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Nonlinear section model for analysis of RC circular tower structures weakened by openings

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Abstract. This paper presents the section model for analysis of RC circular tower structures based on nonlinear material laws. The governing equations for normal strains due to the bending moment and the normal force are derived in the case when openings are located symmetrically in respect to the bending direction. In this approach the additional reinforcement at openings is also taken into account. The mathematical model is expressed in the form of a set of nonlinear equations which are solved by means of the minimization of the sums of the second powers of the residuals. For minimization the BFGS quasi-Newton and/or Hooke-Jeeves local minimizers suitably modified are applied to take into account the box constraints on variables. The model is verified on the set of data encountered in engineering practice. The numerical examples illustrate the effects of the loading eccentricity and size of the opening on the strains and stresses in concrete and steel in the cross-sections under consideration. Calculated results indicate that the additional reinforcement at the openings increases the resistance capacity of the section by several percent.

Key words: tower; structure; reinforced concrete; opening; box-constrained optimization.

1. Introduction

Determination of the normal strains and stresses in cross-sections of tower structures has been analysed as a theoretical problem as well as a practical one. The ultimate load analysis of a shell with the circular cross-section weakened by one opening is presented in the monograph by Pinfold (1984). The similar approach is also used in Nieser and Engel (1986), DIN 1056 code (1984) and CICIND code (1998). The generalized linear section model for analysis for RC chimneys weakened by openings was proposed in Lechman and Lewinski (1994, 2001). Despite the generality of the referred papers there are no appropriate analytical formulae describing elasto-plastic behaviour of the circular tower structures and covering the majority of important problems encountered in

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engineering practice. In particular, the assumption of the central layout of steel reinforcement in the wall of tower structures commonly used, may not be justified. Furthermore, the effect of the additional steel bars at openings on the resistance capacity of the section, should be examined. In this paper, the governing equations for the normal strains and stresses due to the bending moment and the normal force are derived in the case when openings are located symmetrically in respect to the bending direction. The normal tensile stresses in concrete are neglected, and the reinforcing steel can be continuously spaced at l layers $(l \in N)$. The constitutive equations for both concrete and steel are assumed to be nonlinear, while the concrete is described as an elasto-plastic material in compression and brittle in tension. Furthermore, the strains are assumed to be small and their distribution across the section to be linear. The numerical iterative technique is applied for the solution of the obtained equations based on the modified BFGS and Hooke-Jeeves methods (see e.g. Bazaraa *et al.* 1993, Bertsekas 1997, Fletcher 1970, 1987, Hooke and Jeeves 1961, Stachurski and Wierzbicki 2001).

2. Derivation of equations for the section with one or two openings

The annular cross-section, described by the outer radius – R and the inner radius – r, is assumed to be weakened by one or two openings. The locations of the openings are determined by couples of the angular coordinates $(0,\alpha_1),(\alpha_2,\pi), 0 \le \alpha_1 \le \alpha_2 \le \pi$. The reinforcing steel spaced in a general case continuously at l layers can be replaced by a continuous ring of equivalent area located on the reference circumference of radius r_s. The section under consideration is subject to the normal force

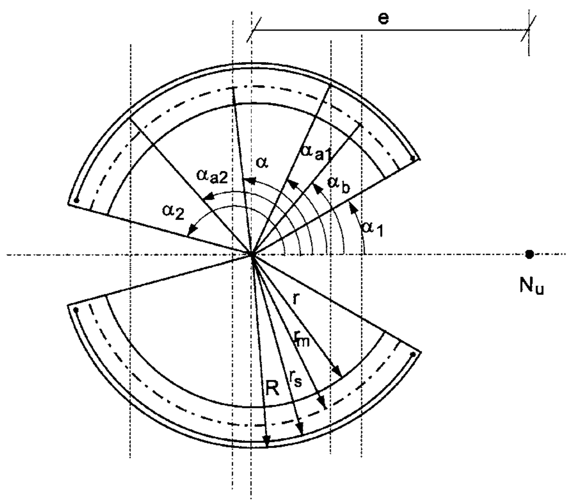

Fig. 1 The cross-section weakened by two openings

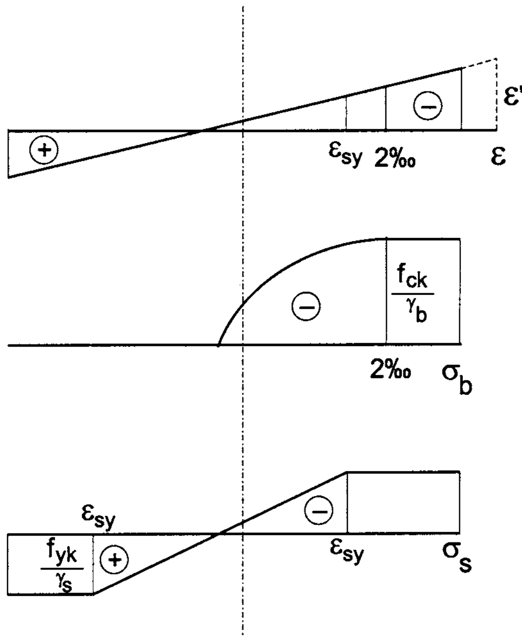

Fig. 2 Distribution of strains ε, stresses in concrete σ_b and in steel σ_s across the section

N and the bending moment M. The eccentricity of the normal force e is obtained as a resultant force of the weight of the tower above the considered section and the wind pressure measured from the geometrical center of the cross-section (Fig. 1, Fig. 2).

