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Effects of modelling on the earthquake response of asymmetrical multistorey buildings

David P. Thambiratnam †

School of Civil Engineering, Queensland University of Technology, Brisbane, Queensland 4001, Australia

Abstract. Responses of asymmetrical multistorey buildings to earthquakes are obtained by the quasi-static code approach and real time dynamic analysis, using two different structural models. In the first model, all vertical members are assumed to be restrained at the slab levels and hence their end rotations, about horizontal axes, are taken as zero. In the second model this restriction is removed and the rotation is assumed to be proportional to the lateral stiffness of the member. A simple microcomputer based procedure is used in the analyses, by both models. Numerical examples are presented where results obtained from both the models are given. Effects of modelling on the response of three buildings, each with a different type and degree of asymmetry, are studied. Results for deflections and shear forces are presented and the effects of the type of model on the response are discussed.

Key words: asymmetry; multistorey building; torsional coupling; core; twisting; earthquake loads; quasi-static method; dynamic analysis; modelling; degrees of freedom.

1. Introduction

Most multistorey buildings are asymmetric as they have either an unsymmetrical elevation and/or plan or an unsymmetrical distribution of vertical members or an unsymmetrical mass distribution in the floors. When asymmetric buildings are subjected to lateral loads caused by winds or earthquakes, there will be coupling between the lateral and torsional components of the building response and the building is said to be torsionally unbalanced. A torsionally balanced building is one which has at each floor level coincident centres of mass and stiffness which lie on a common vertical axis. However, the conditions for torsional balance are so restrictive so as to never actually occur. Due to this torsional coupling, lateral load analysis of asymmetric buildings is complicated and the true response can deviate considerably from that predicted and allowed for in design. Structural asymmetry has been identified as a cause for the failure or poor performance of multistorey buildings during recent earthquakes (Chandler 1991). Hence it is important to correctly model and analyse and adequately design multistorey buildings for earthquake loads. Research on the lateral load analysis of asymmetric buildings has been pursued by many (Balendra et al. 1984, Cheung and Tso 1987, Thambiratnam and Irvine 1989 and Stafford Smith and Cruvellier 1990); while response of buildings to earthquakes in particular has also been treated extensively by many including Chandler et al.

† Assoc. Professor

(1986, 1991), Kan and Chopra (1981), Bozorgiua and Tso (1986) and Tso and Dempsey (1980). Most of the models proposed so far are either too simple and pertain to single storey buildings or tedious and require extensive computational effort. Since the results of these studies are model dependent and ambiguous they will not be applicable to all asymmetric buildings. Then there are also computer packages, usually employing the finite element method, which can do "almost anything". But the author has found that some of the more popular programs take several hours to analyse typical multistorey buildings. This might render them unsuitable to practising engineers with limited time and computer resources. Hence there is a need for a simple model(s) which can be used to analyse several types of, or all asymmetric buildings.

According to seismic design codes, dual design criteria are used for the earthquake resistant design of buildings. These are: (a) buildings should be able to resist minor/moderate earthquakes without structural damage and (b) buildings should be able to resist major earthquakes without collapse or loss of life but with some (selective) structural damage permitted. In Australia where only moderate earthquakes can be expected, design for criterion (a), involving elastic analysis, is usually considered sufficient (Irvine and Hutchinson 1991), unless the building concerned is one of great significance.

Structural engineers have three methods of analysing buildings subjected to earthquakes. These are, in order of preference: (i) real time dynamic analysis, (ii) response spectrum method and (iii) quasi-static code approach (Standards Association of Australia 1979, Irvine and Hutchinson 1991). The first two methods, though more accurate, will usually require an elaborate finite element model of the structure and will therefore be expensive and time consuming. For this reason, the third method is commonly used by practising engineers. But it has been found that, even in the case of a symmetric building, the quasi-static code approach can grossly underestimate the earthquake response by as much as 200% (Berg 1988, Park and Paulay 1975). For an asymmetric building, when the quasi-static code approach is used on a plane frame taken at a particular section of the building, it is difficult to calculate and provide for the effects of horizontal torsion and hence the true response to earthquakes can be quite different to that anticipated and provided for. Moreover the code formulae for lateral loads depend on the fundamental period of the building, which in turn is calculated (estimated) by using different empirical formulae in different codes. It is therefore not possible to obtain the same response when different codes are used in the design of a building.

In order to provide for the effects of asymmetry, in either a static or a dynamic analysis, a three dimensional model is required and a search has been on for such a model which is easy to use. For important buildings subjected to mild or moderate earthquakes, it is preferable to carry out a real time dynamic analysis or a response spectrum analysis for which too a (simple) three dimensional model is required. The author developed a simple microcomputer based procedure to carry out lateral load analysis of asymmetric buildings using the shear beam model (Thambiratnam and Irvine 1989). In this model, it was assumed that there was no rotation of the vertical members at the slab junctions. Later, this restriction was removed and the work was extended to accommodate other types of buildings (Thambiratnam and Thevendran 1992). Rotations at the ends of the vertical members, about (local) horizontal axes, were assumed to be proportional to the respective lateral stiffnesses of the members. These two models can be used along with the code provisions to obtain either a quasi-static analysis or a dynamic time history analysis of buildings subjected to earthquakes.

