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The elastic deflection and ultimate bearing capacity of cracked eccentric thin-walled columns

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Abstract. The influence of cracks on the elastic deflection and ultimate bearing capacity of eccentric thin-walled columns with both ends pinned was studied in this paper. First, a method was developed and applied to determine the elastic deflection of the eccentric thin-walled columns containing some model-I cracks. A trigonometric series solution of the elastic deflection equation was obtained by the Rayleigh-Ritz energy method. Compared with the solution presented in Okamura (1981), this solution meets the needs of compatibility of deformation and is useful for thin-walled columns. Second, a two-criteria approach to determine the stability factor φ has been suggested and its analytical formula has been derived. Finally, as an example, box columns with a center through-wall crack were analyzed and calculated. The effects of cracks on both the maximum deflection and the stability coefficient φ for various crack lengths or eccentricities were illustrated and discussed. The analytical and numerical results of tests on the columns show that the deflection increment caused by the cracks increases with increased crack length or eccentricity, and the critical transition crack length from yielding failure to fracture failure ξ_c is found to decrease with an increase of the slenderness ratio or eccentricity.

Key words: crack; thin-walled column; elastic deflection; stability factor; eccentricity; ratio of slenderness.

1. Introduction

Thin-walled structures are widely used in civil and mechanical engineering fields. With the development of new construction techniques and materials, one of the most important considerations is the effect of the initiation and propagation of cracks. The thin-walled steel structures are usually built up with welding seams. There are some unavoidable defects that may be considered cracks in the welding position (Wang and Zhai 1989). In a poor working environment, such as one in which there is low temperature, dynamic load, corrosion media, etc, the presence and growth of cracks is more certain to decrease the bearing capacity and residual life of structures. However, research in this area is very limited and there are no guidelines available for structural engineers to predict and

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control the crack effects. Therefore, it is absolutely essential to estimate quantitatively the influence of cracks on the behavior of deformation and failure of thin-walled structures.

The load carrying capacity of thin-walled structures is usually governed by the buckling manners. Moreover, the buckling of thin-walled members is susceptible to the imperfection of geometry and load. It is generally assumed that the presence of cracks can account for geometrical imperfections and thus reduce the load carrying capacity of a thin-walled structure (Estekanchi and Vafai 1999). EI Naschi (1974) considered the buckling problem of a cracked shell for the first time, and the progress of research in this area was briefly reviewed in literature (Estekanchi and Vafai 1999). Recently, some studies have begun to pay attention to Finite Element (FE) modeling for the buckling analysis of cracked plates and shells. A general FE element model for the analysis has been proposed, verified and applied (Vafai and Estekanchi 1999). With this FE model, Estekanchi *et al*. (1999) have investigated the sensitivity of the buckling load of cylindrical shells to the crack length and orientation. Also, Javidruzi *et al*. (2004) have studied the vibration, buckling and dynamic stability of cracked cylindrical shells. An interesting result of the above studies is that the initiation of cracks not only decreases the limit load of compressed shells but also causes buckling for tensile shells. However, the concepts used in the above studies, that the cracks in a shell structure cause geometrical imperfections, is not appropriate, because both the deformation before buckling and the failure mechanism in the limit state are possibly changed due to the presence of cracks. Obviously, a theoretical study on the deflection and ultimate bearing capacity of thin-walled cracked structures is still necessary.

The object of the present study is to describe the deformation and failure behavior of eccentric thin-walled columns containing model-I cracks with both ends pinned. A common method for determining the elastic deflection of cracked members is to resolve such a boundary-value problem that the local deformation due to cracking is considered as a boundary condition. For example, the elastic deflection of a rectangular eccentric column with an edge crack has been analyzed by Okamura (1981), where the rotation of the cracked cross section was simulated by a very short spring. The disadvantages of the above model are that the smooth condition of the deflecting curve could not be met and the model is not applicable to thin-walled members because sometimes the crack extension direction is vertical to the plane of flexure. This method was still applied in Wang's study (Wang and Chase 2003) on the buckling of a cracked column, however. In order to describe the elastic deformation of eccentric thin-walled columns containing model-I cracks, the Rayleigh-Ritz energy method was used and a trigonometric series solution of the elastic deflection equation was obtained by Li (2000). This solution fulfils the compatibility of deformation and is useful for the thin-walled cracked column.

