Structural Engineering and Mechanics, Vol. 2, No. 4 (1994) 395-402
DOI: http://dx.doi.org/10.12989/sem.1994.2.4.395

Geometrically non-linear dynamic analysis of plates by an improved finite element-transfer matrix method on a microcomputer

YuHua Chen†

Department of Physics, SuZhou University, SuZhou, JiangSu 215006, China

Abstract. An improved finite element-transfer matrix method is applied to the transient analysis of plates with large displacement under various excitations. In the present method, the transfer of state vectors from left to right in a combined finite element-transfer matrix method is changed into the transfer of generally incremental stiffness equations of every section from left to right. Furthermore, in this method, the propagation of round-off errors occurring in recursive multiplications of transfer and point matrices is avoided. The Newmark-β method is employed for time integration and the modified Newton-Raphson method for equilibrium iteration in each time step. An ITNONDL-W program based on this method using the IBM-PC/AT microcomputer is developed. Finally numerical examples are presented to demonstrate the accuracy as well as the potential of the proposed method for dynamic large deflection analysis of plates with random boundaries under various excitations.

Key words: Finite element; transfer matrix method; dynamic analysis; large displacement; microcomputer.

1. Introduction

In numerical analysis of engineering structures, the microcomputer is playing an increasingly important role in China. The most powerful and most widely used numerical method in structural analysis is the finite element method (FEM). The disadvantage of FEM, however, is that, in the case of complex structures, a large number of nodes must be used, with the result that large amounts of microcomputer computation time are needed. Such disadvantages become rather serious, especially in the transient analysis of the geometrically non-linear structures under various excitations. This entails such direct integration methods as the Newmark-β (Newmark 1959) method or the Wilson-θ (Bathe and Wilson 1976) method on the microcomputer.

The combined finite element-transfer matrix method (FETM) was proposed for the first time by Dokainish (1972) for free plate vibration problems. This method has the advantage of reducing stiffness matrix size to much smaller than that obtained with the FEM method and was successfully applied to various linear and non-linear problems (McDaniel and Eversole 1977, Ghiatti and Sestieri 1979, Ohga, *et al.* 1983, 1984, Ohga and Shigematsu 1987, 1988, Chen and Xue 1991). However in the FETM method, the submatrix $[K_{LR}]_i$ must be a square matrix in order to derive the inverse matrix of the submatrix $[K_{LR}]_i$ of the stiffness matrix $[K]_i$ for strip i. Therefore for strip i, the number of degrees of freedom on the left boundary must be the same as on

† Lecturer

the right. In addition to this, the transformation of state vectors is employed to avoid propagation of round-off errors occuring in recursive multiplications of transfer and point matrices (Ohga and Shigematsu 1987, Chen and Xue 1991). For these reasons, various techniques (Ghiatti and Sestieri 1979, Muctno and Pavelic 1980, Sankar and Hoa 1980) for treating more complicated structures are presented, even though research into this problem has been as yet insufficient.

The purpose of this paper is to present an improved finite element-transfer matrix (IFETM) method for geometrically non-linear dynamic analysis of plates with random boundaries under various excitations. In the present method, because the transfer of incremental state vectors from left to right in the FETM method is transformed into the transfer of generally incremental stiffness equations in every section from left to right, the inverse matrix of sub-matrix $[K_{LR}]$ of the FETM method becomes the inverse matrix of sub-matrix $[K_{LL}]$ of the IFETM method. It is well known that $[K_{LL}]$ is always a square matrix whether the structures are rectangular plates or not. Since the numerical solution of a two point boundary value problem in the FETM method is converted into the numerical solution of an initial value problem in the present method, the propagation of round-off errors occuring in recursive multiplications of the transfer and point matrices is avoided. The Newmark-β method is used for time integration, but other integration methods, such as the Wilson-θ method and the Houbolt method, may be used. The modified Newton-Raphson method is employed for equilibrium iteration in each time step. The ITNONDL-W program based on this method using a microcomputer is developed.

