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Stochastic analysis of fluid-structure interaction systems by Lagrangian approach

Alemdar Bayraktar† and Ebru Hançer‡

Karadeniz Technical University, Department of Civil Engineering, 61080, Trabzon, Turkey

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Abstract. In the present paper it is aimed to perform the stochastic dynamic analysis of fluid and fluid-structure systems by using the Lagrangian approach. For that reason, variable-number-nodes two-dimensional isoparametric fluid finite elements are programmed in Fortran language by the authors and incorporated into a general-purpose computer program for stochastic dynamic analysis of structure systems, STOCAL. Formulation of the fluid elements includes the effects of compressible wave propagation and surface sloshing motion. For numerical example a rigid fluid tank and a dam-reservoir interaction system are selected and modeled by finite element method. Results obtained from the modal analysis are compared with the results of the analytical and numerical solutions. The Pacoima Dam record S16E component recorded during the San Fernando Earthquake in 1971 is used as a ground motion. The mean of maximum values of displacements and hydrodynamic pressures are compared with the deterministic analysis results.

Key words: fluid-structure interaction; stochastic dynamic analysis; Lagrangian approach; fluid finite element.

1. Introduction

Gravity dams, arch dams, underground power plants, harbors, fluid tanks etc. are the application fields of fluid-structure interaction problems (Mackerle 1999). In fluid-structure interaction systems, the structure affects the behavior of the fluid as well as the fluid affecting the behavior of structure under a dynamic effect. Consequently, hydrodynamic pressures in the fluid and additional loads in the structure due to hydrodynamic pressures occur. Most fluid-structure interaction analyses are based on simplifying assumptions (e.g. inviscid flow) which allow one of two approaches (Olson and Bathe 1983): (1) displacements are the variables in the solid, pressures are the variables in fluid (Eulerian approach); (2) displacements are the variables in both the fluid and solid (Lagrangian approach). Since the variables in fluid and solid are different in Eulerian approach, a special-purpose computer program is required for the solution of the coupled systems (Fenves and Chopra 1984, Greeves and Dumanoğlu 1989). In the Lagrangian approach, the behavior of the fluid and structure is expressed in terms of displacements. For that reason, compatibility and equilibrium are

† Professor, Corresponding author, E-mail: alemdar@ktu.edu.tr
‡ Research Assistant, E-mail: ebruhancer@hotmail.com

automatically satisfied at the nodes along the interfaces between the fluid and structure. This makes a Lagrangian displacement-based fluid finite element very desirable, which can be readily incorporated into a general-purpose computer program for structural analysis, because special interface equations are not required. It is seen from the literature review that different types of fluid elements are used in the solution of the fluid and fluid-structure interaction problems (Olson and Bathe 1983, Akkaş *et al.* 1979, Bathe and Hann 1979, Zienkewics and Bettess 1978, Hamdi *et al.* 1978, Malkus 1976, Deshpande *et al.* 1981, Wilson and Khavati 1983). In the fluid element formulation proposed by Wilson and Khalvati (1983), reduced integration technique as well as rotational constraints are used to eliminate all unnecessary zero-energy modes. Also, the formulation includes the fluid free surface sloshing motion. Fluid element proposed by Wilson and Khalvati (1983) was used by many researchers to determine the dynamic responses of fluid and fluid-structure interaction problems (Greeves 1990, Calayır and Dumanoğlu 1993, Calayır 1994, Bayraktar 1995, Bayraktar *et al.* 1996). In these papers, loads due to earthquake forces are considered as deterministic. However, seismic actions have essentially stochastic characteristics, therefore they should be considered as random loads (Lin 1967). Dynamic responses of fluid-structure interaction systems subjected to random loads have been investigated by Araujo and Awruch (1998) and Di Paola and Zingales (2003). In these papers, Eulerian approach is used to determine the fluid response.

The focus of the present paper is to determine the stochastic dynamic response of fluid and fluid-structure systems by using the Lagrangian (displacement-based) fluid finite elements. For that reason, variable-number-nodes two-dimensional fluid finite elements proposed by Wilson and Khalvati (1983) were programmed in FORTRAN language by the authors and incorporated into a general-purpose computer program for stochastic dynamic analysis of solid systems, STOCAL (Button *et al.* 1981). The program STOCAL is modified for the stochastic dynamic analysis of fluid-structure interaction systems based on the Lagrangian approach and named as STOCALF. Stationary and ergodic assumptions are made for the stochastic processes and results are obtained using the program STOCALF.

2. Formulation

2.1 Finite element formulation of fluid systems

The formulation of the fluid system based on Lagrangian approach is given according to (Wilson and Khalvati 1983, Calayır and Dumanoğlu 1993). Fluid is assumed to be linear-elastic, inviscid and irrational. For this fluid, the relation between pressure and volumetric strain is given by

$$P = \beta\varepsilon_v \tag{1}$$

where P is pressure, β is the bulk modulus of the fluid, and ε_v is the volumetric strain which can be expressed in terms of the displacements. For two-dimensional problems, ε_v can be expressed as follows;

$$\varepsilon_v = \frac{\partial u_{fy}}{\partial y} + \frac{\partial u_{fz}}{\partial z} \tag{2}$$

where u_{fy} and u_{fz} are the components of the displacement in the y and z directions, respectively.

