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Tuned liquid column dampers with adaptive tuning capacity for structural vibration control

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Abstract. The natural frequencies of a long span bridge vary during its construction and it is thus difficult to apply traditional tuned liquid column dampers (TLCD) with a fixed configuration to reduce bridge vibration. The restriction of TLCD imposed by frequency tuning requirement also make it difficult to be applied to structure with either very low or high natural frequency. A semi-active tuned liquid column damper (SATLCD), whose natural frequency can be altered by active control of liquid column pressure, is studied in this paper. The principle of SATLCD with adaptive tuning capacity is first introduced. The analytical models are then developed for lateral vibration of a structure with SATLCD and torsional vibration of a structure with SATLCD, respectively, under either harmonic or white noise excitation. The non-linear damping property of SATLCD is linearized by an equivalent linearization technique. Extensive parametric studies are finally carried out in the frequency domain to find the beneficial parameters by which the maximum vibration reduction can be achieved. The key parameters investigated include the distance from the centre line of SATLCD to the rotational axis of a structure, the ratio of horizontal length to the total length of liquid column, head loss coefficient, and frequency offset ratio. The investigations demonstrate that SATLCD can provide a greater flexibility for its application in practice and achieve a high degree of vibration reduction. The sensitivity of SATLCD to the frequency offset between the damper and structure can be improved by adapting its frequency precisely to the measured structural frequency.

Key words: semi-active tuned liquid column damper; adaptive tuning; lateral vibration; torsional vibration; harmonic excitation; white noise excitation; liquid column pressure; parametric study.

1. Introduction

The rapid development of urban area has resulted in a large number of tall buildings and long span cable-supported bridges over the world, but at the same time wind-induced vibration problems of these slender structures become serious. This is particularly true for a long span bridge under construction: excessive vibration in the lateral or torsional direction could occur under gust winds due to the lack of continuous rigidity. Tuned liquid column damper (TLCD) has been proved to be an effective device in suppressing the lateral vibration and torsional vibration of structures (Sakai *et al.* 1989, Xu *et al.* 1992, Gao *et al.* 1997, Xue *et al.* 2000, Xu and Shum 2003). However, the

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natural frequency of a long span bridge varies during its construction, and it is thus difficult to apply TLCD with a fixed configuration to the bridge during its construction or it is not economical to design a series of TLCDs with different configurations. Furthermore, the lateral frequency of a slender structure could be very low, and using tuned liquid column dampers to suppress the lateral vibration of the structure would require the TLCD with an impossible long liquid length. On the other hand, the first torsional frequency of a long span bridge is often higher than that in either the vertical or lateral direction. The application of TLCD on the bridge for reducing torsional vibration may require very short liquid length in order to have a proper frequency tuning. Consequently, a larger number of liquid containers are required, leading to a higher cost of installation and maintenance. It is clear that the restriction of TLCD imposed by frequency tuning requirement make it difficult to be applied to the structure with either very low or high natural frequency. As such, a new semi-active tuned liquid column damper (SATLCD) with adaptive tuning capacity is studied in this paper as an alternative solution for suppressing the lateral or torsional vibration of structure in these circumstances.

Aiming at facilitating frequency tuning, Hitchcock *et al.* (1997a, 1997b) proposed a variation of TLCD, termed the liquid column vibration absorber (LCVA) whose natural frequency can be adjusted by changing its length of liquid column together with its geometric configuration. Nevertheless, its adaptability is still limited due to the nature of passive control. Semi-active tuned liquid column dampers have been investigated by Kareem (1994), Haroun *et al.* (1996), Yalla *et al.* (2001) and others, but most of these studies aim to maintain the optimal damping of TLCD for suppressing lateral vibration of structures. There is little work relevant to the improvement of TLCD frequency-tuning adaptability by the use of semi-active control technology.

The objective of this paper is thus to develop a new semi-active tuned liquid column damper (SATLCD) with adaptive tuning capacity for reducing either lateral or torsional vibration of a structure with very low or high frequency. The principle of SATLCD with adaptive tuning capacity is first introduced. The analytical models are then developed for lateral vibration of a structure with SATLCD and torsional vibration of a structure with SATLCD, respectively, under either harmonic or white noise excitation. The non-linear damping property of SATLCD is linearized by an equivalent linearization technique. Extensive parametric studies are finally carried out in the frequency domain to find the beneficial parameters by which the maximum vibration reduction can be achieved.

2. Mathematical models

To facilitate the frequency tuning of liquid column inside a TLCD to the natural frequency of a structure, two air chambers are formed at the two ends of the TLCD (see Fig. 1 and Fig. 2). Net external pressure in the two air chamber acts on the liquid column to change the restoring force of liquid column. The pressure is regulated according to the displacement of the liquid column in a prescribed way. As a result, the natural frequency of liquid motion inside the TLCD can be increased or decreased accordingly so as to tune it to the natural frequency of the structure. The proposed pressure system with a closed loop control installed on a TLCD is shown in Fig. 1 for lateral vibration control and Fig. 2 for torsional vibration control. The net pressure inside the two air chambers, sensed by the pressure transducer, is to be forced to follow or track the desired pressure determined by a computer. Any deviation from the desired pressure is fed back to the computer to

