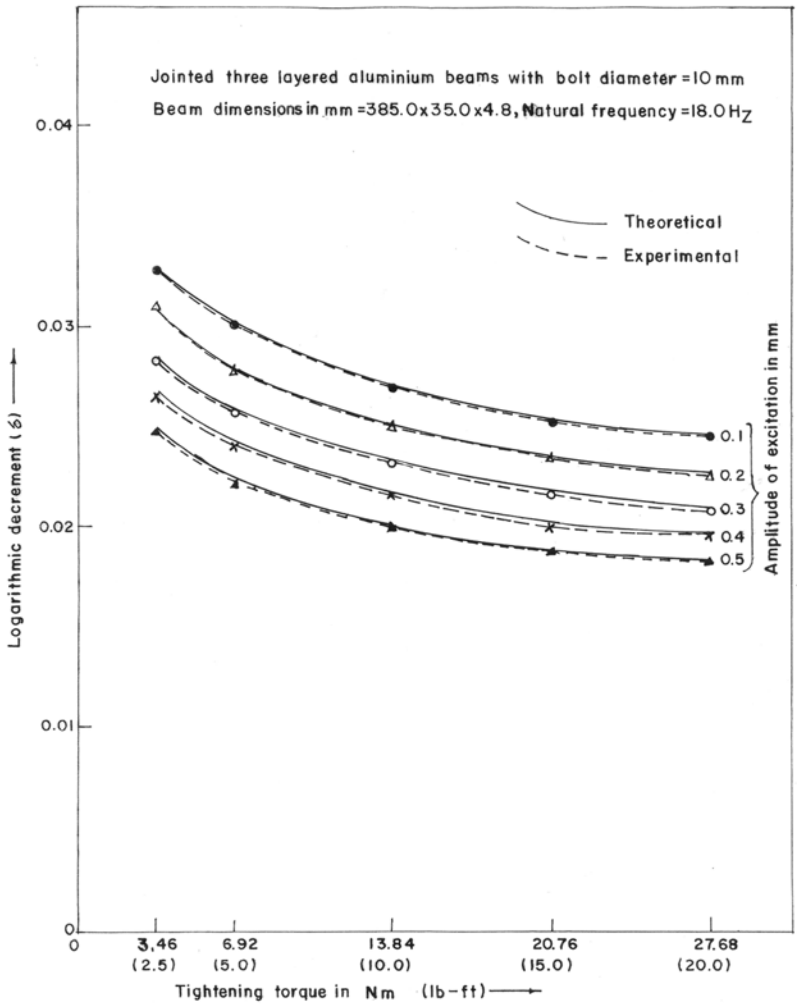

Fig. 10 Variation of logarithmic decrement (δ) with applied tightening torque on the connecting bolts