Some numerical example of non-linear dynamic problems are also given and their results compared with those obtained with the ordinary finite element method and other methods.

2. Direct integration method

Proceeding as in Chen and Xue (1991), which is concerned with transient analysis of geometrically non-linear system, we obtain linearizing incremental equilibrium equations of strip i in Fig. 1 from time t to $t+\Delta t$.

$$[M]_i\{\Delta\ddot{U}\}_i+[C]_i\{\Delta\dot{U}\}_i+[K_T]_i\{\Delta U\}_i=\{\Delta R\}_i+\{\Delta N\}_i \tag{1}$$

in which $[M]_i$ and $[C]_i$ are mass and damping matrices of strip i, $[K_T]_i$ is the tangent stiffness matrix of strip i at time t defined in Zienkiewicz (1977), $\{\Delta\ddot{U}\}_i$, $\{\Delta\dot{U}\}_i$, $\{\Delta U\}_i$ and $\{\Delta R\}_i$ are respectively incremental acceleration, velocity, displacement and external load vectors in time interval Δt from time t to $t+\Delta t$. $\{\Delta N\}_i$ represents the incremental internal force vectors in time interval Δt from time t to $t+\Delta t$ produced by adjacent strips on the left and right of strip i.

As described previously, the Newmark-β method is used for time integration. We gain by a series of operations

$$\{\Delta\ddot{U}\}_i=\frac{1}{\beta\cdot\Delta t^2}\{\Delta U\}_i-\frac{1}{\beta\cdot\Delta t}\{\dot{U}_t\}_i-\frac{1}{2\beta}\{\ddot{U}_t\}_i \tag{2}$$

$$\{\Delta\dot{U}\}_i=\frac{\gamma}{\beta\cdot\Delta t}\{\Delta U\}_i-\frac{\gamma}{\beta}\{\dot{U}_t\}_i-\left(\frac{\gamma}{2\beta}-1\right)\Delta t\{\ddot{U}_t\}_i \tag{3}$$

where β and γ are parameters capable of obtaining integration accuracy and stability. When $\beta=1/6$ and $\gamma=1/2$, this method reduces to the linear acceleration method and when $\beta=1/4$ and $\gamma=1/2$, to the constant average acceleration method.

Substituting Eq. (2) and Eq. (3) into Eq. (1), we have

$$[H]_i\{\Delta U\}_i=\{\Delta G\}_i+\{\Delta N\}_i \tag{4}$$

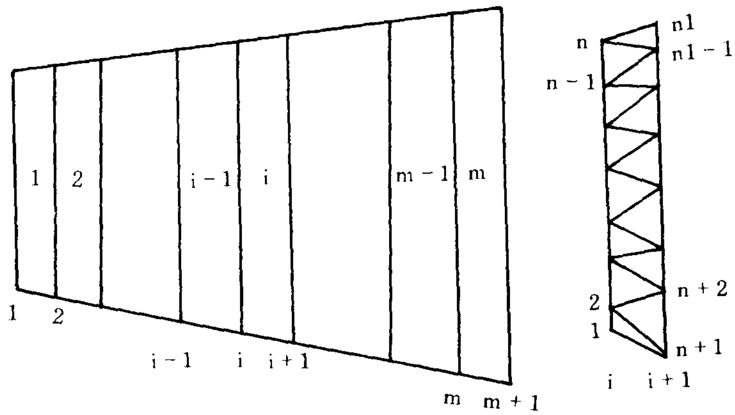

Fig. 1 Subdivision of structure into strips and finite elements.

where

$$[H]_i = \frac{1}{\beta\cdot\Delta t^2}[M]_i + \frac{\gamma}{\beta\cdot\Delta t}[C]_i + [K_T]_i \tag{5}$$

$$\{\Delta G\}_i = \{\Delta R\}_i + [M]_i\left(\frac{1}{\beta\cdot\Delta t}\{\dot{U}_t\}_i + \frac{1}{2\beta}\{\ddot{U}_t\}_i\right) +$$

$$[C]_i\left(\frac{\gamma}{\beta}\{\dot{U}_t\}_i + \left(\frac{\gamma}{2\beta} - 1\right)\Delta t\{\ddot{U}_t\}_i\right) \tag{6}$$

