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Probabilistic analysis of peak response to nonstationary seismic excitations

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Abstract. The main objective of this study is to examine the accuracy of the complete quadratic combination (CQC) rule with the modal responses defined by the ordinates of the uniform hazard spectra (UHS) to evaluate the peak responses of the multi-degree-of-freedom (MDOF) systems subjected to nonstationary seismic excitations. For the probabilistic analysis of the peak responses, it is considered that the seismic excitations can be modeled using evolutionary power spectra density functions with uncertain model parameters. More specifically, a seismological model and the Kanai-Tajimi model with the boxcar or the exponential modulating functions were used to define the evolutionary power spectral density functions in this study. A set of UHS was obtained based on the probabilistic analysis of transient responses of single-degree-of-freedom systems subjected to the seismic excitations. The results of probabilistic analysis of the peak responses of MDOF systems were obtained, and compared with the peak responses calculated by using the CQC rule with the modal responses given by the UHS. The comparison seemed to indicate that the use of the CQC rule with the commonly employed correlation coefficient and the peak modal responses from the UHS could lead to significant under- or over-estimation when contributions from each of the modes are similarly significant.

Key words: peak response; probability; nonstationary process; transient response.

1. Introduction

The peak responses of single-degree-of-freedom (SDOF) systems subjected to the seismic excitations can be succinctly represented by the response spectra; and the peak responses of linear multi-degree-of-freedom (MDOF) systems can be estimated using the complete quadratic combination (CQC) rule (Rosenblueth and Elorduy 1969, Der Kiureghian 1981, Chopra 1996) with the modal peak responses given by the spectra.

If the uniform hazard spectra (UHS), (i.e., the response spectra at different frequencies having the same probability of exceedance P_e), are used to represent the seismic demand due to all possible seismic excitations, it is desirable that the use of this modal combination rule with the peak modal responses given by the UHS will lead to the peak responses of the MDOF systems having same probability of exceedance P_e. This is because it is hoped that the use of probability-consistent peak

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responses in design will likely result the designed structures having a consistent reliability level. Unfortunately, such a desirable probability consistency is not always achieved and in occasions the differences could be very large (Hong and Wang 2002) if the UHS are considered and the CQC rule with the correlation coefficient given by Der Kiureghian (1981) is employed. The conclusions given by Hong and Wang (2002) were based on the results obtained by considering the seismic excitation as a Gaussian stationary process. It provided a first step in identifying the possible weakness associated with the use of the CQC rule with the UHS. However, the seismic excitation is inherently nonstationary. To further verify this possible weakness, a systematic probabilistic analysis of the peak responses must be carried out by considering the seismic excitations as nonstationary processes.

In this study, results of probabilistic analysis of the peak responses of MDOF systems are presented for assessing the adequacy of the CQC rule with peak modal responses given by the UHS. For the analysis, the seismic excitations are characterized by evolutionary power spectral density (EPSD) functions that consist a time modulating function and a power spectral density (PSD) function of a Gaussian stationary process. More specifically, the boxcar or the exponential modulating functions are used as the time modulating function, and the Kanai-Tajimi PSD function or the Fourier amplitude spectrum of a seismological model are employed to characterize the stationary process. Further, parameters for the EPSD functions are considered to be uncertain. This is aimed at representing the uncertainty in seismic excitations from all potential earthquakes. Detailed formulation, analysis procedure and results are presented in the following sections.

2. Ground motion and structural response

2.1 Stationary ground motion

The seismic excitation modeled as a stationary process can be characterized by a PSD function $G_x(\omega)$ (Tajimi 1960, Hu and Zhou 1962, Clough and Penzien 1975, Brune 1970, Boore 1983). In this study, we only consider two often used PSD functions, namely the Kanai-Tajimi PSD function (Tajimi 1960) and the PSD function based on the seismological model of radiated spectra (Brune 1970, Boore 1983).

The Kanai-Tajimi PSD function of the ground acceleration $G_{x1}(\omega)$ is given by Tajimi (1960):

$$G_{x1}(\omega) = \frac{\omega_g^4 + 4\xi_g^2\omega_g^2\omega^2}{(\omega_g^2 - \omega^2)^2 + 4\xi_g^2\omega_g^2\omega^2}G_0 \tag{1}$$

where ω (rad/sec) is the frequency, G_0 is the intensity of the white noise; ω_g (rad/sec) is the filter frequency that determines the dominant range of input frequencies; and the damping coefficient ξ_g is a parameter that influences the shape of the PSD function.

The PSD function based on the seismological model of radiated spectra can be expressed by the following form (Boore 1983, Quek *et al.* 1990, 1991):

$$G_{x2}(\omega) = 2\pi\bar{A}^2(\omega)/I \tag{2}$$

where $\bar{A}(\omega) = \frac{CM_0}{R}\frac{\omega^2}{1+(\omega/\omega_c)^2}e^{-\kappa\omega}$ is the equivalent-stationary averaged Fourier amplitude of ground acceleration at rock sites due to seismic excitation. $C = R_\phi \cdot F_S \cdot P_r / 4\pi\rho\beta_w^3$ is a scaling factor, in which R_ϕ, F_S and P_r are the factors accounting for the radiation pattern, free surface effect and partition of energy into horizontal components, respectively; ρ is the density of the medium, β_w is the seismic wave velocity; $M_0 = 10^{(1.5\, M_{rec} + 9.05)}$ (N · m) is the seismic moment where M_{rec} is recorded earthquake magnitude; R is the hypocentral distance; $\kappa = \kappa_R R + \kappa_c$ characterizes the attenuation of the seismic wave with the attenuation constants κ_R and κ_c; $\omega_c = 2\pi \cdot (0.49\beta_w) \cdot (\Delta\sigma/M_0)^{1/3}$ is the corner circular frequency, in which $\Delta\sigma$ is the stress drop. In Eq. (2), $I = \int_0^\infty |A(t)|^2 dt$, in which $A(t)$ is the time modulating function which will be discussed in next section.

2.2 Time modulating function

The time modulating function, $A(t)$, is taken as a boxcar function (Vanmarcke 1976) or an exponential function (Shinozuka and Sato 1967).

The boxcar modulating function takes the form

$$A(t) = \begin{cases} A_0 & \text{for } 0 \le t \le T_0 \\ 0 & \text{for } t < 0 \text{ and } t > T_0 \end{cases} \tag{3}$$

where T_0 is the strong ground motion duration and A_0 is a scaling factor. For simplicity and without loss of generality, A_0 is taken as unit in the study.

The exponential modulating function $A(t)$ can be written as

$$A(t) = A_0(e^{-b_1 t} - e^{-b_2 t}), \quad b_2 > b_1 \tag{4}$$

where the values of the shape parameters, b_1 and b_2, can be calculated for any earthquake record having a strong motion duration T_0, and a rise time fraction, ε (Quek *et al.* 1990). In this study, the scaling factor A_0 is chosen to be $1/\mathrm{Max}(e^{-b_1 t} - e^{-b_2 t})$ such that the maximum value of the exponential modulating function equals unit as well.

