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An efficient finite element modeling of dynamic crack propagation using a moving node element

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Abstract. The objective of this study was to develop a simple and efficient numerical modeling technique for dynamic crack propagation using the finite element method. The study focused on the analysis of a rapidly propagating crack in an elastic body. As already known, discrete crack tip advance with the stationary node procedure results in spurious oscillation in the calculated energy terms. To reduce the spurious oscillation, a simple and efficient moving node procedure is proposed. The procedure does require neither remeshing the discretization nor distorting the original mesh. Two different central difference schemes are also evaluated and compared for a dynamic crack propagation problem.

Key words: finite element method; crack propagation; dynamic fracture.

1. Introduction

Inherent flaws in engineering materials can lead to a catastrophic failure due to unstable crack propagation. By eliminating the conditions and/or manufacturing defects, catastrophic failure can be prevented. In many structural components absolute prevention of crack can not be guaranteed. For such structures, catastrophic failure can be minimized by a crack arrest system. To ensure the proper incorporation of a crack arrest system, the effects of a propagating crack must be understood. These effects can be understood through the study of dynamic fracture mechanics. Dynamic fracture mechanics can be applied to problems involving bodies containing cracks in which inertia forces play an important role. Dynamic crack propagation can be divided into two categories: crack initiation and crack propagation. A third category sometimes used is crack arrest, which can be included in the crack propagation phase (Kanninen 1977).

Dynamic fracture mechanics problems have dealt with bodies that contain rapidly moving cracks. Many numerical dynamic fracture analyses for rapid crack propagations have been studied. For example, Kobayashi, et al. (1976) analyzed two fracturing Homalite-100 plates by dynamic finite element and dynamic photoelastic analyses. These analyses used the process of discrete crack-tip advances with a fixed mesh. The restraining nodal force was suddenly released when the crack-tip reached the next adjacent node. As indicated by Malluck and King (1978), the above procedure has inherent problems. The sudden release of a node induces an unwanted high-frequency of dynamic motion and the crack tip location within the nodal spacing can

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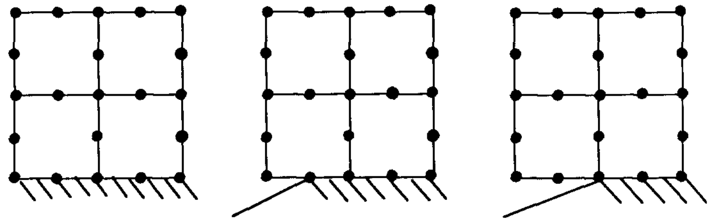
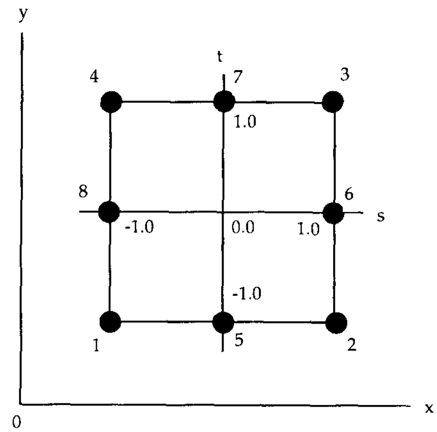


(a)



(b)

Fig. 1 (a) Modeling of a propagating crack using a stationary node element; (b) An eight noded regular isoparametric element.

not be determined. To overcome this problem Malluck and King incorporated a mechanism for energy release in their finite element analysis. The nodal reaction force of the crack tip was gradually reduced until the crack tip propagated from one node to the next node. Kobayashi, et al. (1977) also incorporated a nodal force release mechanism to depict a more gradual transit of the crack tip between adjacent nodes. However, the assumed nodal release force was a somehow arbitrary choice. As indicated in Nishioka and Atluri (1986), computational procedures for crack propagation problems can be grouped into two types; stationary mesh procedure and moving mesh procedure. The above analyses may be categorized as a stationary mesh procedure. To predict the continuing propagation of a crack in a discrete model closely, moving mesh procedures were introduced. Nishioka and Atluri (1980) analyzed the propagating crack using a moving singular element. The singular element is larger than surrounding regular element. As a result, it is necessary to remesh the neighborhood of the crack tip after the crack tip moves beyond the singular element. Kwon and Akin (1989) developed a procedure for modeling the crack propagation problem using a node moving along the edge of the element. In this procedure the element is not distorted; therefore, remeshing is not required. A modification of the latter procedure was incorporated into this study.

