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The buried arch structural system for underground structures

Enrique Hernández-Montes†

University of Granada, Campus de Fuentenueva, 18072 Granada, Spain

Mark Aschheim‡

Civil Engineering Department, Santa Clara University, 500 El Camino Real, Santa Clara, CA 95053, USA

Luisa María Gil-Martín‡†

University of Granada, Campus de Fuentenueva, 18072 Granada, Spain

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Abstract. In many cases, underground structures are built using conventional above-grade structural systems to carry gravity load. This paper proposes the use of underground arches, termed "buried arches", to support gravity loads, wherein the horizontal thrust of the arch is equilibrated by soil pressure. Because the horizontal soil pressure increases with depth, the depth of the arch may be reduced as the depth below grade increases. Critical to the success of such an approach is a proper accounting of creep and shrinkage for arches made of reinforced concrete. This paper addresses the influence of equilibrium, creep, and shrinkage as they affect the design of the arch from a theoretical perspective. Several examples illustrate the use of buried arches for the design of underground parking structures.

Key words: reinforced concrete; arches; creep; shrinkage; underground structures.

1. Introduction

Arches and funicular shapes have the ability of span large distances without developing large flexural moments. Due to this fact, arches among other equilibrium shapes are always attractive and a deeper view of their capabilities is always worthy. Recently, Chai and Kunnath (2003) proposed a solution for the shape of submerged funicular arches. Behaviour of shallow-buried reinforced concrete arches under severe dynamics loads was studied by Krauthhammer (1992). Gil-Martín *et al.* (2001) developed a simplified transverse seismic analysis of buried structures that allows static lateral forces to be applied accounting for soil-structure interaction.

† Associate Professor, E-mail: emontes@ugr.es
‡ Associate Professor, Corresponding author, E-mail: maschheim@scu.edu
‡† Associate Professor, E-mail: mlgil@ugr.es

This paper develops the concept of using buried arches to provide structural support to underground structures. The arches are considered to span a distance L and carry dead loads arising from the structural concrete and concrete fill together with live load that may be due to vehicles (in the case of parking structures) and people. The central idea of this paper is that the configuration of the arch can be determined to equilibrate the horizontal resultant of the earth pressure. The governing equations of equilibrium are developed both for cases in which creep and shrinkage are neglected and for cases in which these time-varying quantities are considered. A cross section illustrating a practical example is provided in Fig. 1.

Various load conditions generally must be considered in design. The geometry of the arch is established so that it carries pure compression under a particular design load combination, designated C_o. Other load combinations, designated C_i, generally may induce flexural moments in combination with axial compression. The alignment of the arch may be determined by recognizing that the reaction forces are tangent to the alignment of the arch at any cross section under the design loading C_o. At the ends of the arch, these reaction forces are transmitted to the soil through a retaining wall, as shown in Fig. 1. The reaction force R can be resolved into its horizontal and vertical components, H and V, respectively, as shown in Fig. 2. Because design loading C_0 and the

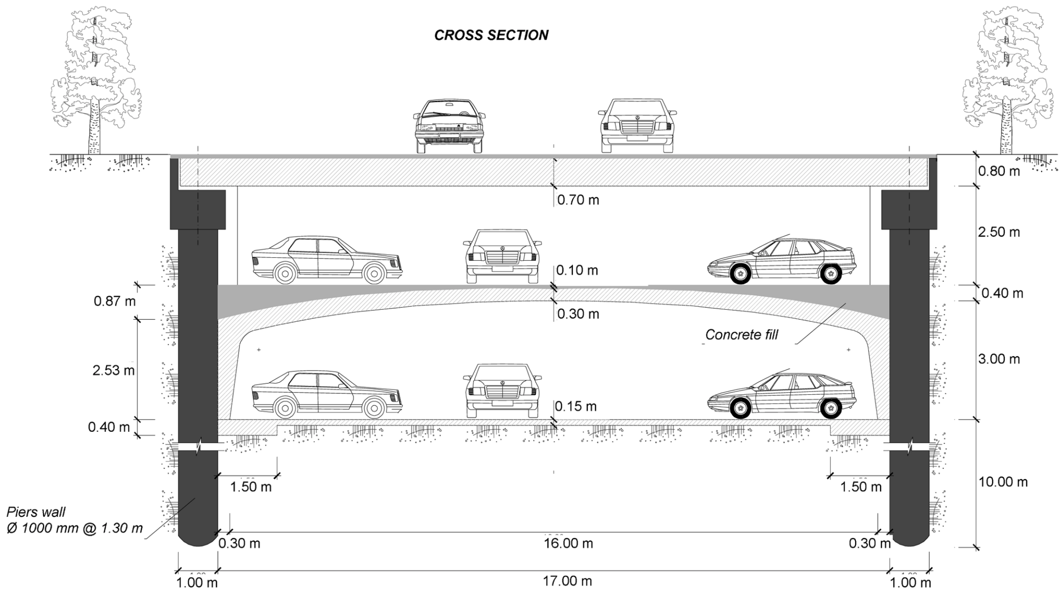

Fig. 1 Parking under construction in Granada (Spain), in which the retaining wall is a pier wall, composed of 1-m diameter piers spaced at 1.5 m on center

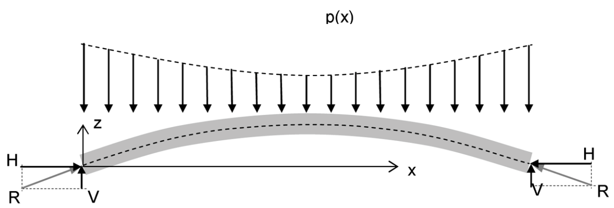

Fig. 2 Arch reactions in equilibrium with applied loads

configuration of the arch are symmetric, the reactions forces are also symmetric. For a design gravity loading $p(x)$, the vertical reaction, V, can be determined from vertical equilibrium. Because the height of the arch (h) has only a small influence on the dead load, V is nearly independent of the height of the arch. However, the height of the arch has a large effect on the horizontal component of the reaction, H, which increases as the height of the arch, h, decreases. The height must be selected so that H equilibrates a soil reaction whose value is bounded between the active and passive conditions. In many cases, it will be convenient to assume that the design loading C_o is equilibrated by the at-rest pressure of the soil, and then to verify that other load combinations result in lateral forces that are within an acceptable range defined by the active and passive conditions.