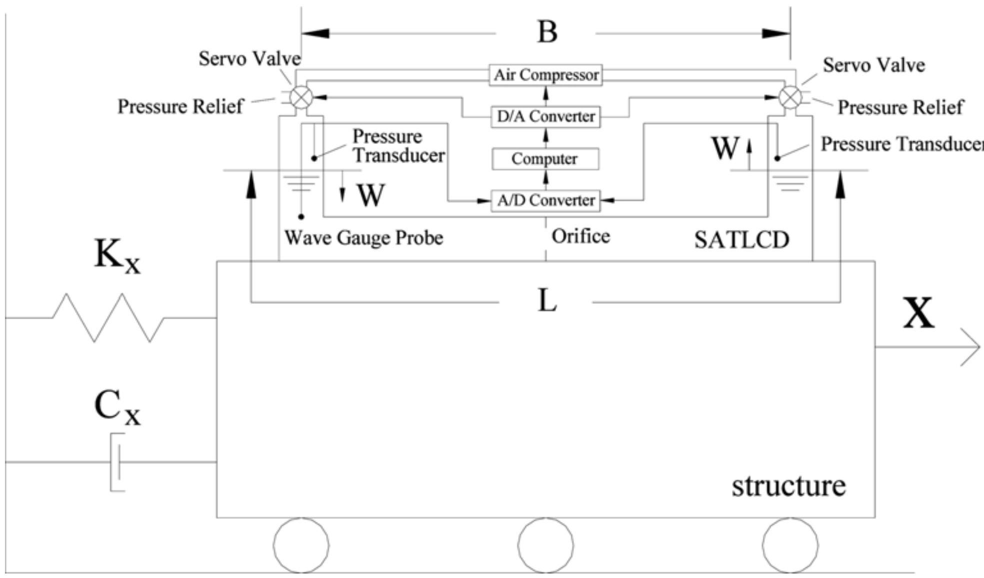


Fig. 1 Lateral vibration of SATLCD-structure system

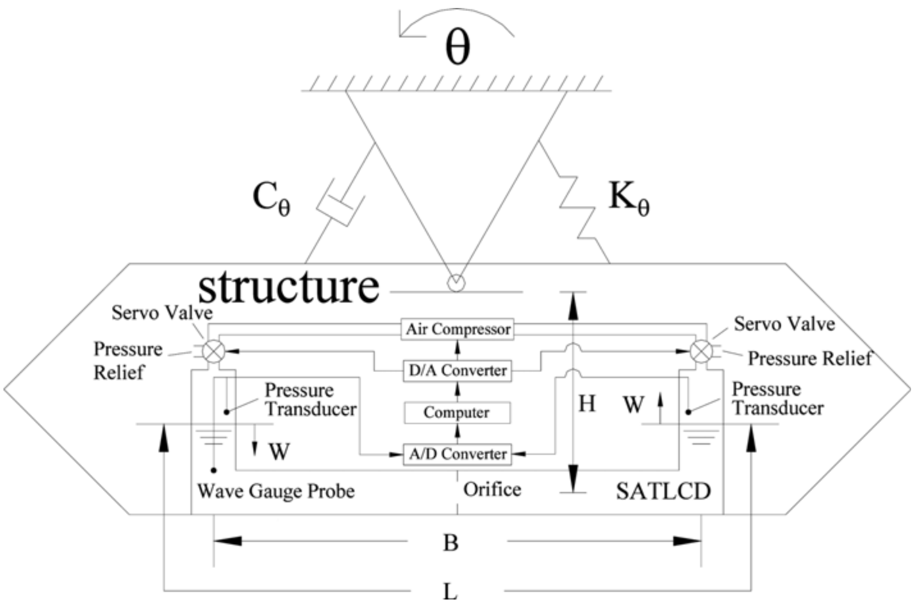


Fig. 2 Torsional vibration of SATLCD-structure system

take corrective action to adjust the control valve. Thus, the control system is continually monitoring and correcting pressure deviation to maintain the desired pressure acting on the liquid column.

2.1 Lateral vibration of SATLCD-structure system

Let us consider a SATLCD installed in a structure subjected to lateral vibration (Fig. 1). The

SATLCD is a U-shaped container of uniform rectangular cross-section filled with liquid and two air chambers filled with compressed air of static pressure P_o. In consideration of dynamic equilibrium condition and the interaction between the structure and the liquid column in the SATLCD, the equation of motion of a structure equipped with the SATLCD for lateral vibration control is

$$\rho AL\ddot{W} + \frac{1}{2}\rho A\delta|\dot{W}|\dot{W} + 2\rho AgW = -\rho AB\ddot{x} - u \tag{1}$$

$$m_s(1+\mu)\ddot{x} + C_x\dot{x} + K_x x = -\rho AB\ddot{W} + F_s \tag{2}$$

under the condition

$$W \le \frac{L-B}{2} - \frac{d}{2} \tag{3}$$

where m_s is the mass of structure; C_x and K_x are the lateral damping coefficient and the lateral stiffness of structure, respectively; x is the lateral displacement of structure; W is the relative motion of liquid column inside the SATLCD to the container; g is the acceleration due to gravity; d is the thickness of liquid column; δ is the head loss coefficient of the SATLCD governed by the opening ratio of orifice; ρ is the liquid density in the SATLCD; A is the cross-sectional area of liquid column in the SATLCD; B is the horizontal length of liquid column; L is the total length of liquid column; μ is the mass ratio of the liquid column to the structure; F_s is the external force acting on the structure. The control force $u(t)$ in the SATLCD is given by

$$u(t) = S \cdot W(t) \tag{4}$$

S is the constant displacement feedback gain of the SATLCD. The direction of the control force $u(t)$ is in the same (opposite) direction as the liquid motion W when the constant displacement feedback gain is positive (negative). The circular natural frequency of liquid motion in the SATLCD, ω_d, is then determined by

$$\omega_d^2 = \frac{2\rho Ag + S}{\rho AL} \tag{5}$$

For a targeted frequency of liquid damper, it is easily seen from Eq. (5) that the liquid column length of the SATLCD is given by

$$L = \frac{2g}{\omega_d^2} + \frac{S}{\rho A\omega_d^2} \tag{6}$$

Clearly, the liquid column length can be increased or decreased by adjusting the constant displacement feedback gain while keeping its frequency unchanged. The SATLCD is therefore more flexible than the traditional TLCD in which S is equal to zero and there is no way for changing the liquid column length L. The frequency-tuning ratio, which could affect the performance of the SATLCD, is defined as

$$\Delta\gamma = \frac{\omega_d}{\omega_x} \tag{7}$$

where ω_x is the circular lateral frequency of the structure without control. Once the frequency and length of the liquid column are decided, the required constant displacement feedback gain of the SATLCD can be determined by

$$S = m_d\left[(\Delta\gamma\omega_x)^2 - \frac{2g}{L}\right] \tag{8}$$

where m_d is the mass of the liquid column. The desired control force acting on the liquid column can be provided by regulating the air pressure in the right chamber with respect to the air pressure in the left chamber to obtain a net pressure $P(t)$ as

$$P(t) \cdot A = S \cdot W(t) \tag{9}$$

The air pressure in the left chamber P_L and in the right chamber P_R is determined, respectively, by

$$P_L = P_o - \frac{P(t)}{2} \qquad P_R = P_o + \frac{P(t)}{2} \tag{10}$$

Eq. (9) provides a way of determining the required net pressure P at time t if the displacement feedback gain S and the liquid displacement inside the SATLCD at time t are known. Eq. (5) shows that the natural frequency of liquid motion inside the SATLCD is determined not only by the length of liquid column but also the feedback gain S. It can also be seen from Eq. (8) that the constant displacement feedback gain can be obtained if the structural natural frequency, the length of liquid column, the frequency-tuning ratio and the mass of liquid column are known. Eq. (9) also shows that the net pressure P required for the feedback force is linearly proportional to the liquid displacement inside the SATLCD.