The main objective of this study is to examine different numerical modeling techniques for studying dynamic crack propagation. The main emphasis is placed on reducing the spurious oscillation in the calculated energy terms which resulted from the stationary node procedure (Fig. 1(a)). An eight noded regular element (Fig. 1(b)) was used in the procedure. To reduce the spurious oscillation a moving node procedure (Fig. 2(a)) using a moving node (Fig. 2(b)) was

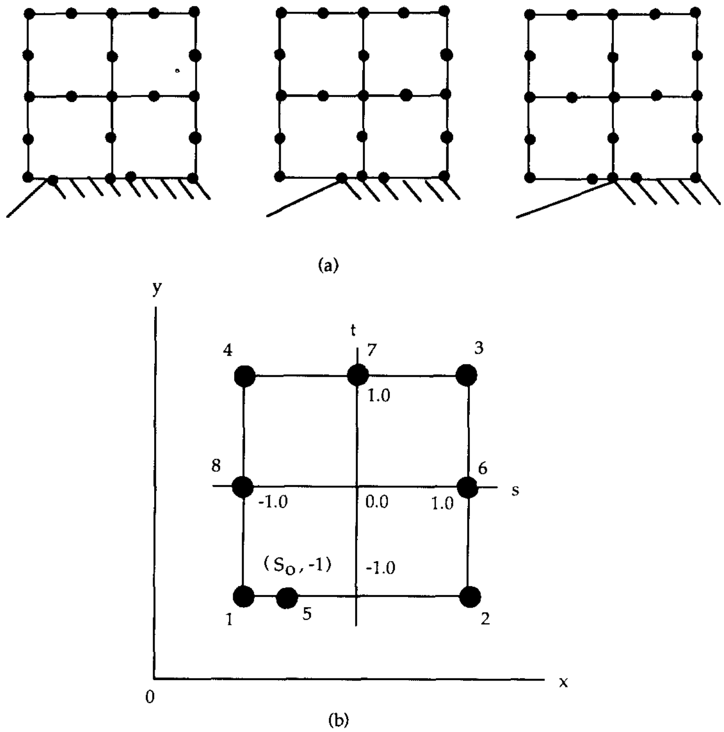


Fig. 2 (a) Modeling of a propagating crack using a moving node element;
(b) An eight noded isoparametric element with a moving mid-node.

incorporated into the finite element model.

The study focused on the analysis of a rapidly propagating crack in a linearly elastic body. Inertia force was formulated using a diagonal mass matrix and the numerical time integration was accomplished using two different central difference techniques. Crack propagation was simulated by the sequential movement (at prescribed time intervals) of the nodes along the edge of the finite element model. Both stationary and moving node techniques were used to model the crack tip movement. Crack opening displacement, work, strain energy, and kinetic energy were calculated and a comparative analysis was conducted from the results. In addition, two different central difference techniques are compared in view of the dynamic crack propagation problem. The range of movement of the moving mid-node within the element is also studied. Too close distance between the moving mid-node and a corner node causes unwanted oscillation of high frequency and large magnitude.