2.3 Nonstationary responses to nonstationary seismic excitation

The seismic ground motion is inherently a nonstationary process which may be characterized by the EPSD function (Priestly 1967), $G_x(\omega, t)$,

$$G_x(\omega, t) = |A(t)|^2 G_x(\omega) \tag{5}$$

where $G_x(\omega)$ equals $G_{x1}(\omega)$ for the Kanai-Tajimi model and $G_{x2}(\omega)$ for the seismological model.

For a linear SDOF system with the natural frequency ω_0 and the damping ratio ξ subjected to a nonstationary ground motion with the EPSD function $G_x(\omega, t)$, the relative displacement $R(t)$ of the system is also nonstationary with the EPSD function, $G_R(\omega, t)$, given by:

$$G_R(\omega, t) = |M(\omega, t)|^2 G_x(\omega) \tag{6}$$

where $M(\omega, t) = \int_0^t h(\tau)A(t-\tau)e^{-i\omega\tau}d\tau$, in which

$$h(\tau) = \frac{1}{\omega_0\sqrt{1-\xi^2}}e^{-\xi\omega_0\tau}\sin(\omega_0\sqrt{1-\xi^2}\,\tau) \tag{7}$$

Similarly, for an n-degree-of-freedom system with ω_i and ξ_i denoting, respectively, the frequency and damping ratio of the i-th mode, the EPSD function of its response can be expressed as:

$$G_R(\omega, t) = \sum_i^n\sum_j^n C_iC_jM_i(\omega, t)M_j^*(\omega, t)G_x(\omega) \tag{8}$$

where C_i is the effective participation factor of the i-th mode; the asterisk denotes the complex conjugate; $M_i(\omega, t) = \int_0^t h_i(\tau)A(t-\tau)e^{-i\omega\tau}d\tau$, in which $h_i(t)$ is the response of i-th mode which is defined in Eq. (7) with ξ and ω_0 replaced by ξ_i and ω_i.

3. Probabilistic analysis of peak structural response

3.1 Variability of parameters

It is considered that the seismic excitation for all potential earthquakes can be characterized by the EPSD function with uncertain parameters in stationary PSD function, as well as in time modulating function.

Probabilistic characterizations of the model parameters (G_0, ω_g, ξ_g) in Eq. (1) for the Kanai-Tajimi PSD function, taken basically from Lai (1982) and used by Hong and Wang (2002), are shown in Table 1 and, are adopted in the present study. Note that this characterization was based on more than 100 strong ground motion records for soil sites. The statistics of the strong ground motion duration T_0 and the rise time fraction adopted from Lai (1982) and Quek *et al.* (1991) for defining the time modulating function are also shown in the table.

A study of the required model parameters in Eq. (2) for the seismological model was carried out in Quek *et al.* (1991) based on the results of 54 acceleration records from 8 earthquakes for rock sites. To consider the uncertainty in M_{rec}, a bias factor γ_M was introduced, and M_0 was rewritten as $M_0 = 10^{(1.5\gamma_M M_{rec}+9.05)}$ (N · m). It is noted that instead of M_{rec} and R, the local magnitude M_L and the epicentral distance R_{epi} were used in Quek *et al.* (1991) for the analysis because these values are

Table 1 Statistics of model parameters in Kanai-Tajimi PSD function

Model parameter	Mean	Standard deviation	Distribution type
ω_g (rad/sec)	19.06	8.139	Gamma
ξ_g	0.316	0.135	Lognormal
G_0 (cm^2/sec^3)	35.32	86.675	Lognormal
T_0 (sec)	10.09	9.081	Gumbel
ε	0.159	0.092	Uniform

Table 2 Values of parameters in Fourier Amplitude of seismological model

Model parameter	Value	Model parameter	Value
ρ (kg/m^3)	2700	κ_R	5.007×10^{-8}
β_w (m/sec)	3200	κ_c	0.0293
R_ϕ	0.63	τ_R	0.118×10^{-3}
P_r	0.71	τ_M	6.3
F_s	2.0	τ_c	−31.06

Table 3 Statistics of model parameters in Fourier Amplitude of seismological model

Model parameter	Mean	Standard deviation	Distribution type
γ_M	0.969	0.076	Lognormal
$\Delta\sigma$ (N/m^3)	1.413×10^8	2.67×10^8	Lognormal
κ_0 (sec/rad)	0.0	0.0065	Normal
τ_0 (sec)	0.0	4.31	Normal
ε	0.159	0.092	Uniform
M_L	6.116	0.582	Normal
R_{epi} (m)	40196	27179	Lognormal

readily available for all records considered. Further, it was suggested in Quek *et al.* (1991) that κ and T_0 can be evaluated through the use of $\kappa = \kappa_R R_{epi} + \kappa_c + \kappa_0$ and $T_0 = \tau_M M_L + \tau_R R_{epi} + \tau_c + \tau_0$. The values of these parameters κ_R, κ_c, τ_m, τ_R, and τ_c determined from the regression analysis and the values of ρ, β_w, R_ϕ, P_r, and F_S are shown in Table 2 (Quek *et al.* 1991). The remaining parameters (γ_M, $\Delta\sigma$, κ_0, τ_0, M_L, R_{epi}, ε) are treated as random variables with the statistics shown in Table 3 (Quek *et al.* 1991).

3.2 Probability of non-exceedance

For the structural response characterized by Eq. (6) or Eq. (8), the probability that the peak response is within the prescribed barriers $\pm r$ during (0, t), $L(r, t)$, can be evaluated using (Vanmarcke 1976):

$$L(r, t) = \exp\left(-\int_0^t \alpha(r, \tau) d\tau\right) \tag{9}$$

where $\alpha(r, t) = 2\nu(r, t)\dfrac{1 - \exp(-\sqrt{\pi/2}\,\delta_e(t) r/\sqrt{\lambda_{R,0}(t)})}{1 - \exp(-r^2/\lambda_{R,0}(t))}$ represents the decay rate; the shape factor, $\delta_e(t) = (1 - \lambda_{R,1}^2(t)/(\lambda_{R,0}(t)\lambda_{R,2}(t)))^{1.2/2}$, measures the variability in the frequency content of the response; $\nu(r,t) = \nu_0(t)\exp(-r^2/2\lambda_{R,0}(t))$ is the r-level crossing rate; and $\nu_0(t) = \sqrt{\lambda_{R,2}(t)/\lambda_{R,0}(t)}/\pi$ is the zero up-crossing rate. In the above, $\lambda_{R,k}(t)$ representing the k-th moment of the PSD function of the response is given by

$$\lambda_{R,k}(t) = \mathrm{Re}\left[\int_0^{\infty} \omega^k G_R(\omega, t)d\omega\right], \quad k = 0, 1, 2, \ldots \tag{10}$$

where $G_R(\omega, t)$ is given by Eq. (6) for SDOF systems and by Eq. (8) for MDOF systems.