In this paper, the series solution was used to analyze the effects of crack size on the maximum elastic deflection of eccentric H-column and box column under different load and eccentricity states, where the double edge and center crack models were applied respectively to simulate the tensile flanges that contain a model-I through-wall crack. The stability factor $\varphi-\lambda$ curves of cracked eccentric columns were classified as two types: yielding dominant failure and compound failure from yielding to fracture. A method for determining the ultimate bearing capacity of cracked eccentric columns was suggested by use of double failure criteria, i.e., yielding criterion and fracture criterion. Accordingly, the analytical formula of stability coefficient φ was derived. In addition, the box columns with a center through-wall crack were calculated as an example of the use of the above φ formula. The influences of cracking on the stability coefficient φ were discussed for various crack lengths or eccentricities and the critical transition crack lengths ξ_0 was found.

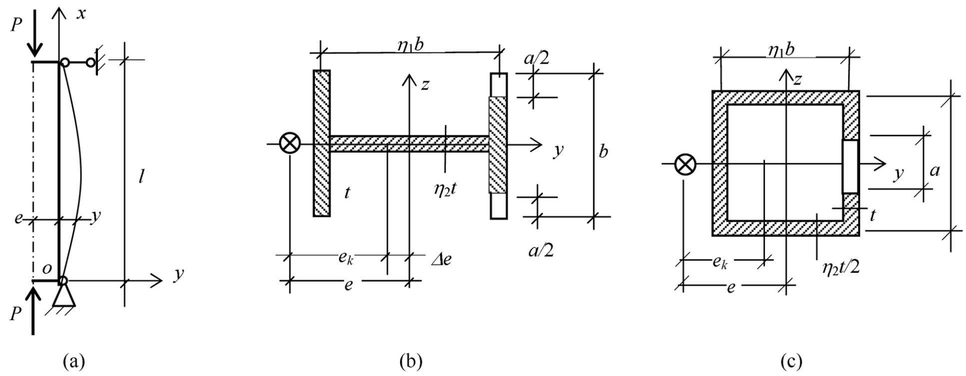

Fig. 1 Analysis model of eccentric thin-walled columns with model-I cracks

2. Analysis model

The analysis and discussion in this paper are based on the following assumptions: (1) the columns have both ends pinned and the eccentricity of the axial concentric loads applied at both ends are equal; (2) the loads are applied along the longitudinal symmetrical plane of the column and the lateral stiffness is sufficient to prevent buckling; (3) the deflection of the column is linear elastic; and (4) the model-I crack is located at the tensile edge of a certain cross section of the column. The analysis model according to these assumptions is as shown in Fig. 1, where plane *xoy* is the longitudinal symmetrical plane of the column, and e and e_k are the eccentric distance of the gross and the cracked cross-section, respectively.

Assuming the longitudinal coordinate of the cracked cross-section is x_k and, therefore, that the deflection is $y_k = y(x_k)$, the maximum tensile stress (assuming the eccentric distance is far enough) and the compress stress in the cracked cross-section can be written, respectively, as follows:

$$\sigma_k = \alpha\sigma_E \cdot \frac{A_0}{A_k} \cdot \left(\varepsilon_k \frac{y_k}{e_k} + \varepsilon_k - 1\right) \tag{1}$$

$$\sigma_y = \alpha\sigma_E \frac{A_0}{A_k} \cdot \left(\varepsilon_y \frac{\delta}{e_1} + \varepsilon_y + 1\right) \tag{2}$$

where A_0 and A_k are the areas of gross and cracked cross-section, respectively; $\varepsilon_k = e_k A_k / W_k$ and $\varepsilon_y = e_k A_k / W_y$, refer to the relative eccentricity of the cracked edge and to another edge, respectively, in which W_k and W_y are the section modulus of the cracked cross-section at the cracked edge and at the another edge, respectively; and $\alpha = P/P_E = \sigma/\sigma_E$, in which P_E and σ_E are Euler's critical load and stress for an ideal column, respectively. It should be noted that all the parameters with down marking of k in the above equations are associated with the crack length.