2. Mathematical derivation

2.1. Spatial discretization

In the elastodynamic problem of a continuous body, the equation of motion is written as

$$[M]\{\ddot{u}\}+[C]\{\dot{u}\}+[K]\{u\}=\{F\} \tag{1}$$

with a finite element discretization. In this study eight noded isoparametric elements were used for the finite element mesh of a two-dimensional body. The shape functions for the regular eight noded isoparametric element (Fig. 1(b)) are given in many textbooks and they are not repeated here. The shape functions for the eight noded isoparametric element with a moving mid-node (Kwon and Akin 1989) are

$$H_1=\frac{1}{4}(1-s)(1-t)-\frac{1-s_o}{2}H_5-\frac{1}{2}H_8 \tag{2a}$$

$$H_2=\frac{1}{4}(1+s)(1-t)-\frac{1+s_o}{2}H_5-\frac{1}{2}H_6 \tag{2b}$$

$$H_3=\frac{1}{4}(1+s)(1+t)-\frac{1}{2}H_6-\frac{1}{2}H_7 \tag{2c}$$

$$H_4=\frac{1}{4}(1-s)(1+t)-\frac{1}{2}H_7-\frac{1}{2}H_8 \tag{2d}$$

$$H_5=\frac{1}{2(1-s_o^2)}(1-s^2)(1-t) \tag{2e}$$

$$H_6=\frac{1}{2}(1+s)(1-t^2) \tag{2f}$$

$$H_7=\frac{1}{2}(1-s^2)(1+t) \tag{2g}$$

$$H_8=\frac{1}{2}(1-s)(1-t^2) \tag{2h}$$

The shape functions in Eqs. (2a)-(2h) become those for the regular eight noded isoparametric element if s_o is set to zero. For a moving crack problem, s_o is computered from the crack tip location within the element. For example, let the distance between the two corner (nodes 1 and 2 in Fig. 2(b)) be L_e and the distance from node 1 in Fig. 2(b) to the crack tip be L_c. Then s_o is $(2L_c/L_e)$-1 and s_o must be between -1 and 1. Because the moving mid-node travels in both the physical and natural coordinate systems, the determinant of the Jacobian matrix becomes constant as long as the element does not have any distortion.

The diagonalized mass matrix was used for computational efficiency. The diagonalized mass matrix was obtained from the consistent mass matrix using the following procedure (Cook 1981).

(1) Compute the diagonal coefficients of the consistent mass matrix using eight noded shape functions.
(2) Determine the total mass of the element, m.
(3) Add the diagonal coefficients associated with translational degrees of freedom only to obtain a number, $e=\Sigma m_{ii}$.
(4) Multiply each diagonal coefficient, m_{ii}, by m/e.
(5) Set all the off-diagonal coefficients to zeros.

2.2. Direct time integration

This study used two different forms of the central difference techniques for time integration of the equation of motion. The first method is summarized below (Bathe 1982)

A. Initial calculations.
(1) Compute mass $[M]$, damping $[C]$, and stiffness matrices $[K]$.
(2) Compute $\{a\}^0=[M]^{-1}[\{R\}^0-[C]\{\upsilon\}^0-[K]\{d\}^0]$.

where $\{d\}^0$ and $\{\upsilon\}^0$ are the initial displacement and velocity vectors, respectively, and $\{a\}^0$ is the initial acceleration vector.
(3) Compute the fictitious displacement vector $\{d\}^{-\Delta t}=\{d\}^0-(\Delta t)\{\upsilon\}^0+(\Delta t^2/2)\{a\}^0$.
(4) Compute $[\hat{M}]=(1/\Delta t^2)[M]+(1/2\Delta t)[C]$.
B. For each time increment, repeat the following steps:
(1) Compute $\{\hat{R}\}^t=\{R\}^t-([K]-(2/\Delta t^2)[M])\{u\}^t-((1/\Delta t^2)[M]-(1/2\Delta t)[C]\{d\}^{t-\Delta t}$.
(2) Compute $\{d\}^{t+\Delta t}=[\hat{M}]^{-1}\{\hat{R}\}^t$.
(3) Compute $\{a\}^t=(1/\Delta t^2)[\{d\}^{t+\Delta t}-2\{d\}^t+\{d\}^{t-\Delta t}]$.
(4) Compute $\{\upsilon\}^t=(1/2\Delta t)[\{d\}^{t+\Delta t}-\{d\}^{t-\Delta t}]$.
The second technique, called summed form, (Park 1977) is as follows:
A. For each time increment, repeat the following steps.
(1) $\{a\}^t=[M]^{-1}(\{R\}^t-[C]\{\upsilon\}^t-[K]\{d\}^t)$.
(2) $\{\upsilon\}^{t+\Delta t/2}=\{\upsilon\}^{t-\Delta t/2}+\Delta t\{a\}^t$.
(3) $\{d\}^{t+\Delta t}=\{d\}^t+\Delta t\{\upsilon\}^{t+\Delta t/2}$.