Since the parameters of the EPSD function of ground motion are uncertain as discussed in the previous section, $G_R(\omega, t)$ that is conditioned on these parameters is also uncertain. This uncertainty can be incorporated in evaluating the unconditional probability that the peak response exceeds the prescribed barriers $\pm r$, $P_e(r)$, by using:

$$P_e(r) = \int_{\Omega}(1 - L(r, t|\mathbf{x}))f_{\mathbf{x}})(\mathbf{x})d\mathbf{x} \tag{11}$$

where Ω denotes the domain of a set of random variables $\mathbf{X}$ which is $(G_0, \omega_g, \xi_g, T_0, \varepsilon)$ for the Kanai-Tajimi model and is $(\gamma_M, \Delta\sigma, \kappa_0, \tau_0, M_L, R_{epi}, \varepsilon)$ for the seismological model; and $f_X(\mathbf{x})$ represents the joint probability density function of $\mathbf{X}$. Note that in Eq. (11), $L(r, t|\mathbf{x})$ is used to replace $L(r, t)$ to emphasize that it is dependent on the value of $\mathbf{X}$.

3.3 Fractile of peak response

Given an EPSD function and structural characteristics, the probability $P_e(r)$, and the $(1 - P_{Te})$-fractile of peak response r_T (i.e., $P_e(r_T) = P_{Te}$) can be evaluated by solving the integral equation (iteratively) shown in Eq. (11). For the case of stationary excitations, the calculation steps including the use of the first order reliability method (FORM) were given by Hong and Wang (2002). For the nonstationary case, the analysis steps for evaluating $P_e(r)$ can be stated as follows:

1) Give an initial value r;

2) Evaluate $P_e(r) = \int_{\Omega}(1 - L(r, t|\mathbf{x})f_{\mathbf{x}})(\mathbf{x})d\mathbf{x} = \int_{g \le 0} \phi(z)f_{\mathbf{x}}(\mathbf{x})dzd\mathbf{x}$ using FORM where $g = z - \Phi^{-1}(1 - L(r, t|\mathbf{x}))$, $\Phi^{-1}(\bullet)$ is the inverse of the standard normal distribution function $\Phi(\bullet)$, and $P_e(r)$ is approximated by $\Phi(-\beta)$. For each iteration, this calculation involves:

2a) For the value of $\mathbf{X}$, $\mathbf{x}$, dictated by FORM, calculate the moments $\lambda_{R,k}(t)$, $k = 0, 1, 2$, defined in Eq. (10) with $G_R(\omega, t)$ defined in Eq. (8) by the Gaussian numerical integration method (with 20 points);

2b) Evaluate $L_R(r, t|\mathbf{x})$ defined in Eq. (9) by using the Gaussian numerical integration method (with 20 points).

For estimating r_T, the above steps should be modified to include:

3) If $|P_{Te} - P_e(r)|$ is less than a specified tolerance, convergence is achieved and r is assigned to r_T; otherwise, calculate a new r, using the Quasi-Newton method, and repeat Step 2).

If the SDOF systems with a range of natural frequency of vibration and damping ratios are considered, the fractiles of the peak responses of these SDOF systems can be used to form the response spectra. These response spectra can be viewed as the UHS since they have equal probability of exceedance P_{Te}, and the seismic risk was reflected by considering the uncertainty in the parameters of EPSD function.

The fractiles of peak responses of the MDOF systems can also be estimated using the probabilistic analysis steps. However, for practical applications, the peak response of a MDOF system is commonly evaluated using the CQC rule with peak modal responses obtained from a response spectrum. According to the CQC rule, the response of a MDOF system r_0 can be calculated using (Der Kiureghian 1981, Chopra 2000),

$$r_0 = \left(\sum_i \sum_j \rho_{ij} C_i C_j r_{i0} r_{j0}\right)^{1/2} \tag{12}$$

where r_{i0} is the i-th modal peak response with effective participation factor equal to one (i.e., response given by response spectrum), and the correlation coefficient of the responses ρ_{ij} is given by Der Kiureghian (1981),

$$\rho_{ij} = \frac{2\sqrt{\xi_i \xi_j}((\omega_i + \omega_j)^2(\xi_i + \xi_j) + (\omega_i^2 - \omega_j^2)(\xi_i - \xi_j))}{4(\omega_i - \omega_j)^2 + (\omega_i + \omega_j)^2(\xi_i + \xi_j)^2} \tag{13}$$

4. Numerical results

4.1 Uniform hazard spectra

Consider the responses of the systems subjected to the nonstationary seismic excitation characterized by the evolutionary Kanai-Tajimi PSD function with the statistics of the random variables $X = (G_0, \omega_g, \xi_g, T_0, \varepsilon)$ shown in Table 1. The probability levels of the non-exceedance of the UHS, $(1 - P_{Te})$, are chosen to be 0.368, 0.57, and 0.841 which are consistent with those used by Hong and Wang (2002) and will facilitate the comparison of the results between nonstationary and stationary cases. The fractiles of the peak responses of a set of SDOF systems are calculated and shown in Fig. 1(a) for the boxcar modulating function, and in Fig. 1(b) for the exponential modulating function. For the results shown in the figure and the subsequent numerical analysis in this study, the damping ratios of 0.05 are employed. A simple analysis of the fractiles of the peak