Because the restraint effect of web plate on flange plate is not significant and can be neglected, the tensile flange plate with model-I cracks of the H-column (shown in Fig. 1b) and the box column (shown in Fig. 1c) can be analyzed according to the double edge crack plate (shown in Fig. 2a) and the center crack plate (shown in Fig. 2b), respectively. It has been recognized in linear elastic fracture mechanics that the critical failure state of a cracked body is dominated by the stress intensity factor (SIF). When a plate with model-I cracks is subjected to tensile stress σ_k, the general

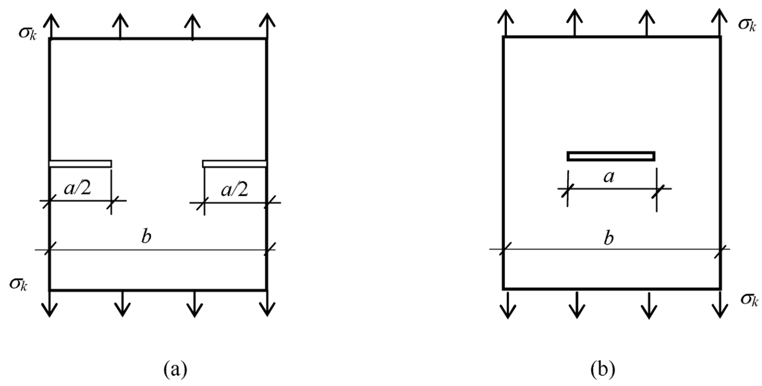

Fig. 2 The crack model of flange plates

expression of SIF can be written as

$$K_I = \sigma_k \sqrt{\pi a^*} f_k(\xi) \tag{3}$$

where $\xi = a/b$ is the relative crack length, a^* is a characteristic length parameter that is related to the configuration of the cracked body and uncorrelated to crack length, and $f_k(\xi)$ is the configuration correction factor of the SIF and may be looked up in SIF handbooks or determined computationally.

3. Series solution of deflection

3.1 Objective functions and energy variation equations of deflection

According to the boundary conditions of the column shown in Fig. 1(a), the objective function of the deflection can be assumed to take the trigonometric series form

$$y(x) = e \sum_{m=1}^{n} C_m \sin \frac{m\pi x}{l} \tag{4}$$

where C_m (m = 1, 2, ..., n) may be called the deflection coefficients and can be determined by means of Rayleigh-Ritz's energy variational calculations. When $\varepsilon_k(y_k/e_k + 1) \leq 1$ and thus the tensile stress is $\sigma_k \leq 0$, the closed crack has no influence on the deflection of the column, so $C_m = C_m(\alpha, \varepsilon)$. When $\varepsilon_k(y_k/e_k + 1) > 1$ and thus $\sigma_k > 0$, the opened crack has some influence on the deflection of the column, so $C_m = C_m(\alpha, \varepsilon, \xi)$. Where $\varepsilon = eA_0/W_0$ is the relative eccentricity of the gross cross section of the column.

By using the deflection function given by Eq. (4), the total potential energy of the cracked thin-walled column may be written as

$$\Pi(C_1, C_2, \ldots, C_m, \ldots, C_n) = U_0 + U_k - U_P \tag{5}$$

where U_0 is the elastic strain energy of the loaded column assumed to be uncracked, U_k is the change in elastic strain energy caused by introducing the crack in the column, and U_P is the work performed by external forces. From the principle of minimum potential energy, it can be known that $\delta\Pi = 0$ that is

$$\frac{\partial \Pi}{\partial C_m} = \frac{\partial U_0}{\partial C_m} + \frac{\partial U_k}{\partial C_m} - \frac{\partial U_P}{\partial C_m} = 0 \tag{6}$$

The deflection coefficients C_m ($m = 1, 2, ..., n$) in Eq. (4) can be determined by Eq. (6), and the deflection curve can then be obtained.