Each technique has its own advantages and disadvantages. The diagonal mass matrix is essential for both techniques for efficient computation. However, the first technique can not take advantage of the diagonalized mass matrix if a damping matrix exists, which is generally not a diagonal matrix. The first technique does not require computing the velocity and acceleration to obtain the displacement. When needed, the technique computes the displacement, velocity and acceleration at the same time stage.

On the other hand, the second technique is simpler in coding and non-diagonal damping matrix does not ruin its computational efficiency as long as the mass matrix is diagonal. However, the technique requires computations of displacement, velocity and acceleration all the time. It computes velocity at different time stages from displacement and acceleration. In order to compute dynamic energy release rate, both displacement and velocity should be known at the same time stage. As a result, an interpolation technique is used to obtain the velocity at the same time stage as the displacement. The second technique does not have the self-consistent starting in contrast to the first technique. The velocity at the fictitious time ($t=-\Delta t/2$) is assumed to be the same as the initial velocity. Both central difference schemes are conditionally stable. The time step Δt must be controlled from the smallest period of the system.

2.3. Fracture dynamics aspects

A common criterion for determining the dynamic propagation of a crack is

$$G=R(\dot{a}) \tag{3}$$

where R is the energy dissipation rate required for crack growth and G is the dynamic energy release rate given by

$$G=\frac{1}{b\dot{a}}\left(\frac{\partial W}{\partial t}-\frac{\partial U}{\partial t}-\frac{\partial T}{\partial t}\right) \tag{4}$$

where U is the strain energy, T is the kinetic energy, W is the work done on the structure by external loads, a is the crack length, b is the plate thickness, t is time, and superimposed dot denotes the temporal derivative.

The dynamic energy release rate G and the dynamic stress intensity factor K_D are connected through Freund's formula:

$$K_D=\left\{\frac{EG}{1-\nu^2}A(\dot{a})\right\}^{1/2} \tag{5}$$

where E is Young's modulus, ν is Poisson's ratio, and A is the function that is dependent on crack speed. A is given as

$$A(\dot{a})=\frac{\left(\frac{\dot{a}}{C_2}\right)^2\left(1-\frac{\dot{a}^2}{C_1^2}\right)^{1/2}}{(1-\nu)\left\{4\left(1-\frac{\dot{a}^2}{C_1^2}\right)^{1/2}\left(1-\frac{\dot{a}^2}{C_2^2}\right)^{1/2}-\left(2-\frac{\dot{a}^2}{C_2^2}\right)^2\right\}} \tag{6}$$

where C_1 and C_2 are the longitudinal and shear wave speeds (Kanninen 1977).

3. Results and discussion

The example problem for dynamic crack propagation is Broberg's problem (Broberg 1960). The problem consists of a two-dimensional plate with a dynamically propagating central crack from no crack condition. The material is isotropic with elastic modulus of 206.7 GPa, Poisson's ratio of 0.3, and mass density of 8000 Kg/m^3. Because of symmetry, a quarter of the plate is shown in Fig. 3. The plate is subjected to a uniform tensile traction of 68.9 MPa. The finite element mesh is composed of a uniform mesh of eight noded isoparametric elements. The crack speed is constant and 1/3 of the dilatational wave speed of the material.

Fig. 4 compares the crack opening displacements obtained using stationary mesh and moving node techniques. The displacement at the center of the crack is compared in the figure. The two techniques do not show much difference in their solutions of crack opening displacements. However, the two techniques result in very different strain and kinetic energy calculations as shown at the following paragraphs.