In the present derivation the following assumptions are introduced:

(i) distribution of strain across the section is plane,
(ii) the tensile strength of concrete is ignored,
(iii) the reinforcement in both the tension and compression zone is taken into account,
(iv) the shell is thin compared with its diameter,
(v) elasto-plastic stress/strain relationships for concrete and steel are used,
(vi) the ultimate strain for concrete is defined as −0.0035 or −0.002, while for reinforcement as 0.005 (tension) and −0.005 (compression).

The stress-strain relationships for concrete in compression are assumed as (Fig. 1 and Fig. 2):

$$\sigma_b = \frac{f_{ck}}{\gamma_b}\varepsilon_b(1 + 0.25\varepsilon_b) \qquad \text{for} \quad -2 \le \varepsilon_b \le 0 \tag{1}$$

$$\sigma_b = -\frac{f_{ck}}{\gamma_b} \qquad \text{for} \quad -3.5 \le \varepsilon_b \le -2 \tag{2}$$

where

σ_b : compressive stress in concrete,
ε_b : strain in concrete, expressed per mille, [‰],
f_{ck} : the characteristic strength of concrete in compression,
γ_b : partial safety factor for concrete.

The material law for steel in tension and compression is given by (Fig. 2):

$$\sigma_s = \frac{f_{yk}}{\varepsilon_{ss}}\varepsilon_s \quad \text{for} \quad -\varepsilon_{sy} \le \varepsilon_s \le \varepsilon_{sy} \tag{3}$$

$$\varepsilon_{ss} = \frac{f_{yk}}{E_s} \qquad \varepsilon_{sy} = \frac{\varepsilon_{ss}}{\gamma_s}$$

$$\sigma_s = \frac{f_{yk}}{\gamma_s} \quad \text{for} \quad \varepsilon_{sy} \le \varepsilon_s \le 5 \tag{4}$$

$$\sigma_s = -\frac{f_{yk}}{\gamma_s} \quad \text{for} \quad -5 \le \varepsilon_s \le -\varepsilon_{sy} \tag{5}$$

where:

ε_s : strain in steel per mille,
f_{yk} : the yield stress of steel,
γ_s : partial safety factor for steel,
E_s : modulus of elasticity of steel.

Let us consider the section under combined compression and bending. Due to the Bernoulli assumption we obtain:

$$\varepsilon_b = \frac{\cos\varphi - \cos\alpha}{\rho_R - \cos\alpha}\dot{\varepsilon}$$
$$\varepsilon_s = \frac{\rho\cos\varphi - \cos\alpha}{\rho_R - \cos\alpha}\dot{\varepsilon} \tag{6}$$

where:

$\dot{\varepsilon}$: the maximum compressive strain in concrete at the wall edge,
α : the angle describing the location of the neutral axis ($\alpha_1 \le \alpha \le \alpha_2$),
φ : angular coordinate,
r_m : mean radius of the ring (equals to the centroidal concrete radius r_c),
ρ : coefficient, $\rho = \dfrac{r_s}{r_m}$,
ρ_R : coefficient, $\rho_R = \dfrac{R}{r_m}$.

There exist eight possible forms of the stress distribution in the section:

(i) elastic phase of the concrete and steel
(ii) plastic phase of the concrete, elastic phase of the steel
(iii) plastic phase of the concrete and the compressive steel, elastic phase of the tensile steel
(iv) plastic phase of the concrete and the tensile steel, elastic phase of the compressive steel
(v) elastic phase of the concrete and the compressive steel, plastic phase of the tensile steel
(vi) elastic phase of the concrete and the tensile steel, plastic phase of the compressive steel
(vii) elastic phase of the concrete, plastic phase of the compressive and tensile steel
(viii) plastic phase of the concrete and steel.

Let us consider the case (viii). The equilibrium equation of the normal forces in the cross-section weakened by one or two openings takes the following form

$$2\left(\int_{\alpha_1}^{\alpha_b}\sigma_b^{pl}dA_b + \int_{\alpha_b}^{\alpha}\sigma_b^{e}dA_b\right) + 2\left(\int_{\alpha_1}^{\alpha_{a1}}\sigma_s^{pl}dA_s + \int_{\alpha_{a1}}^{\alpha_{a2}}\sigma_s^{e}dA_s + \int_{\alpha_{a2}}^{\alpha_2}\sigma_s^{pl}dA_s\right)$$
$$+ 2F_{ad1}\sigma_s^{pl}(\alpha_1) + 2F_{ad2}\sigma_s^{pl}(\alpha_2) + N = 0 \tag{7}$$

where:

α_b : the angle determining the depth of the plastifying zone of the concrete,
α_{s1} : the angle determining the depth of the plastifying zone of the compressive steel,
α_{s2} : the angle determining the depth of the plastifying zone of the tensile steel,
F_{ad1} : the area of the additional reinforcement at the opening specified by α_1,
F_{ad2} : the area of the additional reinforcement at the opening specified by α_2,
σ_b^{pl} : the stress function of the concrete in the plastic range given by (2),
σ_b^{e} : the stress function of the concrete in the elastic range given by (1),
σ_s^{pl} : the stress function of the steel in the plastic range given by (4) or (5),
σ_s^{e} : the stress function of the steel in the elastic range given by (3),
dA_b : the element of the concrete area,
dA_s : the element of the steel area.