In order to avoid displacements in the soil, temporary supports must be placed during construction to equilibrate the soil pressure, until the time that the forms supporting the arches are removed. Because we typically design for the at-rest pressure, little or no movement of the soil is required to equilibrate the arch thrust. As will be shown in an example, for arches at or near the top of the structure, it is sometimes necessary to design for pressures greater than the at-rest pressure. However, in these cases the arch height, h, is large enough that the horizontal reaction is insensitive to movement at the supports, and thus the arch can accommodate the small movements that may be needed to develop the required resistance of the soil.

This paper introduces the idea that the value of h used for design of the arch should be selected to

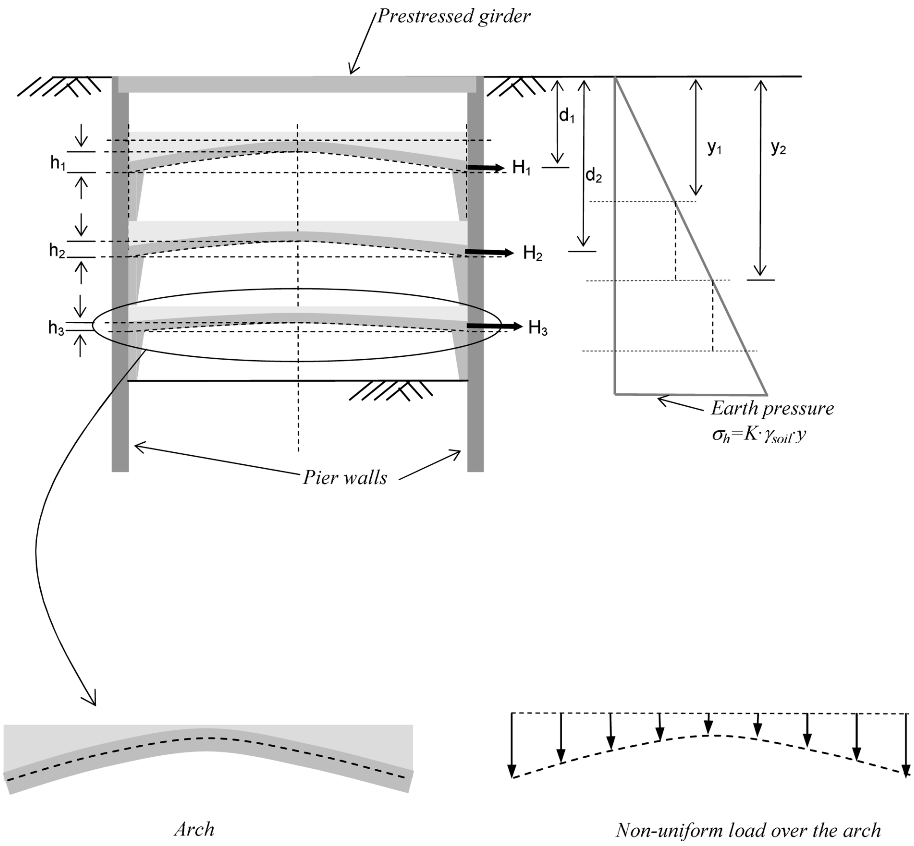

Fig. 3 Arch system to equilibrate the horizontal earth pressure and non-uniform load over the arch

equilibrate the earth pressure. Two issues affecting the choice of h are addressed: (1) because concrete fill is used to obtain a finish floor surface, the load on the arch is initially unknown, as it depends on the choice of h, and (2) consideration of creep, shrinkage, and initial axial strain of the concrete arch creates an "inverse problem", where the final value of the equilibrating horizontal reaction is specified, but the initial values of h and H are unknown. Issues related to the sequence of construction will be addressed in another paper; feasible construction sequences exist, and are being used to construct the parking structure of Fig. 1.

The structural configuration that results from these considerations consists of a series of arches whose depth may vary over the height of the structure in order to equilibrate the soil pressure resultants (Fig. 3). This structural system is especially useful in urban regions, particularly for construction near historic structures, which typically have shallow foundations and are sensitive to the disturbances to subsurface stress states that often result from conventional construction.

2. Equilibrium configuration neglecting initial and long term deformations

Because our primary concern is to equilibrate the soil pressure corresponding approximately to at-rest conditions, the design load combination (C_o) consists of the service (unfactored) dead loads. This means that the arch is designed to have only axial forces present under service dead loads in the final equilibrium position. Other loading conditions (C_i), such as factored dead and live loads, will generate flexural moments, shear, and axial forces along the length of the arch. The arch cross-sections must be reinforced to have sufficient strength to carry the factored flexural moments, axial forces, and shears. In addition, the soil reactions under all loading conditions must be between the active and passive limits.

The design loading C_o consists of service dead loads. Because a level finish floor is needed, either non-structural concrete fill must be added over a continuous (wide) arch, or a floor system must be provided between and above the system of arches. Either case results in a non-uniform distributed load being applied to the arch. The first case is illustrated in Fig. 3. The horizontal coordinate x is adopted as independent variable, which varies from 0 to the span of the arch, L. The free body diagram of a differential length dx of the arch is shown in Fig. 4. The solution for uniform load is well-known, and can be found in Leonard (1988) among others.