To enforce the rotational constraint, the following rotation is defined by

$$w = \frac{1}{2}\left(\frac{\partial u_{fy}}{\partial z} - \frac{\partial u_{fz}}{\partial y}\right) \tag{3}$$

where w is rigid body rotation about the axis normal to the plane. The relation between the stress and stiffness associated with this rotation is given by

$$P_w = \alpha w \tag{4}$$

where P_w and α are the rotational stress and stiffness (constraint parameter), respectively.

Using the Eqs. (1)-(4), the total strain energy of the fluid system can be expressed as follows;

$$\Pi_e = \frac{1}{2}\int \mathbf{e}^T \mathbf{C}_f \mathbf{e} dV \tag{5}$$

where $\mathbf{e}$ is a vector of strains given by $\mathbf{e}^T = [\varepsilon_v \ \ w]$ and $\mathbf{C}_f$ is a diagonal matrix whose elements are given by the elasticity parameters in Eqs. (1) and (4). Using the finite element method, Eq. (5) may be expressed as

$$\Pi_e = \frac{1}{2}\mathbf{u}_f^T \mathbf{K}_f \mathbf{u}_f \tag{6}$$

where $\mathbf{K}_f$ and $\mathbf{u}_f$ are the stiffness matrix and the nodal displacement vector of the fluid system, respectively.

An important behavior of fluid systems is the ability to displace without a change in volume. For reservoir and storage tanks, this movement is known as sloshing waves in which the displacement is in the vertical direction. The increase in the potential energy of the system due to the free surface motion can be written as

$$\Pi_s = \frac{1}{2}\int \rho g u_{fs}^2 dA \tag{7}$$

where ρ and g are the mass density of the fluid and the gravitational acceleration, respectively, and u_{fs} is the free surface vertical displacement of the fluid. Using the finite element method, the free surface potential energy, Eq. (7), is expressed in terms of the vertical node displacements at the surface as

$$\Pi_s = \frac{1}{2}\mathbf{u}_{fs}^T \mathbf{S}_f \mathbf{u}_{fs} \tag{8}$$

where $\mathbf{S}_f$ and $\mathbf{u}_{fs}$ are the free surface stiffness matrix and the free surface vertical displacement vector of the fluid system, respectively.

Finally, to complete the energy contributions, the kinetic energy of the fluid must be considered. This energy is given by

$$T = \frac{1}{2}\int \rho(\dot{u}_{fy}^2 + \dot{u}_{fz}^2) dV \tag{9}$$

where $\dot{u}_{fy}$ and $\dot{u}_{fz}$ are the components of the velocity in the y and z directions, respectively. Using the finite element method, Eq. (9) can be written in the form

$$T = \frac{1}{2}\dot{\mathbf{u}}_f^T \mathbf{M}_f \dot{\mathbf{u}}_f \qquad (10)$$

where $\mathbf{M}_f$ and $\dot{\mathbf{u}}_f$ are the mass matrix and the nodal velocity vector of the fluid system, respectively.

The direct application of Lagrange's equation (Clough and Penzien 1975) to Eqs. (6), (8) and (10) yields the following set of equations

$$\mathbf{M}_f \ddot{\mathbf{u}}_f + \mathbf{K}_f^* \mathbf{u}_f = \mathbf{F}_f \qquad (11)$$

where $\mathbf{K}_f^*$ and $\mathbf{F}_f$ are the system stiffness matrix including the free surface stiffness and time-varying nodal forces vector for the fluid system, respectively.

2.2 Stochastic analysis formulation of fluid-structure interaction systems

Equations of motion for fluid system, Eq. (11), have similar form with that of the structure when Lagrangian approach is used. But, it requires a different sensitivity to determine interface condition of the system. At the interface of the fluid-structure system, only the displacements in normal direction to the interface are assumed to be compatible in the structure as well as in the fluid. This condition is imposed by the constraint equations (Bathe 1982).

In matrix form, the equations of motion of a fluid-structure system based on Lagrangian approach with N degrees of freedom are

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = -\mathbf{M}\boldsymbol{\delta}\ddot{u}_g(t) \qquad (12)$$

where $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$ are $n \times n$, positive definite, mass, damping and stiffness matrices; $\mathbf{u}(t)$, $\dot{\mathbf{u}}(t)$, $\ddot{\mathbf{u}}(t)$ are the vectors of displacement, velocity and acceleration, respectively. $\boldsymbol{\delta}$ is the direction vector that links the mass terms to the ground acceleration, $\ddot{u}_g(t)$.

As seen from Eq. (12), equation of motion of fluid-structure system based on Lagrangian approach is the same form with that of the structural systems. Since the formulation of the stochastic dynamic analysis of structural systems has been well known for many years, only the final equations will be given in this study. Detailed formulations for stochastic dynamic analysis are given in (Lin 1967, Clough and Penzien 1975, Yang 1986, Manolis and Koliopoulos 2001).

Here we assume that the natural frequencies and mode shapes of the fluid-structure system have been calculated from the equations of motion for a freely vibrating undamped system previously (Clough and Penzien 1975).