Taking into account the relationships (1-6) and the equation $dA_b + dA_s = r_m t d\phi$, after integration and rearrangement of (7) we obtain

$$\begin{aligned} & -\frac{1-\mu}{\gamma_b}(\alpha_b - \alpha_1) \\ & + \frac{1-\mu}{\gamma_b}\frac{\dot{\varepsilon}}{\rho_R - \cos\alpha}\left[X_1(\alpha,\alpha_b) + \frac{\dot{\varepsilon}}{4(\rho_R - \cos\alpha)}X_2(\alpha,\alpha_b)\right] \\ & + \mu\frac{f_{yk}}{f_{ck}}\left[-\frac{1}{\gamma_s}(\alpha_{a1} - \alpha_1) + \frac{1}{\varepsilon_{ss}}\frac{\dot{\varepsilon}}{\rho_R - \cos\alpha}X_3(\alpha_{a1},\alpha_{a2}) + \frac{1}{\gamma_s}(\alpha_2 - \alpha_{a2})\right] \\ & + \frac{f_{yk}}{f_{ck}}\frac{1}{\gamma_s r_m t}(-F_{ad1} + F_{ad2}) + \frac{N}{2 r_m t f_{ck}} = 0 \end{aligned} \tag{8}$$

where:

$$\begin{aligned} X_1(\alpha,\alpha_b) &= \sin\alpha - \sin\alpha_b - \cos\alpha(\alpha - \alpha_b) \\ X_2(\alpha,\alpha_b) &= \left(\frac{1}{2} + \cos^2\alpha\right)(\alpha - \alpha_b) + \frac{1}{4}(\sin 2\alpha - \sin 2\alpha_b) - 2\cos\alpha(\sin\alpha - \sin\alpha_b) \\ X_3(\alpha_{a1},\alpha_{a2}) &= \rho(\sin\alpha_{a2} - \sin\alpha_{a1}) - \cos\alpha(\alpha_{a2} - \alpha_{a1}) \end{aligned} \tag{9}$$

t : the thickness of the cross-section $t = R - r$,
μ : the ratio of areas, steel to concrete,
$d\varphi$: the element of the angle measured from the axis in the compressive zone.

The equilibrium equation of the bending moments in the section under consideration takes in turn the following form

$$\begin{aligned} & 2\left(\int_{\alpha_1}^{\alpha_b}\sigma_b^{pl} r_m \cos\phi\, dA_b + \int_{\alpha_b}^{\alpha}\sigma_b^{e} r_m \cos\phi\, dA_b\right) \\ & + 2\left(\int_{\alpha_1}^{\alpha_{a1}}\sigma_s^{pl} r_s \cos\phi\, dA_s + \int_{\alpha_{a1}}^{\alpha_{a2}}\sigma_s^{e} r_s \cos\phi\, dA_s + \int_{\alpha_{a2}}^{\alpha_2}\sigma_s^{pl} r_s \cos\phi\, dA_s\right) \\ & + 2F_{ad1}\sigma_s^{pl}(\alpha_1) r_m \cos\alpha_1 + 2F_{ad2}\sigma_s^{pl}(\alpha_2) r_m \cos\alpha_2 + M = 0 \end{aligned} \tag{10}$$

Taking into account the relationships (1-6), integrating and rearranging (10) yields

$$\begin{aligned} & -\frac{1}{2}\frac{1-\mu}{\gamma_b}(\sin\alpha_b - \sin\alpha_1) + \frac{1}{2}\frac{1-\mu}{\gamma_b}\frac{\dot{\varepsilon}}{\rho_R - \cos\alpha}\left[Y_1(\alpha,\alpha_b) + \frac{\dot{\varepsilon}}{4(\rho_R - \cos\alpha)}Y_2(\alpha,\alpha_b)\right] \\ & + \frac{1}{2}\mu\frac{f_{yk}}{f_{ck}}\left[-\frac{1}{\gamma_s}\rho(\sin\alpha_{a1} - \sin\alpha_1) + \frac{1}{\varepsilon_{ss}}\frac{\dot{\varepsilon}}{\rho_R - \cos\alpha}Y_3(\alpha_{a1},\alpha_{a2}) + \frac{1}{\gamma_s}\rho(\sin\alpha_2 - \sin\alpha_{a2})\right] \\ & + \frac{1}{2}\frac{f_{yk}}{f_{ck}}\frac{\rho}{\gamma_s r_m t}(-F_{ad1}\cos\alpha_1 + F_{ad2}\cos\alpha_2) + \frac{M}{4 r_m^2 t f_{ck}} = 0 \end{aligned} \tag{11}$$

