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The torsional buckling analysis for cylindrical shell with material non-homogeneity in thickness direction under impulsive loading

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Abstract. This study considers the buckling of orthotropic cylindrical thin shells with material non-homogeneity in the thickness direction, under torsion, which is a power function of time. The dynamic stability and compatibility equations are obtained first. Applying Galerkin's method then applying Ritz type variational method to these equations and taking the large values of loading parameters into consideration, analytic solutions are obtained for critical parameter values. Using those results, the effects of the periodic and power variations of Young's moduli and density, ratio of Young's moduli variations, loading parameters variations and the power of time in the torsional load expression variations are studied via pertinent computations. It is concluded that all these factors contribute to appreciable effects on the critical parameters of the problem in question.

Key words: torsion; buckling; non-homogeneous orthotropic material; cylindrical shell; critical torsional loads; dynamic factor.

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1. Introduction

Non-homogeneous materials are of considerable technical and engineering importance. These materials have properties that vary as a function of position in the body. Non-homogeneous materials can frequently be found in nature as well as in man-made structures. However, typically non-homogeneous materials seem to be those with elastic constants varying continuously in different spatial directions. Continuous non-homogeneity is a direct generalization of homogeneity in theory; besides, material non-homogeneity becomes essential and must sufficiently be considered in a number of practical situations. In all the referenced works, and in most of available solutions to elastic non-homogeneity, it is assumed that the material is isotropic or orthotropic, the Poisson's ratio is constant, and the Young's moduli or density is either an exponential or a power function of a spatial variable (Massalas *et al.* 1981, Khoroshun and Kozlov 1988, Guiterrez *et al.* 1998, Wang *et al.* 2000, Sofiyev 2002).

There are many studies about the buckling of homogeneous cylindrical shells under torsion (static or dynamic torsional buckling) and some of these are mentioned as (Donnell 1933, Sachenkov and Baktieva 1978, Tan 2000, Park 2001, Sofiyev *et al.* 2003). But, one such problem, not considered

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till today, is the torsional buckling of non-homogeneous orthotropic cylindrical shells under loads, which is a power function of time.

The aim of the present research is to study the buckling problem for a non-homogeneous orthotropic cylindrical shells, of constant Poisson's ratio and non-uniform Young's moduli and density in the form of a continuously function of the thickness coordinate, subjected to torsion varying as a power function of time, by using the Ritz type variational method.

2. Problem formulation

Consider a cylindrical shell, with length L, thickness h and radius R, which is made of an orthotropic non-homogeneous material with immovable simple supports along the whole circumference of the ends. The origin of a coordinate system is located within the midpoint of length the reference surface of the shell with x, y and z measured along the longitudinal, circumferential, and radial directions, respectively. The axes of orthotropy are parallel to the x and y-axes (see Fig. 1).

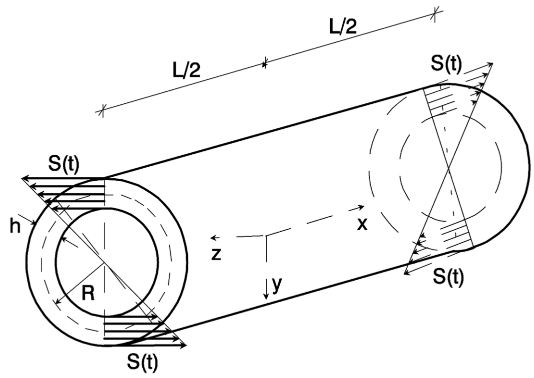

Fig. 1 Geometry and the coordinate system of a cylindrical thin shell

Assume that the Young's moduli and density of the material are continuous functions of the coordinate in the thickness direction. Hence, the Young's moduli and density can be expressed as functions of $\overline{z} = z/h$ as follows:

$$E_j(\overline{z}) = E_{0j}[1+\mu\varphi_1(\overline{z})],\ G(\overline{z}) = G_0[1+\mu\varphi_1(\overline{z})],\ \rho(\overline{z}) = \rho_0[1+\mu\varphi_2(\overline{z})],\quad j = 1,2 \quad (1)$$

where E_{01}, E_{02} and G_0 are the Young's moduli of the homogeneous orthotropic material and its shear modulus, respectively, ρ_0 is the density of the homogeneous material and μ is the variation coefficient of the Young's moduli and density satisfying $0 \le \mu < 1$, $\varphi_i(\overline{z})$, ($i = 1, 2$) are continuous functions corresponding to the variations of the Young's moduli and density, which satisfy $|\varphi_i(\overline{z})| \le 1$.