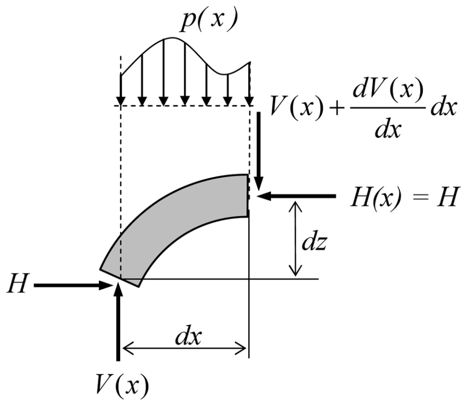

Fig. 4 Free body diagram for an arch for a C_0 load case (no bending moment)

For an arbitrarily varying load, equilibrium of forces over the differential length gives:

$$\frac{dH(x)}{dx} = 0 \rightarrow H(x) = H(x=0) \equiv H = E_{soil}$$
$$\frac{dV(x)}{dx} + p(x) = 0 \tag{1}$$

where $H(x)$ is the horizontal component of the internal reaction at any point in the arch, H is the value of $H(x)$ at the ends of the arch, and E_{soil} is the resultant of the earth pressure to be equilibrated by the horizontal component of the arch reaction, H. $V(x)$ is the vertical component of the internal reaction at any point in the arch, and $p(x)$ is the value of the load over the arch, as function of x.

In order to not generate bending moments in the arch, the geometric alignment of the arch must satisfy the following constraint:

$$\frac{dz(x)}{dx} = \frac{V(x)}{H(x)} \tag{2}$$

Taking derivatives with respect to x and considering that $H(x)$ is constant over the length of the arch, the differential equation that defines the alignment of the arch is obtained:

$$\frac{d^2z(x)}{dx^2} = \frac{1}{H}\frac{dV(x)}{dx} = \frac{-p(x)}{H} \tag{3}$$

where it is recognized that $H(x) = H$ over the length of the arch.

Because the local curvature of the arch may be approximated by the first term of Eq. (3), the curvature of the arch must be proportional to the load $p(x)$. The special problem here is that $p(x)$ depends on the equilibrium configuration of the arch, which is not known *a priori*. Let us consider the structural system of Fig. 5, which consists of a single, wide arch with non-structural concrete fill added to create a level floor. If we assume that the unit weight of the concrete fill is equal to the unit weight of the structural concrete, the expression for $p(x)$ is:

$$p(x) = \gamma_c \cdot (a + h - z(x)) \tag{4}$$

where γ_c is the unit weight of the concrete, a is the thickness of the concrete at midspan, and h is the height of the arch (Fig. 5). Because the weight of the concrete fill is assumed equal to the weight of the structural concrete, the thickness of the arch need not be represented in Eq. (4). (The arch thickness will be considered in a subsequent formulation that addresses creep and shrinkage). Because the arch is assumed here to be of constant thickness, the height of the arch can be

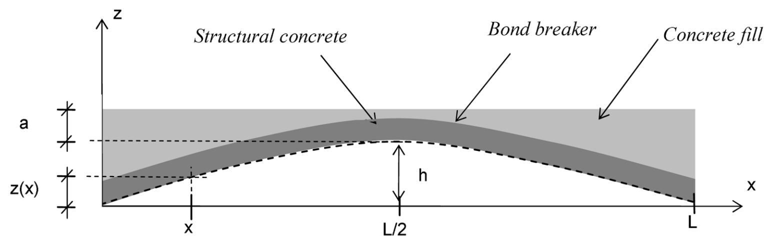

Fig. 5 Load system

measured from either the soffit of the arch or from the centreline of the arch alignment. Thus, the differential equation that defines the alignment of the constant thickness arch and the boundary conditions are:

$$\frac{d^2z(x)}{dx^2} = -\frac{\gamma_c(a + h - z(x))}{H} \qquad (5)$$

$$z(0) = 0, \quad z(L) = 0$$

With this derivation, which neglects creep and shrinkage, H is set equal to the resultant of the earth pressure, E_{soil}. L is the span of the arch. In this paper, the resultant of the earth pressure corresponding to at-rest conditions is typically used to equilibrate the horizontal reaction H of the arch under the design load combination C_o. The thickness of the concrete fill plus the depth of the arch, a, is established before the design of the arch itself.

The mathematical novelty of Eq. (5) is that it cannot be solved because while a relationship between the height of the arch, h, and the horizontal thrust, H, exists, this relationship is not known *a priori*, because h is a function of the equilibrium configuration, which depends on H. Eq. (5) can be solved analytically. The method of solution is to solve for $z(x)$ subject to a constraint on z at $L/2$; that is:

$$z\left(\frac{L}{2}\right) = h \qquad (6)$$

The analytical solution of Eq. (5) is:

$$z(x) = -(a + h)\frac{\exp\left(-\sqrt{\frac{\gamma_c}{H}}x\right)\left(\exp\left(\sqrt{\frac{\gamma_c}{H}}x\right) - 1\right)\left(\exp\left(\sqrt{\frac{\gamma_c}{H}}x\right) - \exp\left(\sqrt{\frac{\gamma_c}{H}}L\right)\right)}{1 + \exp\left(\sqrt{\frac{\gamma_c}{H}}L\right)} \qquad (7)$$

If the constraint of Eq. (6) is imposed on Eq. (7), the height of the arch h can be determined. Then, the geometry of the arch is completely defined by Eq. (7). For a given H, the value of h may be determined by imposing the constraint of Eq. (6) on Eq. (7), leading to:

$$h = \frac{a}{2}\exp\left(-\sqrt{\frac{\gamma_c}{H}}\frac{L}{2}\right)\left(\exp\left(-\sqrt{\frac{\gamma_c}{H}}\frac{L}{2}\right) - 1\right)^2 \qquad (8)$$