The modal force corresponding to the jth mode can be obtained by

$$P_j(t) = \phi_j^T \mathbf{M}\boldsymbol{\delta}\ddot{u}_g(t) \qquad (13)$$

The Fourier transform of the Eq. (13) in the frequency domain may be written as

$$P_j(\omega) = \phi_j^T \mathbf{M}\boldsymbol{\delta}A(\omega) \qquad (14)$$

where $A(\omega)$ is the Fourier transform of the ground acceleration.

For all the modal forces taken into account,

$$\mathbf{P}(\omega) = \mathbf{\Phi}^T\mathbf{M}\delta A(\omega) \tag{15}$$

in which $\mathbf{\Phi}$ is the matrix of the modal shapes.

For the jth mode, the relationship between the modal coordinates $Y_j(t)$ and modal forces may be expressed in the frequency domain as follow

$$Y_j(\omega) = H_j(\omega)P_j(\omega) \tag{16}$$

where $H_j(\omega)$ is the complex frequency response function for jth mode and can be defined by

$$H_j(\omega) = \frac{1}{(\omega_j^2 - \omega^2 + 2i\xi_j\omega_j)} \tag{17}$$

Here ω_j and ξ_j are the natural frequency and the damping ratio corresponding jth mode.

For all modes taken together

$$\mathbf{Y}(\omega) = \mathbf{H}(\omega)\mathbf{\Phi}^T\mathbf{P}(\omega) \tag{18}$$

where $\mathbf{H}(\omega)$ is the diagonal matrix of the $H_j(\omega)$ from Eq. (17).

A structural response $u_j(t)$ in Eq. (12) may be expressed in terms of modal coordinates as,

$$u_j(t) = \sum_{r=1}^{N}\Psi_{jr}Y_r(t) \tag{19}$$

where N is the number of modes which are considered to contribute to the response, Ψ_{jr} is the contribution of the jth mode to the $u_j(t)$, $Y_r(t)$ is the modal coordinate. The Fourier transform of Eq. (19) gives

$$U_j(\omega) = \mathbf{\Psi}_j^T\mathbf{Y}(\omega) \tag{20}$$

Substituting Eq. (18) into Eq. (20)

$$U_j(\omega) = \mathbf{\Psi}_j^T\mathbf{H}(\omega)\mathbf{\Phi}^T\mathbf{P}(\omega) \tag{21}$$

For the response displacements $u_i(t)$ and $u_j(t)$, the cross power spectral density function, $S_{ij}(\omega)$, may be obtained as (Lin 1967, Clough and Penzien 1975)

$$S_{ij}(\omega) = \lim_{T\to\infty}\frac{1}{n}\sum_{k=1}^{n}\frac{2U_{i,k}(\omega)U_{j,k}^*(\omega)}{T}, \qquad n\to\infty \tag{22}$$

where * shows complex conjugate and T is the duration of the process. By substituting Eq. (21) into Eq. (22)

$$S_{ij}(\omega) = \mathbf{\Psi}_i^T\mathbf{H}(\omega)\mathbf{\Phi}^T\mathbf{S}(\omega)\mathbf{\Phi}\mathbf{H}(\omega)\mathbf{\Psi}_j \tag{23}$$

where $\mathbf{S}(\omega)$ represents the spectral density function of the input.

If a single ground acceleration record is used for the input, then by substituting Eq. (15) into Eq. (21) and using Eq. (22), the cross power spectral density function, $S_{ij}(\omega)$, can be simplified as (Button *et al.* 1981)

$$S_{ij}(\omega) = S_{in}(\omega)\sum_{r=1}^{N}\sum_{s=1}^{N}\Psi_{ir}\Psi_{js}H_{ir}(\omega)H_{js}^{*}(\omega) \tag{24}$$

where ω is the frequency, $S_{in}(\omega)$ is the power spectral density function of the input.

The basic equation to obtain the autocorrelation function or the cross-correlation function of the stationary output of a multi-degree of freedom system is the modal cross-correlation function given by (Clough and Penzien 1975)

$$R_{rs}(t_1 - t_2) = \int_{-\infty}^{\infty} S_{in}(\omega)H_r(\omega)H_s^{*}(\omega)e^{i\omega(t_1 - t_2)}d\omega \tag{25}$$

where t is the time. The cross-correlation function for the displacements in the nodal points i and j can be expressed as (Clough and Penzien 1975)

$$R_{ij}(\tau) = \sum_{r=1}^{N}\sum_{s=1}^{N}\Psi_{ir}\Psi_{js}R_{rs}(\tau) \tag{26}$$

where $\tau = t_1 - t_2$

Statistics related to the structural behaviour can be determined for a stationary process using the zeroth, the first and the second spectral moments of the output process. Spectral moments, which can be expressed in terms of power spectral density function and frequency, may be determined as follow (Button *et al.* 1981)

$$\lambda_{m,\,ij} = 2\int_{0}^{\infty}\omega^{m}S_{ij}(\omega)d\omega, \qquad m = 0, 1, 2 \tag{27}$$

Here, m = 0, 1, 2 is the zeroth, the first and the second spectral moments, respectively. Substituting Eq. (24) into Eq. (27), $\lambda_{m,\,ij}$ may be obtained as (Button *et al.* 1981)

$$\lambda_{m,\,ij} = \sum_{r=1}^{N}\sum_{s=1}^{N}\Psi_{ir}\Psi_{js}\lambda_{m,\,rs} \tag{28}$$

where

$$\lambda_{m,\,rs} = 2Re\int_{0}^{\infty}\omega^{m}S_{in}(\omega)H_{ir}(\omega)H_{js}^{*}(\omega)d\omega \tag{29}$$

in which $\lambda_{m,\,rs}$ is the cross-spectral moment of the normal coordinates with modes r and s. Re shows the real part.