3.2 Strain energy and external forces work

In the final deformation state, assuming that the deflection due to both the loading and cracking is resisted when the crack disappears, the elastic strain energy of the loaded, uncracked column obtained by elastic bend theories is as follows:

$$U_0 = \frac{EI}{2}\int_0^l \left(\frac{d^2y}{dx^2}\right)^2 dx = \frac{\pi^2 e^2 P_E}{4l}\sum_{m=1}^{n} C_m^2 m^4 \tag{7}$$

From the theories of linear fracture mechanics, we know that the change in elastic strain energy caused by introducing the crack in the column is

$$U_k = -\frac{1}{E}\int_0^A K_I^2(A)dA = -\frac{A_0}{E}\int_0^{\xi} K_I^2(\xi)d\xi \tag{8}$$

by substituting from Eq. (3) into (8) together with Eq. (4), the expression of U_k is obtained as

$$U_k = -\frac{\pi^3 P_E a^* \alpha^2}{\lambda^2}\left[g_1(\xi, \varepsilon)\left(\sum_{m=1}^{n} C_m \sin\frac{m\pi x_k}{l}\right)^2 + 2g_2(\xi, \varepsilon)\sum_{m=1}^{n} C_m \sin\frac{m\pi x_k}{l} + g_3(\xi, \varepsilon)\right] \tag{9}$$

where

$$\left.\begin{aligned} g_1(\xi, \varepsilon) &= \int_0^{\xi}\left[\left(\frac{A_0}{A_k}\right)\left(\frac{e}{e_k}\right)\varepsilon_k f_k(\xi)\right]^2 d\xi \\ g_2(\xi, \varepsilon) &= \int_0^{\xi}\left(\frac{A_0}{A_k}\right)^2\left(\frac{e}{e_k}\right)\varepsilon_k(\varepsilon_k - 1)f_k^2(\xi)d\xi \\ g_3(\xi, \varepsilon) &= \int_0^{\xi}\left[\left(\frac{A_0}{A_k}\right)(\varepsilon_k - 1)f_k(\xi)\right]^2 d\xi \end{aligned}\right\} \tag{10}$$

and λ is the slenderness of the column. All parameters of A_0/A_k, e/e_k and ε_k that appeared in the above equations are all the function of the relative crack length ξ and are related to the configuration of the cross-section of the columns.

The work performed by external forces is:

$$U_p = \frac{P}{2}\int_0^l \left(\frac{dy}{dx}\right)^2 dx - 2Pe\left(\frac{dy}{dx}\right)\bigg|_{x=0} = \frac{\pi^2 e^2 P}{4l}\sum_{m=1}^{n} m^2 C_m^2 + \frac{2\pi e^2 P}{l}\sum_{m=1}^{n} C_m m \tag{11}$$

3.3 The series solution of equations of deflection

Substituting from Eqs. (7), (9) and (11) into Eq. (6), the equations of the deflection coefficients are given by

$$C_m = 4\alpha \frac{m + \dfrac{\pi^2 a^* r\alpha}{e^2\lambda}\sin\dfrac{m\pi x_k}{l}\left[g_1(\xi,\varepsilon)\sum_{j=1}^{n} C_j \sin\dfrac{j\pi x_k}{l} + g_2(\xi,\varepsilon)\right]}{\pi m^2(m^2-\alpha)} \quad (m = 1, 2, \ldots, n) \tag{12}$$

where r is the gyration radius of the gross cross-section.