The foregoing results can be applied to analyze an existing underground parking structure (Fig. 6) located in the Revolution Square in Moscow, that was designed by Yurkevich Engineering Bureau (1999). In this design, each arch of the 4-story underground structure has the same height. The height of each arch, h, is 1.0 m and the span L is 17.0 m. The total thickness of the arch and fill at midspan is 0.4 m. The arch over the third underground level is considered further. For this arch, the horizontal component of the reaction acts at a depth of 7.25 m, and it is desired that this horizontal component equilibrate the lateral pressure developed over a tributary soil thickness 3 m in height. Based on a unit weight of 25 kN/m^3, the service level dead loads are calculated to develop a horizontal reaction of 487 kN per meter of width, as may be determined by solving for H in Eq. (8). The lateral pressure can be assumed to be given by $\sigma_h = K\gamma_{soil} \cdot y$ (Fig. 3), where K = the pressure coefficient, and y = the height below grade. The unit weight of the cohesionless soil, γ_{soil}, is assumed

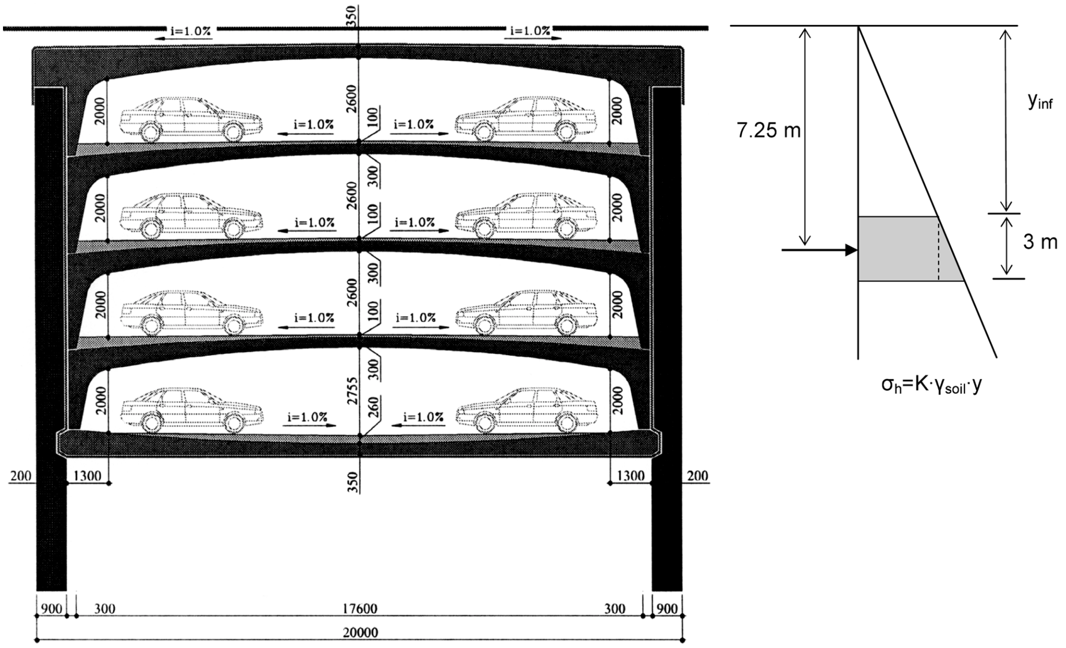

Fig. 6 Underground parking structure at the Revolution Square, Moscow. Courtesy of Professional Engineer Pavel Yurkevich

to be 20 kN/m^3. For the horizontal component of the arch reaction to equilibrate the resultant of the soil pressure tributary to the arch, the coefficient *K* must be 1.14, based on equilibrium. It can be seen in Fig. 6 that equilibrium of forces and moments can be used to establish two unknowns, the coefficient *K* and the lower limit of tributary depth, y_{inf}. This value of *K* is between the active and passive pressure coefficients, given by 0.33 and 3.00 for cohesionless soils having a friction angle of 30 degrees, according to the Rankine theory. Because the arches above the third level have the same depth and nominal loading while the active and passive soil pressure resultants are reduced, the horizontal reactions for the stories above correspond to higher soil pressure coefficients, approaching the passive value.

3. The inverse problem: consideration of initial and long-term deformations

In the previous section, creep, shrinkage, and the initial axial deformation were not considered. The horizontal component of the reaction did not change over time, and the values of *H* and *h* associated with equilibrium were therefore constant, with the geometry of the arch given by Eq. (7).

McGrath and Mastroianni (2002) conducted full-scale field tests on two 8.5-m span reinforced concrete arch culverts. They showed that for typical structural design, material parameters (related to concrete and soil properties) may be approximated without seriously degrading the accuracy of predicted design forces.

Due to creep under sustained compression, shrinkage, and the initial elastic deformation, the arch shortens, such that its final length is $L_1 = L_0 - \Delta L_0$, where L_0 is the initial length of the arch along its centreline, L_1 is the final length of the arch, and ΔL_0 is the change in length due to creep, shrinkage, and initial elastic axial deformation. Note that L, the span of the arch, does not change in the construction process due to the fact that temporary supports are used in order to avoid soil displacements during construction. As the cribbing that supports the arch during curing is removed, loading from the arch replaces the force applied from the temporary supports to the soil. At this point, the initial alignment of the arch is given by $z_0(x)$. Recognizing that ds_0 and ds_1 are the differential lengths along the arch centerline for the initial and final configurations, respectively, the initial length of the arch can be obtained by parametric integration as x ranges from 0 to L:

$$\begin{aligned} ds_0 &= \sqrt{dz_0^2 + dx^2} = \sqrt{z_0'^2 + 1}\,dx; \quad ds_1 = \sqrt{dz_1^2 + dx^2} = \sqrt{z_1'^2 + 1}\,dx \\ L_0 &= \int ds = \int_0^L \sqrt{z_0'^2 + 1}\,dx; \quad L_1 = \int ds = \int_0^L \sqrt{z_1'^2 + 1}\,dx \end{aligned} \tag{9}$$

where $z_1(x)$ is the final alignment of the arch after initial and long term deformation.