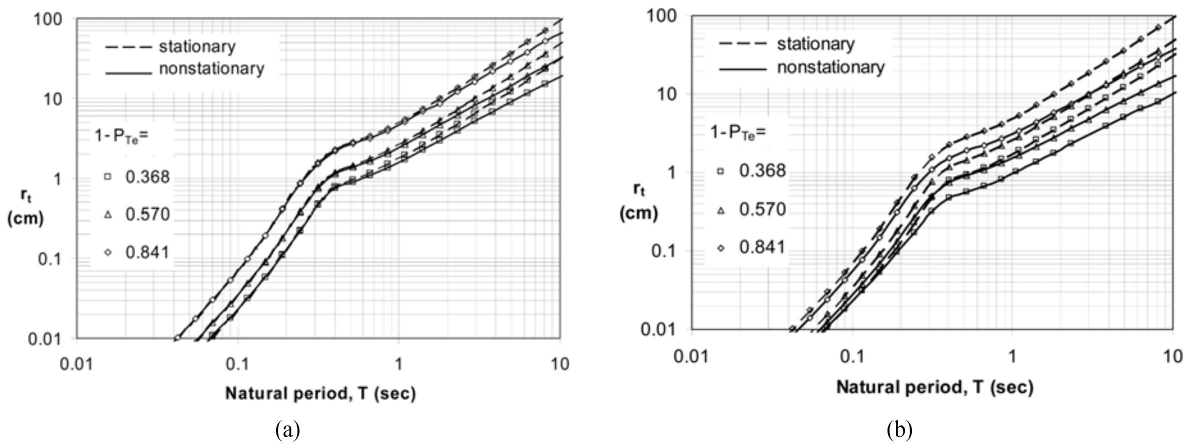
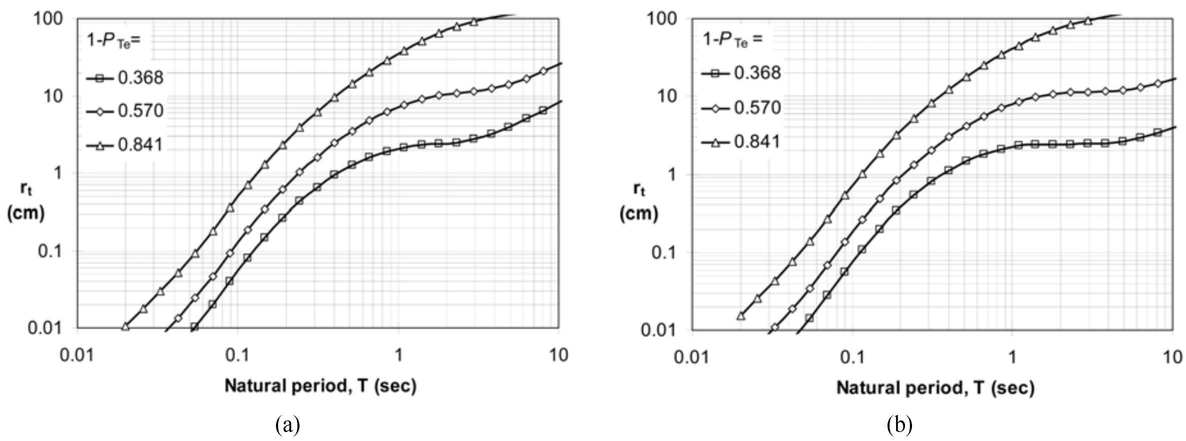


Fig. 1 Peak responses of SDOF systems to the evolutionary Kanai-Tajimi PSD function. (a) Boxcar modulating function, (b) Exponential modulating function

responses shown in Fig. 1 suggests that the ratios of the fractiles and the ratios of the logarithmic of fractiles vary with natural vibration periods. Also, the logarithmic of these ratios are slightly different as well. These imply that the peak responses at different natural periods are not fully correlated. Therefore, the peak responses cannot be represented accurately by the product of or the logarithmic of product of a deterministic function (i.e., traditional response spectrum) of the natural frequency and a random variable (i.e., the peak ground acceleration or the peak ground velocity), which provides justification of the use of the UHS. These findings are similar to the ones given in Hong and Wang (2002) where the responses and the seismic excitations are treated as stationary processes. Further, comparison of the results shown in Figs. 1(a) and 1(b) indicates that the peak responses for the boxcar modulating function are larger than those for the exponential modulating function. This is due to the way the modulating functions are normalized.

For comparison purpose, the fractiles of the peak responses obtained by considering stationary excitation with the Kanai-Tajimi PSD function (Hong and Wang 2002) is also shown in Fig. 1. It indicates that the fractiles of the peak responses to the stationary excitation are larger than those to the nonstationary excitation modeled as the time-modulated Kanai-Tajimi PSD function. This is expected because that the nonstationary excitation is equal to the stationary excitation multiplied by the modulating function with magnitude less than or equal to one. The comparison also suggests that the nonstationarity of the ground motion affects the peak response of flexible systems more significantly than that of rigid systems. This can be explained because for a rigid system whose fundamental period is several times shorter than the duration of strong ground motion its response reaches stationarity much faster than that of a flexible system.

Similar analysis was carried out for the nonstationary seismic excitation characterized by the EPSD function of the seismological model. In the analysis, the deterministic parameters shown in Table 2 and the statistics of random variables shown in Table 3 are used. The obtained results are presented in Figs. 2(a) and 2(b). Again, it appears that the peak responses at different natural periods are not fully correlated since they do not have exactly the same shape. However, unlike the results shown in Fig. 1, the peak responses to the exponentially modulated nonstationary excitation are not always less than those to the nonstationary excitation modulated by the boxcar modulating function. This is because that the EPSD function for the seismological model shown in Eq. (5) with



(a) (b)

Fig. 2 Peak responses of SDOF systems to the evolutionary PSD function of seismological model. (a) Boxcar modulating function, (b) Exponential modulating function

$G_x(\omega)$ substituted by $G_{x2}(\omega)$ given in Eq. (2) is independent of the magnitude of the modulating functions.

4.2 Peak response of two-degree-of-freedom system

As described previously, the peak responses of MDOF systems can be calculated using the CQC rule with the modal peak responses from the response spectra. For a two-degree-of-freedom (2DOF) system with modal frequencies ω_1 and ω_2, the calculated peak response denoted by r_a is given by,

$$r_a = ((C_1 r_{10p})^2 + 2\rho_{12} C_1 C_2 r_{10p} r_{20p} + (C_2 r_{20p})^2)^{1/2} \tag{14}$$

where r_{10p} and r_{20p} denote the peak modal responses with the specified probability of exceedance P_{Te}; ρ_{12} is the correlation coefficient shown in Eq. (13); C_1 and C_2 represent the effective participation factors due to the first mode and the second mode. The peak modal responses r_{i0p}, i = 1, 2, are obtained directly from the UHS at ω_i.

To carry out a systematic assessment of the adequacy of the CQC rule for the 2DOF systems with ranges of values of C_1 and C_2, we use a parameter ζ, $\zeta = |C_2 r_{20p}|/r_{ABS}$, in which $r_{ABS} = |C_1 r_{10p}| + |C_2 r_{20p}|$, to represent the contribution of the second mode to the system. Thus, Eq. (14) can be rewritten as follows:

$$r_a = ((1-\zeta)^2 + \mathrm{sgn}(C_1 C_2) 2\rho_{12}(1-\zeta)\zeta + \zeta^2)^{1/2} r_{ABS} \tag{15}$$

where sgn( ) returns the sign of the argument.

By assigning different values of ζ from 0 to 1, different percentage of the contribution due to each mode to the system can be obtained. In particular, if ζ equals zero or one, the system is a SDOF system. If ζ equals 0.5, it implies that the contribution of the first mode to the system response is equally important as that of the second mode.

Note that since $G_R(\omega, t)$ shown in Eq. (8) for a 2DOF system can be expressed as:

$$G_R(\omega, t) = \left(\sum_{i=1}^{2}\sum_{j=1}^{2} \mathrm{sgn}(C_i C_j) C_i' C_j' M_i(\omega, t) M_j^*(\omega, t) G(\omega)\right) r_{ABS}^2 \tag{16}$$

where $C_i' = (1-\zeta)/r_{10p}$ and $C_2' = (1-\zeta)/r_{20p}$. Since the quantity inside of the parenthesis on the left side of Eq. (16) is independent of r_{ABS}, $\alpha(r, t)$ in Eq. (9) is independent of r_{ABS}. In other words, given the value of ζ the probability that the peak response of the 2DOF system exceeds a fraction of or a multiple of r_{ABS} can be evaluated using Eq. (11) without the actual value of the r_{ABS}.