where:

$$Y_1(\alpha, \alpha_b) = \frac{1}{2}(\alpha - \alpha_b) + \frac{1}{4}(\sin 2\alpha - \sin 2\alpha_b) - \cos\alpha(\sin\alpha - \sin\alpha_b)$$

$$Y_2(\alpha, \alpha_b) = (1 + \cos^2\alpha)(\sin\alpha - \sin\alpha_b) - \frac{1}{3}(\sin^3\alpha - \sin^3\alpha_b)$$
$$- \cos\alpha\left[\alpha - \alpha_b + \frac{1}{2}(\sin 2\alpha - \sin 2\alpha_b)\right] \tag{12}$$

$$Y_3(\alpha_{a1}, \alpha_{a2}) = \rho\left\{\rho\left[\frac{1}{2}(\alpha_{a2} - \alpha_{a1}) + \frac{1}{4}(\sin 2\alpha_{a2} - \sin 2\alpha_{a1})\right] - \cos\alpha(\sin\alpha_{a2} - \sin\alpha_{a1})\right\}$$

In all other cases (i)-(vii) the general form of the equilibrium equations is similar to (8) and (11). However, terms $\frac{f_{yk}}{f_{ck}}\frac{1}{\gamma_s r_m t}F_{ad1}$ and $\frac{f_{yk}}{f_{ck}}\frac{1}{\gamma_s r_m t}F_{ad2}$ occurring in Eq. (8) and $\frac{1}{2}\frac{f_{yk}}{f_{ck}}\frac{\rho}{\gamma_s r_m t}F_{ad1}\cos\alpha_1$ and $\frac{1}{2}\frac{f_{yk}}{f_{ck}}\frac{\rho}{\gamma_s r_m t}F_{ad2}\cos\alpha_2$ occurring in Eq. (11) take appropriate modified forms, when the additional reinforcement at one or two openings is in the elastic state. The conditions of the strain continuity for the concrete and the compressive and tensile steels are expressed, respectively:

$$\frac{\cos\alpha_b - \cos\alpha}{\rho_R - \cos\alpha}\dot{\varepsilon} = -2 \tag{13}$$

$$\frac{\rho\cos\alpha_{a1} - \cos\alpha}{\rho_R - \cos\alpha}\dot{\varepsilon} = -\varepsilon_{sy} \tag{14}$$

$$\frac{\rho\cos\alpha_{a2} - \cos\alpha}{\rho_R - \cos\alpha}\dot{\varepsilon} = \varepsilon_{sy} \tag{15}$$

Thus, the problem is described by the set of the five nonlinear equations given in the form (8), (11), (13)-(15) with the unknown variables α, $\dot{\varepsilon}$, α_b, α_{a1}, α_{a2}. For the cases (i)-(vii), conditions (13) -(15) take modified forms. In order to solve them effectively numerical methods are used. In a similar way one can analyze the section wholly in compression.

The presented model may be generalized for the cross-section weakened by more than two openings located symmetrically. Let us consider the annular cross-section weakened by m openings situated symmetrically with respect to the bending direction. By this assumption, the locations of the openings are determined by couples of the angular coordinates $(\alpha_1, \alpha_2), (\alpha_3, \alpha_4), \ldots,$ (α_{m-1}, α_m), $0 \le \alpha_1 \le \alpha_2 \le \ldots \le \alpha_{m-1}, \le \alpha_m \le \pi$. Using the principle of mathematical induction one can obtain the analogous set of equations for each interval $\langle \alpha_{i-1}, \alpha_i \rangle$ that takes similar form as (8)-(12).

3. The ultimate limit state

The resistance capacity of the cross-section is reached when either ultimate strain in concrete ε_{bu} or in steel ε_{su} is reached anywhere in that section. The problem under consideration is mathematically described by the following set of equations resulting from those derived above

$$
\begin{aligned}
&-\frac{1-\mu}{\gamma_b}(\alpha_b-\alpha_1)\\
&+\frac{1-\mu}{\gamma_b}\frac{\dot{\varepsilon}}{\rho_R-\cos\alpha}\left[X_1(\alpha,\alpha_b)+\frac{\dot{\varepsilon}}{4(\rho_R-\cos\alpha)}X_2(\alpha,\alpha_b)\right]\\
&+\mu\frac{f_{yk}}{f_{ck}}\left[-\frac{1}{\gamma_s}(\alpha_{a1}-\alpha_1)+\frac{1}{\varepsilon_{ss}}\frac{\dot{\varepsilon}}{\rho_R-\cos\alpha}X_3(\alpha_{a1},\alpha_{a2})+\frac{1}{\gamma_s}(\alpha_2-\alpha_{a2})\right]\\
&+\frac{f_{yk}}{f_{ck}}\frac{1}{\gamma_s r_m t}(-F_{ad1}+F_{ad2})+\pi\cdot n_u = 0
\end{aligned}
\tag{16}
$$

where $n_u = \dfrac{N}{2\pi r_m t f_{ck}}$ denotes the normalized ultimate normal force.