It may be worthwhile to mention that passively pressurized TLCD has been successfully applied to suppress ship oscillation in lateral direction (Kagawa *et al.* 1989). A passively pressurized TLCD uses the differential pressure between two closed air chambers to control the oscillation of liquid motion but it is of the passive nature rather than the semi-active control presented in this paper. The experimental results of using differential pressure between two chambers to actively control a hydraulic damper can be found in Jelali and Kroll (2003). These results make the proposed SATLCD prospective to be implemented in real civil engineering structures.

2.2 Torsional vibration of SATLCD-structure system

In consideration of the dynamic interaction between the structure and the liquid column of the SATLCD under torsional vibration (Fig. 2), the equation of motion of a structure equipped with the SATLCD can be derived as

$$\rho AL\ddot{W} + \frac{1}{2}\rho A\delta|\dot{W}|\dot{W} + 2\rho AgW = -\rho AB\left(H + \frac{L-B}{2}\right)\ddot{\theta} - \rho ABg\theta - u \tag{11}$$

$$(I_s + I_d)\ddot{\theta} + C_\theta\dot{\theta} + (K_\theta + \rho ALg\overline{H})\theta = -\rho AB\left(H + \frac{L-B}{2}\right)\ddot{W} - \rho ABgW + M_s \tag{12}$$

with the condition

$$W \le \frac{L-B}{2} - \frac{d}{2} \tag{13}$$

where I_s is the mass moment of inertia of the structure; C_θ and K_θ are the torsional damping coefficient and the torsional stiffness of the structure, respectively; I_d denotes the total mass moment of inertia of liquid column in the SATLCD about the elastic center of the structure; θ is the torsional displacement of the structure; H is the vertical distance between the center line of the horizontal part of SATLCD and the elastic center of the structure; $\bar{H}$ is the distance between the mass center of liquid inside the SATLCD and the elastic center of the structure; M_s is the external moment acting on the elastic center of the structure; the control force u is given by Eq. (4); and the circular natural frequency of liquid motion in the SATLCD is determined by Eq. (5).

The frequency-tuning ratio, which could affect the performance of the SATLCD for torsional vibration reduction, is defined as

$$\Delta\gamma = \frac{\omega_d}{\omega_\theta} \tag{14}$$

where ω_θ is the circular torsional frequency of the structure without control. By substituting Eq. (14) into Eq. (5) and after some manipulations, the constant displacement feedback gain of the SATLCD is given by

$$S = m_d\left[(\Delta\gamma\omega_\theta)^2 - \frac{2g}{L}\right] \tag{15}$$

The required external pressure acting on the liquid inside the SATLCD is determined by Eq. (9) and Eq. (10).

2.3 Equation of motion of SATLCD-structure system in matrix form

The equation of motion of the coupled SATLCD-structure system subjected to either lateral or torsional excitation can be written in a matrix form as

$$\mathbf{M}\ddot{\mathbf{y}} + \mathbf{C}\dot{\mathbf{y}} + \mathbf{K}\mathbf{y} = \mathbf{P} \tag{16}$$

where the vector $\mathbf{y}$ is the displacement vector; $\mathbf{P}$ is the external excitation vector; $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are the mass, damping, and stiffness matrices of the coupled SATLCD-structure system, respectively. For the case of lateral vibration of the system,

$$\mathbf{y} = [x \;\; W]^T \qquad \mathbf{P} = [F_s \;\; 0]^T \tag{17}$$

$$\mathbf{M} = \begin{bmatrix} m_s + m_d & \alpha m_d \\ \alpha m_d & m_d \end{bmatrix}; \quad \mathrm{C} = \begin{bmatrix} 2m_s\omega_x\xi_x & 0 \\ 0 & \frac{1}{2}\rho A\delta|\dot{W}| \end{bmatrix}; \quad \mathbf{K} = \begin{bmatrix} K_x & 0 \\ 0 & 2\rho A g + S \end{bmatrix} \tag{18}$$

For the case of torsional vibration of the system,

$$\mathbf{y} = [\theta \;\; W]^T \qquad \mathbf{P} = [M_s \;\; 0]^T \tag{19}$$

$$\mathbf{M} = \begin{bmatrix} I_s + I_d & G \\ G & m_d \end{bmatrix}; \quad \mathrm{C} = \begin{bmatrix} 2I_s\omega_\theta\xi_\theta & 0 \\ 0 & \frac{1}{2}\rho A\delta|\dot{W}| \end{bmatrix}; \quad \mathbf{K} = \begin{bmatrix} K_\theta + mg & m_d g\alpha \\ m_d g\alpha & 2\rho Ag + S \end{bmatrix} \tag{20}$$

where

$$m_d = \rho AL; \quad \alpha = \frac{B}{L}; \quad G = m_d\alpha\left(H + \frac{L-B}{2}\right); \quad m = m_d\overline{H}; \quad \overline{H} = H - \frac{(L-B)^2}{4L} \tag{21}$$