The present study investigates the earthquake response of multistorey buildings, which are asymmetric for one reason or another, using the two different models and the simplified procedures developed by the author. In these multistorey buildings, asymmetry has been introduced either by an unsymmetrical elevation and/or plan, or by an unsymmetrical distribution of vertical members. For such asymmetrical buildings, torsional coupling becomes significant and this paper examines the effects of two different types of modelling on their response to earthquakes.

2. Analytical models

The models and the simple procedures developed by Thambiratnam and Irvine (1989) and Thambiratnam and Thevendran (1992) for lateral load analysis of asymmetric buildings are used in this paper to study the response of the asymmetric buildings to earthquakes. Consider the asymmetric building of n storeys shown in Figure 1. A reference column R is chosen and a common vertical axis along this column originating at R_o on the ground floor intersects each floor at R_i($i = 1$ to n storeys). R_i is taken as the origin of the x–y axes at each floor level. Thus the position P of a vertical member r at any floor level is defined by the coordinates x_r, y_r as shown in Figure 2a. In Figure 1, the reference column R is chosen at the bottom left hand corner in the plan for convenience.

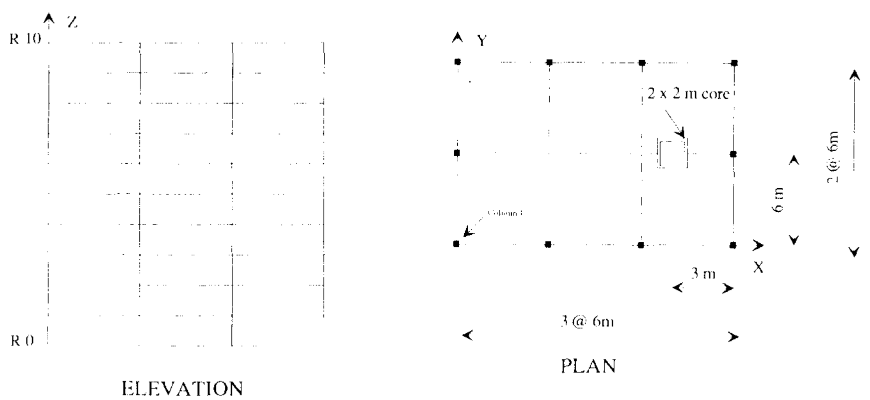

Fig. 1 10 Storey Asymmetric Building

2.1. Model (a) – Three degrees of freedom at ends of vertical members

The following assumptions are made in both the three dimensional models used in this paper:

(i) Floors are treated as rigid diaphragms each of which possesses two horizontal translational and one rotational (about the vertical axis) degrees of freedom. These are shown as u_i, v_i, θ_i in Figure 2b with respect to the reference point R_i.

(ii) Kinetic energies of vertical members are neglected. However these can be included, if necessary, by lumping column masses on to the floors between which the columns span. For buildings with shear walls, point masses can be added when working out the mass moments of inertia for floors.

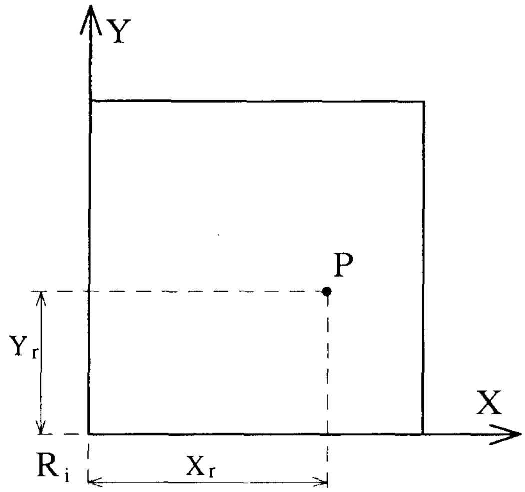

Fig. 2a Reference Axes at Floor Level "i"

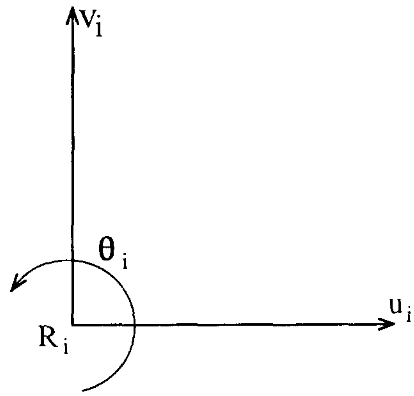

Fig. 2b Degrees of Freedom at Floor Level "i"

(iii) In the first model, all vertical members are assumed to be restrained at the floor levels, i.e. a shear beam model is assumed; while in the second model rotations about horizontal axes are allowed and these are assumed to be proportional to the lateral stiffnesses of the members.

Because of assumption (i), it is possible to express the end displacements w_{ri} of a vertical member (such as P in Figure 2a), in terms of those at the reference point R_i, in the form

$$w_{ri} = C_r \, w_{Ri} \tag{1}$$

where $(w_{ri})^T = (u_{ri}, v_{ri}, \theta_i)$, $(w_{Ri})^T = (u_i, v_i, \theta_i)$ is the displacement vector at R_i and the transformation matrix C_r is given by

$$C_r = \begin{bmatrix} 1 & 0 & -y_r \\ 0 & 1 & x_r \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

Similarly, the centroidal velocities of the slabs can also be expressed in terms of those at the nodes on the reference column. The reference column will have a node with three degrees of freedom at each floor level. Since the potential energies of all vertical members and the kinetic energies of all slabs can be expressed in terms of the nodal degrees of freedom on the R axis, the structure stiffness and mass matrices and the equivalent load vector can be considered to be "consistently lumped" on this axis. The equations of motion are derived using Lagrange's equations, and in the absence of damping will be

$$M\ddot{w}_R + Kw_R = P_R \tag{3}$$

Where M and K are the structure mass and stiffness matrices respectively, P_R the nodal load vector, w_R, and $\ddot{w}_R$ are the vectors of nodal displacements and nodal accelerations respectively. The orders of all the matrices and the vectors in the above equation are $3n$, where n is the number of storeys.