Fig. 5 shows the work done on the plate during crack propagation. There is a similar trend between the stationary node and the moving node elements with the exception that there is a slight increase in the work for the moving node element. The slight increase in the work was caused by a slightly larger crack opening displacement for the moving node element than for the stationary node element.

Fig. 6 shows the strain energy variation during crack growth. The stationary node element produces spurious oscillation. The oscillation is caused by the instant release of the crack tip during the process of discrete crack tip advance. As shown in Fig. 6(a), there is a rapid build up in the strain energy until the node is released. Upon nodal release, there is a rapid reduction in the strain energy followed by a rapid increase until the next nodal release. The oscillation amplitude of the strain energy increases during crack propagation. Employing a moving node element makes the crack tip movement more continuous. As a result, the oscillation of the strain energy is greatly reduced (Fig. 6(b)).

Fig. 7 shows the calculated kinetic energy during crack growth. The delayed nodal release,

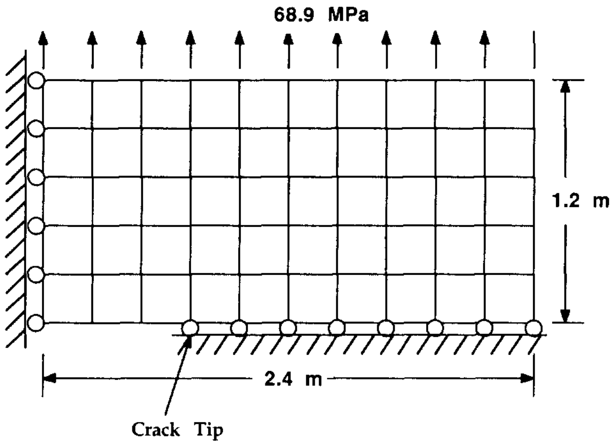


Fig. 3 Geometry of a quarter of the plate for Broberg's problem.

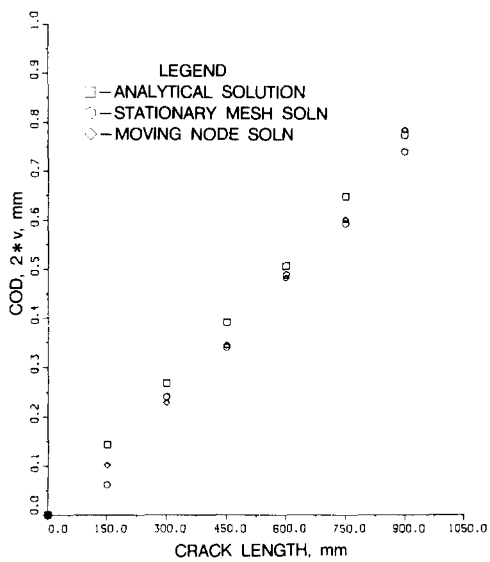


Fig. 4 Crack opening displacement at the center of crack.

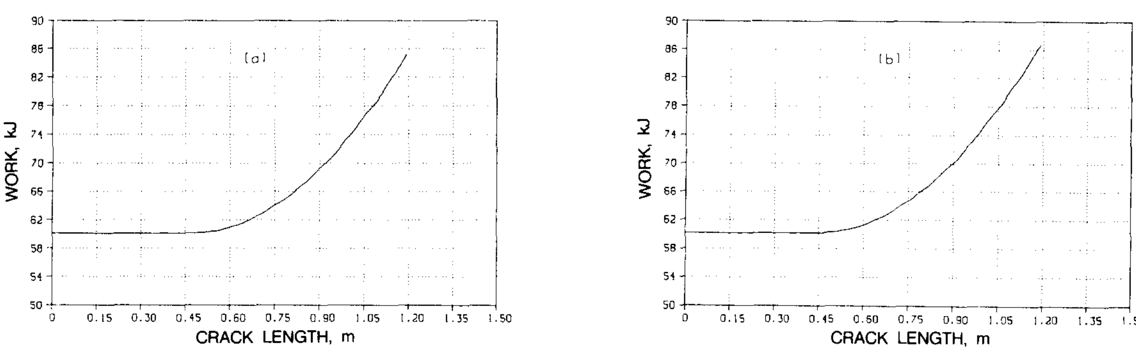


Fig. 5 Work vs crack length: (a) Stationary node element; (b) Moving node element.