$$
\begin{aligned}
&-\frac{1}{2}\frac{1-\mu}{\gamma_b}(\sin\alpha_b-\sin\alpha_1)+\frac{1}{2}\frac{1-\mu}{\gamma_b}\frac{\dot{\varepsilon}}{\rho_R-\cos\alpha}\left[Y_1(\alpha,\alpha_b)+\frac{\dot{\varepsilon}}{4(\rho_R-\cos\alpha)}Y_2(\alpha,\alpha_b)\right]\\
&+\frac{1}{2}\mu\frac{f_{yk}}{f_{ck}}\left[-\frac{1}{\gamma_s}\rho(\sin\alpha_{a1}-\sin\alpha_1)+\frac{1}{\varepsilon_{ss}}\frac{\dot{\varepsilon}}{\rho_R-\cos\alpha}Y_3(\alpha_{a1},\alpha_{a2})+\frac{1}{\gamma_s}\rho(\sin\alpha_2-\sin\alpha_{a2})\right]\\
&+\frac{1}{2}\frac{f_{yk}}{f_{ck}}\frac{\rho}{\gamma_s r_m t}(-F_{ad1}\cos\alpha_1+F_{ad2}\cos\alpha_2)+\pi\cdot m_u = 0
\end{aligned}
\tag{17}
$$

where $m_u = \dfrac{M}{4\pi r_m^2 t f_{ck}}$ denotes the normalized ultimate bending moment.

The conditions of the ultimate limit state are expressed by

$$\frac{\cos\alpha_1-\cos\alpha}{\rho_R-\cos\alpha}\dot{\varepsilon} = \varepsilon_{bu} \tag{18}$$

$$\frac{\rho\cos\alpha_2-\cos\alpha}{\rho_R-\cos\alpha}\dot{\varepsilon} = \varepsilon_{su} \tag{19}$$

Additionally, the continuity conditions (13)-(15) remain valid.

The resulting set of seven equations (in case (viii)) can be easily solved analytically. The unknowns are: α, $\dot{\varepsilon}$, α_b, α_{a1}, α_{a2}, n_u and m_u.

4. Description of the optimization algorithm used for solving the sets of the derived equations

4.1 Formulation of the problem

The set of the derived nonlinear equations takes the general form

$$F_i(x) = 0, \quad i = 1, \ldots, n \tag{20}$$

The number of equations is equal to the number of unknown variables in any problem considered in this paper and depends on the states of the steel and the concrete.

Formulating the optimization problem the authors have decided to use the least squares approach, i.e., to minimize the sum of the second powers of the F_i functions. The resulting optimization problem is as follows

$$\begin{aligned} &\min_{x \in R^n} \quad f(x) = \sum_{i=1}^{n} F_i^2(x) \\ &\text{s.t.} \quad x_i^L \le x_i \le x_i^U, \quad i = 1, \ldots, n \end{aligned} \tag{21}$$

Box constraints in the problem follow from the mechanical interpretation of the unknowns. Functions F_i are nonlinear and therefore the optimization problem is also nonlinear. The problem has some unpleasant numerical properties. The minimized functions are very flat in the major part of the feasible region. This feature makes the numerical estimation of the gradient unreliable in some situations.

The authors have used two solvers developed by themselves in the standard ANSI C language. The first solver implements the local BFGS quasi-Newton minimizer with the numerical gradient estimation and the second the Hooke-Jeeves direct search method (see e.g. Bazaraa *et al*. 1993, Bertsekas 1997, Fletcher 1970, 1987, Hooke and Jeeves 1961, Stachurski and Wierzbicki 2001). The BFGS and Hooke-Jeeves methods are the unconstrained optimization methods. However, in this implementation box constraints have been introduced on the parameters and the algorithm rules have been modified appropriately to ensure feasibility. The BFGS method reaches only a certain level of accuracy (perhaps due to the numerical gradient estimation). For this reason, the nongradient routine has been prepared in order to overcome this difficulty and to have a tool for comparison purposes.

5. Numerical examples

The set of nonlinear equations given in the form (8)-(15) has been reformulated as an optimization problem. In fact, the sum of the second powers of their residuals is minimized. The resulting optimization problems have been solved by means of the modified BFGS quasi-Newton and/or Hooke-Jeeves direct search methods.

The presented approach enables the determination of strains and stresses in the sections under consideration by the interactive analysis. For presentation of the proposed Eqs. (8)-(15) one particular design with a single opening was chosen. Figs. 3-8 show the numerically calculated strains and stresses in the concrete and in the steel for the following values of the basic parameters: $f_{ck} = 20$ MPa, $f_{yk} = 410$ MPa, $\gamma_b = 1.5$, $\gamma_s = 1.15$, $\mu = 1.0\%$ and $F_{ad1} = F_{ad2} = 0$. The values of the strains ε_b and stresses σ_b in the concrete as well as in the compressive steel ε_{s1} and σ_{s1} and in the tensile one ε_{s2} and σ_{s2} are plotted directly as a function of the eccentricity ratio e/R and size of the opening $2\alpha_1$.