The shell is subjected to a torsion applied along the edges, varying as a power function of time in the form: $N_{11}^0 = N_{22}^0 = 0$, $2N_{12}^0 = -(S_1 + S_0 t^{\alpha})$. Here N_{11}^0, N_{22}^0 and N_{12}^0 are the membrane forces for the condition with zero initial moments, S_1 is static torsional load, S_0 is the torsional loading parameter, t is time and $\alpha \geq 1$ is the power expressing the time dependence of the torsional loading.

The dynamic stability and compatibility equations for non-homogeneous orthotropic cylindrical shells for the stress function ϕ and the displacement w, after some mathematical operations, can be obtained as (see Sofiyev 2002)

$$c_{12}\phi,_{xxxx} + (c_{11} - 2c_{31} + c_{22})\phi,_{xxyy} + c_{21}\phi,_{yyyy} - c_{13}w,_{xxxx} - (c_{14} + 2c_{32} + c_{23})w,_{xxyy} - c_{24}w,_{yyyy} + \phi,_{xx}/R - (S_1 + S_0 t^{\alpha})w,_{xy} = \rho_t w,_{tt} \tag{2}$$

$$b_{22}\phi,_{xxxx} + (b_{12} + 2b_{31} + b_{21})\phi,_{xxyy} + b_{11}\phi,_{yyyy} - b_{23}w,_{xxxx} - (b_{13} - 2b_{32} + b_{24})w,_{xxyy} - b_{14}w,_{yyyy} = -w,_{xx}/R \tag{3}$$

where a comma denotes partial differentiation with respect to the corresponding coordinates b_{ij}, c_{ij}, i, $j = 1, 2, 3, 4$ and are given in Sofiyev (2002). Expressions a_{ij}^k, $k = 0, 1, 2$ included by b_{ij} and c_{ij} are defined as follows:

$$a_{jj}^k = \frac{E_{0j}h^{k+1}}{1 - \nu_{12}\nu_{21}}\int_{-1/2}^{1/2}\bar{z}^k[1 + \mu\varphi_1(\bar{z})]d\bar{z}, \quad j = 1, 2, \qquad a_{12}^k = \nu_{21}a_{11}^k = \nu_{12}a_{22}^k = a_{21}^k,$$

$$a_{33}^k = 2G_0h^{k+1}\int_{-1/2}^{1/2}\bar{z}^k[1 + \mu\varphi_1(\bar{z})]d\bar{z}, \qquad \rho_t = \rho_0 h\int_{-1/2}^{1/2}[1 + \mu\varphi_2(\bar{z})]d\bar{z} \tag{4}$$

3. Solution of the eigenvalue problem

Assuming the cylindrical shell to have simply supports at the ends, the solution of equation set (2, 3) is sought in the following form:

$$w = \xi(t)\sin(\pi x/L)\sin n(y + \gamma x)/R, \quad \phi = \zeta(t)\sin(\pi x/L)\sin n(y + \gamma x)/R \tag{5}$$

where, n is the wave number in the direction of the y axis, γ tangent of the angle between the waves and x axes, $\xi(t)$ and $\zeta(t)$ are the time dependent amplitudes.

Boundary conditions are satisfied when they integrated from 0 to $2\pi R$, if $x = \pm L/2$.