Eq. (4) is an equation with unknown variables $\{\Delta U\}_i$ as well as $\{\Delta N\}_i$ and can be solved by the IFETM method described in the following. After the $\{\Delta U\}$ of the total structure is solved, the displacements, velocities and accelerations in time $t+\Delta t$ are given as follows:

$$\{U_{t+\Delta t}\}_i = \{\Delta U\}_i + \{U_t\}_i \tag{7}$$

$$\{\dot{U}_{t+\Delta t}\}_i = \frac{\gamma}{\beta\cdot\Delta t}\{\Delta U\}_i + \left(1 - \frac{\gamma}{\beta}\right)\{\dot{U}_t\}_i + \left(1 - \frac{\gamma}{2\beta}\right)\Delta t\{\ddot{U}_t\}_i \tag{8}$$

$$\{\ddot{U}_{t+\Delta t}\}_i = \frac{1}{\beta\cdot\Delta t^2}\{\Delta U\}_i - \frac{1}{\beta\cdot\Delta t}\{\dot{U}_t\}_i + \left(1 - \frac{1}{2\beta}\right)\{\ddot{U}_t\}_i \tag{9}$$

It should be pointed out that the $\{\Delta U\}$ represents only approximate incremental displacements.

3. Improved finite element-transfer matrix method

Without losing generality, we consider the plate shown in Fig. 1 to be divided into m strips, with each strip subdivided into a finite element. The vertical sides dividing or bordering the strips are called sections. It is apparent that the right of section i is the left of strip i.

Let $\{\Delta U\}_i^L$, $\{\Delta N\}_i^L$ and $\{\Delta U\}_i^R$, $\{\Delta N\}_i^R$ be the left and right incremental displacement and force vectors of section i from time t to $t+\Delta t$.

We assume that the generalized stiffness equations which relate the incremental force vectors to the incremental displacement vectors on the left of section i are given by

$$\{\Delta N\}_i^L = [T]_i \{\Delta U\}_i^L + \{\Delta E\}_i \qquad (i>=2) \tag{10}$$

3.1. Transfer at section i

The deflections are continuous across section i, so that we obtain

$$\{\Delta U\}_i^L = \{\Delta U\}_i^R \tag{11}$$

Without losing generality, we suppose that there is no concentrated external load acting on section i. Due to the continuity of force at section i, we obtain

$$\{\Delta N\}_i^L = -\{\Delta N\}_i^R \tag{12}$$

Substituting Eq. (11) and Eq. (12) into Eq. (10), we obtain

$$\{\Delta N\}_i^R = -[T]_i \{\Delta U\}_i^R - \{\Delta E\}_i \tag{13}$$

Eq. (13) describes the relation between the incremental internal force vectors and the incremental displacement vectors on the right of section i.

3.2. Transfer in strip i

Eq. (4) is rearranged and repartitioned. We obtain

$$\begin{bmatrix} [H_{LL}] & [H_{LR}] \\ [H_{RL}] & [H_{RR}] \end{bmatrix}_i \cdot \begin{Bmatrix} \{\Delta U\}_i^R \\ \{\Delta U\}_{i+1}^L \end{Bmatrix} = \begin{Bmatrix} \{\Delta N\}_i^R \\ \{\Delta N\}_{i+1}^L \end{Bmatrix} + \begin{Bmatrix} \{\Delta Q\} \\ \{\Delta \bar{Q}\} \end{Bmatrix}_i \tag{14}$$

in which $[H_{LL}]$, $[H_{LR}]$, $[H_{RL}]$ and $[H_{RR}]$ are the submatrices of matrix $[H]$ in Eq. (5); $\{\Delta Q\}$ and $\{\Delta \bar{Q}\}$ are respectively the generalized incremental external force vectors on the left and the right of strip i, obtained from $\{\Delta G\}$ in Eq. (6).