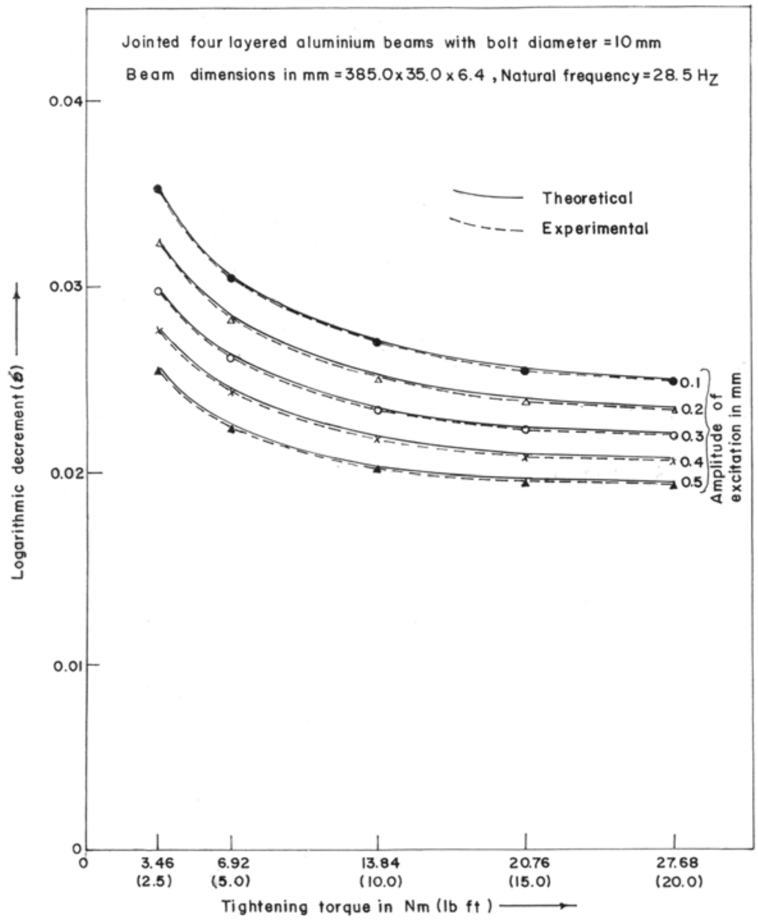

Fig. 11 Variation of logarithmic decrement (δ) with applied tightening torque on the connecting bolts

6. Conclusions

From the theoretical analysis as well as numerical and experimental results, the following salient points have been observed. They are discussed below and the conclusions have been drawn accordingly.

(1) The static bending stiffness of the layered and jointed structure is smaller than that of an equivalent solid one and increases with increase in the tightening torque on the connecting bolts and remains constant beyond a limiting value of the tightening torque, i.e., 10.370 N m (7.50 lb ft). Moreover, with the increase in the tightening torque, the interface pressure increases and the cantilever beam tends to behave like a solid one thereby increasing the static bending stiffness. On comparison, it is found that the static bending stiffness for layered and jointed specimens with uniform intensity of pressure is less than that of the jointed specimens with same diameter on the connecting bolts arranged otherwise. This is because of the presence of more number of holes for the connecting bolts on the specimens with uniform intensity of interface pressure.

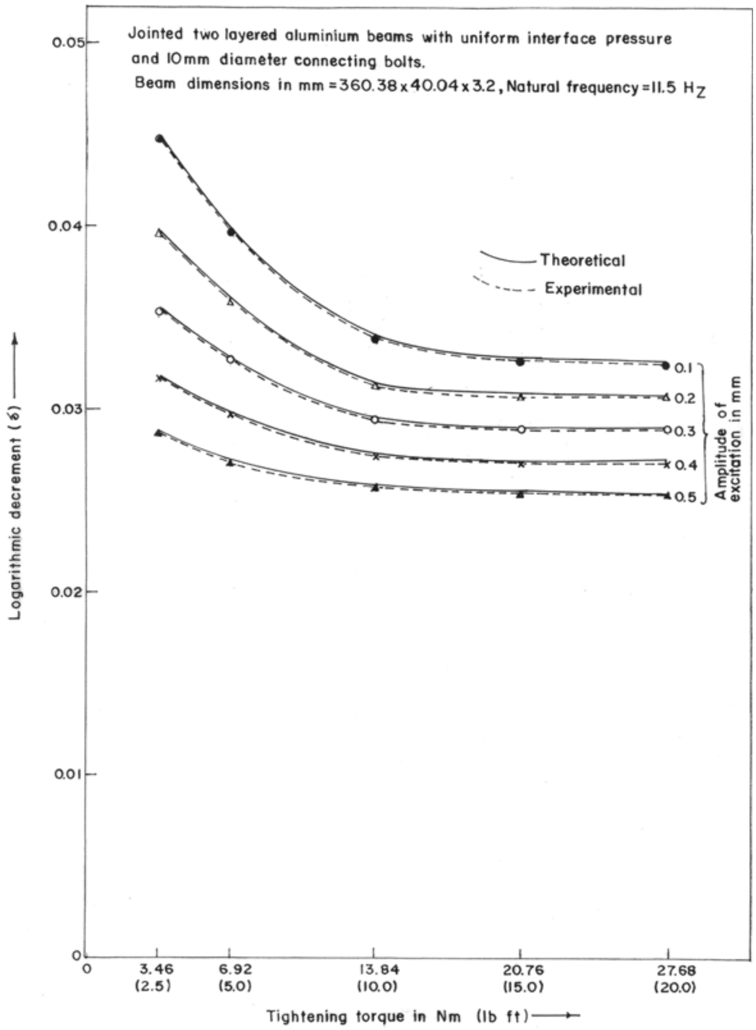

Fig. 12 Variation of logarithmic decrement (δ) with applied tightening torque on the connecting bolts

(2) The natural frequency of first mode vibration of the layered and jointed structure is found to be smaller than that of its equivalent solid one and increases with an increase in the tightening torque on the connecting bolts due to higher static bending stiffness. The static bending stiffness increases because of higher interface pressure due to tightening torque. However, the frequency remains constant beyond a limiting value of the tightening torque, i.e., 10.370 N m (7.5 lb ft).