Substituting from Eq. (12) into (4), the deflection equation of the cracked column is finally given by

$$y(x) = \frac{4\alpha e}{\pi}\sum_{m=1}^{n}\frac{m + \dfrac{\pi^2 a^* r\alpha}{e^2\lambda}\sin\dfrac{m\pi x_k}{l}\left[g_1(\xi,\varepsilon)\dfrac{y_k}{e} + g_2(\xi,\varepsilon)\right]}{m^2(m^2-\alpha)}\sin\frac{m\pi x}{l} \tag{13}$$

where it should be noted that y_k is still the functions of the coefficients C_m.

The analytical results (Li 2000) show that the accuracy of the first term of the series expressed by Eq. (13) is 1.5% for a typical thin-walled column. For an engineering application, Eq. (13) can be therefore simplified as

$$y(x) = \frac{4\alpha e}{\pi(1-\alpha)}\left[1 + \frac{\pi^2 a^* r\alpha}{e^2\lambda}\sin\frac{\pi x_k}{l}\left[g_1(\xi,\varepsilon)\frac{y_k}{e} + g_2(\xi,\varepsilon)\right]\right]\sin\frac{\pi x}{l} \tag{14}$$

Assuming that the crack is located on the intermediate cross-section of the column shown in Fig. 1, which is the most dangerous section, i.e., $x = x_k = l/2$, the maximum deflection can be determined by solving Eq. (14), that is:

$$\delta = y_k = y(l/2) = \frac{4e\alpha}{\pi}\cdot\frac{1 + \beta g_2(\xi,\varepsilon)\alpha}{1 - \alpha - \dfrac{4}{\pi}\beta g_1(\xi,\varepsilon)\alpha^2} \tag{15}$$

where $\beta = \pi^2 a^* r / e^2\lambda$.

4. Ultimate bearing capacity of cracked eccentric columns

4.1 Stability coefficients according to SIF criteria

According to the fracture criteria of stress intensity factor criteria, the SIF K_I increases linearly with increased tensile stress. When K_I exceeds a critical value K_c, the crack propagates unstably. Thus, fracture occurs when

$$K_I(a) = \sigma_k\sqrt{\pi a^*/2}\,f(\xi) = K_c \tag{16}$$

Substituting from Eq. (1) into (16), it can be seen that the dimensionless parameter of the ultimate compress capacity of the column is

$$\alpha = \frac{\gamma_k \lambda^2}{\varepsilon_k \frac{\delta}{e_k} + \varepsilon_k - 1} \tag{17}$$

where

$$\gamma_k = \frac{\sqrt{2} K_c}{\pi^{5/2} E \sqrt{a^*} f(\xi)} \cdot \frac{A_k}{A_0} \tag{18}$$

Substituting from Eq. (15) into (17) and neglecting the α^3 term, the ultimate compress capacity α, can be solved and, therefore, the formula of stability coefficient φ_k (= $\sigma_c / f_y = \alpha\pi^2 E / f_y \lambda^2$) based on K criteria is given by

$$\varphi_k = \frac{\pi^2 E}{2 f_y \lambda^2} \frac{\sqrt{(\gamma_k\lambda^2 - \varepsilon_k + 1)^2 + \frac{16}{\pi}\gamma_k\lambda^2\left[\varepsilon_k \frac{e}{e_k} + \beta g_1(\xi, \varepsilon) \cdot \gamma_k \lambda^2\right]} - \gamma_k\lambda^2 - \varepsilon_k + 1}{\frac{4}{\pi}\left[\varepsilon_k \frac{e}{e_k} + \beta g_1(\xi, \varepsilon) \cdot \gamma_k\lambda^2\right] - \varepsilon_k + 1} \tag{19}$$

where f_y is the yielding strength of the material.