2.4 Equivalent linearization technique

It is noted that damping terms in Eq. (1) and Eq. (11) for liquid motion are non-linear and therefore the equation of the SATLCD-structure system is also nonlinear. To carry out an extensive parametric study in the frequency domain, the equivalent linearization technique is applied to nonlinear damping force of liquid motion in Eq. (1) and Eq. (11) in terms of the principle of equivalent energy dissipation. As a result, the nonlinear damping terms in Eq. (1) and Eq. (11) can be replaced by an equivalent damping coefficient. If external excitation is a harmonic excitation, the equivalent damping ratio can be calculated (Gao *et al.* 1997) by

$$\xi = \frac{2\delta}{\omega_d L}\overline{W}\tilde{\omega} \tag{22}$$

where $\overline{W}$ is the amplitude of liquid motion in the SATLCD; and $\tilde{\omega}$ is the circular frequency of the applied harmonic excitation. If external excitation is a stationary random excitation, the equivalent damping coefficient can be calculated (Xu *et al.* 1992) by

$$\xi = \frac{1}{\sqrt{2\pi}}\frac{\delta}{\omega_d L}\sigma_{\dot{w}} \tag{23}$$

where $\sigma_{\dot{w}}$ is the standard deviation of the liquid velocity in the SATLCD. Since the equivalent damping ratio described by either Eq. (22) or Eq. (23) depends on the liquid motion, iterations are generally required.

3. Parametric studies on lateral vibration reduction

Because of lacking structural continuity from pylon to pylon, the lateral stiffness of a long span cable-stayed bridge during construction when using a double cantilever technique is much lower than that after the bridge is completed. If the traditional TLCD is used in such a case, it will require a very long liquid column length that may not be practical. As such, the semi-active tuned liquid column dampers with adaptive tuning capacity studied in this paper can be used. The performance of the SATLCD is therefore investigated for the lateral vibration reduction of a structure under two different kinds of loading: harmonic and white noise excitations. The dynamic parameters of a structure studied herein are $m_s = 4.6905 \times 10^6$ kg, $K_x = 3.6504 \times 10^5$ N/m, $\xi_x = 0.01$, and these values are selected from the first lateral vibration mode of a cable-stayed bridge under construction. The lateral circular natural frequency of the structure (ω_x) is 0.2790 rad/s. It should be noted that the use of traditional TLCD for the lateral vibration suppression of the concerned structure would require a liquid column length of 252 m, which is impossible in practice. To evaluate the SATLCD performance, the mass ratio μ is defined as the ratio of the total mass of liquid inside the SATLCD to the mass of the structure. For the sake of comparison, the structural response ratio R is introduced as:

$$R = \frac{\text{Maximum structural response with control}}{\text{Maximum structural response without control}} \quad \text{for harmonic excitation} \tag{24}$$

$$R = \frac{\text{Standard deviation of structural response with control}}{\text{Standard deviation of structural response without control}} \quad \text{for white noise excitation} \tag{25}$$

The smallest value of structural response ratio is often used as the indication of the effectiveness of the SATLCD. The harmonic force amplitude is 6000 N and the power spectral density function of white noise force excitation is 2.5×10^8 N^2s/rad.

3.1 Time domain analysis

The linear equation of motion of liquid inside a SATLCD is obtained by the linearization of its nonlinear damping. To make sure the validity of the linear equation of motion of a coupled SATLCD-structure system, the Wilson-θ method is used to find the numerical solution of the nonlinear equations of the SATLCD-structure system described by Eqs. (1) and (2) for lateral vibration in the time domain. The obtained steady state responses of both structure and water under the harmonic excitation and the standard deviation responses of both structure and water under white noise excitation in the time domain are compared with those obtained by the corresponding linear equations in the frequency domain, and the comparative results are summarized in Tables 1 and 2. In the computation, the time history of white noise force excitation is generated using an algorithm proposed by Shinozuka (1972). It is modelled as a band-limited Gaussian white noise with a bandwidth of 25 Hz. The duration of the simulated time history of force excitation is 1310.7s with an interval of 0.02s. The parameters of the SATLCD are $\Delta\gamma = 1$, $\delta = 15$, $\alpha = 0.7$, $L = 22$ m, and $\mu = 0.03$. It can be seen from the tables that the results from the linear equation in the frequency domain are almost the same as those from the nonlinear equation in the time domain. This implies

Table 1 Comparisons of linear and nonlinear responses of SATLCD-structure system under harmonic excitation

	Displacement response ratio	Maximum water displacement (m)	Maximum pressure (Pa)
Nonlinear	0.2435	0.6694	11987.64
Linear	0.2447	0.6698	11995.53

Table 2 Comparisons of linear and nonlinear responses of SATLCD-structure system under white noise excitation

	Displacement response ratio	Standard deviation of water displacement (m)	Standard deviation of pressure (Pa)
Nonlinear	0.5298	0.4324	7742.99
Linear	0.5509	0.4131	7397.41

that the nonlinearity of liquid motion resulting from nonlinear damping is relatively weak. The linear equation can be used to describe the motion of the SATLCD-structure system in the frequency domain. Since the real implementation of SATLCD is also in the time domain, the successful analysis of the SATLCD-structure system in the time domain demonstrated here throws light on its real implementation.

3.2 Effect of liquid length ratio

The effect of liquid length ratio, α, which is the ratio of the horizontal length to the total length of liquid column of the SATLCD, on the lateral displacement response ratio is depicted in Fig. 3. The other parameters of the SATLCD used in computation are $\Delta\gamma = 1$, $\delta = 15$, $L = 22$ m and $\mu = 0.02$, 0.03 and 0.04. Note that the liquid column length selected is 22 m only, which is much shorter than 252 m required by the traditional TLCD. Fig. 3 demonstrates that SATLCD can reduce the displacement response of the structure significantly if its parameters are selected properly. When the liquid length ratio equals to 0.7, the lateral displacement reduction achieved by the SATLCD ($\mu = 0.02$) reaches the level of 66% for the structure under harmonic excitation and 38% under white noise excitation. The SATLCD with a larger value of liquid length ratio α can achieve larger reduction in displacement response but at the cost of relatively larger value of pressure required for the feedback control force. For a given value of liquid length ratio α, SATLCD with larger liquid mass can achieve larger reduction in displacement response but the required pressure for the feedback control force decreases as the mass of liquid inside the container increases.