(3) It has been found that the interface pressure distribution between the contacting layers jointed by connecting bolts having spacing of 3.5 times its diameter increases with decrease in the distance between the consecutive connecting bolts and attains uniformity throughout the contacting surfaces for a particular spacing of the connecting bolts which has been found to be 2.00211 times the diameter of the bolt. Any further decrease in the distance between consecutive connecting bolts will result in decrease of logarithmic damping decrement as the

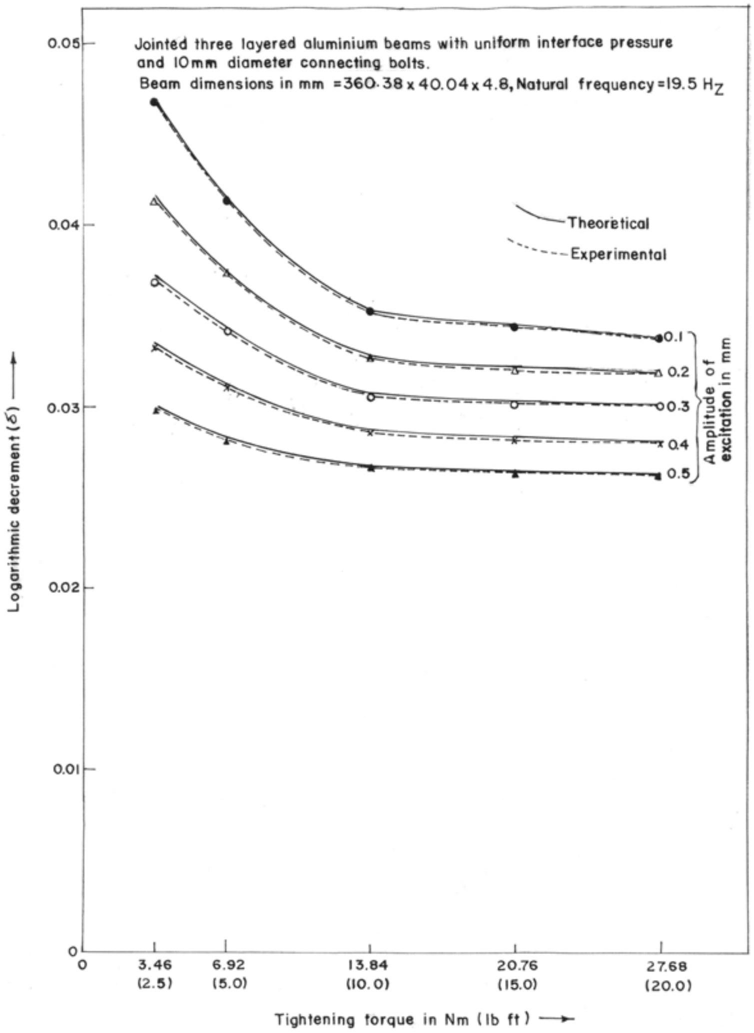

Fig. 13 Variation of logarithmic decrement (δ) with applied tightening torque

layered and jointed cantilever beam behaves like a solid one.

(4) The following influencing parameters play a vital role on the damping capacity of layered structures jointed with connecting bolts. They are: (a) tightening torque on the connecting bolts, (b) number of layers, (c) amplitude of excitation, (d) frequency of excitation, and (e) arrangement of connecting bolts.

(a) The logarithmic damping decrement increases with increase in tightening torque and reaches a peak value at a particular torque as established by Masuko *et al.* (1973) and this limiting torque at which the logarithmic decrement reaches its peak value is so small that it is not possible to examine practically in actual applications and is always less than 3.46 N m (2.5 lb ft). This increase of the logarithmic decrement in the lower range of tightening torque is due to combined effect of low interface pressure with high dynamic slip ratio. However, the same logarithmic damping decrement decreases with further increase in tightening torque on the connecting bolts due to higher interface