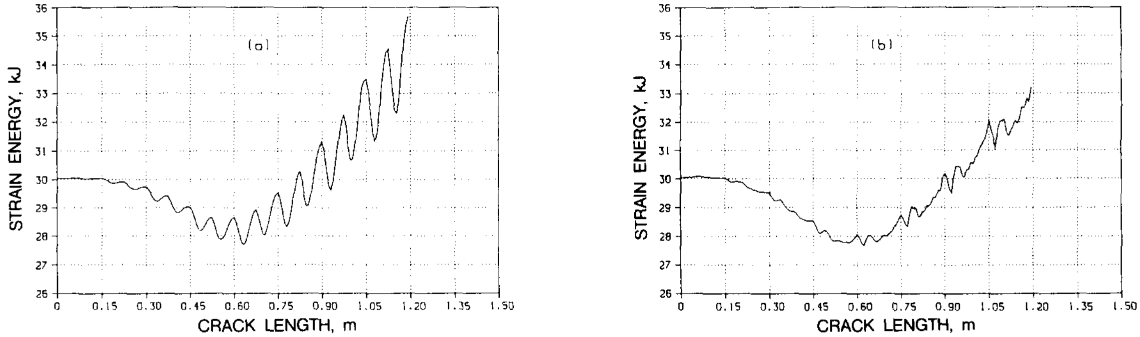


Fig. 6 Strain energy vs crack length: (a) Stationary node element; (b) Moving node element.

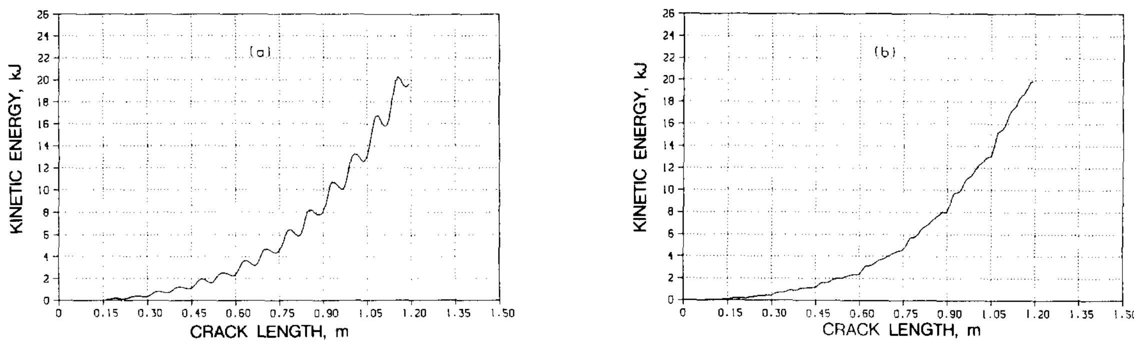


Fig. 7 Kinetic energy vs crack length: (a) Stationary node element; (b) Moving node element.

caused by the stationary node, results in a rapid decrease in the kinetic energy prior to the release of the node. After that, an rapid increase follows by a rapid reduction until the next nodal release. This trend is a reversal of that seen for strain energy. The oscillation amplitude of the kinetic energy increases during crack growth as it does for the strain energy. Employing a moving node element almost eliminates the oscillation because the moving node better represents a moving crack tip.

3.1. Numerical time integration

Based on the present study, it is noted that the first central difference method is very sensitive to computational round off errors. As the time increment decreases, the central difference does not converge while running the model in single precision. The model should be run at double precision to achieve convergence. On the other hand, the second method, called "summed form" converges during single precision runs. Comparison of the convergent results reveals no difference between the two central difference techniques. The "summed form" simplifies the computation, and as a result, it saves time and money.