The cross-correlation coefficients related with the spectral moments can be expressed as

$$\rho_{m,\,rs} = \frac{\lambda_{m,\,rs}}{\sqrt{\lambda_{m,\,rr}\lambda_{m,\,ss}}}, \qquad m = 0, 1, 2 \tag{30}$$

Substituting Eq. (30) into Eq. (28) gives

$$\lambda_{m,\,ij} = \sum_{r=1}^{N}\sum_{s=1}^{N} \Psi_{ir}\Psi_{js}\rho_{m,\,rs}\sqrt{\lambda_{m,\,rr}\lambda_{m,\,ss}}, \qquad m = 0,\,1,\,2 \tag{31}$$

$\lambda_{m,\,rr}$ shows the spectral moments of the response of a single degree system with the frequency ω_n and the damping coefficient ξ_r to the specified input.

The mean of maximum value is considered to be the most important parameter in the analysis of structures affected by dynamic loads. In stochastic analysis the mean of maximum value μ is the mean value of all maximum values. The mean of maximum value depends on the peak factor and on the root-mean-square response, which can be expressed as (Der Kiureghian 1980)

$$\mu = p\sqrt{\lambda_0} \tag{32}$$

and its standard deviation can be given as

$$\sigma = q\sqrt{\lambda_0} \tag{33}$$

where λ_0 is the spectral moment, p and q are the peak factors, which are the functions of the duration of the motion and the mean zero crossing rate, respectively.

Frequency of occurrence is the average number of times in which the line ($y(t) = 0$) is crossed by the response in a unit of time. For a Gaussian process, the mean zero-crossing rate is given as,

$$v = \frac{1}{\pi}\frac{\sigma_{\dot{y}}}{\sigma_y} = \frac{1}{\pi}\sqrt{\frac{\lambda_2}{\lambda_0}} \tag{34}$$

Frequency of occurrence will be equal to $v/2$, because the zero level is crossed two times for each cycle. Therefore, frequency of occurrence may be obtained as (Der Kiureghian 1980)

$$f_0 = \frac{v}{2} = \frac{1}{2\pi}\sqrt{\frac{\lambda_2}{\lambda_0}} \tag{35}$$

where λ_0, λ_2 are the zeroth and the second spectral moments, respectively.

For any stochastic analysis it is valuable to be able to calculate the probability of occurrence of a particular value of the selected structural response quantity, and this has been achieved by Vanmarcke (1975) through a cumulative probability distribution function for the first crossing time of a symmetric barrier for a zero-mean stationary Gaussian process.

3. Numerical applications

In this study, it is aimed to perform the stochastic dynamic analysis of fluid and fluid-structure systems by using the Lagrangian approach. A rigid fluid tank and a dam-reservoir interaction system are selected as numerical applications. These systems are modeled by the finite element method.

3.1 Modal analysis of rigid fluid tank

A rectangular rigid fluid tank is firstly selected to evaluate the correctness of the frequencies

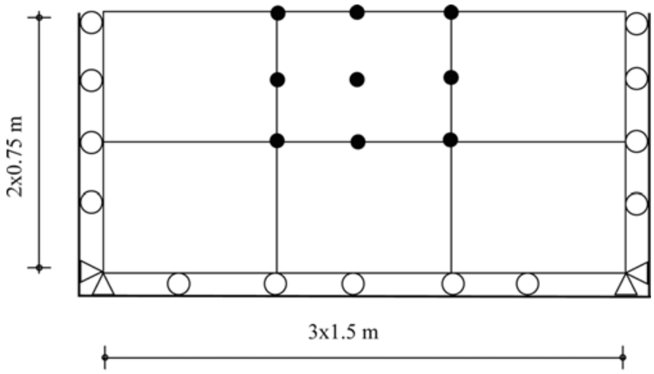

Fig. 1 The finite element model of the rigid fluid tank

obtained from program STOCALF. The finite element model of the rigid fluid tank is given in Fig. 1. In the finite element model, nine-noded isoparametric quadrilateral fluid finite elements, in which surface sloshing is taken into account, is used. Although the nodes in the fluid domain can make horizontal and vertical displacement, the vertical and horizontal exterior edge of the rigid tank can only make vertical and horizontal displacement, respectively.