3.3 Effect of head-loss coefficient

To investigate the effect of head-loss coefficient on the performance of SATLCD in reducing lateral displacement response of the structure, head-loss coefficient is taken as a variable. The other parameters of SATLCD are $\Delta\gamma = 1$, $\alpha = 0.7$, $\mu = 0.03$. Fig. 4 shows the variation of the lateral displacement response ratio with the head-loss coefficient for four different water lengths $L = 20$ m, 22 m, 24 m, and 26 m. The corresponding thickness of liquid column (d) is selected as 1.0 m, 1.2 m,

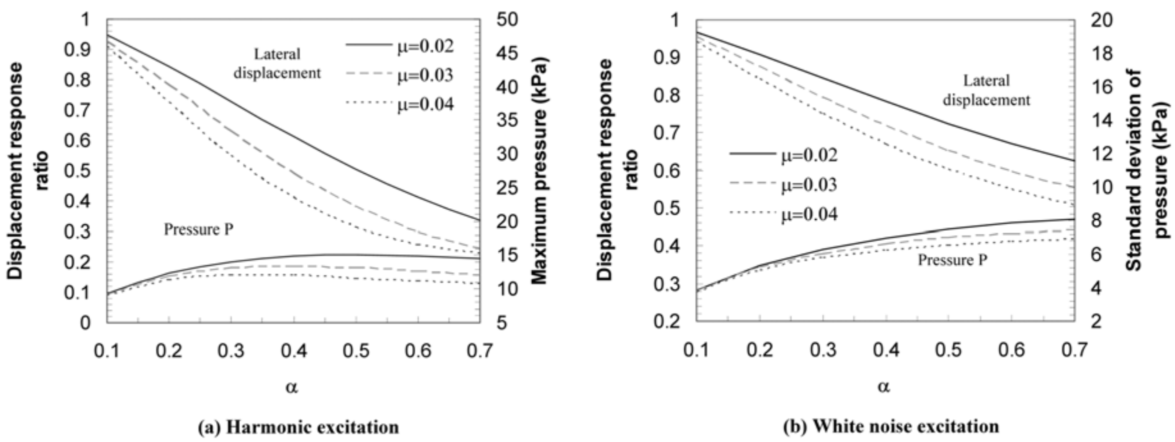


Fig. 3 Effect of liquid length ratio on lateral vibration reduction

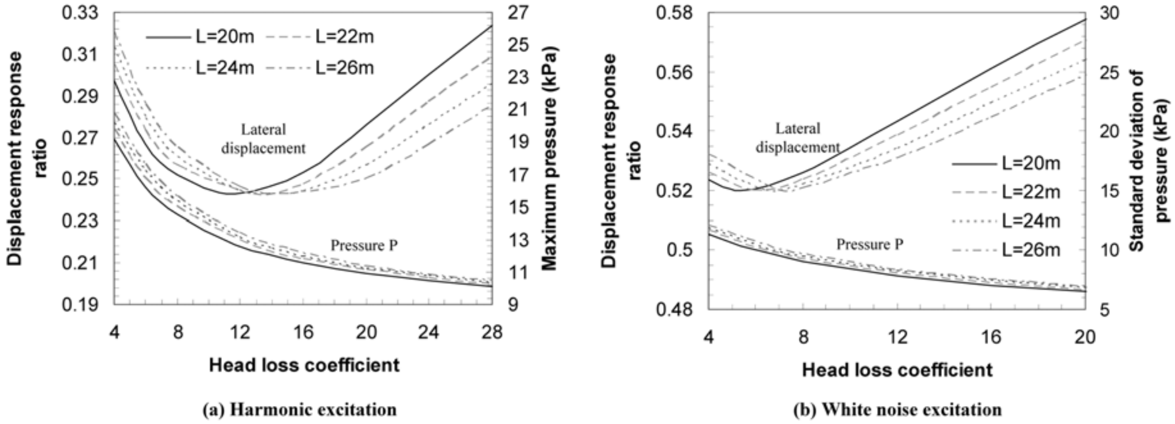
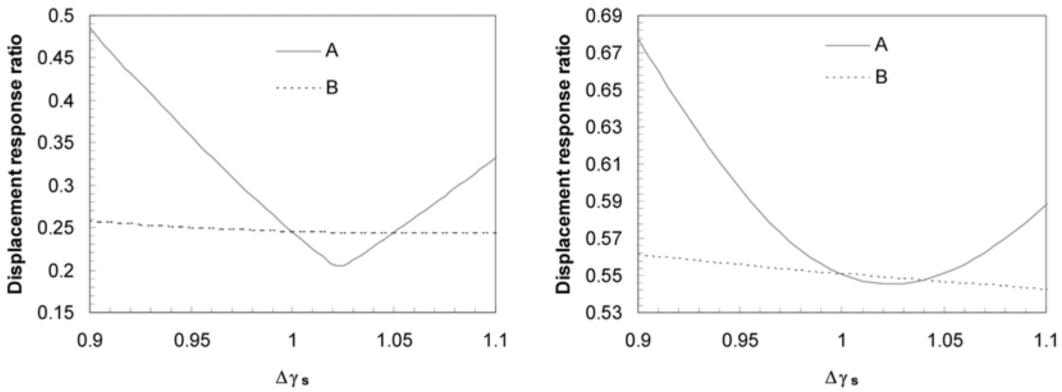


Fig. 4 Effect of head loss coefficient on lateral vibration reduction

1.4 m, and 1.6 m, respectively, to retain a reasonable geometric size of the damper. To keep the same mass ratio of 0.03 for each case, the width of the liquid column, which is in the direction perpendicular to the elevation of the damper, is then determined accordingly. Clearly, SATLCD provides a great flexibility for selecting different liquid lengths for a given structure. It can be seen from Fig. 4 that the effectiveness of SATLCD is affected by head loss coefficient. There exists an optimal head loss coefficient to achieve the maximum lateral displacement reduction. The effectiveness of the optimized SATLCD is almost the same for different liquid lengths, and the corresponding optimal head loss coefficient increases with the increasing length of liquid. Fig. 4 also indicates that the pressure required for feedback control force increases as the head loss coefficient decreases. It is seen that the maximum lateral displacement reduction achieved by the optimized SATLCD reaches the level of 75% for the structure under harmonic excitation and 48% under white noise excitation.