3.2. Limit of travel distance of moving node

The distance the moving node can travel is limited by β. β is the fractional distance of the element length. It represents the minimum distance the moving node should be away from

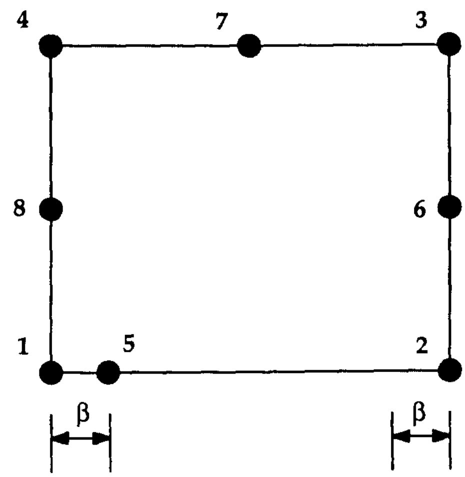


Fig. 8 Minimum distance from the mid-node to a corner node.

Table 1 Fractional minimum distance the moving mid-node from the end node

NSTEP*	10	25	50	100
β	1/5	1/10	1/10	1/10

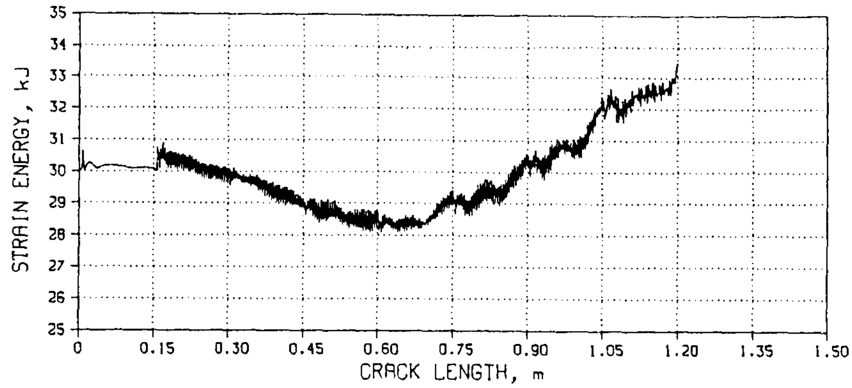
*NSTEP is the number of time steps for the crack to advance one half the element length.

a corner node (Fig. 8). If the mid-node is located within the minimum distance from a corner node, the numerical result shows a high frequency solution with similar magnitude in energy terms (Fig. 9). If the mid-node is too close to a corner node, the stiffness terms associated with the mid-node becomes so dominant that the numerical solution becomes very unstable. A numerical solution of very high frequency and large amplitude is resulted as shown in Fig. 10.

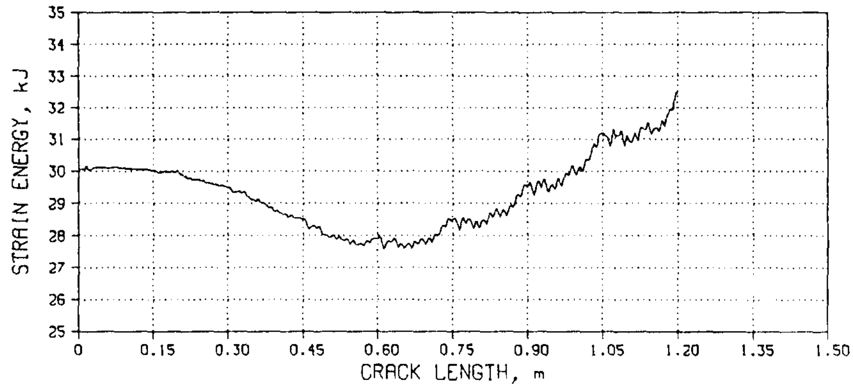
The minimum distance β is relatively constant for various time step sizes which satisfy the stability criterion as seen in Table 1. The mid-node can travel about 90 percent of the element length for most time step sizes except for a relatively large time step size. As a result, the crack tip stays at a corner node until it propagates beyound the minimum distance. Because the minimum distance is small compared to the element size, oscillation caused by such a small jump of the crack tip movement is negligible.