Element matrices are computed using 2×2 reduced integration orders. The bulk modulus and the mass density of the fluid, and the gravitational acceleration are taken as 207×10^7 N/m^2, 1000 kg/m^3, 9.81 m/s^2, respectively. The sloshing, volume change and rotational modes occur in modal analysis of fluids when the Lagrangian approach is used. The rotational frequencies increase with the increasing rotational constraint parameter. As a result of this, the frequencies related with volume change go towards the first range in the frequency table. This causes a decrease in the number of frequencies to be found in the modal analysis. In case of the nine-noded fluid element it is offered that the rotational constraint parameter can be chosen as 100 times of the bulk modulus (Wilson and Khalvati 1983, Calayır and Dumanoğlu 1993). In this study, the value of rotational constraint parameter is taken as 100 times of the bulk modulus. The first 9 frequencies calculated from the program STOCALF for the rigid fluid tank is shown in Table 1. The first 5 frequencies are surface sloshing and the others are volume change frequencies. It can be seen from Table 1 that the frequencies obtained from the program STOCALF are very close to the frequencies calculated using the same finite element model by Calayır (1994) and Bayraktar (1995). It is thought that the small differences between the results are due to the solution technique of the program STOCALF. Because the mass values of each degree of freedom in the program STOCALF are calculated with hand and given as input from outside, and a different solution technique is used in modal analysis. The first surface sloshing frequency for the rigid fluid tank is obtained as 0.368 Hz with the analytical solution given by Lamb (1932). The first volume change frequency from analytical

Table 1 The first 9 frequencies of rigid fluid tank

Frequency (Hz)	1	2	3	4	5	6	7	8	9
Calayır (1994)	0.368	0.398	0.571	0.650	0.668	239.800	287.700	394.500	490.100
Bayraktar (1995)	0.368	0.398	0.571	0.650	0.668	239.800	287.700	394.500	490.100
STOCALF	0.296	0.364	0.522	0.544	0.596	236.981	285.071	387.334	529.268

solution (Olson and Bathe 1983) is obtained as 239.791 Hz. It can be seen from Table 1 that the frequencies obtained from analytical solutions are very close to the frequencies obtained from the program STOCALF.

3.2 Stochastic analysis of fluid-structure interaction systems

The dimensions of the dam and the finite element model of the dam-reservoir interaction system selected for the determination of the stochastic dynamic responses of fluid-structure interaction systems subjected to earthquakes are given in Figs. 2 and 3, respectively. The dam and the reservoir are represented by eight-noded and nine-noded isoparametric quadrilateral solid and fluid finite elements, respectively. There are two unknown displacements at each nodal point in the dam and reservoir. Element matrices for fluid and solid systems are computed using 2×2 and 3×3

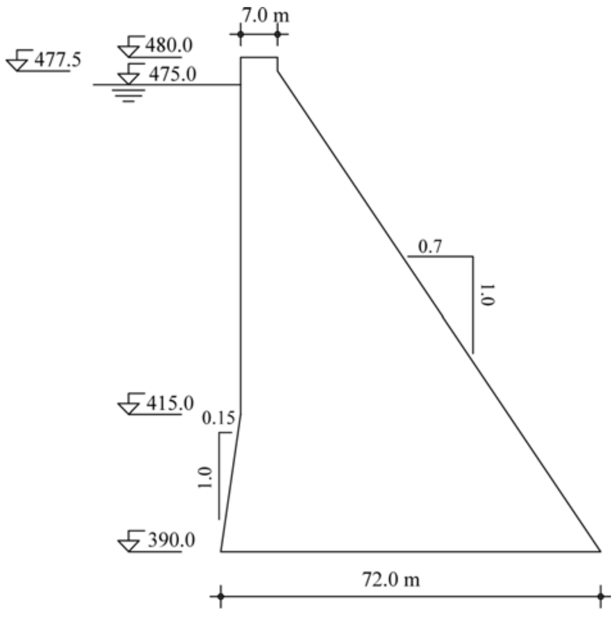

Fig. 2 The dimensions of the dam

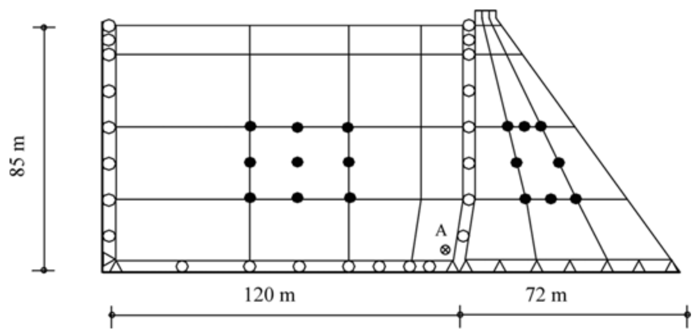

Fig. 3 The finite element model of the dam-reservoir interaction system

Table 2 The first 10 frequencies of dam-reservoir interaction system

Frequency (Hz)	1	2	3	4	5	6	7	8	9	10
Bayraktar (1995)	0.075	0.078	0.111	0.126	0.133	0.150	0.177	3.944	4.983	7.343
STOCALF	0.059	0.077	0.098	0.113	0.135	0.154	0.173	3.738	4.801	7.149

integration orders, respectively (Bathe 1982). The normal components of the displacements of the reservoir-dam interfaces are to be continuous. This condition is accomplished by using short and axially almost rigid truss elements in the normal direction of the interfaces. Plane strain conditions are taken into account in the calculations. The dam material is assumed to be linear-elastic, homogeneous and isotropic. For the dam, the elasticity modulus, the unit weight and Poisson's ratio are chosen as 35×10^9 N/m^2, 24×10^3 N/m^3, 0.20, respectively. For the reservoir, the bulk modulus and the mass density are taken as 207×10^7 N/m^2, 1000 kg/m^3, respectively. The value of rotational constraint parameter is taken as 100 times of the bulk modulus. The selection of the mode number in the modal analysis that is based on the Lagrangian approach is very important. The number of surface sloshing modes, which vary with the finite element model of the reservoir, becomes very much. These modes take place in the first ranges of the frequency table. The effects of these modes on the behavior of dams are very little. For that reason, the first 20 modes are taken into account in this study (Calayır and Dumanoğlu 1993).