2.2. Model (b) – Five degrees of freedom at each end of a vertical member

Additional assumptions made in the second model are:

(vi) Kinetic energies corresponding to bending rotations of vertical members are not considered.

(vii) Bending rotations α_{ri}, β_{ri} at the ends of a vertical member r at level i about the x and y axes are proportional to its lateral stiffness about these axes and are given by

$$\alpha_{ri} = a_{rx}\,\alpha_i\,; \quad \beta_{ri} = a_{ry}\,\beta_i \tag{4}$$

where α_i, β_i are the corresponding rotations of the reference column R and a_{rx} a_{ry} are the ratios of the second moments of area of the member r to that of member R about the x and y axes respectively.

In this model there are 5 degrees of freedom at each end of a vertical member and hence the vector of nodal displacements is given by $(\delta_{ri})^T = (u_{ri}, v_{ri}, \theta_i, \alpha_{ri}, \beta_{ri})$. This vector can be related to the vector of nodal displacements $(\delta_{Ri})^T = (u_i, v_i, \theta_i, \alpha_i, \beta_i)$ in the form

$$(\delta_{ri})^T = C_r(\delta_{Ri})^T \tag{5}$$

The transformation matrix C_r in equation (5) is given by:

$$C_r = \begin{bmatrix} 1 & 0 & -y_r & 0 & 0 \\ 0 & 1 & x_r & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & a_{rx} & 0 \\ 0 & 0 & 0 & 0 & a_{ry} \end{bmatrix} \tag{6}$$

For the second model all five degrees of freedom at the ends of vertical members contribute to the potential energy and hence to the stiffness matrix; but the contributions from the bending rotations to the kinetic energy and hence to the mass matrix are not considered. Hence the order $(5n)$ of the stiffness matrix K is higher than that $(3n)$ of the mass matrix (M) by $(2n)$, twice the number of floors in the building. The degrees of freedom pertaining to the bending rotations $(\gamma_R = \alpha_R, \beta_R)$, are eliminated by condensation prior to solving the equations of motion.

For a four storey building M and K for either model are given by

$$M = \begin{bmatrix} M_1 & 0 & 0 & 0 \\ 0 & M_2 & 0 & 0 \\ 0 & 0 & M_3 & 0 \\ 0 & 0 & 0 & M_4 \end{bmatrix} \tag{7}$$

$$K = \begin{bmatrix} K_{R1}+K_{R2} & -K_{R2} & 0 & 0 \\ -K_{R2} & K_{R2}+K_{R3} & -K_{R3} & 0 \\ 0 & -K_{R3} & K_{R3}+K_{R4} & -K_{R4} \\ 0 & 0 & -K_{R4} & K_{R4} \end{bmatrix} \tag{8}$$

where M_i is the mass matrix for storey i and K_{Ri} is the stiffness matrix for vertical members between the i and $(i-1)$ storeys (please see Appendix). In the quasi-static code approach, the static loads acting at the slab centroids are replaced by an equivalent set of static loads at the nodes on the reference axis. In the real time dynamic analysis, the ground acceleration components $\ddot{u}_g$ and $\ddot{v}_g$ specified in the x and y directions are used to obtain the inertial load vector P_i at level i in the form

$$P_i = -m_i \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -y_{Gi} & x_{Gi} \end{bmatrix} \begin{Bmatrix} \ddot{u}_g \\ \ddot{v}_g \end{Bmatrix} \tag{9}$$

where m_i is the slab mass and x_{Gi}, y_{Gi} are the centroidal coordinates of the slab. The load vector P_R is formed from the various P_i acting at all levels.

The equations of motion (3) can be conveniently programmed on a microcomputer to obtain (Corderoy and Thambiratnam 1993):

(a) free vibration analysis of the building with $P_R = 0$;
(b) earthquake response of the building by the quasi-static code approach; and
(c) real time dynamic analysis.

For case (b), the fundamental period T_1 determined from case (a) can be used in preference to that determined from the formulae given in the code. For case (c) direct numerical integration of equation (3) is carried out using a linear variation of acceleration over the time interval Δt (Clough and Penxien 1975). The time step Δt is chosen to be $<< T_1/10$, to ensure that contributions from a reasonable number of the earlier modes are included in the analysis.

3. Numerical examples and discussion

Numerical examples are treated in this paper to illustrate the effects of modelling on the earthquake response of multistorey buildings. Results for the two models obtained by the quasi-static code approach and real time dynamic analysis are presented. Three multistorey buildings, each with a different type of asymmetry, are considered: (i) building with set-backs, (ii) L shaped in plan with a core and (iii) rectangular in plan with an eccentric core. The buildings are of reinforced concrete for which Young's modulus and Poisson's ratio are assumed to be $30\text{x}10^6\ kN/m^2$ and 0.20 respectively. The earthquake is assumed to act in the y direction in all three examples.