The differential equation for the final state is:

$$\begin{aligned} \frac{d^2 z_1(x)}{dx^2} &= \frac{\gamma_c(a + h_1 - z_1(x))}{H_1} \\ z_1(0) &= 0, \quad z_1(L) = 0 \end{aligned} \tag{10}$$

where h_1 is the final height of the arch and H_1 the final value of the reaction.

If creep and shrinkage are considered, both H and h will change over time. Let us call H_0 and H_1 the initial and final values of the horizontal reaction. In order to solve this problem, the transition from the initial to the final state has to be accounted for. Therefore, when creep and shrinkage and initial axial deformation are considered, the horizontal component of the initial reaction, H_0, is an unknown in the design process while the horizontal component of the final reaction, H_1, is a known value, because H_1 is determined to equilibrate a desired level of resultant earth pressure, E_{soil}, under the design load condition C_0. This problem is an inverse problem, wherein the final reaction is known (H_1) and the initial design values are unknown (H_0 and h_0).

As in the previous section, the constraint on height is imposed on the analytical solution at $L/2$:

$$z_1\left(\frac{L}{2}\right) = h_1 \tag{11}$$

The initial shape of the arch is not known *a priori* and this causes the initial value of the horizontal reaction to be an unknown. An additional constraint can be imposed on the analytical solution, given as follows:

$$\begin{aligned} L_0 &= \frac{L_1}{1 - \varepsilon_{c\sigma} - \varepsilon_{cs}} \\ L_1 &= \int_0^L \sqrt{z_1'(x)^2 + 1}\,dx \end{aligned} \tag{12}$$

where $\varepsilon_{c\sigma}$ is the strain due to stress (initial deformation plus creep) and ε_{cs} is the strain due to shrinkage. Creep is defined as strain under constant load, so it can be calculated over the initial or final compression of the arch if the compression is approximately invariant. The calculation process can be broken into several steps if the compression varies significantly.

Continuing with the analysis of the underground parking structure in Moscow, we might assume that h_0 = 1 m is the initial height of each arch. In this case, the depth of the structural arch is 300 mm and the shrinkage and creep values of strain, estimated according to Eurocode 2 (1991), are $\varepsilon_{c\sigma}$ = 0.000168 and ε_{cs} = 0.000496. (These values are associated with an assumed characteristic compressive strength of concrete of 25 N/mm^2 and an initial horizontal reaction H_0 = 487 kN per meter of width, relative humidity of 60%, onset of creep due to loading at 20 days and evaluation at 10,000 days). For these values, L_0 = 17.176 m and L_1 = 17.165 m, resulting in a final horizontal reaction H_1 = 499 kN per meter of width and final height h_1 = 0.97 m. These results are based on a first order approximation in which the initial values of the horizontal reaction are used to estimate creep deformations. To check the validity of this assumption, the final value of the horizontal reaction may be used to estimate the final values of h_1 and H_1 considering creep and shrinkage. The final values of h_1 and H_1 calculated this way are nearly identical to those determined based on the initial values of the horizontal reaction, indicating the validity of the first order assumption. Had significant differences emerged, a better approximation could have been obtained by considering the creep deformations in several (or more) discrete intervals of time.

4. Example

To further illustrate the concepts presented here, analyses are applied to the design of a parking garage structure. This structure (illustrated in Fig. 1) is under construction in Granada, Spain. A linear distribution of horizontal earth pressure was assumed according to the CIRIA Report 104 (1991). This report was used to design the wall system as a propped retaining wall, recognizing that the wall is acted upon by the passive resistance of the soil below the excavation, the arches, which act as props, and the active pressure from the opposing wall of the excavation.

The intensity of the soil pressure was obtained from a site-specific geotechnical report. The at-rest horizontal earth pressure is assumed to be given by $10 \cdot y$ kN/m^2, where y is the depth (m) below grade. This pressure is located between the active ($6 \cdot y$ kN/m^2) and passive ($67 \cdot y$ kN/m^2) values associated with the site soils. The design of the arch geometry used the at-rest pressure resultant to equilibrate the dead load thrust of the arch, recognizing that there is substantial capacity remaining to equilibrate the additional thrust associated with live loads and overloads associated with ultimate strength design.

The location of the horizontal reaction of the arch was established in the design at 4.7 m below grade. Because this reaction must equilibrate the soil pressure resultant tributary to the arch, the depth over which the lateral earth pressure acts is determined to be 7.05 m (because $2/3 \cdot 7.05 = 4.7$). Over this depth, the at-rest earth pressure resultant is 249 kN per meter of width and the passive pressure resultant is 1660 kN per meter of width. The final value of the horizontal reaction is set equal to H_1 = 400 kN per meter of width as a design decision, to obtain a practical value for the height of the uppermost arch.

Using a = 0.40 m, H_1 = 400 kN per meter of width, a unit weight of concrete of 25 kN/m^3, and a span of 17 m in the analytical solution obtained from Eq. (10), the final height (h_1) of the arch is determined to be 1.30 m, with L_1 = 17.30 m. Using the same values of relative humidity, concrete strength, and time of evaluation as in the previous example, L_0 can be calculated from Eq. (12). Imposing the condition that the length of $z_0(x)$ from 0 to L is L_0 (Eq. 9), H_0 and h_0 are obtained from Eq. (7) and the equality of L_0 in Eq. (9).