The curves presented in Figs. 2-6 indicate that the obtained relationships are strongly nonlinear after the yield stress in the steel has been reached. As it is apparent in Figs. 1-4 a single opening can result in increasing the strains and stresses in the concrete and compressive steel by more than

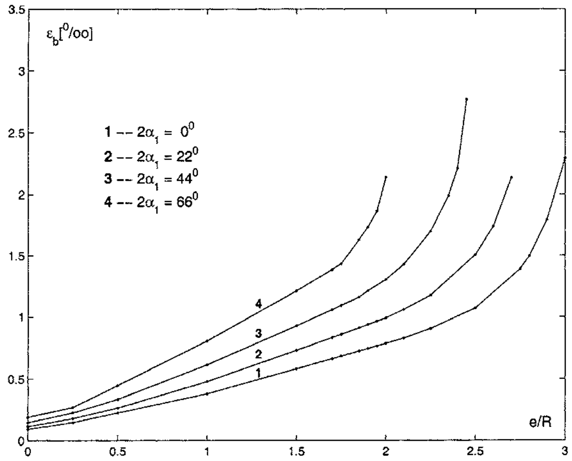

Fig. 3 Strains in concrete ε_b versus eccentricity ratio *e/R* and size of opening $2\alpha_1$

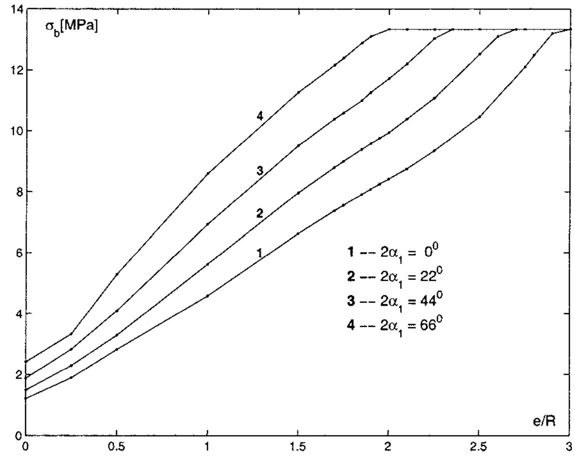

Fig. 4 Stresses in concrete σ_b versus eccentricity ratio *e/R* and size of opening $2\alpha_1$

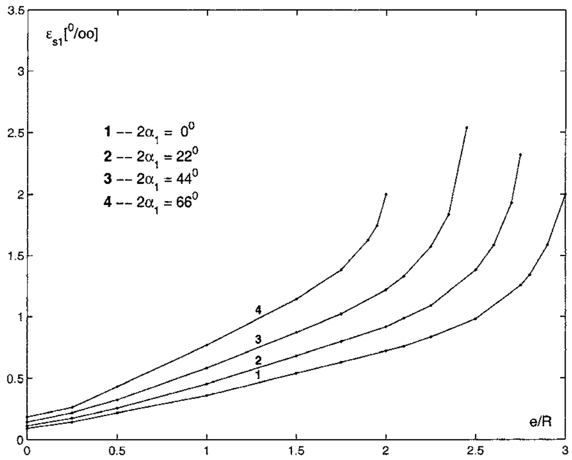

Fig. 5 Strains in compressive steel ε_{s1} versus eccentricity ratio *e/R* and size of opening $2\alpha_1$

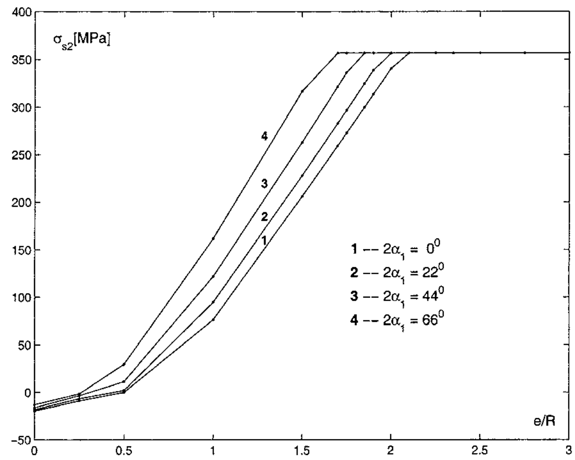

Fig. 6 Stresses in compressive steel σ_{s1} versus eccentricity ratio *e/R* and size of opening $2\alpha_1$

100%. The effect of the additional lumped reinforcement at openings was examined under the assumption that the area of the additional steel bars at the sides of the openings is equal to that which would have passed through the openings. The results of calculations collected in Table 1 show that the additional reinforcement (Add.+) at a single opening reduces the strains and stresses in the concrete and steel in the plastic state by about 7-8% (RD-relative difference), whereas, its influence in the elastic case is negligible.