Consider that a set of 2DOF systems with the same fundamental period, $T_1 = 0.2$ sec, and different period ratios, $T_2/T_1 = 1.2$, 1.5 and 2.0, subjected to the nonstationary seismic excitations with modulated Kanai-Tajimi PSD functions. To verify the accuracy of using the CQC rule with peak modal responses from the UHS to assess the peak responses of the 2DOF systems, the peak responses of the systems, r_a, are calculated by using Eq. (15) with r_{10p} and r_{10p} obtained from Figs. 1(a) and 1(b), and are compared with the $(1 - P_{Te})$-fractile of the peak responses, r_T, obtained using the probabilistic analysis procedure described in the previous section. The values of the ratio, $\eta = r_a/r_T$, are shown in Figs. 3(a) and 3(b) for the boxcar modulating function, and in Figs. 3(c) and 3(d) for the exponential modulating function. The results shown in Figs. 3(a) and 3(c) are obtained

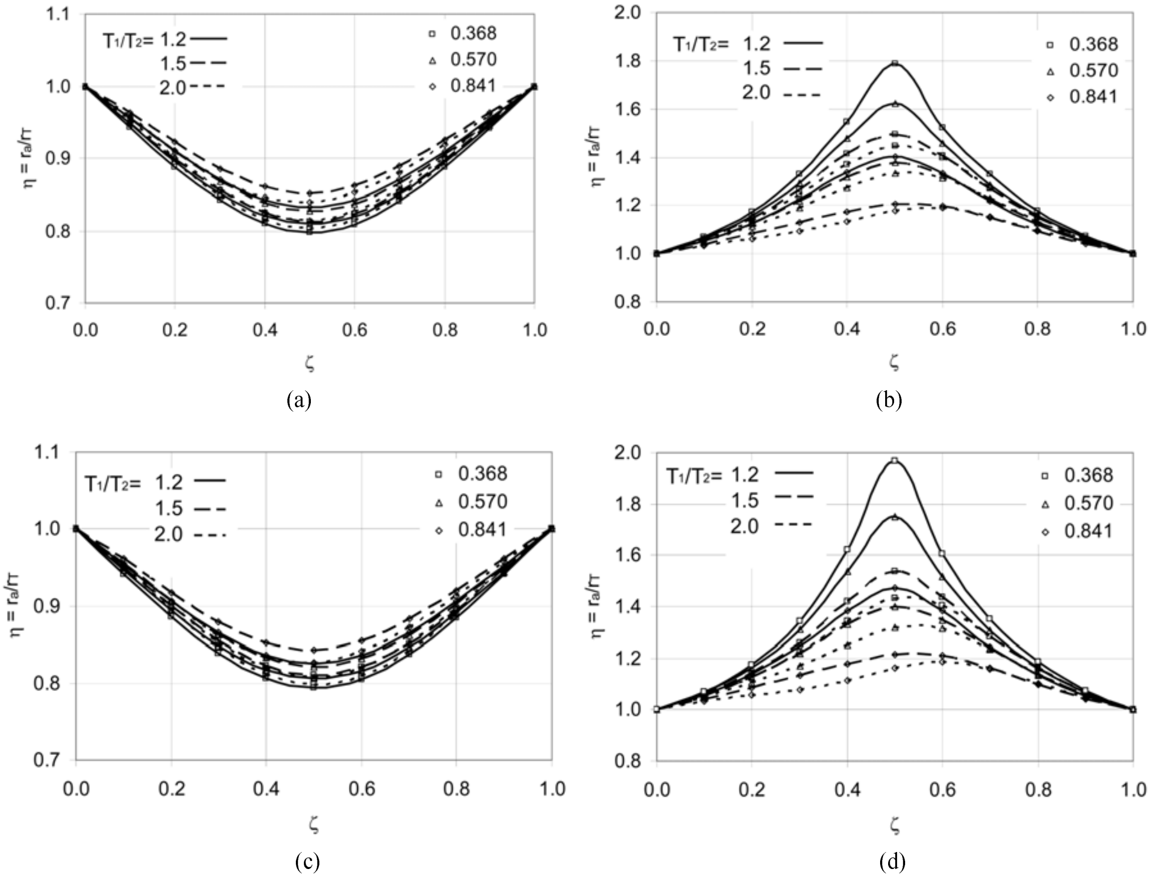


Fig. 3 Ratios of r_a to r_T calculated using the evolutionary Kanai-Tajimi PSD function. (a) Effect of T_1/T_2 for T_1 = 0.2s, sgn(C_1C_2) > 0, boxcar modulating function, (b) Effect of T_1/T_2 for T_1 = 0.2s, sgn(C_1C_2) < 0, boxcar modulating function, (c) Effect of T_1/T_2 for T_1 = 0.2s, sgn(C_1C_2) > 0, exponential modulating function, (d) Effect of T_1/T_2 for T_1 = 0.2s, sgn(C_1C_2) < 0, exponential modulating function

when two modes displace in the same direction (i.e., sgn(C_1C_2) > 0), while those in Figs. 3(b) and 3(d) are for two modes displacing in the opposite direction (i.e., sgn(C_1C_2) < 0). The figures indicate that the ratios vary significantly with ζ. The peak responses, r_a and r_T, are almost identical when ζ is close to 0 or 1; otherwise they are different. This suggests that when the response is dominated by one mode the use of the CQC rule with the UHS, even under nonstationary excitation, is adequate. However, if the contributions of both modes to the system response are significant, the use of the CQC rule with the UHS could lead r_a to be significantly different than r_T. The results also indicate that the error of the peak response (i.e., absolute value of $1 - \eta$) decreases as the probability of non-exceedence, $(1 - P_{Te})$, increases. This implies that the accuracy of the CQC rule is improved for high probability of non-exceedance levels. For these 2DOF systems, the CQC rule under- and over-estimates the peak responses of the systems for sgn(C_1C_2) > 0 and sgn(C_1C_2) < 0, respectively. The ratio attains the lowest value for sgn(C_1C_2) > 0 and the highest value for sgn(C_1C_2) < 0 when ζ is within about 0.4 to 0.6. It is noted that the highest value of η almost reaches 2 in Fig. 3(d). In such a case the error of the peak response evaluated by the CQC rule with the modal response from the UHS is about 100%. Further, the results show that the ratio of the modal period affects the value of η. In Figs. 3(b) and 3(d), it appears that when $\mathrm{sgn}(C_1C_2) < 0$ differences between r_a and r_T increase as the vibration modes become closer. However, as shown in Figs. 3(a) and 3(c), such trend can not be found if $\mathrm{sgn}(C_1C_2) > 0$. Comparison between Figs. 3(a) and 3(b), and Figs. 3(c) and 3(d) shows that the errors appear to be larger for the cases with $\mathrm{sgn}(C_1C_2) < 0$ than for the cases with $\mathrm{sgn}(C_1C_2) > 0$.