4.2 Stability coefficients according to edge yielding criteria

After derivation similar to that in the above section, the formula of stability coefficient φ_y based on edge yielding criteria can be obtained as follows:

$$\varphi_y = \frac{\pi^2 E}{2 f_y \lambda^2} \frac{\sqrt{(\gamma_y\lambda^2 - \varepsilon_y - 1)^2 + \frac{16}{\pi}\gamma_y\lambda^2\left[\varepsilon_y \frac{e}{e_k} + \beta g_1(\xi, \varepsilon) \cdot \gamma_y \lambda^2\right]} - \gamma_y\lambda^2 - \varepsilon_y - 1}{\frac{4}{\pi}\left[\varepsilon_y \frac{e}{e_k} + \beta g_1(\xi, \varepsilon) \cdot \gamma_y\lambda^2\right] - \varepsilon_y - 1} \tag{20}$$

where

$$\gamma_y = \frac{f_y}{\pi^2 E} \cdot \frac{A_k}{A_0} \tag{21}$$

Eq. (20) may be regarded as a development of Perry's column formula.

4.3 The $\varphi - \lambda$ curves of columns in terms of the two-criteria approach

For generally eccentric thin-walled columns containing model-I cracks, there are two possible kinds of failure mechanisms in states of ultimate bearing capacity: yielding at the compression edge and fracturing at the tensile edge. Which kind of failure will occur under the compression load depends on the compound states of the slenderness λ of columns, the eccentricity ε of the loads, the relative crack length ξ and the material properties such as the yielding strength f_y, fracture toughness K_c and elastic modulus E. Therefore, the equations of the $\varphi-\lambda$ curves of the cracked eccentric thin-walled columns should be expressed according to

$$\varphi = \min(\varphi_y, \varphi_k) \tag{22}$$

The above approach of determining the $\varphi-\lambda$ curves of the columns may be called the two-criteria approach. Generally speaking, the higher the values of the slenderness λ, the eccentricity ε and the relative crack length ξ, the greater the possibility of column fracture failure.

5. Example and analysis

5.1 Introduction of problem

The mechanical properties of the steel 16 Mn calculated here are as follows: yielding strength $f_y = 390$ N/mm^2, fracture toughness $K_c = 4282$ Nmm$^{1..5}$, and elastic modulus $E = 206\times10^5$ N/mm^2.

The box column shown in Fig. 1(c) will be numerically computed and discussed. The geometric properties of the gross section are as follows: $b = 160$ mm, $t/b \le 0.1$, $\eta_1 = 1.0$, $\eta_2 = 2.0$, $A_0 = 4bt$, $r = \sqrt{I/A} = b/\sqrt{6}$, and $\varepsilon = eA_0/W_0 = 3e/b$. The dimensionless characteristics of the cracked section and relative eccentricity can be given by

$$A_0/A_k = 4/(4-\xi) \tag{23}$$

$$\varepsilon_k = \frac{4(4-\xi)\varepsilon - 6\xi}{16-10\xi+3\xi} \tag{24}$$

$$\varepsilon_y = \left(1-\frac{\xi}{2}\right)\varepsilon_k \tag{25}$$

$$\frac{e}{e_k} = \frac{(4-\xi)\varepsilon}{(4-\xi)\varepsilon-6\xi} \tag{26}$$

The configuration correction factor of the SIF may be looked up in Wang and Chase (2003); it is

$$f(\xi) = [1-0.025\xi^2+0.06\xi^4]\sqrt{\xi\sec\left(\frac{\pi\xi}{2}\right)} \tag{27}$$

with 0.1% accuracy. The parameter a^* in expression (3) is b/π.

The values of functions $g_1(\xi, \varepsilon)$ and $g_2(\xi, \varepsilon)$ can be obtained by substituting from Eqs. (23)-(27) into (10) and thus the deflection and the stability coefficient are finally determined.