As a rule the calculations have been initiated with case (i), assuming elastic state of both materials. Then, they were proceeded in an interactive way. If the solution does not satisfy the task requirements, the solution scheme is modified appropriately, until the satisfactory accuracy has been reached. Solutions were accepted when the minimized sum of the second powers of the residua was smaller than 10^{-4} and otherwise rejected. However, in many cases the solution accuracy was of

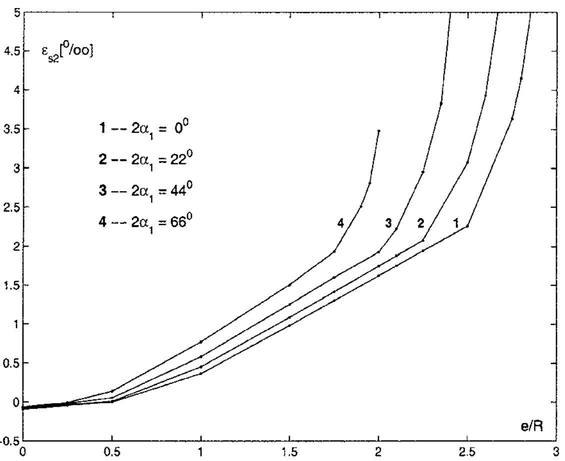

Fig. 7 Strains in tensile steel ε_{s2} versus eccentricity ratio *e/R* and size of opening $2\alpha_1$

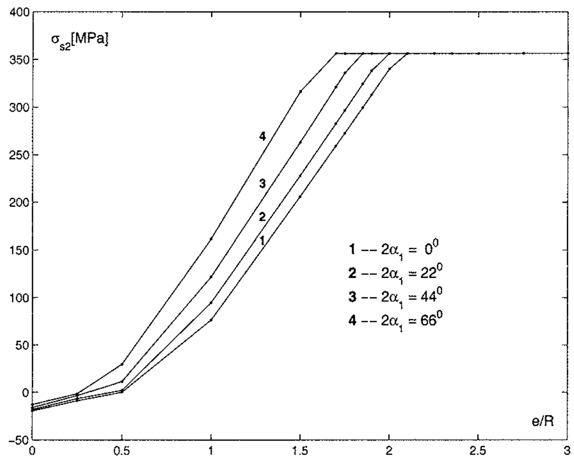

Fig. 8 Tensile stresses in steel versus eccentricity ratio *e/R* and size of opening $2\alpha_1$

Table 1 Effect of the additional reinforcement at the opening on the strains and stresses in the concrete and in the steel

	e/R = 1.0			*e/R* = 2.0			*e/R* = 2.35		
	Add.+		RD [%]	Add.+		RD [%]	Add.+		RD [%]
ε_b [‰]	−0.6141	−0.6143	0.027	−1.257	−1.305	3.8	−1.841	−1.986	7.9
σ_b [MPa]	−6.93	−6.93		−11.50	−11.72	1.9	−13.25	−13.33	0.6
ε_{s1} [‰]	−0.583	−0584	0.022	−1.177	−1.221	3.8	−1.703	−1.836	7.8
σ_{s1} [MPa]	−122.50	−122.50		−247.16	−256.50	3.8	−356.52	−356.52	
ε_{s2} [‰]	0.5802	05803	0.021	1.876	1.925	2.6	3.531	3.826	8.3
σ_{s2} [MPa]	121.84	121.86	0.02	356.52	356.52		356.52	356.52	

several orders better (even 10^{-15}). In extreme cases (large values of *e/R*), the resulting accuracy was worse. For this reason the range of argument *e/R* was restricted for each curve in Figs. 1-6. In most cases satisfactory results were obtained by means of the BFGS method. However, in some situations, the Hook-Jeeves direct search algorithm found solutions with better accuracy. In the authors' opinion, it may result from the insufficient accuracy in the numerical gradient estimation.

As the next problem the resistance capacity of the sections was analyzed. Using the derived Eqs. (16)-(19) one can obtain the interaction curves with the designed values of the normalized, cross-sectional forces n_u and m_u for the section weakened by one or two diametrically opposite openings (Fig. 9, Fig. 10). The comparison presented in Table 2 indicates that the resistance capacity of the section determined by the values of n_u, m_u increases due to the additional lumped reinforcement at a single opening by more than 10% depending on the opening size and the ultimate values ε_{bu} and ε_{su}.

In Table 3 the design values of the normalized, cross-sectional forces n_u and m_u obtained by the authors and those given according to DIN 1056, are compared. The resulting differences do not exceed 7%. In the authors' opinion, the differences result from the inaccuracies of reading the DIN diagrams. Furthermore, they may be attributed to the differences in the used models.