By expanding Eq. (14) and using a series of additional operations, we obtain

$$\{\Delta U\}_i^R = -([H_{LL}] + [T])_i^{-1} [H_{LR}]_i \{\Delta U\}_{i+1}^L + ([H_{LL}] + [T])_i^{-1} (\{\Delta Q\} - \{\Delta E\})_i \tag{15}$$

and

$$\{\Delta N\}_{i+1}^L = [T]_{i+1} \{\Delta U\}_{i+1}^L + \{\Delta E\}_{i+1} \tag{16}$$

where

$$[T]_{i+1} = [H_{RR}]_i - [H_{RL}]_i ([H_{LL}] + [T])_i^{-1} [H_{LR}]_i \tag{17}$$

$$\{\Delta E\}_{i+1} = [H_{RL}]_i ([H_{LL}] + [T])_i^{-1} (\{\Delta Q\} - \{\Delta E\})_i - \{\Delta \bar{Q}\}_i \tag{18}$$

Eq. (16) represents the relationships for the incremental internal force vectors and the incremental displacement vectors on the left of section $i+1$ in time interval Δt from time t to $t+\Delta t$.

3.3. Transfer of entire structure

Using Eq. (17) and Eq. (18), $[T]$ and $\{\Delta E\}$ are transfered from the left of the second section to the right of total structure. Hence we have

$$\{\Delta N\}_{m+1}^L = [T]_{m+1} \{\Delta U\}_{m+1}^L + \{\Delta E\}_{m+1} \tag{19}$$

By considering boundary conditions, the known incremental force or displacement variables on the right hand boundary of the total structure are substituted into Eq. (19) to determine the unknown incremental force or displacement variables. After the incremental force and displacement vectors on the right hand boundary of the total structure are solved, the incremental force and displacement vectors at any section i are calculated by Eq. (15) and Eq. (13).

3.4. The method of determining $[T]_2$ and $\{\Delta E\}_2$

For strip 1, by expanding Eq. (14), we have

$$[H_{LL}]_1\{\Delta U\}_1^R+[H_{LR}]_1\{\Delta U\}_2^L=\{\Delta N\}_1^R+\{\Delta Q\}_1 \tag{20}$$

$$[H_{RL}]_1\{\Delta U\}_1^R+[H_{RR}]_1\{\Delta U\}_2^L=\{\Delta N\}_2^L+\{\Delta \bar{Q}\}_1 \tag{21}$$

It is obvious that $\{\Delta U\}_1^R$ and $\{\Delta N\}_1^R$ may be determined by using the left hand boundary conditions of the total structure.

3.4.1. Displacement boundary condition

It is obvious that $\{\Delta U\}_1^R$ is known to be in a displacement boundary condition, hence by Eq. (21), we obtain

$$[T]_2=[H_{RR}]_1 \tag{22}$$

$$\{\Delta E\}_2=[H_{RL}]_1\{\Delta U\}_1^R-\{\Delta \bar{Q}\}_1 \tag{23}$$

3.4.2. Force boundary condition

It is obvious that $\{\Delta N\}_1^R$ is known to be in a force boundary condition, hence $\{\Delta U\}_1^R$ is obtained from Eq. (20). Substituting the $\{\Delta U\}_1^R$ into Eq. (21), we have

$$[T]_2=[H_{RR}]_1-[H_{RL}]_1[H_{LL}]_1^{-1}[H_{LR}]_1 \tag{24}$$

$$\{\Delta E\}_2=[H_{RL}]_1[H_{LL}]_1^{-1}(\{\Delta N\}_1^R+\{\Delta Q\}_1)-\{\Delta \bar{Q}\}_1 \tag{25}$$