Substituting expressions (5) in the equation set (2, 3) and eliminating $\zeta(t)$, applying Galerkin's method in the ranges $-L/2 \leq x \leq L/2$ and $0 \leq y \leq 2\pi R$ and when it is taken in consideration that $n\gamma << 1$ and $n^4 >> m_2^4$ are provided for a certain wave number n of the cylindrical shells in medium length, the following equation is obtained:

$$\xi,_{\tau\tau}(\tau) + \frac{t_{cr}^2}{\rho_t R^2}[(c_{24}b_{11} - c_{21}b_{14})b_{11}^{-1}R^{-2}n^4 + m_2^4 b_{11}^{-1}n^{-4} - m_2 n(S_1 + S_0 t_{cr}^{\alpha}\tau^{\alpha})]\xi(\tau) = 0 \tag{6}$$

where $m_2 = \pi R/L$ and $\tau = t/t_{cr}$, t_{cr} being the critical time and $0 \leq \tau \leq 1$ the dimensionless time parameter.

An approximating function will be chosen as $\xi(\tau) = A\exp(\beta\tau)[\tau(\beta+2)/(\beta+1) - \tau^2]$ satisfying the initial conditions $\xi(0) = 0$, $\partial\xi(1)/\partial\tau = 0$. Here A is constant. The β coefficient for torsion given as a power function of time, it can be shown by numerical computations that correspond to: $\beta = 2$ when $\alpha = 1, 2$; $\beta = 3$ when $\alpha = 3, 4$.

Applying the Ritz type variational method to differential Eq. (6) and minimizing characteristic equation according to the wave number n, an equation is obtained. After solving this equation for $S_1 = 0$ and for large values of S_0, and after some mathematical operations, the following expressions are found for the static critical torsional load, dynamic critical torsional load and dynamic factor, respectively (see Sofiyev *et al*. 2003):

$$S_{crs} = 1.6 \times (5/3)^{3/8} \times m_2^{1/2} b_{11}^{-1} R^{-5/4} (c_{24}b_{11} - c_{21}b_{14})^{5/8}$$

$$S_{crd} = S_0 t_{cr}^{\alpha} = 4 b_{11}^{-1} B_2^{-1}(\alpha) B_0(\alpha) R^{-5/4} (c_{24}b_{11} - c_{21}b_{14})^{5/8} m_2^{1/2} \Omega^{3\alpha/(6+4\alpha)}$$

$$K_d = S_{crd}/S_{crs} = 2.5 \times 0.6^{3/8} \times B_0(\alpha) B_2^{-1}(\alpha) \Omega^{3\alpha/(6+4\alpha)} \tag{7}$$

where $B_q(\alpha)$, $q = 0, 1, 2$ given in Sofiyev (2002) and the following definition apply:

$$\Omega = 4^{-2/\alpha} \times S_0^{2/\alpha} B_1(\alpha) [B_2(\alpha)]^{2/\alpha} [B_0(\alpha)]^{-(\alpha+2)/\alpha} b_{11}^{(2+\alpha)/\alpha} R^{(6\alpha+5)/(2\alpha)} \times \\ (c_{24}b_{11} - c_{21}b_{14})^{-(2\alpha+5)/4\alpha} m_2^{-(2\alpha+1)/\alpha} \rho_t/3 \tag{8}$$

When $\mu = 0$, $\alpha = 1$, the appropriate formulas for a cylindrical shell made of a homogeneous isotropic material are found as a special case (see Sachenkov and Baktieva 1978).

4. Numerical computations and results

For the numerical computations, the material properties are given in Table 1:

Table 1 The material properties of the Glass/epoxy and Graphite/epoxy composites

Materials	E_{01} (N/m^2)	E_{02} (N/m^2)	ρ_0 (kg/m^3)	ν_{12}	ν_{21}
Glass/epoxy	5.38×10^7	1.793×10^7	2004	0.25	0.0833
Graphite/epoxy	1.724×10^8	7.79×10^6	1530	0.35	0.016

In Table 2, the values of the dynamic critical torsional load and dynamic factor of cylindrical thin shells made of the Glass/epoxy and Graphite/epoxy material versus the power α and loading parameter S_0 are presented, when Young's moduli and density function are given linearly and parabolic. When Young's moduli and density varies linearly and parabolic functions the effect on critical parameters is bigger in parabolic state. When the Young's moduli and density function are negative, the effect to the critical parameters are more. For both materials properties, the effect of Young's moduli and density variations to the critical parameters are the same as percentage. There is a very big difference between the critical parameter values for glass/epoxy and graphite/epoxy composites.