Even though the span to depth ratio of the arch is 17.00/1.30 = 13.1, initial elastic deformations and long-term deformations associated with creep and shrinkage of the arch must be considered, because these deformations will reduce the height of the arch and therefore will affect the horizontal component of the reaction.

Using the Eurocode 2 (1991) formulation for creep and shrinkage, the strain due to creep including the initial elastic deformation, $\varepsilon_{c\sigma}$, under H_l is 0.000125 at 10,000 days. The thickness of the arch is equal to 300 mm. The strain due to shrinkage (ε_{cs}) is estimated to be 0.000472 at 10,000 days. Applying the constraint of Eq. (12), the initial length of the arch must be L_0 = 17.31 m (h_0 = 1.32 m), so that the final length will be L_1 = 17.30 m. The initial horizontal reaction and height are therefore H_0 = 396 kN per meter of width and h_0 = 1.32 m, respectively. Because these values are very close to the final values, the estimated creep strains can be based on the final configuration. Alternatively, the initial configuration values can be used as illustrated in the previous example, where the initial configuration was shown on the construction documents, as long as there is agreement between the results computed using either set of values.

The horizontal reaction H_1 at each end of the arch is used to equilibrate the earth pressure resultant. Fig. 7 shows the influence of the height of the arch on the horizontal reaction. Plotted on Fig. 7 is the variation of the initial and final horizontal reactions with the initial height of the arch. It can be seen that for an initial value of the height of the arch h_0 = 0.6 m (a = 0.4 m), the horizontal reaction per meter of width has an initial value H_0 = 750 kN (point A) and a final value H_1 = 820 kN (point B). If the initial value of the height of the arch is 0.9 m, the initial and final values of the horizontal reaction per meter of width are almost invariant, equal to approximately 550 kN.

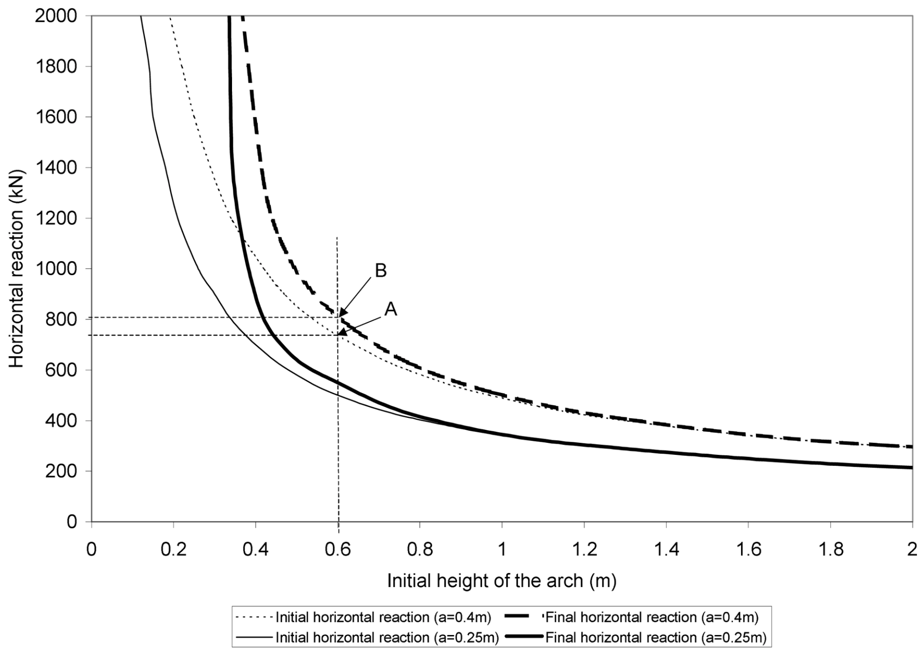

Fig. 7 Effect of the height of the arch on the horizontal thrust at the base of the arch, for a span of 17 m, concrete unit weight of 25 kN/m^3 and minimum thickness of 0.4 m at the crown

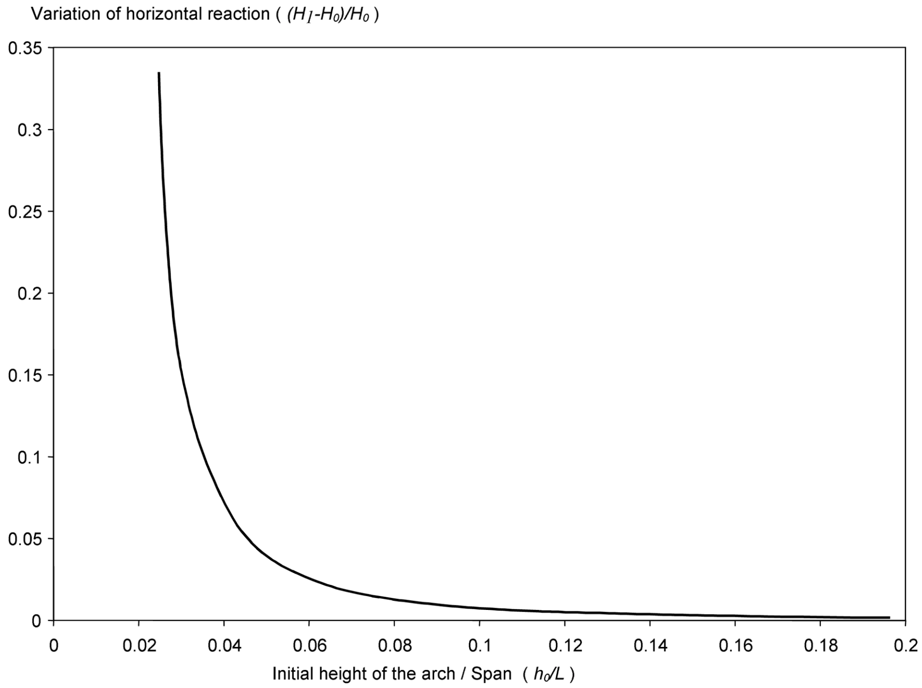

Fig. 8 Influence of creep and shrinkage. Weight of the concrete of 25 kN/m^3, 0.4 m of minimum thickness, and arch thickness of 0.35 m, and span of 17 m