Lateral shear loads due to an earthquake are obtained from the formulae in $AS2121$ (Standards Association of Australia 1979) for use in the quasi static method. The following values for the various factors were used: Zone Factor $Z=1$, Importance Factor $I=1$, Horizontal Force Factor $K=1$, Seismic Response Factor $C=1/15\sqrt{T_1}$, where T_1 is the fundamental period of vibration of the building, and Site-structure Resonance Factor $S=1.5$.

For the dynamic analysis, the earthquake loading is specified by the following ground acceleration.:

$$\ddot{v}_g = A\sin 2\pi t/T \tag{10}$$

where the amplitude A and the period T are given by

$$A=0.20g,\ T=2secs \qquad 0<t<2secs$$

$$A=0.15g,\ T=3secs \qquad t>2secs \tag{11}$$

Using the code formulae, the earthquake load is applied in the form of a triangular loading at the slab centroids (Zeman and Irvine 1986).

3.1. Example 1: Three storey set-back building

A three storey building with two set-backs as shown in Figure 3 is considered. All the columns in this building are $0.30m \times 0.30m$ in cross section and the storey height is $3m$. All slabs have a weight density of $5kPa$.

(a) Quasi-static code approach

Deflected shapes of column 1 (the reference column), in the y direction, obtained from the two models, are shown in Figure 4. The $3dof$ model being much stiffer gives very small deflections, in comparison to the deflections obtained from the $5dof$ model. The enhanced deflections obtained with the $5dof$ model are probably due to the bending rotations at the ends of the vertical members. The deflected shape obtained with the $3dof$ model is typical for shear buildings. The deflections of column 1 are reduced by the anti-clockwise twisting of the building about a vertical axis – a consequence of asymmetry. However, with other columns, positioned to the right of the mass centroid, the deflections will be enhanced by torsional coupling.

Variation with height, of the nodal values, of the shear force F_y (in the y directions) in column 1 is shown in Figure 5. For this particular column, both models give the same shear force values at each level. However this may not be true with other columns as the shear response depends on the rotation of the building about a vertical axis which will modify the horizontal deflections. Furthermore, in the case of the $5dof$ model, the shear force in the y direction will also depend on the end rotations of the vertical member about the x axis.

(b) Real time dynamic analysis

Deflection time histories for column 1, in the y direction, at the roof level are shown in