3.2.1 Frequencies

The first 10 frequencies obtained from the modal analysis of dam-reservoir interaction system by using the program STOCALF and Bayraktar (1995) are given in Table 2. For dam-reservoir interaction system, the first 7 frequencies are surface sloshing, and the others are volume change frequencies. As can be seen from the tables that the frequencies obtained from both programs are close to each other.

3.2.2 Displacements

A stationary assumption is made for the stochastic dynamic analysis where the statistical parameters are independent of time. Also ergodic assumptions are made for the stochastic processes. The Pacoima Dam record S16E component recorded during the San Fernando earthquake in 1971 is used as a ground motion (Fig. 4). The earthquake motion continued up 13.5 s is applied to the dam-reservoir system in horizontal direction. The power spectral density function of Pacoima Dam record is determined with the Fourier transforms of the autocorrelation function as shown in Fig. 5. A damping ratio of 5% and a time interval of 0.005 s are adopted for calculating displacements.

The stochastic and deterministic dynamic analyses of dam-reservoir interaction system are performed by the programs STOCALF and MULSAPF (Bayraktar 1995), respectively. To show the correctness of the results of the stochastic dynamic analysis, the time history of the horizontal displacement on the upstream crest point of the dam obtained from deterministic dynamic analysis is depicted in Fig. 6. Taking the average of 18 maximum horizontal displacements from Fig. 6, the mean of maximum horizontal displacement is calculated as 3.48 cm. The mean of maximum horizontal displacement obtained from stochastic dynamic analysis is 3.44 cm. These values are close to each other. The frequency of occurrence values of the horizontal displacements on the upstream face of the dam is shown in Fig. 7. From Fig. 7, the frequency of occurrence value of the horizontal displacement at the upstream crest is 4.55 Hz, with period of 0.22 s. The period of

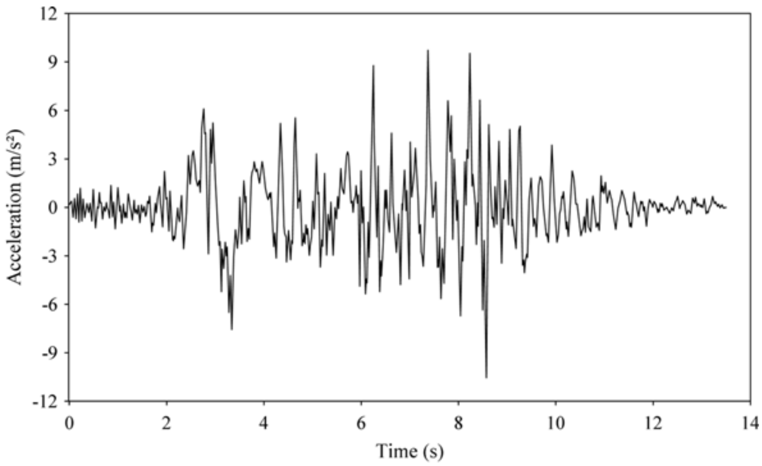

Fig. 4 Acceleration record of S16E component of San Fernando earthquake in 1971

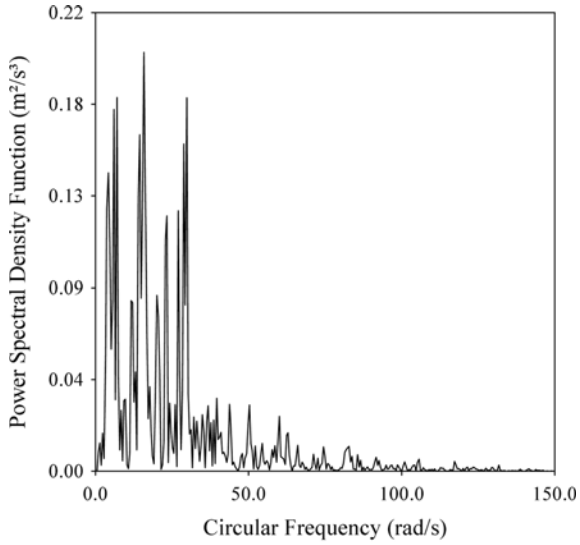

Fig. 5 Power spectral density function of S16E component of San Fernando earthquake in 1971

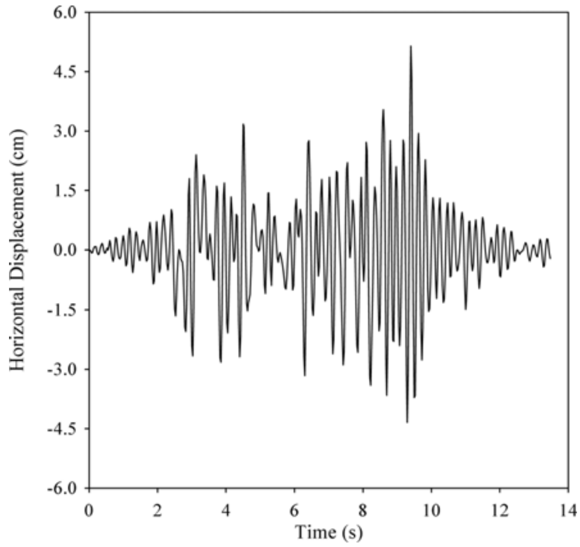

Fig. 6 Time history of the horizontal displacement at the upstream crest point of the dam

maximum horizontal displacement is 0.23 s from Fig. 6. As can be seen, the periods obtained for both programs are very close to each other. This situation shows the accuracy of the stochastic dynamic analysis results.