The effect of the fundamental natural vibration period T_1 on η is investigated by considering another set of 2DOF systems with T_2/T_1 = 2.0 and T_1 equal to 0.2, 0.5 or 1.5 sec. The obtained η are shown in Figs. 4(a) and 4(b) for the boxcar modulating function and in Figs. 4(c) and 4(d) for the exponential modulating function. Again, the values of η vary significantly with ζ. The use of the CQC rule with the UHS seems to be adequate for assessing the peak responses of the systems whose peak responses are dominated by one mode, while it leads to error for the systems having equally significant contributions from both modes to the peak responses. The trend observed in

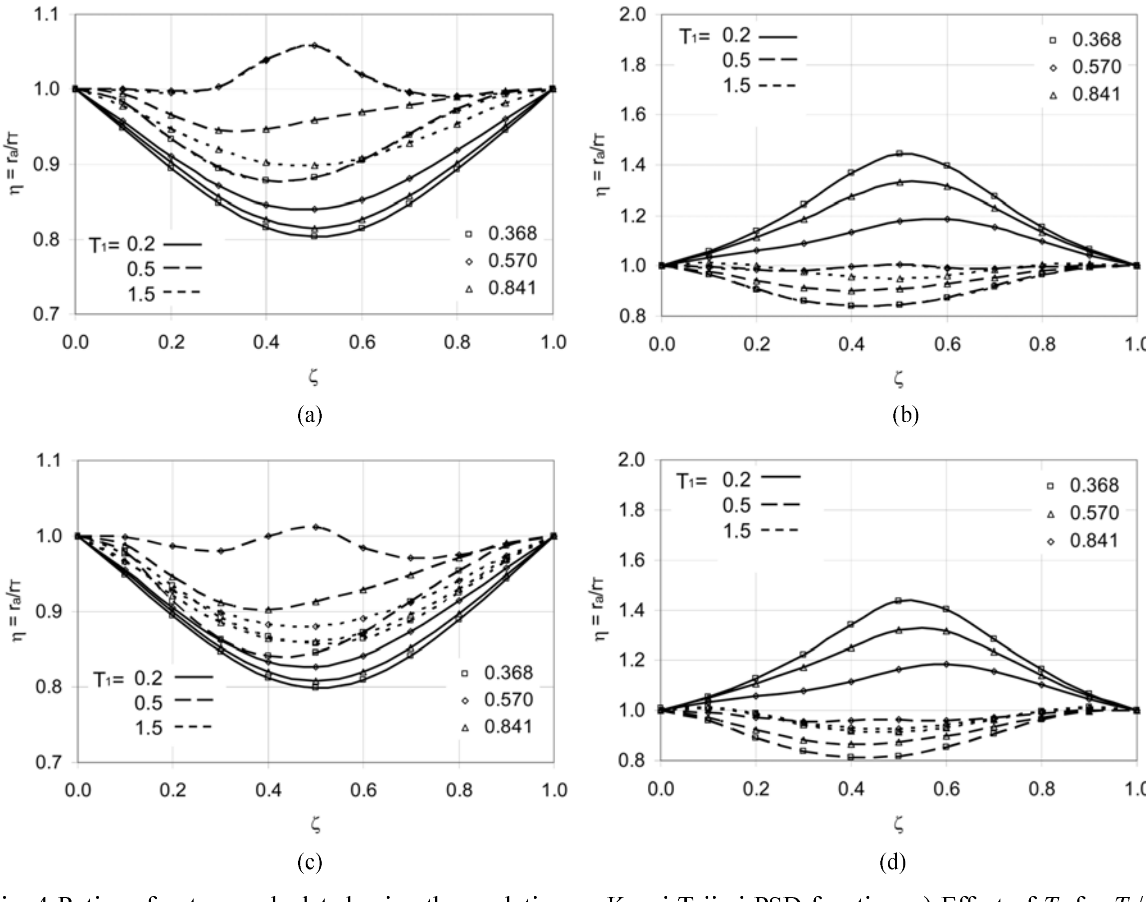


Fig. 4 Ratios of r_a to r_T calculated using the evolutionary Kanai-Tajimi PSD function. a) Effect of T_1 for T_1/T_2 = 2.0, $\mathrm{sgn}(C_1C_2) > 0$, boxcar modulating function, b) Effect of T_1 for T_1/T_2 = 2.0, $\mathrm{sgn}(C_1C_2) < 0$, boxcar modulating function, c) Effect of T_1 for T_1/T_2 = 2.0, $\mathrm{sgn}(C_1C_2) > 0$, exponential modulating function, d) Effect of T_1 for T_1/T_2 = 2.0, $\mathrm{sgn}(C_1C_2) < 0$, exponential modulating function

above that the CQC rule under- and over-estimates the peak response of the system for sgn(C_1C_2) > 0 and sgn(C_1C_2) < 0, respectively, does not hold for this set of systems. It appears that this trend depends on whether T_1 is larger or smaller than the mean value of the filter period T_g($T_g = 2\pi/\omega_g$ = 0.33 sec), as observed for the case of stationary excitation (Hong and Wang 2002).

To investigate the influence of treating the excitations as nonstationary processes versus as stationary processes on η, the values of η for 2DOF systems under both types of excitations are compared in Figs. 5(a) and 5(b) for T_1 = 0.2 sec and T_1/T_2 = 1.2, and in Figs. 5(c) and 5(d) for T_1 = 0.2 sec and T_1/T_2 = 2.0. For the stationary case, the Kanai-Tajimi PSD function is employed; while for the nonstationary case the boxcar or exponential modulating function together with the Kanai-Tajimi PSD function is used. The results shown in Figs. 5(a) and 5(c) are for sgn(C_1C_2)>0 and those in Figs. 5(b) and 5(d) are for sgn(C_1C_2) < 0. The results suggest that when the excitation is considered as a nonstationary process the variations of η are similar to those obtained when the seismic excitation is treated as a stationary process. However, considering the excitation as a nonstationary process defined by an EPSD function may lead to η deviate from unit more significantly. The results also show that the type of the modulating function (boxcar or exponential)