3.4 Effect of frequency offset ratio

The natural frequency of liquid column damper is usually designed to be the same as that of the structure in order to have maximum vibration energy dissipation. However, the natural frequency of the structure may vary from its original value due to uncertainties in structural parameters. This uncertainty could lead to an offset in frequency between damper and structure which deteriorates the damper's control performance significantly. Gao *et al.* (1997) indicated that the use of multiple tuned liquid column dampers could increase its robustness against the offset in frequency tuning. Due to the nature of passive dampers, the adaptability is still limited. The feature of the SATLCD studied herein can have greater adaptability against the change in structural frequency. If the online information of structural natural frequency is available, the damper can then be adjusted to the updated structural natural frequency simply by changing the feedback gain of the damper. The sensitivity of SATLCD to offset in frequency tuning between damper and structure is therefore investigated. The change in structural frequency is achieved by altering the structural stiffness. The frequency offset ratio ($\Delta\gamma_s$) is defined as the ratio of the updated structural natural frequency to the original structural natural frequency. The performance of SATLCD without and with self-updating of its natural frequency from the measured structural natural frequency is represented by the curves



(a) Harmonic excitation **(b) White noise excitation**

Fig. 5 Effect of frequency offset ratio on lateral vibration reduction

A and B, respectively, shown in Fig. 5. The parameters of the SATLCD are $\alpha = 0.7$, $\delta = 15$, $L = 22$ m and $\mu = 0.03$. It is seen from the figure that for SATLCD without updating of its natural frequency (curve A), the displacement response ratio increases and the control performance deteriorates significantly as the frequency offset ratio deviates from a value of 1.02. On the contrary, the performance of SATLCD with updating of its natural frequency (curve B) is almost insensitive to the change in frequency offset ratio. Hence, the adaptability of SATLCD can be much enhanced by self-adjusting of its natural frequency precisely to the measured structural natural frequency.

4. Parametric studies on torsional vibration reduction

The performance of SATLCD in reducing the torsional vibration of a structure is investigated in this section. The structural dynamic parameters are $I_s = 2.2099 \times 10^8$ kgm^2, $C_\theta = 7.7036 \times 10^6$ Nms/rad, and $K_\theta = 6.7135 \times 10^8$ Nm/rad, which are selected with reference to the first torsional vibration mode of the cable-stayed bridge during construction. The mass of structure is 4.6905×10^6 kg. The torsional circular natural frequency of the structure, ω_θ, is 1.7430 rad/s. It is noted that the use of traditional TLCD for the torsional vibration suppression of the concerned structure would require a short liquid (water in most cases) length of 6.458 m only. This would in turn require a large number of TLCD containers, leading to a higher cost of installation. The performance of SATLCD with adaptive tuning capacity is thus studied for the reduction of torsional vibration of the structure under two different kinds of loading: harmonic excitation and white noise excitation. The harmonic moment amplitude is 2.0×10^5 Nm and the power spectral density function of the moment is 10×10^{10} N^{2}m^{2}s/rad.

4.1 Effect of distance from SATLCD to rotational axis

The effect of the distance from the SATLCD to the rotational axis of the structure on the torsional displacement response ratio is depicted in Fig. 6. The parameters of the SATLCD are $\Delta\gamma = 1$, $\delta = 50$, $\alpha = 0.7$, and $\mu = 0.015$. Four different SATLCD, with water lengths of 15 m, 20 m, 22 m and 24 m, are studied. The thickness of the corresponding liquid column d is selected as 0.7 m, 1.0 m,

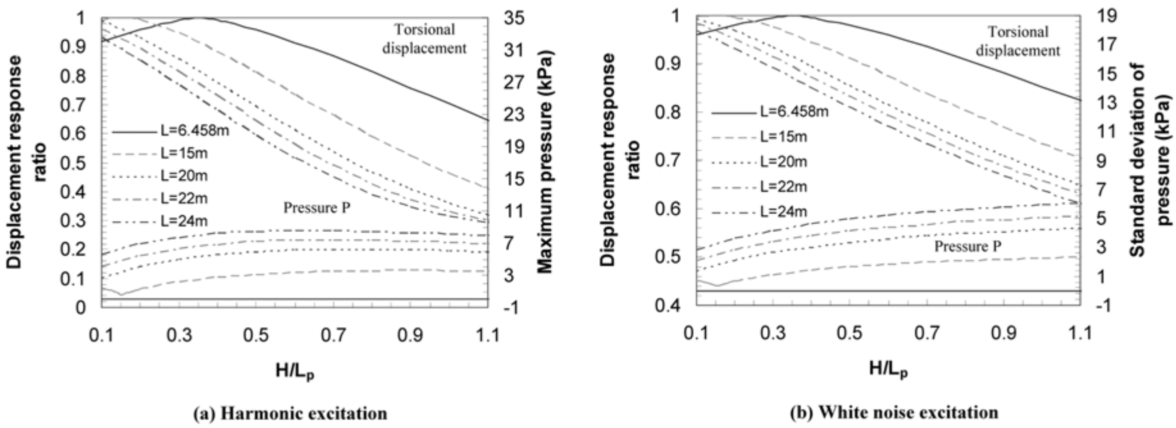


Fig. 6 Effect of distance from SATLCD to rotational axis on torsional vibration reduction