5.2 Effects of crack on deflection

The ratio of the maximum deflection change caused by cracking to the maximum deflection of an uncracked column, $\Delta\delta/\delta_0$, can be adopted to describe the effects of cracking on deflection. The curves of $\Delta\delta/\delta-\xi$ for various eccentricity ε are illustrated in Fig. 3, where the dimensionless load is $\alpha = 0.6$ and the slenderness of the column is $\lambda = 100$. From Fig. 3 it can be found that, in the same

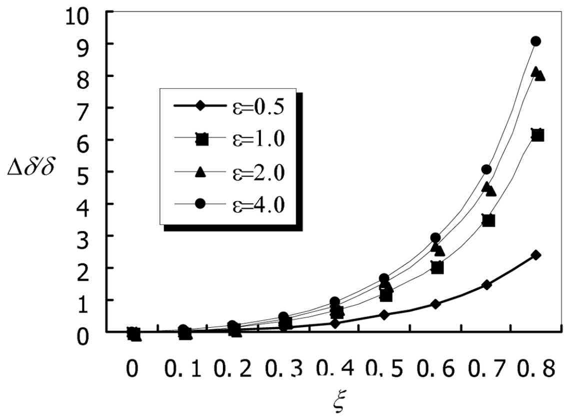

Fig. 3 The curves of the $\Delta\delta/\delta-\xi$ for different eccentricity values

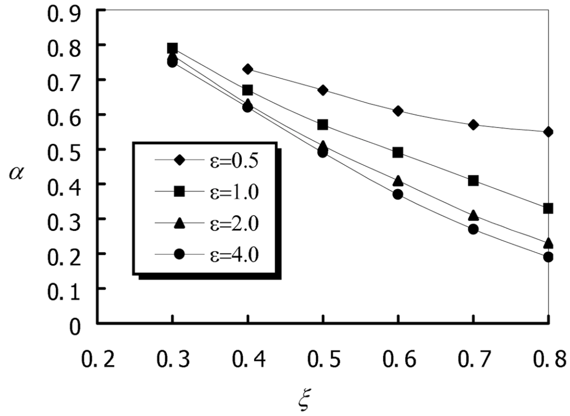

Fig. 4 The curves of $\xi-\alpha$ under the condition of $\Delta\delta/\delta = 1\%$

situation of eccentricity, the ratio of $\Delta\delta/\delta_0$ increases monotonously with the increasing of crack length and, in a certain state of cracking, the higher the value of relative eccentricity, the higher the value of $\Delta\delta/\delta_0$. In other words, the effect of cracking on the deflection of the column increases with an increase in the crack length or the eccentricity.

There is a limit of load-crack states, i.e., $\xi-\alpha$, for a column, in which the effects of cracking on the deflection are so few that the cracking can be neglected (for example, $\Delta\delta/\delta \le 1\%$). The $\xi-\alpha$ state curves under the condition of $\Delta\delta/\delta = 1\%$ are shown in Fig. 4 for various eccentricity values ε, where the slenderness of the column is also $\lambda = 100$. According to the curves in Fig. 4, if the point related to the state $\xi-\alpha$ is under the curve, the effect parameter of cracking on deflection, $\Delta\delta/\delta$, is less than 1% and can be neglected for engineering applications. On the other hand, the parameter $\Delta\delta/\delta$, is higher than 1%. It can be observed that the loads from the crack effects are clearly lower for a longer crack or higher eccentricity.