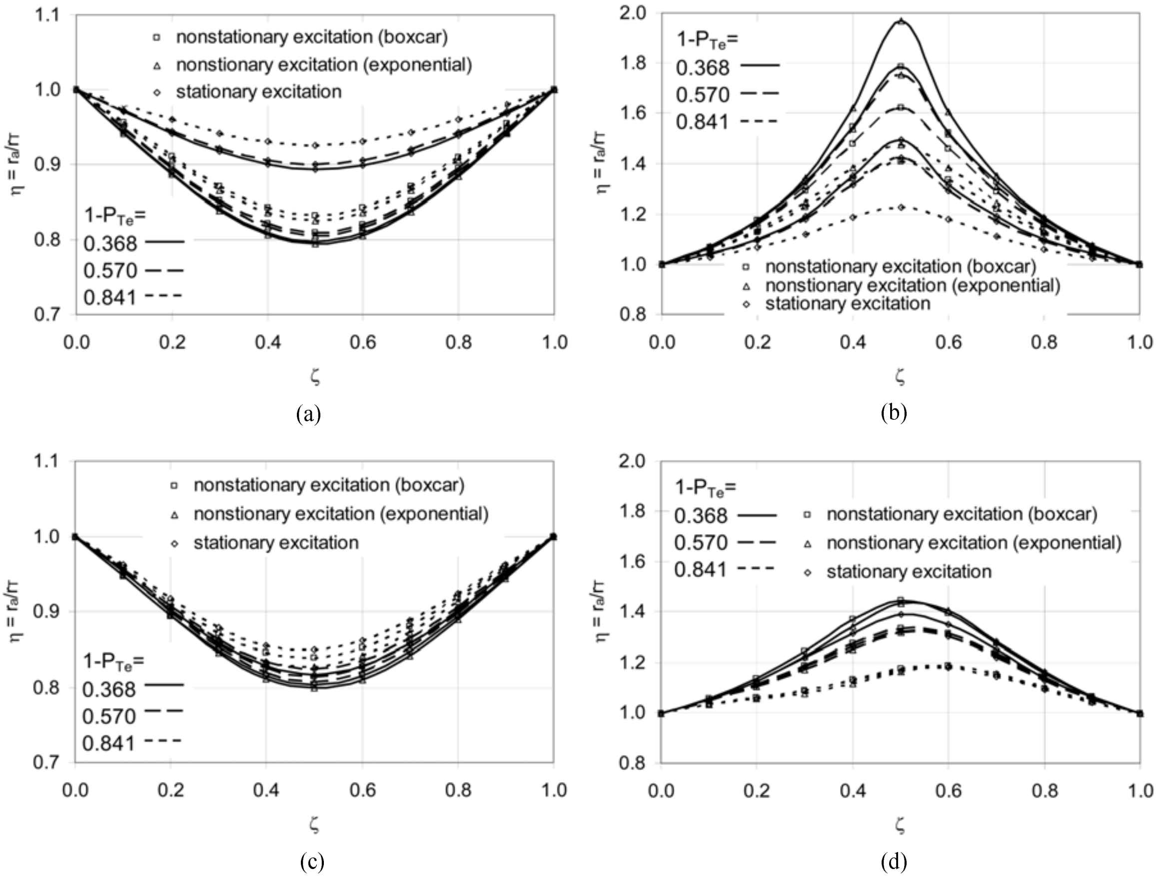


Fig. 5 Comparison between the ratios of stationary cases and nonstationary cases (with the Kanai-Tajimi PSD function). (a) T_1 = 0.2s, T_1/T_2 = 1.2, sgn(C_1C_2) > 0, (b) T_1 = 0.2s, T_1/T_2 = 1.2, sgn(C_1C_2) < 0, (c) T_1 = 0.2s, T_1/T_2 = 2.0, sgn(C_1C_2) > 0, (d) T_1 = 0.2s, T_1/T_2 = 2.0, sgn(C_1C_2) < 0

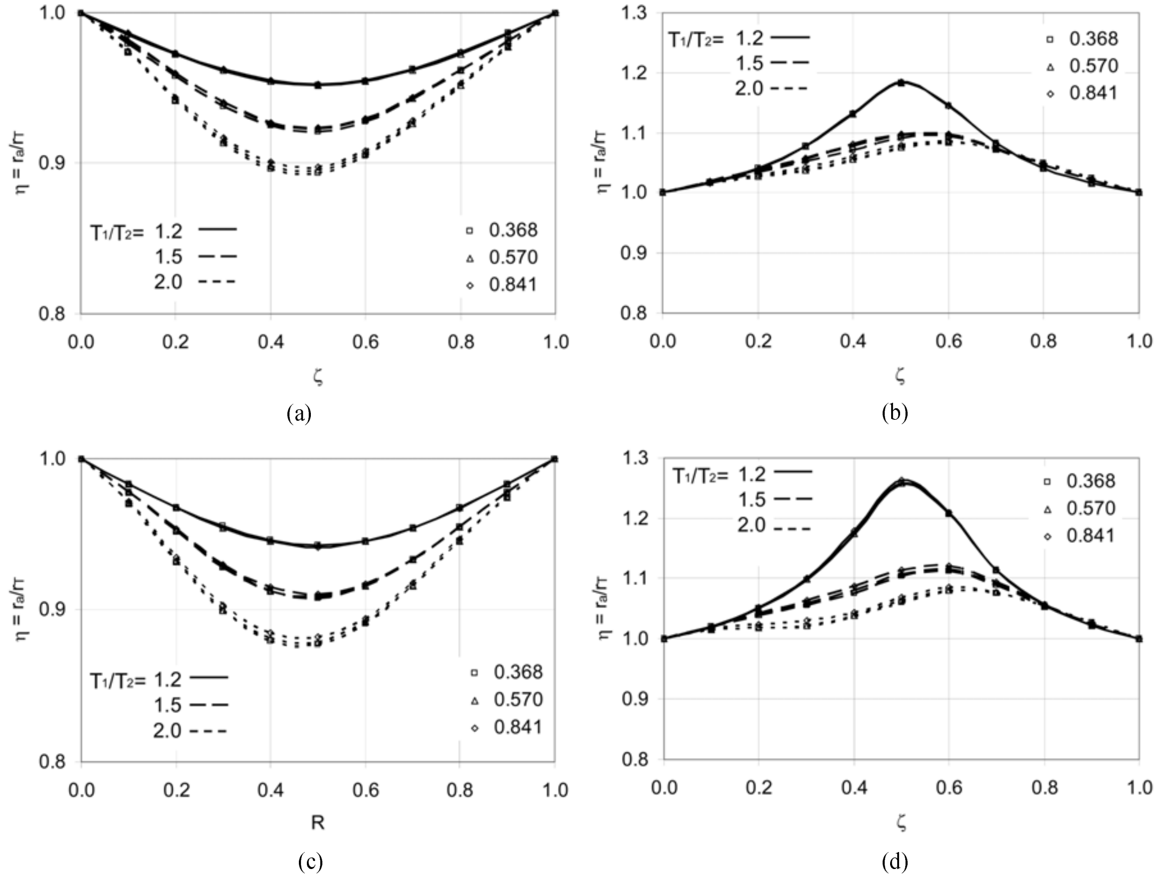


Fig. 6 Ratios of r_a to r_T calculated using the EPSD function for seismological model. (a) Effect of T_1/T_2 for T_1 = 0.2s, sgn(C_1C_2) > 0, boxcar modulating function, (b) Effect of T_1/T_2 for T_1 = 0.2s, sgn(C_1C_2) < 0, boxcar modulating function, (c) Effect of T_1/T_2 for T_1 = 0.2s, sgn(C_1C_2) > 0, exponential modulating function, (d) Effect of T_1/T_2 for T_1 =0.2s, sgn(C_1C_2) < 0, exponential modulating function

employed to modulate the seismic excitation does not affect the value of η considerably.