1.2 m and 1.4 m, respectively, to retain a reasonable geometric size of the damper. To keep the same mass ratio of 0.015 for each case, the width of the liquid column, which is in the direction perpendicular to the elevation of the damper, is then determined accordingly. The performance of the traditional passive TLCD is also included for the comparison with the SATLCD. The parameters of the passive TLCD are $\Delta\gamma = 1$, $\delta = 50$, $\alpha = 0.7$, and $\mu = 0.015$. The water length of the passive TLCD is 6.458 m, denoted L_p. The thickness of the liquid column is selected as 0.35 m. The width of the liquid column is then determined based on the mass ratio of 0.015. Clearly, SATLCD provides a great flexibility for selecting different liquid lengths for a given structure. It can be seen that the reduction of torsional displacement response is significantly affected by the value of H. The reduction in torsional displacement response is hardly achieved if the ratio H/L_p is near to a value of 0.35 for the case of the passive TLCD and a value of 0.15 for the case of the SATLCD with water length of 15 m. Similar finding has been reported and explained by Xue *et al.* (2000) for the case of TLCD in reducing the torsional vibration of a structure. This phenomenon is attributed to the zero interaction between the structure and the TLCD when the ratio H/L_p approaches to $\alpha/2$ (Xu and Shum 2003). For the case of SATLCD under harmonic or white noise excitation, when the excitation frequency equals to the structural frequency, the condition for the zero interaction between the structure and the SATLCD can be derived as follows:

$$\frac{H}{L_p} = \frac{\alpha}{2} - T(1-\alpha); \quad T = \frac{S}{4\rho A g} \tag{26}$$

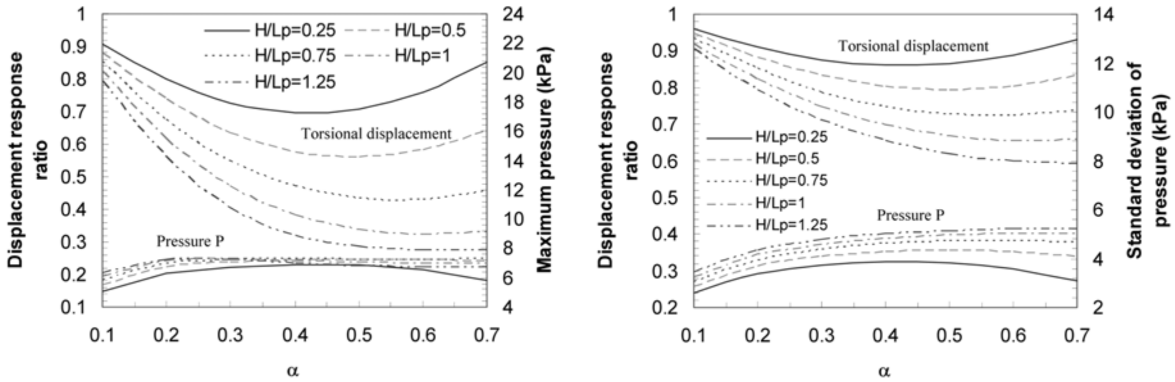
It is noted that Eq. (26) can be deduced to the condition for the case of the passive TLCD by setting the feedback displacement gain S to be zero. Due to the fact that all the concerned SATLCDs for torsional vibration control in this study have a positive constant displacement feedback gain S, it is easily seen from Eq. (26) that the H/L_p ratio for the zero interaction between the structure and the SATLCD is smaller than that for the TLCD. Furthermore, Eq. (15) indicates that T increases with the increasing water length of a SATLCD, and hence the ratio H/L_p for the zero interaction between the structure and the SATLCD decreases with the increasing water length of the SATLCD. For the SATLCD with water length L of 20 m, 22 m and 24 m, the corresponding value of the H/L_p ratios is smaller than that with water length of 15 m. This feature further facilitates the application of SATLCD in practice. It is also seen from the figure that the effectiveness of SATLCD in reducing the torsional displacement increases with the increasing ratio of H/L_p but the required pressure for the feedback control force also increases. For a given H/L_p, the SATLCD with a larger value of L can achieve larger reduction in displacement response but at the expense of larger value of pressure required for the feedback control force. Fig. 6 also depicts that for H/L_p ratio beyond a value of 0.35, the performance of the SATLCD in reducing the torsional displacement is more effective than that of the passive TLCD and it is further enhanced with the increasing ratio of H/L_p. The performance of SATLCD is less affected by the value of H than that of the passive TLCD, but it is still important to select a proper ratio of H/L_p for the SATLCD so as to avoid the zero interaction between the structure and SATLCD.

4.2 Effect of liquid length ratio

The effect of the liquid length ratio, α, of the horizontal length to the total length of liquid column of the SATLCD on torsional displacement response ratio is depicted in Fig. 7 for five different distance ratios H/L_p. The parameters of the SATLCD are $\Delta\gamma = 1$, $\delta = 50$, $L = 22$ m and $\mu = 0.015$. The liquid length ratio α varies within the range from 0.1 to 0.7. It is seen from Fig. 7 that the reduction of torsional displacement is dependent on the ratio of horizontal length to total length of liquid column. For the ratio H/L_p equal to 0.25, 0.5 or 0.75, an optimal value of liquid length ratio α exists for a maximum reduction of torsional displacement. It is also seen that increase in the ratio H/L_p leads to more reduction in torsional displacement response. When the ratio H/L_p increases to 1 or above, the torsional displacement response reduces monotonically with increasing liquid length ratio α: the larger the length ratio, the greater is the torsional displacement response reduction.

4.3 Effect of head-loss coefficient

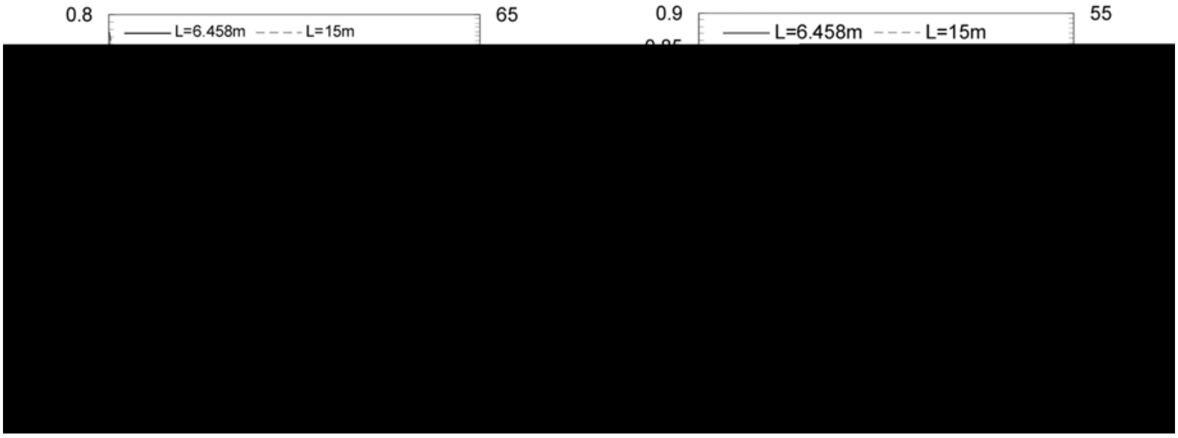
To investigate the effect of head-loss coefficient on the performance of SATLCD in reducing torsional displacement response of the structure, the head-loss coefficient is taken as a variable. The other parameters of the SATLCD are $\Delta\gamma = 1$, $\alpha = 0.7$, $H/L_p = 1$, and $\mu = 0.015$. Fig. 8 shows the variation of the torsional displacement response ratio with the head-loss coefficient for five different