Fig. 8 shows the influence of initial elastic deformation and long term deformations associated with creep and shrinkage on the horizontal reaction for a = 0.4 m. If a limit of the variation of the horizontal reaction under service dead loads of 10% is imposed, the minimum height for an arch of 17 m span is h_0 = 0.595 m, which corresponds to an initial horizontal reaction H_0 of 741 kN per meter of width. If this horizontal component of the arch reaction is used to equilibrate earth pressure resultants associated with a tributary height of 3 m, each arch can equilibrate at-rest pressures to a depth of horizontal resultant of 23 m below grade. This depth corresponds to 7 to 8 underground parking levels. Because the resultants at higher levels are smaller, the depth of the arch will increase and the variation of horizontal reaction due to creep and shrinkage will decrease as one approaches grade level. Because the design is not restricted to equilibrating at-rest pressures (any reliable values between active and passive pressures may be used, provided that safety is assured for all relevant load combinations), there is greater latitude in design.

In any design process many parameters may be varied, leading to many possible solutions. In order to further illustrate the influence of creep and shrinkage on design, a structure with ten underground stories is now considered (a = 0.4 m, L = 17 m, weight of concrete of 25 kN/m^3, with each horizontal resultant spaced at intervals of 3 m below grade. The first and second column in Table 1(a) shows the arch number and the depth of application of the horizontal reaction. Columns 3 to 5 present the horizontal resultant for active, at-rest and passive pressure resultants associated with the tributary depth given in Column 6. The tributary depth refers to the depth of the lower edge of the horizontal earth pressure distribution that is tributary to the earth pressure resultant acting at depth d. The tributary depths are determined from the expressions of equilibrium for force and

Table 1(a) Initial and final values for arches at different depths

Arch no.	d (m)	E_{soil} (kN)			Tributary depth (m)	For at-rest conditions		
		Active	At rest	Passive		h_0 (m)	H_1 (kN)	h_1 (m)
1	3	61	101	679	4.5	13.26	101	13.25
2	6	98	164	1098	7.28	5.14	164	5.13
3	9	173	289	1933	10.52	2.06	289	2.04
4	12	204	340	2275	13.37	1.63	342	1.61
5	15	283	472	3162	16.52	1.04	482	1.01
6	18	310	517	3462	19.40	0.93	532	0.90
7	21	393	654	4385	22.52	0.69	694	0.64
8	24	417	694	4653	25.42	0.64	745	0.59
9	27	502	837	5606	28.52	0.52	950	0.44
10	30	523	873	5846	31.43	0.49	1004	0.42

moment, under the constraint that the moment at the location of a horizontal resultant is zero (Fig. 3), leading to the following system of nonlinear equations:

$$\begin{aligned} \sum_{i=1}^{j} H_i &= K \cdot \gamma_{soil} \frac{y_j^2}{2} \cdot w; \quad \text{for} \quad j = 1 \text{ to } n \\ \sum_{i=1}^{j} H_i \cdot d_i &= K \cdot \gamma_{soil} \frac{y_j^3}{3} \cdot w; \quad \text{for} \quad j = 1 \text{ to } n \end{aligned} \tag{13}$$

where i = number of the arch under consideration, n = number of arches over the height of the structure, y_j = depth to the lower edge of the horizontal earth pressure distribution that is tributary to the earth pressure resultant acting at depth d_i, and w = the tributary width.

The at-rest pressure is used for the design of the arch. The initial height of the arch, final horizontal component of the arch reaction accounting for elastic deformation, creep and shrinkage, and the final value of the height of the arch, are given in columns 7, 8, and 9, respectively. One can observe in Table 1(a) that the height of the arch decreases as the depth of the arch below grade increases. The corresponding earth pressure resultants increase as the depth below grade increases. The initial and long term deformations cause the height of the arch to decrease. The difference between the initial and final heights is small for relatively large arch heights, but becomes more substantial as the arch height decreases, with increasing depth below grade. The corresponding increase in horizontal resultant can be appreciable, being about 6% of the at-rest value for arch number 7 and increasing to approximately 15% for arch number 10. Fortunately, the passive pressure capacity is substantially greater than the at-rest value; however, any deformations of the soil due to pressures above the at-rest pressure should be considered.

The first arch in Table 1(a) is determined to have an initial height of 13.26 m, indicating that the arch midspan is 10.26 m above grade, and therefore would not be useful as an underground parking garage. A more practical solution can be obtained by (a) using a prestressed girder or a variable depth cast-in-place girder in place of the top arch (Fig. 1 and Fig. 6) or (b) increasing the horizontal reaction of the first arch to 400 kN per meter of width (a value that is intermediate between the at-rest and passive pressure resultants) and increasing the depth of the arch reaction from 3 m to 4 m.