Maximum horizontal displacements on the upstream face of the dam are plotted in Fig. 8. As it is expected, the mean of maximum values of horizontal displacements are smaller than the absolute maximum horizontal displacements obtained from deterministic dynamic analysis. Since the mean of maximum values are obtained by averaging all the maximum response values in stochastic dynamic analysis, it should be expected that the absolute maximum values obtained from deterministic dynamic analysis will be higher than the mean of maximum values.

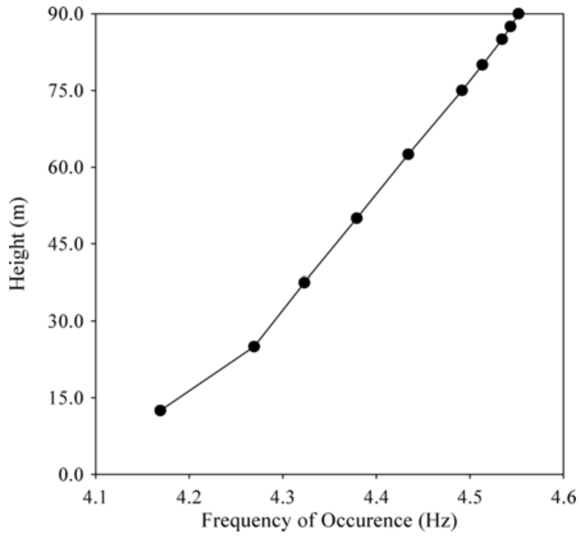

Fig. 7 Frequency of occurrence values of horizontal displacements on the upstream face of the dam

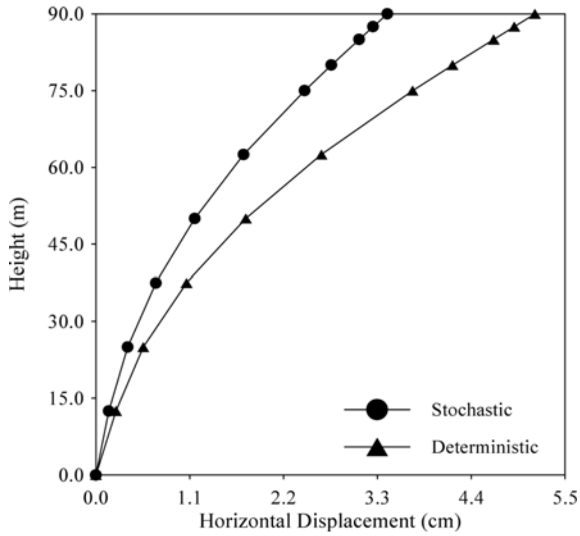

Fig. 8 Horizontal displacements on the upstream face of the dam

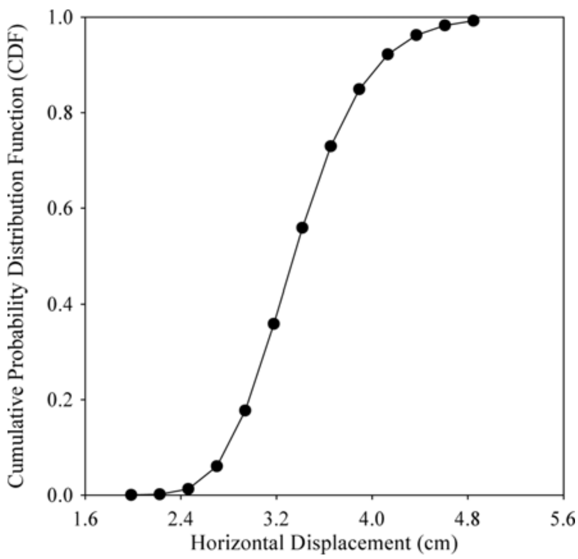

Fig. 9 CDF of the horizontal displacement at the upstream crest point of the dam

An important characteristic of the stochastic dynamic analysis is being informed about the probability of the responses by using cumulative probability distribution functions. The cumulative probability distribution function (CDF) of the horizontal displacement at the crest of the dam is shown in Fig. 9. From Fig. 9, the probability of exceeding the maximum horizontal displacement of 3.44 cm is about 58%. Also probability of occurrence of a maximum value under 2 cm is very low, probability of occurrence of a maximum value under 4.8 cm is 100%.