(a) Harmonic excitation

(b) White noise excitation

Fig. 7 Effect of liquid length ratio on torsional vibration reduction



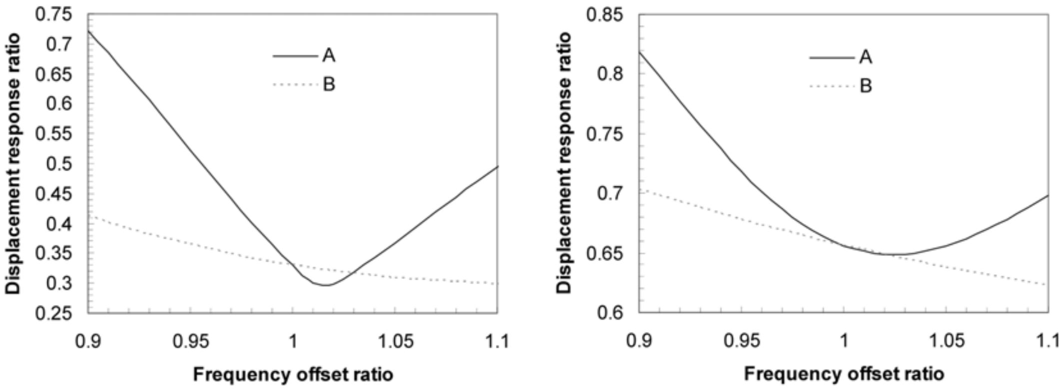
[redacted]

Fig. 8 Effect of head loss coefficient on torsional vibration reduction

water lengths. It can be seen from Fig. 8 that the effectiveness of the SATLCD is affected by the head loss coefficient, and there is an optimal head-loss coefficient for a given water length. The pressure required for the feedback control force of SATLCD increases with the decreasing head loss coefficient. Fig. 8 also shows that the performance of the optimized SATLCD increases with the increasing water length *L*. The corresponding optimal head-loss coefficient and the pressure required for the feedback control force also increase with the increasing water length. However, the SATLCD with a larger value of *L* would require more space for installation. The selection of water length should consider the balance between the control effectiveness and the available space inside the structure.

4.4 Effect of frequency offset ratio

In real situation, it may be difficult to tune the damper frequency precisely to the structural natural frequency because of uncertainties in structural parameters. This could lead to the deterioration of the damper control performance. As already demonstrated in Section 3.3, the feature of the



(a) Harmonic excitation **(b) White noise excitation**

Fig. 9 Effect of frequency offset ratio on torsional vibration reduction

SATLCD can have greater adaptability to the change in structural frequency. The sensitivity of SATLCD to the offset in frequency between damper and structure is investigated herein for the torsional vibration. The change in structural frequency is achieved by altering the structural stiffness. The performance of SATLCD without and with updating of its natural frequency from the measured structural natural frequency is represented by the curves A and B, respectively, shown in Fig. 9. The parameters of the SATLCD are $\alpha = 0.7$, $\delta = 50$, $L = 22$ m and $\mu = 0.015$. The value of H is taken the same value as one in Section 4.3. It is seen from the figure that for SATLCD without updating of its natural frequency (curve A), the displacement response ratio increases and the control performance deteriorates significantly as the frequency offset ratio deviates from a value of 1.02. On the contrary, the performance of SATLCD with updating of its natural frequency (curve B) is less sensitive to the change in frequency offset ratio. The performance of SATLCD in reducing torsional displacement deteriorates slightly when the frequency offset ratio decreases. The adaptability of SATLCD to the offset in frequency between the damper and the structure is enhanced by actively adjusting its natural frequency to the measured structural natural frequency.

5. Conclusions

A semi-active tuned liquid column damper (SATLCD), whose natural frequency can be altered by active control of liquid column pressure, has been studied numerically in this paper. The analytical models for lateral vibration of a structure with SATLCD and torsional vibration of a structure with SATLCD have been developed accordingly. The SATLCD can provide a great flexibility for selecting a liquid length while keeping a proper frequency tuning through the change of air pressure acting on liquid. Another feature of the SATLCD is its adaptability to the change in structural frequency. If the online information of varying structural natural frequency is available, the frequency of the damper can be actively adjusted to the varying structural natural frequency to maintain high vibration control performance. The numerical examples carried out in this study demonstrate that SATLCD can effectively reduce either lateral or torsional vibration of a structure if its parameters are properly selected. There exists an optimal head loss coefficient for maximum reduction in lateral or torsional vibration, and the optimal head loss coefficient increases with the increasing liquid length of SATLCD. For the case of lateral vibration of SATLCD-structure system, the results obtained in the present study reveal that SATLCD with a larger value of liquid length ratio can achieve larger reduction in displacement response but at the expense of larger value of pressure required for the feedback control force. For the case of torsional vibration of SATLCD-structure system, it is found that H/L_p ratio is an important parameter affecting the performance of SATLCD in reducing torsional vibration. For the same value of the H/L_p, the SATLCD with a larger value of liquid length can achieve larger reduction in displacement response. When the ratio H/L_p is increased to 1 and above, the torsional response reduction is increased with the increasing liquid length ratio.

The effectiveness of the SATLCD is assessed through computation simulation based on the developed analytical model in this paper. Detailed experimental investigations of the semi-active control system in pressurized liquid column dampers are required before implementing the SATLCD to real civil engineering structures.

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