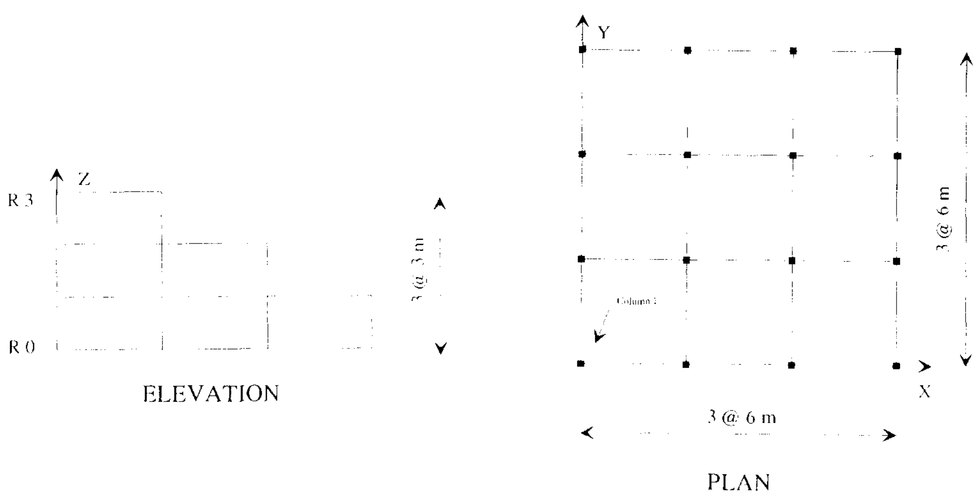

Fig. 3 3 Storey Set-back Building

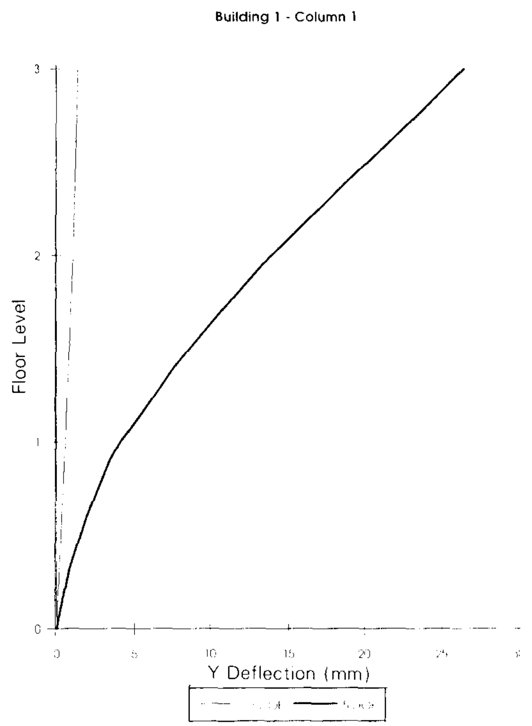

Fig. 4 Deflected Shapes of Column 1 in the Y direction – Example 1

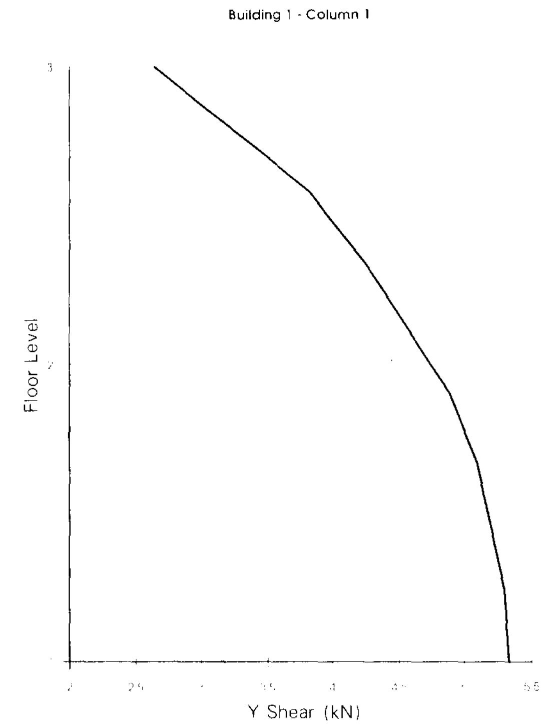

Fig. 5 Shear Force Distribution in Column 1 – Example 1

Figures 6, where it can be seen that the response of the $3dof$ model is very much smaller than that of the $5dof$ model. Time histories of the y direction shear force at this location were analogous. This behaviour leads to the belief that a considerable portion of both the deflection and shear force in the y direction, for the $5dof$ model, could be directly attributed to the end rotations of the vertical member about the x axis and enhanced building rotation about the vertical axis. The effects of modelling are therefore significant in this example. In order to illustrate the $3dof$ model response, the y direction deflection is amplified and shown separately in Figure 7. The periods of the $3dof$ model response(s) match exactly those of the applied earthquake; while the period of the $5dof$ model responses keep on varying during the time of observation. But with time (12secs) the periods of these responses approached and remained at 3secs, which is the period of the applied earthquake.

3.2. *Example 2: Ten storey L shaped building*

In the second example, a 10 storey L shaped building with the prismatic cross-section shown in Figure 8 is considered. All the columns in the building are $0.30m \times 0.30m$ in section and the storey height is $3m$. All the slabs have a weight density of $7kP_a$ and the core has external dimensions of $2.0m \times 2.0m$ with a wall thickness of $0.15m$. Column 1 was chosen to be monitored in this example.

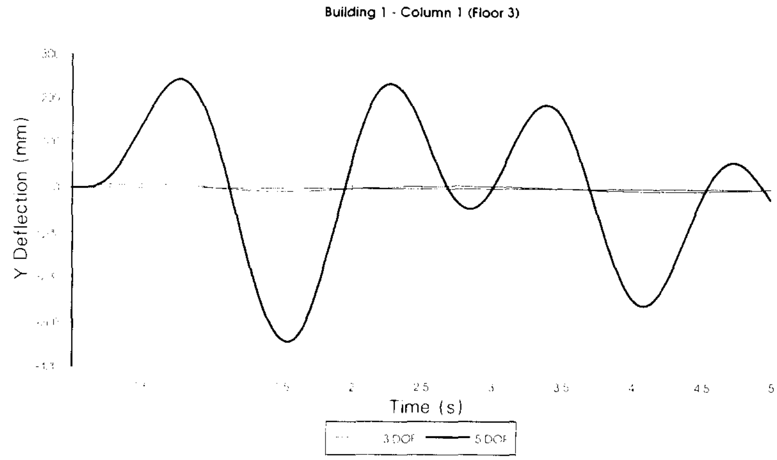

Fig. 6 Deflection Time Histories of Column 1 in the Y direction at Roof Level – Example 1

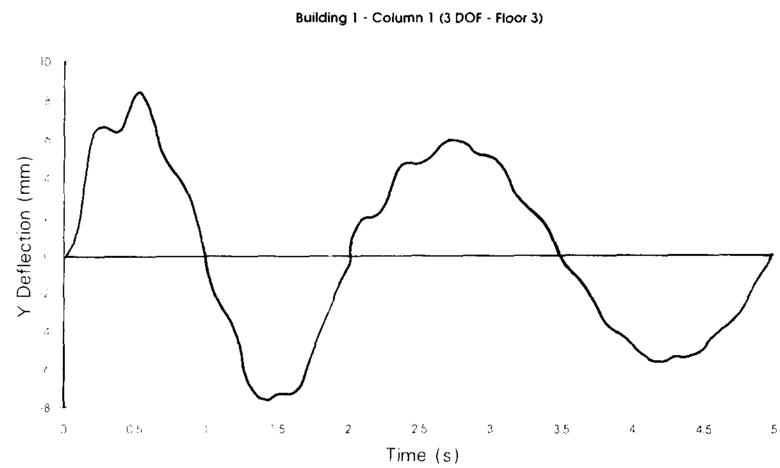

Fig. 7 Deflection Time History of Column 1 in the Y Direction at Roof Level – Example 1 (3dof)