Table 1(b) Initial and final values for arches at different depths

Arch no.	d (m)	E_{soil} (kN)			Tributary depth (m)	For at-rest conditions		
		Active	At rest	Passive		h_0 (m)	H_1 (kN)	h_1 (m)
1	4	108	180	1206	6.00	4.36	180	4.35
2	7	80	133	891	7.91	7.58	133	7.57
3	10	232	387	2593	11.83	1.36	390	1.34
4	13	177	295	1977	14.10	1.99	297	1.97
5	16	349	582	3899	17.76	0.80	602	0.77
6	19	277	462	3095	20.19	1.07	470	1.05
7	22	464	773	5179	23.71	0.56	835	0.52
8	25	379	632	4234	26.24	0.72	659	0.69
9	28	578	963	6452	29.69	0.44	1120	0.37
10	31	482	803	5380	32.28	0.54	876	0.49

The height of the arch satisfying these requirements is 1.00 m (determined by Eq. 7). The design of the remaining arches is shown in Table 1(b), corresponding to at-rest pressures for resultants located at 3 m increments beginning at a depth of 4 m. Table 1(b) indicates that design of the first arch for a value of 400 kN per meter of width is well within the range of 108 and 1206 kN per meter of width. The tributary depths, d, and horizontal resultants shown in Table 1(b) are determined based on the assumption that horizontal earth pressures increase linearly with depth below grade. The tributary depths no longer increase uniformly as for Table 1(a), and the horizontal resultants also do not increase uniformly.

Inspection of Table 1(b) reveals that, as for the first arch, the depth of the second arch associated with the at-rest earth pressure is too large. Instead of using a depth of 7.28 m, the second arch also will be sized to resist a horizontal earth pressure resultant of 400 kN per meter of width as was done for the first arch. The remaining arches can be designed to equilibrate the at-rest earth pressures, with dimensions and resultants as given in Table 1(b). Initial and long term deformations can be seen to cause a decrease in the height of the arch and an associated increase in the horizontal resultant. The increase is about 16% for arch number 9.

Factored combinations of dead and live loads are used to generate an envelope of horizontal resultants as well as factored moments and shears. The factored horizontal resultants should be compared with a reliable value of passive pressure resultant and these factored moments and shears are used for the design of the reinforcement of the arch according to standard methods of ultimate strength design. Table 2 indicates that the factored horizontal resultant, determined using the Eurocode 2 load factors, is well within the reliable values of the passive pressure resultants. (The second column of Table 2 repeats the 8th column of Table 1a). The foregoing creep and shrinkage calculations were made assuming no longitudinal reinforcement was present and therefore represent an upper bound expectation. The calculations can be repeated for a known quantity of longitudinal reinforcement to determine the expected increase in horizontal resultant based on the actual reinforcement provided.

Table 2 Variation of *H* with factored dead and live loads

	Service (unfactored)		Factored			
Arch no.	Dead loads H (kN)	Dead loads V (kN)	Dead loads $1.35 \cdot V$ (kN)	Live loads $1.5 * V$ (kN)	DL+LL V (kN)	DL+LL H (kN)
1	101	687	927	51	978	144
2	164	353	476	51	527	245
3	290	206	278	51	329	463
4	342	181	245	51	296	558
5	482	150	202	51	253	816
6	532	144	194	51	245	907
7	694	125	169	51	220	1220
8	745	127	171	51	222	1305
9	950	114	154	51	205	1708
10	1004	110	149	51	200	1819

5. Conclusions

Proposed in this paper is the use of an underground arch system for supporting gravity loads. The principal feature of this structural system is the use of soil pressure to equilibrate the horizontal thrust of the arch. Ensuring equilibrium places constraints on the height of the arch, while the non-uniform loading associated with obtaining a level finish floor surface places constraints on the alignment of the arch. Both instantaneous and long term deformations must be considered, as they may influence the height of the arch and horizontal reaction that is equilibrated by the soil. Fortunately, the wide separation of active and passive pressures can accommodate increases in the horizontal component of the arch reaction associated with creep and shrinkage.

The governing equations of equilibrium were developed to establish the alignment of the arch, both for cases in which deformations are neglected and cases where instantaneous and long term deformations are considered. Examples were used to illustrate the method of solution and the practicality of the structural system. The analytical results demonstrate that initial and long-term deformations have a greater influence on shallower arches. Because shallower arches are usually indicated at larger depths, the simple system of stacked single-story arches may be limited to the upper stories of the underground structure; alternatives such as multistory arches with suspended floors may be useful at greater depths. Both constant height arches and variable height arches can be used.

References

Chai, Y.H. and Kunnath, S.K. (2003), "Geometry of submerged funicular arches in Cartesian coordinates", *J. Struct. Eng.*, **129**(8), 1087-1092.

CIRIA Report 104 (1991), *Design of Retaining Walls Embedded in Stiff Clays*, Construction Industry Research and Information Association. London.

Eurocode 2 (1991), *Design of Concrete Structures. Part 1-1.: General Rules and Rules for Buildings*, European Committee for Standarization, Bruxelles, Belgium.

Gil-Martín, L.M., Hernández-Montes, E. and De la Fuente-Martín. (2001), “Simplified transverse seismic analysis of buried structures”, *Soil Dynamics and Earthquake Engineering*, **21**(8), 735-740.

Krauthammer, T. and Puglisi, R.D. (1992), “Simplified analysis of buried reinforced-concrete arches under simulated nuclear loads”, *Comput. Struct*., **43**(6), 1029-1039.

Leonard, J.W. (1988), *Tension Structures*. Mc-Graw Hill, New York.

McGrath, T.J. and Mastroianni, E.P. (2002), “Finite-element modelling of reinforced concrete arch under live load”, *Design of Structures 2002. Transportation Research Record* (1814), 203-210.

Yurkevich Engineering Bureau (1999), *Underground Parking-garage in the Revolution Square in Moscow: from Technical Proposal to Realization of the Project*, “Tunel 8, Rochnik № 1. (English version available from http://www.yurkevich.ru/pdf_publications/Rs_eng.pdf or http://www.yurkevich.ru/index-e.html)