3.4.3. Mixture boundary condition

In mixture boundary condition, we suppose $\{\Delta U\}_1^R=[\{\Delta U'\}_1^R, \{\Delta U''\}_1^R]^T$ and the corresponding $\{\Delta N\}_1^R=[\{\Delta N'\}_1^R, \{\Delta N''\}_1^R]^T$. If $\{\Delta U'\}_1^R$ is unknown and $\{\Delta U''\}_1^R$ is known, the corresponding $\{\Delta N'\}_1^R$ is known and $\{\Delta N''\}_1^R$ is unknown. For strip 1, Eq. (14) is rearranged and repartitioned, so we have

$$\begin{bmatrix} [D_{11}] & [D_{12}] & [D_{13}] \\ [D_{21}] & [D_{22}] & [D_{23}] \\ [D_{31}] & [D_{32}] & [D_{33}] \end{bmatrix} \begin{Bmatrix} \{\Delta U'\}_1^R \\ \{\Delta U''\}_1^R \\ \{\Delta U\}_2^L \end{Bmatrix} = \begin{Bmatrix} \{\Delta N'\}_1^R \\ \{\Delta N''\}_1^R \\ \{\Delta N\}_2^L \end{Bmatrix} + \begin{Bmatrix} \{\Delta Q'\}_1 \\ \{\Delta Q''\}_1 \\ \{\Delta \bar{Q}\}_1 \end{Bmatrix} \tag{26}$$

expanding Eq. (26) and solving relations for $\{\Delta N\}_2^L$ and $\{\Delta U\}_2^L$, we obtain

$$[T]_2=[D_{33}]-[D_{31}][D_{11}]^{-1}[D_{13}] \tag{27}$$

$$\{\Delta E\}_2=[D_{31}][D_{11}]^{-1}(\{\Delta N'\}_1^R+\{\Delta Q'\}_1)+[D_{32}]\{\Delta U''\}_1^R-[D_{31}][D_{11}]^{-1}[D_{12}]\{\Delta U''\}_1^R-\{\Delta \bar{Q}\}_1 \tag{28}$$

3.5. Iterative procedure

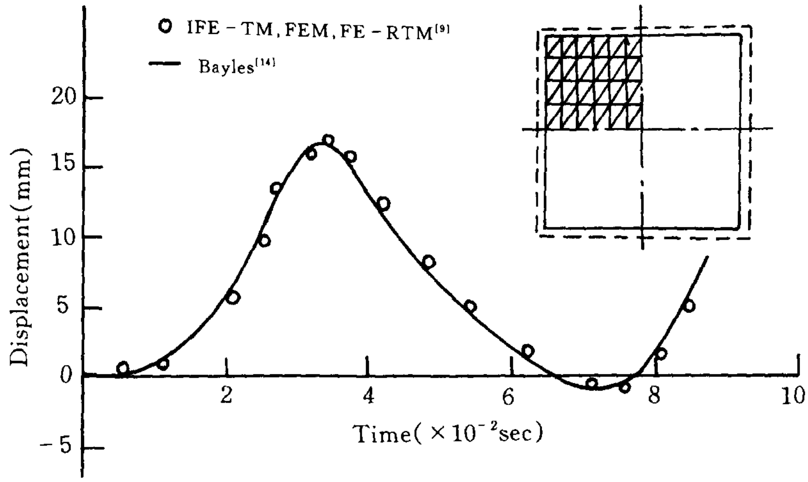

Fig. 2 Central displacement response for a simply supported square plate.

It should be pointed out that because the motion equation linearizing method referred to in Chen and Xue (1991) was employed, the incremental displacement and force vectors solved from the above are only approximations of accurate incremental displacement and force vectors. Hence, a modified Newton-Raphson method is applied to iterate for dynamic equilibrium equations. The iterative scheme is the same as that in Chen and Xue (1991).

4. Numerical examples

To examine the accuracy and the computative efficiency of the IFETM method, we developed the ITNONDL-W program based on this method using an IBM-AT microcomputer. Some numerical results of the square and elliptical plates are compared with those obtained using the ordinary finite element method and other methods.