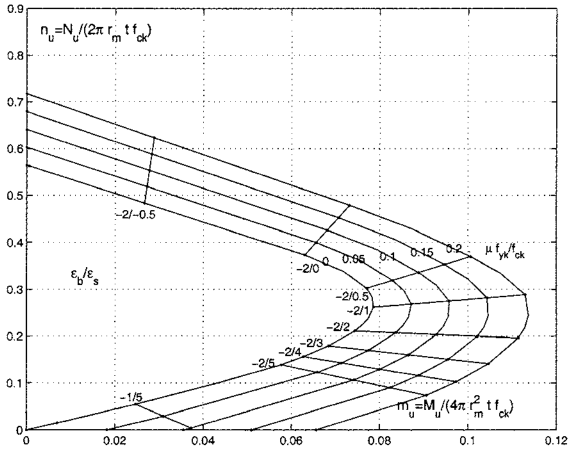

Fig. 9 Interaction diagram with the designed values of the normalized cross-sectional forces n_u and m_u for the section weakened by a single opening: $\alpha_1 = 0.192$ (11^0), $f_{ck} = 12$ MPa, $f_{yk} = 220$ MPa, $F_{ad1} = F_{ad2} = 0$

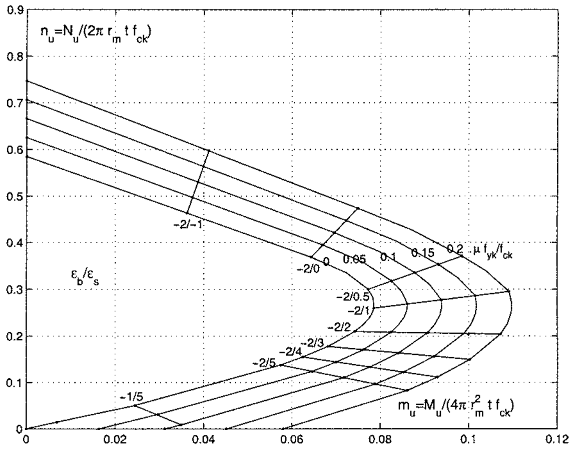

Fig. 10 The corresponding interaction diagram for the section with two diametrically opposite openings: $\alpha_1 = 0.192$ (11^0), $\alpha_2 = 0.9496$ (169^0)

Table 2 Comparison of the capacity of the section with or without the additional reinforcement at the opening ($\alpha_1 = 22\,^o$, $f_{ck} = 20$ MPa, $f_{yk} = 410$ MPa, $\mu = 1\%$, $\gamma_b = 1.5$, $\gamma_s = 1.15$)

$\varepsilon_{bu}/\varepsilon_{su}$	Without additional reinforcement		With additional reinforcement		$\frac{n_u(ad)}{n_u}$ [%]	$\frac{m_u(ad)}{m_u}$ [%]
	n_u	m_u	n_u	m_u		
−2/5	0.0380	0.0803	0.0434	0.0828	114.2	103.1
−2/2	0.1578	0.0948	0.1632	0.0973	103.4	102.7
−2/0	0.4121	0.0563	0.4176	0.0588	101.3	102.9
Two openings of the equal size $\alpha_1 = 33\,^o$						
−2/5	0.0477	0.0358	0.0546	0.0358	114.4	100.0

Table 3 Comparison of the calculated values with those specified in the DIN code ($f_{ck} = 10.5$ MPa, $f_{yk} = 420$ MPa, $\gamma_b = 1.5$, $\gamma_s = 1.15$)

Type of section	α [°]	$\mu\frac{f_{yk}}{f_{ck}}$	$\varepsilon_{bu}/\varepsilon_{su}$	n_u			m_u		
				DIN	Proposed model	RD [%]	DIN	Proposed model	RD [%]
closed		0.2	−2/2	0.260	0.2433	6.8	0.14	0.1342	4.3
1 op.	22	0.2	−2/1	0.305	0.2948	3.5	0.11	0.1065	3.3
1 op.	33	0.3	−2/1	0.30	0.2880	4.2	0.111	0.1106	0.4
2 ops.	22	0.15	−2/1	0.30	0.2883	4.1	0.10	0.0974	2.7
2 ops.	44	0.1	−2/4	0.100	0.1029	2.8	0.059	0.0576	2.4

6. Conclusions

Based on this study the following conclusions can be drawn:

1. General analytical formulae describing the model proved to be a successfull tool for determination and elasto-plastic analysis of strains and stresses in the RC circular tower sections with one or two openings or in the closed ones subjected to arbitrary cross-sectional forces N and M.
2. The authors' implementations of the BFGS and Hooke-Jeeves optimization methods were reliable in finding the numerical solutions with a sufficient accuracy.
3. The presented model works well in most cases encountered in engineering practice.
4. The proposed section model seems to have a wider application field than the previous ones due to the assumptions of non-central layout of reinforcement, additional steel bars at openings and wall edge strains.
5. In the case of a single opening, the calculations indicate that the additional reinforcement results in reducing the strains and the stresses in the concrete and the steel by about 7-8%.
6. The resistance capacity of the section increases due to the additional reinforcement by more than 10% depending on the opening size and the ultimate values ε_{bu}, ε_{su}.
7. The model serves for dimensioning the cross-sections and enables to design strenghtenings of the RC circular tower structures by means of external reinforcement.
8. The obtained equations can be generalized for the section weakened by an arbitrary number of openings located symmetrically in respect to the bending direction.

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