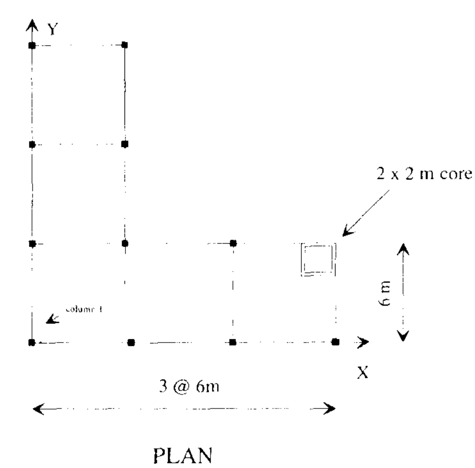

Fig. 8 Plan of L–Shaped Building – Example 2

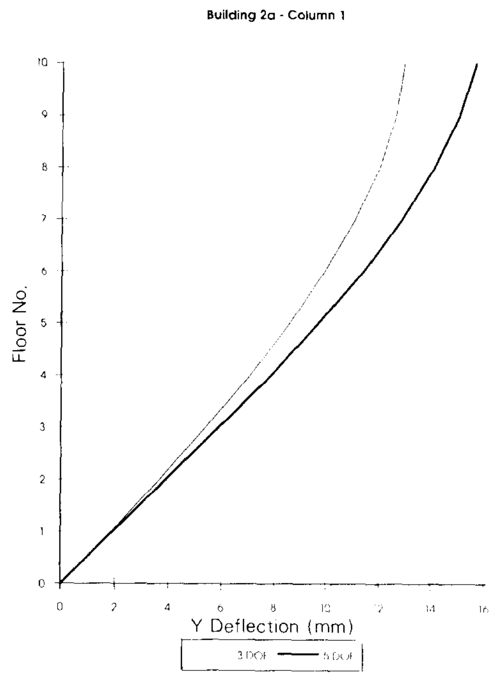

Fig. 9a Deflected Shapes of Column 1 in the Y Direction – Example 2

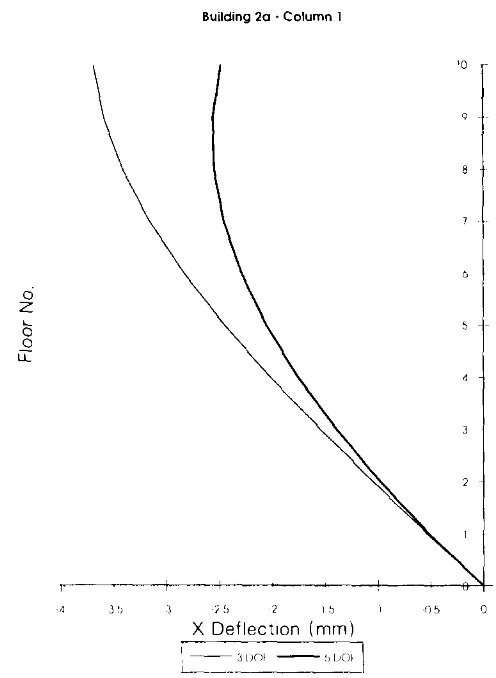

Fig. 9b Deflected Shapes of Column 1 in the X Direction – Example 2

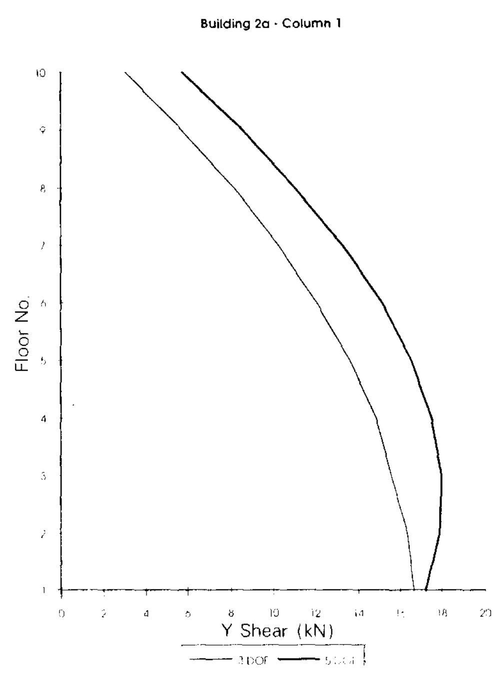

Fig. 10 Shear Force Distribution in Column 1 in the Y Direction – Example 2

(a) Quasi–static code approach

Deflected shapes in the y and x directions of column 1, and the variation of the nodal values of shear force in column 1 in the y direction, are shown in Figures 9a, 9b, and 10 respectively. It can be seen that the deflected shapes obtained with both models are somewhat typical for shear buildings. The geometry of the building coupled with the applied loading has caused the building to twist in the clockwise sense, about a vertical axis. This twisting action gave rise to an increase in the y direction responses and significant deflections and shear forces in the negative x direction.

(b) Real time dynamic analysis

Deflection time histories pertaining to the y deflection and y shear force, for column 1 on the top floor of this structure, are shown in Figures 11 and 12. As observed with the quasi–static analysis, the responses obtained from the $3dof$ model are significant in this example, indicating that the vertical member end rotations about the x axis did not play a dominant role.