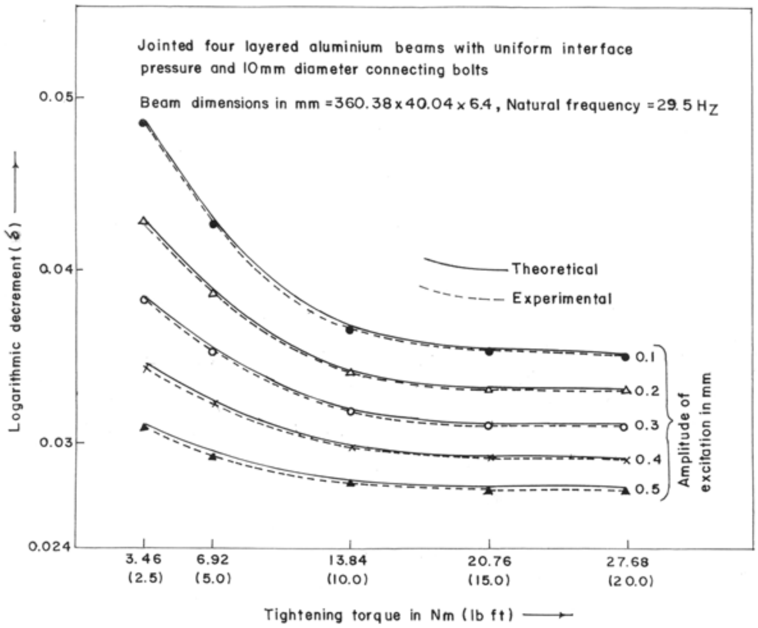

Fig. 14 Variation of logarithmic decrement (δ) with applied tightening torque on the connecting bolts

pressure associated with lower dynamic slip ratio at the interfaces.

(b) Logarithmic damping decrement increases with increase in number of layers in a layered and jointed structure due to increase in interface friction layers which causes increase in energy loss due to interface friction. Although the static bending stiffness increases due to increase in thickness of the multi-layered cantilever specimens but the total energy loss due to interface friction for such beams is more compared to the gain in strain energy. The strain energy increases due to higher static bending stiffness. Moreover, keeping overall thickness of the layered and jointed structure constant, if the number of layers to fabricate the same are increased, the logarithmic decrement for such structures will increase further because of higher dissipated energy due to multi-layered interface friction along with lower static bending stiffness. Thus damping capacity of such structures can be improved considerably.

(c) The logarithmic damping decrement of a layered and jointed structure decreases with an increase in amplitude of excitation due to introduction of higher strain energy into the system compared to that of the dissipated energy due to interface friction. Although the dynamic slip ratio increases with an increase in amplitude of excitation, but the strain energy introduced into the system is more compared to the increase in dissipated energy due to interface friction and the net effect is a decrease in the logarithmic damping decrement.

(d) The logarithmic damping decrement of a layered and jointed structure decreases with increase in natural frequency of vibration of the specimen. This is because of increase in dynamic slip ratio as well as the static bending stiffness with increase in natural frequency of vibration. Although the increase in dynamic slip ratio increases the loss energy due to friction at the interfaces but the increase in strain energy due to increase in static bending stiffness is more, resulting in decrease of logarithmic damping decrement.

(e) The arrangement of the connecting bolts has an influence on the logarithmic damping decrement of layered and jointed structure. The logarithmic damping decrement decreases when the distance between the consecutive bolts increases because of non-contact zone and the same increases with the decrease in the distance between consecutive bolts due to overlapping of influencing zones which increases the interface pressure. Logarithmic decrement attains maximum under the condition of uniform intensity of pressure distribution at the interfaces. Moreover, it has been established that the distance between the consecutive connecting bolts is 2.00211 times the diameter of the connecting bolt under uniform intensity of interface pressure distribution. However, the distance between the consecutive connecting bolts could not be reduced further due to interference of nearby bolt heads for applying tightening torque with the help of the torque wrench. Logarithmic damping decrement will reduce drastically because of very high interface pressure with negligibly small dynamic slip at the interfaces. This type of structure behaves like a solid one although they are layered and jointed.

Finally, it is established that the damping capacity of the layered and jointed structures can be improved considerably under uniform intensity of pressure distribution at the interfaces by using connecting bolts with minimum possible tightening torque on them as well as with a large number of layers. This increase in logarithmic damping decrement may go even up to more than 6 times as compared to that of an equivalent solid beam. Layered and jointed aluminium structures being lighter in weight and having higher damping capacity compared to other metallic structural materials can be effectively used in aerospace structures, aircrafts and space landing vehicles.

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