3.2.3 Hydrodynamic pressures

To control the accuracy of the hydrodynamic pressures obtained from stochastic dynamic analysis of the dam-reservoir interaction system, time history of the deterministic hydrodynamic pressure at the Gauss point A of the fluid element near the upstream heel (Fig. 3) is plotted in Fig. 10. The hydrodynamic pressure is obtained as 526.42 kN/m^2 by taking the average of 18 maximum hydrodynamic pressures from Fig. 10. This is quite close to 523.17 kN/m^2, which is obtained from

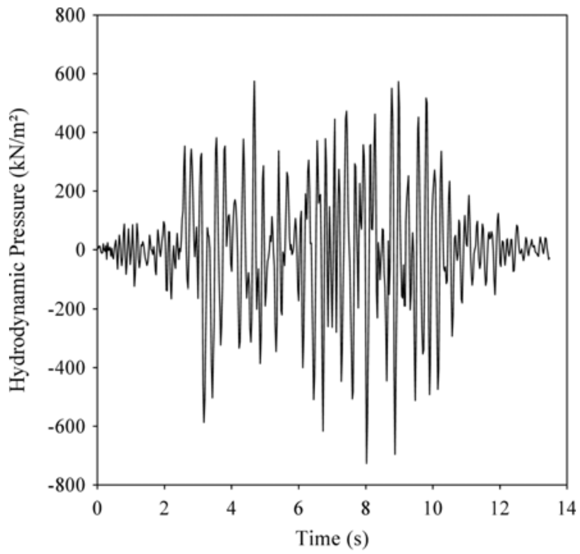

Fig. 10 Time history of the hydrodynamic pressure at Gauss point A

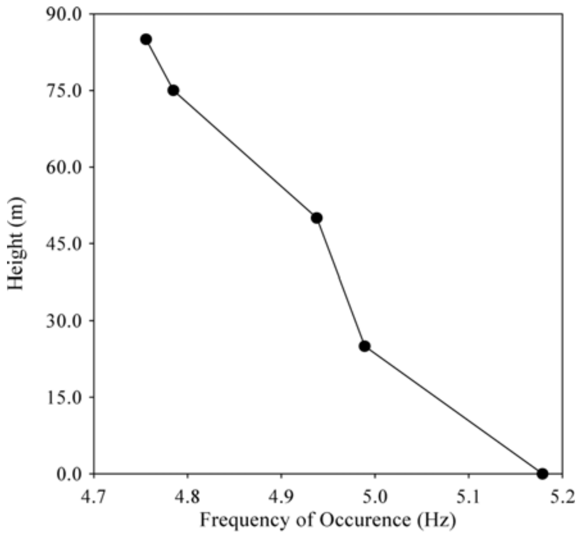

Fig. 11 Frequency of occurrence values of the hydrodynamic pressure on the upstream face of the dam

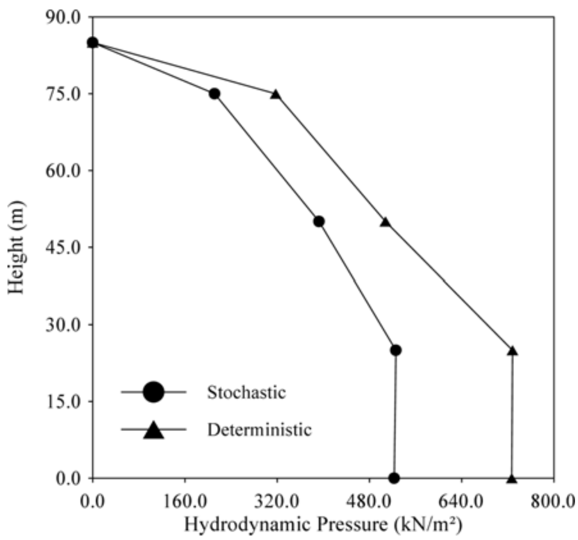

Fig. 12 Hydrodynamic pressure envelopes on the upstream face of the dam

stochastic dynamic analysis. Frequency of occurrence values for the hydrodynamic pressures on the upstream face of the dam is given in Fig. 11. From this figure, the frequency of occurrence value of the hydrodynamic pressure at Gauss point A is 5.179 Hz with period of 0.19 s, which is close to the period of the maximum hydrodynamic pressure, 0.20 s, in Fig. 10.

Hydrodynamic pressures occurring on the upstream face of the dam are calculated using the stochastic and the deterministic dynamic analysis based on the Lagrangian approach. The changing of the hydrodynamic pressures along the dam height is plotted in Fig. 12. Hydrodynamic pressure values are obtained at Gauss points, which are in the fluid elements near the face of the reservoir-dam interaction system. The mean of maximum values of the hydrodynamic pressures are smaller than the absolute maximum hydrodynamic pressures obtained from the deterministic dynamic analysis (Fig. 12).

4. Conclusions

The stochastic dynamic responses of fluid and fluid-structure systems based on Lagrangian approach are investigated in this paper. Variable-number-nodes two-dimensional isoparametric fluid finite elements are programmed in FORTRAN language by the authors and incorporated into a general-purpose computer program for stochastic dynamic analysis of structural systems, STOCAL and named as STOCALF. A rigid fluid tank and dam-reservoir interaction system are selected as numerical examples to evaluate the correctness of the results obtained from the program STOCALF. The frequency values of rigid fluid tank and dam-reservoir interaction system are close to analytical and numerical solutions. Frequency of occurrences of mean of maximum values of displacements and hydrodynamic pressures are compatible with the results of the deterministic dynamic analysis.

As a result, it may be generally stated that the mean of maximum values of the displacements and the hydrodynamic pressures obtained from the stochastic dynamic analysis of fluid-structure systems are smaller than the deterministic results.

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