The above analyses are repeated for the seismic excitations characterized by the EPSD functions based on the seismological model. The obtained values of η for the set of systems with T_1 = 0.2 sec, and T_1/T_2 = 1.2, 1.5, and 2.0 are shown in Figs. 6(a) and 6(b) for the boxcar modulating function and in Figs. 6(c) and 6(d) for the exponential modulating function. Unlike the results shown in Fig. 3, the value of η in Fig. 6 is not substantially affected by the probability of non-exceedance levels. The other observed trends for η by considering the Kanai-Tajimi model seem to be equally applicable for the seismological model. However, as compared to the cases with the Kanai-Tajimi model, the magnitude of the under- and over-estimations are less for the cases with the seismological model especially when ζ is within about 0.4 and 0.6.

The values of η for systems with T_1/T_2 = 2, and T_1 = 0.2, 0.5, and 1.5 sec, are also calculated and shown in Figs. 7(a) and 7(b) for the boxcar modulating function and in Figs. 7(c) and 7(d) for the exponential modulating function. Again, the conclusions drawn from the Fig. 7 are similar to those based on results shown in Fig. 4.

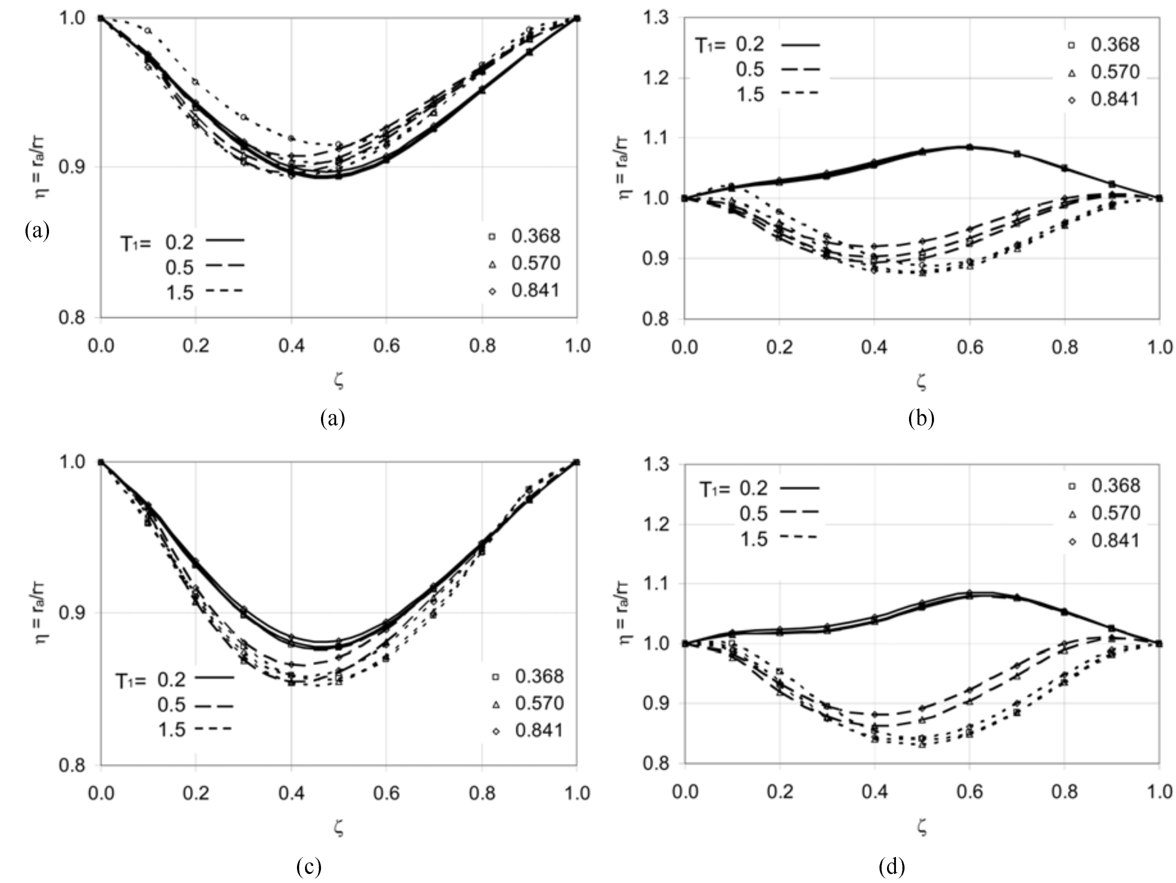


Fig. 7 Ratios of r_a to r_T calculated using the EPSD function for seismological model. (a) Effect of T_1 for T_1/T_2 = 2.0, sgn(C_1C_2) > 0, boxcar modulating function, (b) Effect of T_1 for T_1/T_2 = 2.0, sgn(C_1C_2) < 0, boxcar modulating function, (c) Effect of T_1 for T_1/T_2 = 2.0, sgn(C_1C_2) > 0, exponential modulating function, (d) Effect of T_1 for T_1/T_2 = 2.0, sgn(C_1C_2) < 0, exponential modulating function

5. Conclusions

The adequacy of the use of the complete quadratic combination (CQC) rule with the modal peak responses defined by the ordinates of the uniform hazard spectra (UHS) to calculate the peak responses of multi-degree-of-freedom (MDOF) systems subjected to the nonstationary seismic excitations with uncertain model parameters were assessed. The assessment was based on the ratios of the peak responses obtained from the CQC with the UHS to the fractiles of the MDOF systems. The excitations were characterized by the evolutionary power spectral density (EPSD) functions with the boxcar or the exponential modulating function and the PSD function from the Kanai-Tajimi or the seismological model.

The analysis results show that the use of the CQC rule with the adopted correlation coefficient and the modal responses obtained from the UHS is adequate if the peak responses of MDOF systems are dominated by one of the modes. Otherwise, severe under- or over-estimations may result. More specifically, the over- or under-estimations appear to be

1) Minor or moderate (i.e., less than 10% or 20%) if the peak response of the MDOF system is dominated by one of the modes (i.e., the contribution of the dominant mode is over 90% or 80% of the absolute sum of the effective modal peak response);
2) Severe if the modal contributions to the peak responses of the MDOF system are about equally significant; and
3) Less for the cases when the effective modal participation factors have the same sign than for the cases when they have opposite signs.

The under- or over-estimation is also affected by the modal periods, the probability of non-exceedance levels, the PSD function used (i.e., Kanai-Tajimi versus seismological model), and by the modulating function to a lesser degree. Comparison of the obtained results to the results for stationary cases seem to suggest that the under- or over- estimations for nonstationary excitations are larger than those for stationary excitations. However, the trends of the under- or over-estimations for both types of excitations are similar.

The above conclusions could be used as a guideline for calibrating correlation coefficients that lead to more probability-consistent peak responses of MDOF systems.

Acknowledgements

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