(1) A simply supported square plate subjected to a suddenly applied uniform pressure is selected for the sample problem. The plate chosen is $244\times244\times0.635$ cm with a specific weight of 24.74 KN/m^3, $\nu=0.23$, $E=6.895\times10^4$ Mpa and is subjected to a uniform pressure of 479 N/m^2. In the numerical calculation, a quarter of the plate is divided into 6 strips and each of them subdivided into 8 triangular plate elements as shown in Fig. 2; time step $\Delta t=0.001$ sec is used. Fig. 2 shows a comparison between the IFETM solutions and the FE solutions by using the ADINA program*, th FERTM solutions (Chen and Xue 1991) as well as Bayle's results (Bayles, *et al.* 1972), where the IFETM, FERTM and FE methods are applied to the same mesh pattern and accuracy. A comparison indicates that very little difference exists among the three results. Consequently the plots in Fig. 2 are not distinct and there is very good agreement between the IFETM method solutions and Bayle's results. Table 1 shows comparisons of average computation time for each time step between the IFETM and FE methods in the square plate and elliptical plate examples. It may be observed from Table 1 that computation time using the IFETM method is less than half that using the FE method.

(2) A clamped elliptical plate with half major axis $b=150$ cm, half minor axis $a=100$ cm,

* The ADINA program has been translated for use on a chinese microcomputer.

Table 1 Comparision of average computation time for each time step*

Method by applying	Computation time (sec)	
	Square plate	Elliptical plate
IFETM	82	205
FE	190	302

*Microcomputer AST-386 is used.

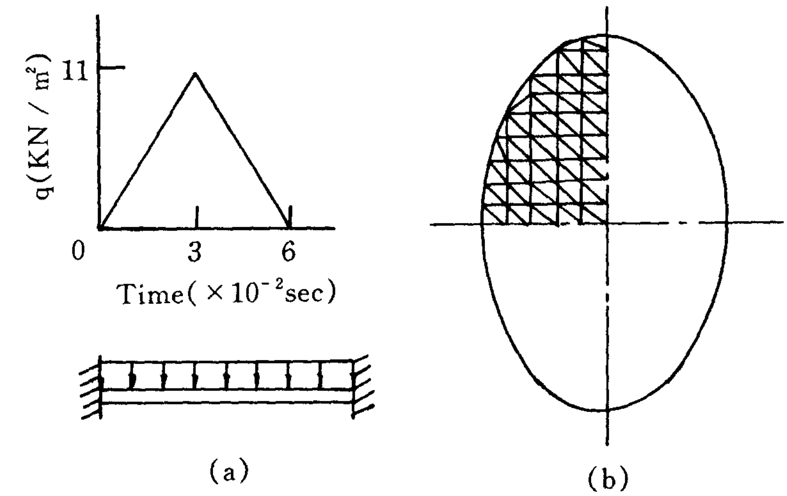

Fig. 3 Load model and elliptical plate model.

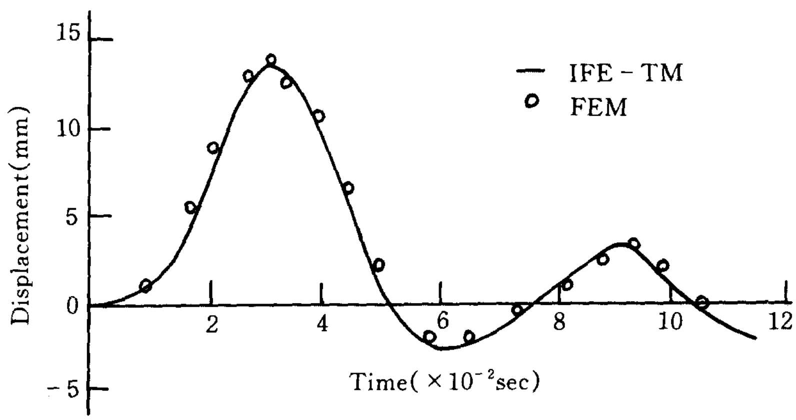

Fig. 4 Central displacement response for a clamped elliptical plate.