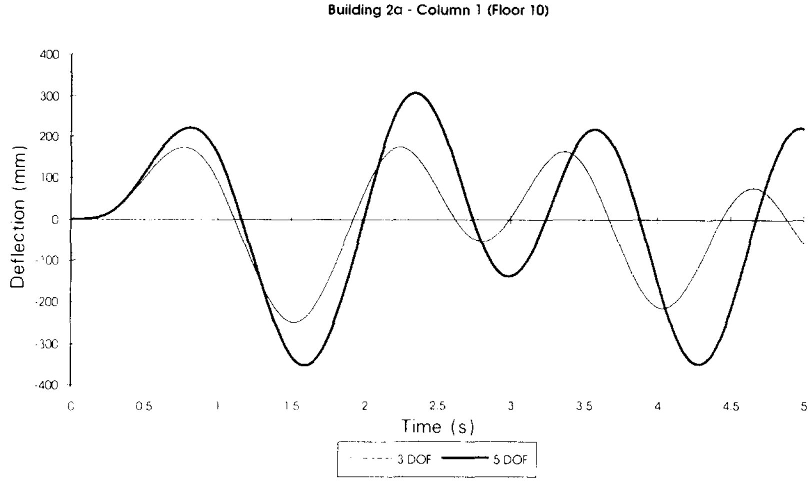

Fig. 11 Deflection Time Histories of Column 1 in the Y direction at Roof Level–Example 2

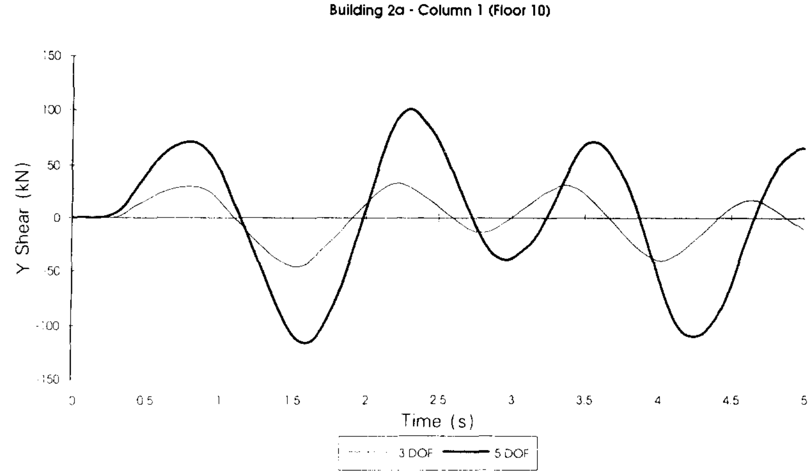

Fig. 12 Shear Force Time Histories of Column 1 in the Y direction at Roof Level – Example 2

The responses are initially in phase; but with the passage of time, they get increasingly out of phase.

3.3. *Example 3: Ten storey rectangular building*

A rectangular ten storey building with a prismatic cross-section as shown in Figure 1 is considered in this example. The core is $2.0m \times 2.0m$ and is $0.2m$ thick. As with example 2, all columns are $0.3m$ square, all slabs have a weight density of 7 kPa and the storey height is $3m$.

(a) Quasi-static code approach

The deflected shape of column 1, in the y direction, (reference column) is depicted in Figure 13. The deflection obtained from the $5dof$ model is substantially more than that of its $3dof$ counterpart. Variations of the shear force in column 1, in the y direction obtained from both the models are shown in Figure 14. The shear forces distributions differ significantly in magnitude and shape with the maximum values occurring at the 1st and 4th floor levels in the $3dof$ and $5dof$ models respectively. This behaviour demonstrates that the response of this asymmetric building is largely model dependant. Other columns displayed analogous behaviour.

(b) Real time dynamic analysis

Deflection time histories of column 1 in the y direction, at the roof level, are shown in Fig-