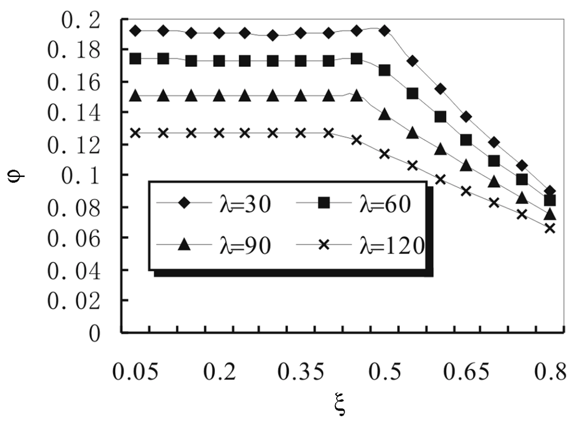

Fig. 5 The $\varphi-\xi$ curves for various slenderness values

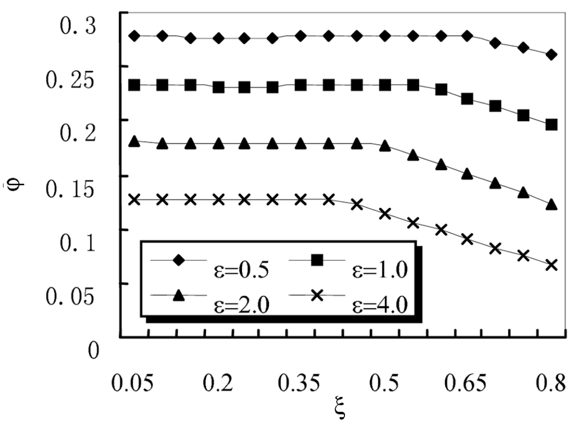

Fig. 6 The $\varphi-\xi$ curves for various eccentricity values

5.3 Effects of crack on ultimate bearing capacity

The curves of stability coefficient φ versus the relative crack length ξ are shown for various slenderness values λ in Fig. 5, where the eccentricity is $\varepsilon = 4$, and for various eccentricity values ε in Fig. 6, where the slenderness is $\lambda = 120$.

From Fig. 5 and Fig. 6, it is obvious that there is always a critical transition value of ξ_c on the $\varphi - \xi$ curves. The stability coefficients φ are insensitive to the varying of crack length when $\xi \le \xi_c$, while they decrease with the increasing of crack length when $\xi > \xi_c$. Therefore, it can be implied that if the relative crack length does not exceed its critical value, that is $\xi \le \xi_c$, the ultimate bearing capacity of the cracked eccentric thin-walled column will be yielding dominant, and if $\xi > \xi_c$, the failure will be fracture dominant.

Fig. 5 and Fig. 6 also show that the critical transition crack length ξ_c decreases with increasing slenderness λ in the column or with increasing eccentricity ε of the loading.

6. Conclusions

A method has been developed and applied to determine the elastic deflection of eccentric thin-walled columns containing some model-I cracks. A trigonometric series solution of the elastic deflection equation was obtained by the Rayleigh-Ritz energy method. In contrast to the solution presented in Okamura (1981), this solution meets the needs of compatibility of deformation and is useful for thin-walled columns.

There are two kinds of failure mechanisms in the states of the ultimate bearing capacity: yielding dominant and fracture dominant. Based on the above deflection solution, a two-criteria approach to determine the stability factor φ has been suggested.

As an example of the use of the above two-criteria method, box columns with both ends pinned and containing a through-wall crack under model I loading were calculated and analyzed. The effects of cracks on both the maximum deflection and the stability coefficient φ for various crack lengths or eccentricity were illustrated and discussed. Finally, the concept of critical transition crack length ξ_c was proposed and the limits of crack influence were described in terms of the load-crack length curve under the equal $\Delta\delta/\delta$ condition, where the parameter $\Delta\delta/\delta$ is a ratio of the deflection change caused by cracking to the deflection of an uncracked column. From results of the analytical and numerical tests on the columns, the following brief conclusions are drawn:

(1) The deflection change caused by cracking increases with decreasing the slenderness ratio or with increasing the eccentricity.

(2) There is always a critical transition value of ξ_c on the curves of $\varphi - \xi$ for various values of slenderness λ or eccentricity ε. The critical transition crack length ξ_c decreases with the increasing of slenderness λ of the column or of the eccentricity of loads.

(3) The ultimate bearing capacity of the cracked eccentric thin-walled column is yielding dominant when $\xi \le \xi_c$, and is fracture dominant when $\xi > \xi_c$.

It should be noted that in the present paper the results from LEFM theory are conservative and residual stresses are not considered. For practical applications to steel structures, both material plasticity and residual stresses are important factors that should be considered. Further investigation on these effects is necessary.

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