thickness $h=1$ cm, modules of elasticity $E=2.0\times10^5$ MPa and poison's ratio $\nu=0.3$ is subjected to a uniform pressure as shown in Fig. 3(a). It is shown in Fig. 3(b) that a quarter of the elliptical plate is divided into 5 substructures which are divided into many triangular plate elements. Fig. 4 compares the dynamic deflection responses at the central point resulting from the employment of both the IFETM and the FE methods, where time step $\Delta t=0.0025$ sec is used. The comparison indicates that results from using the IFETM method coincide completely with those obtained from using the FE method for the same mesh pattern, time step and accuracy. A comparison of computation time shown in Table 1 indicates that a computation efficiency of the IFETM method is higher than that of the FE method.

The two numerical examples described above demonstrate the accuracy and potential of the present method for dynamic large deflection analysis of rectangular and non-rectangular plates under various excitations.

5. Conclusions

In this paper, an improved finite element-transfer matrix method is applied to the transient analysis of geometrically non-linear structures under various excitations. An ITNONDL-W micro-computer program based on this method was developed. Some numerical examples presented in this paper show that the proposed method can be successfully applied to the transient analysis of large deformation plates with random boundaries under various excitations. Like the FETM method, the present method has the same advantage of reducing matrix size to less than that obtained using the ordinary finite element method. The present method, however, has potentially wider application than the FETM method.

References

Bathe, K. J. and Wilson, E. L. (1976), *Numerical Methods in Finite Element Analysis*, Prentice-Hall, Inc., 319-322.

Bayles, D. J., Lowery, R. L. and Boyd, D. E. (1972), "A nonlinear dynamic lumped-parameter model of a rectangular plate", *J. Sound Vibr.*, **21**(3), 329-337.

Chen, YuHua and Xue, HuiYu (1991), "Dynamic large deflection analysis of structures by a combined finite element-Riccati transfer matrix method on a microcomputer", *Comput. Struct.*, **39**(6), 699-703.

Dokainish, M. A. (1972), "A new approach for plate vibration: combination of transfer matrix and finite element technique", *Trans. ASME, J. Engng. Ind.*, **94**(2), 526-530.

Ghiatti, G. and Sestieri, A. (1979), "Analysis of static and dynamic structural problems by a combined finite element-transfer matrix method", *J. Sound Vibr.*, **67**(1), 35-42.

McDaniel, T. J. and Eversole, K. B. (1977), "A combined finite element transfer matrix structural analysis method", *J. Sound Vibr.*, **51**(2), 157-169.

Muctno, H. V. and Pavelic, V. (1980), "An exact condensation procedure for chain-like structures using a finite element-transfer matrix approach", *Trans. ASME, J. Mech. Des.*, 80-C2/DET-123, 1-9.

Newmark, N. M. (1959), "A method of computation for structural dynamic", *J. ASCE*, **85**(EM3), 67-94.

Ohga, M. and Shigematsu, T. (1988), "Large deformation dynamic analysis of plates", *J. ASCE*, **114**(4), 624-637.

Ohga, M. and Shigematsu, T. (1987), "Transient analysis of plate by a combined finite element-transfer matrix method", *Comput. Struct.*, **26**(4), 543-549.

Ohga, M., Shigematsu, T. and Hara, T. (1983), "Structural analysis by a combined finite element-transfer matrix method. *Comput. Struct.*, **17**(3), 321-326.

Ohga, M., Shigematsu, T. and Hara, T. (1984), "A combined finite element transfer matrix method", *J. Engng Mech. Div., Am. Soc. Civ. Engrs*, **110**(9), 1335-1349.

Sankar, S. and Hoa, S. V. (1980), "An extended transfer matrix-finite element for free vibration of plates", *J. Sound Vibr.*, **70**(2), 205-211.

Zienkiewicz, O. C. (1977), *The Finite Element Method*, 3rd edn., McGraw-Hill.