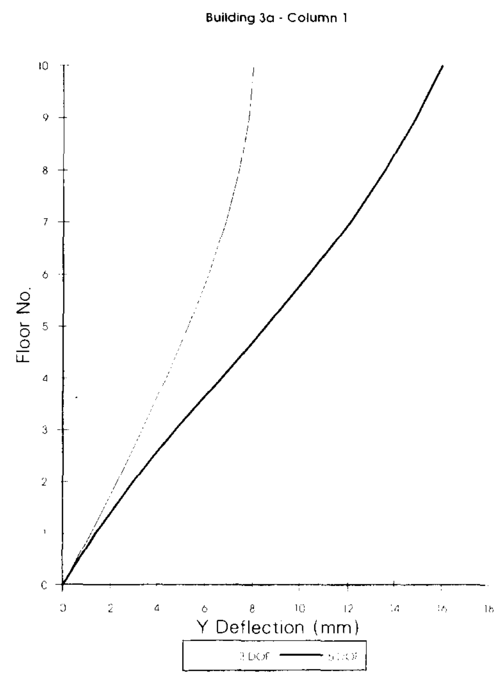

Fig. 13 Deflected Shapes of Column 1 in the Y Direction – Example 3

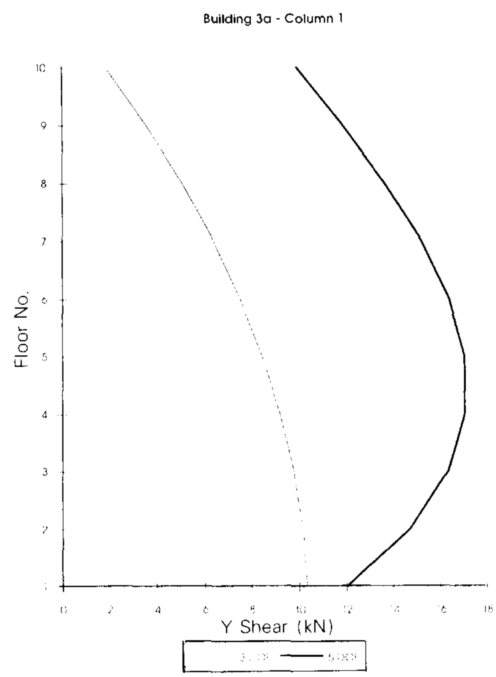

Fig. 14 Shear Force Distributions in Column 1 in the Y Direction – Example 3

ure 15. Response obtained with the *3dof* model is much smaller, as seen in the static analysis and again emphasising model dependency. The time histories of the *y* direction shear force at this level were analogous. The periods of the *3dof* responses follow exactly those of the applied earthquake, while those of the *5dof* response keep on changing during the time of observation. This feature was also observed in example 1.

Both the static and dynamic analyses were performed on three dimensional models. Hence, there is no need to identify or calculate effective eccentricities and separately provide for effects of horizontal torsion, as required if the static analysis was done on a plane frame section of the building.

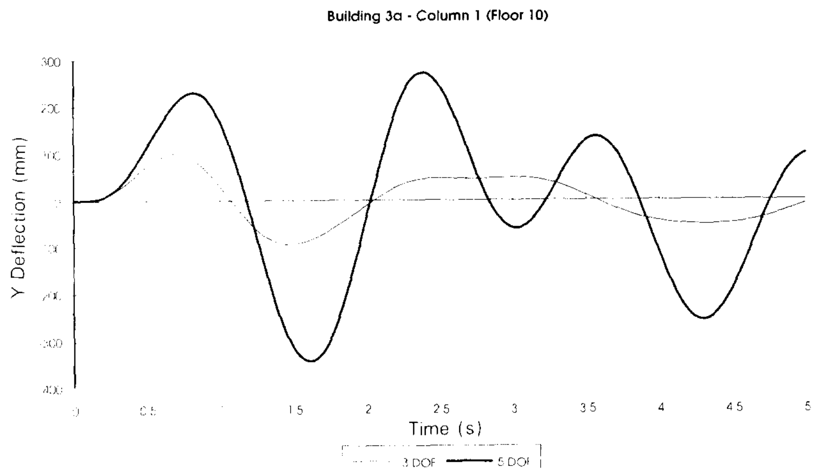

Fig. 15 Deflection Time Histories of Column 1 in the Y Direction at Roof Level – Example 3

4. Conclusions

Effects of modelling on the response of multistorey buildings subjected to earthquakes has been treated in this paper by using a simple microcomputer based procedure. Three buildings are considered in the study so as to illustrate different types of asymmetry and the consequent effects of twisting about the vertical axis on the response of the building. Numerical examples are presented in which results have been obtained by using both the quasi–static code approach and real time dynamic analysis. It is evident that responses are model dependent. When there is no stiffer member such as a core (Example 1), the response is more realistically predicted by the *3dof* model, while the *5dof* model exaggerates the response due to "uncontrolled" rotations at the ends of vertical members. When there is a core or a relatively stiffer member, both models predict responses closer in agreement, with the *5dof* model having larger response values. This indicates that rotations at the ends of vertical members are important and that they must be modelled realistically. Work is underway to achieve this by fine tuning between the two models used in this paper.

Both the static and dynamic analyses give results which agree qualitatively and quantitatively and indicate that the responses are greatly influenced by the degree of asymmetry in the building. The results show that deflections and shear forces not only increase considerably but also change directions, due to the twisting about a vertical axis. It can be seen that quasi–static or real time dynamic analysis performed on a plane frame cannot accurately account for this asymmetry, even though provisions for horizontal torsion are made later on. This can result in gross under estimation of the building response and lead to an inadequate design. Hence the need for a realistic three dimensional model is clearly evident.

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Appendix

In equation (7) the sub matrix M_i appearing in the structure mass matrix M is the mass matrix at level i and is given by:

$$M_i = \begin{bmatrix} m_i & 0 & -m_i y_{Gi} \\ 0 & m_i & m_i x_{Gi} \\ -m_i y_{Gi} & m_i x_{Gi} & J_{Gi} \end{bmatrix} \tag{A1}$$

In the above matrix, x_{Gi}, y_{Gi} are the centroidal coordinates of slab i, and m_i the mass of slab i. J_{Gi} is the centroidal polar moment of inertia of the slab related to its polar moment inertia J_{Ri} about the reference axis R by:

$$J_{Gi} = J_{Ri} - m_i(x_{Gi}^2 + y_{Gi}^2) \tag{A2}$$

In equation (8) the stiffness matrix K_{Ri} representing the stiffness of the vertical members between levels i and $(i-1)$ is given by:

$$K_{Ri} = \sum_r C_r^T k_{ri} C_r \tag{A3}$$

where k_{ri} is the usual 3×3 or 5×5 stiffness matrix for a bar type of element for model (a) or model (b) respectively. The summation r is over all the vertical members between levels i and $(i-1)$.