4. Conclusions

This study focused on the finite element modeling of a rapidly propagating crack. Two procedures were employed in the study. The first procedure, adopting stationary node elements, resulted in spurious oscillation of the energy terms as knowns. For this procedure the propagating crack was represented by discrete jumps at given time intervals. To more closely approximate the propagating crack, a moving node procedure was employed in the model. As shown in the previous section, the moving node procedure significantly reduced or almost eliminated the spu-



(a)



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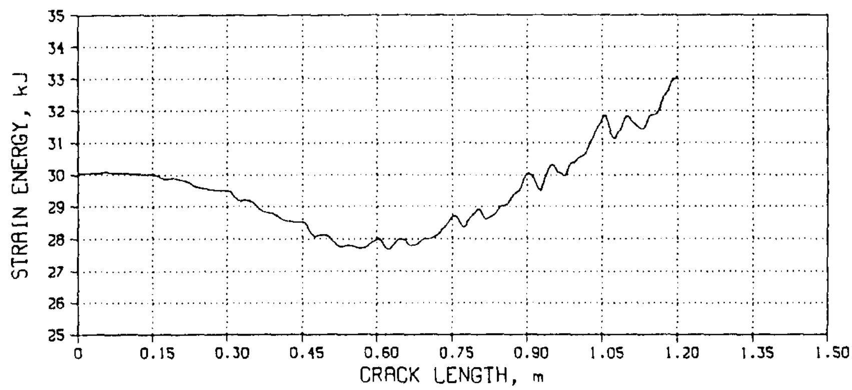

(c)

Fig. 9 Effect of varying β on strain energy for NSTEP=50: (a) $\beta=1/20$; (b) $\beta=1/10$; (c) $\beta=1/5$.

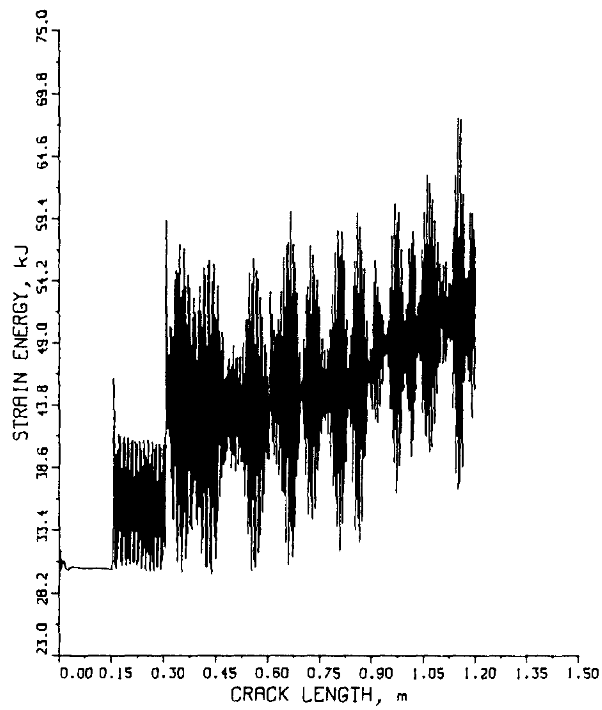

Fig. 10 Strain energy when the moving mid-node is too close to a corner node.

rious oscillation of the energy terms. Unlike the moving mesh procedure, remeshing was not required for this procedure.

The present study also showed the following conclusions: The two forms of the central difference techniques for time integration yielded almost the same results. However, the summed form was less sensitive to computational round off errors. As a result, the summed form was preferred due to its simplicity and efficiency in computational time. The range which the moving node, representing the crack tip, could propagate along the edge of the element was a function of the time step size for relatively large time step sizes. However, for reduced time step sizes, the range was the same. The moving mid-node could travel about 90 percent of the distance between two corner nodes.

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quantity of steel is reduced.

In recent years, increasing use of the ultimate strength design methods has allowed the structural designer to effect savings of material over traditional elastic techniques. However, available experimental data (Levi 1961) has shown that the determination of the effective flange width *be* of [redacted]