Table 2 The variation of the critical parameters for different functions of the Young's moduli and density with different power of time α (L/R = 2.22, R/h = 112.5)

	$\alpha = 1$, $\beta = 2$, $S_0 = 2 \times 10^5$ (N/m×s)		$\alpha =2$, $\beta = 2$, $S_0 = 2 \times 10^8$ (N/m×s^2)		$\alpha = 3$, $\beta = 3$, $S_0 = (2 \times 10^{11}$ N/m×s^3)	
	S_{crd} (N/m)	K_d	S_{crd} (N/m)	K_d	S_{crd} (N/m)	K_d
	Graphite/epoxy (Glass/epoxy) for $\mu = 0$					
	68.3(80.7)	2.57(2.76)	56.9(72.1)	2.14(2.47)	59.1(78.0)	2.22(2.67)
$\varphi_i(z)$, i = 1, 2	Graphite/epoxy (Glass/epoxy) for $\mu = 0.9$					
$\pm\bar{z}$	67.9(80.1)	2.67(2.87)	56.3(71.3)	2.21(2.55)	58.4(77.0)	2.30(2.76)
$\bar{z}^2$	70.7(83.5)	2.39(2.57)	59.7(75.7)	2.02(2.33)	62.6(82.6)	2.12(2.54)
$-\bar{z}^2$	65.8(77.7)	2.79(3.00)	53.9(68.3)	2.28(2.64)	55.5(73.2)	2.35(2.83)

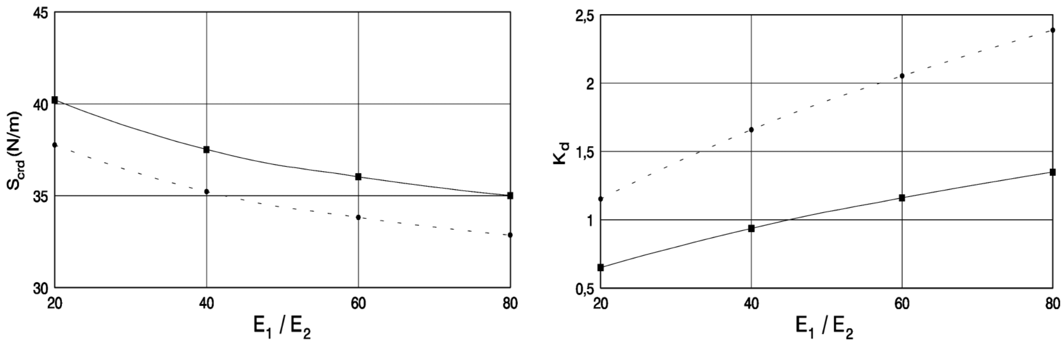

Fig. 2 The variation of the dynamic critical torsional load and dynamic factor with E_1/E_2 ($S = S_0 t$, S_0 = 31600 N/m×s, $\lambda = \pi/6$, $\mu = 0.9$, L/R = 2.22, R/h = 112.5)

The numerical analysis for the cylindrical shell parameters which are taken into consideration show that the torsional loading parameter varies approximately by the following values to become the loading dynamic: a) When $\alpha = 1$, it must be in $4.2 \times 10^4 \le S_0 < 3 \times 10^7$ (N/m×s), b) When $\alpha = 2$, it must be in $3 \times 10^7 \le S_0 < 1.6 \times 10^{10}$ (N/m×s^2), c) When $\alpha = 3$, it must be in $1.6 \times 10^{10} \le S_0 < 6 \times 10^{12}$ (N/m×s^3). Consequently when the loading law changes, the values of loading parameter change, too.

In Fig. 2, the numerical computations were carried out for the following material properties: $E_{01} = 2 \times 10^8$ (N/m^2), $\nu_{12} = 0.2$, $\nu_{21} = \nu_{12} \times E_{02}/E_{01}$, $\rho_0 = 7800$ (kg/m^3). Fig. 2 shows the variation of the dynamic critical torsional load and dynamic factor versus the ratio E_1/E_2, when Young's moduli functions are given as $\varphi_i(\bar{z}) = \cos\lambda\bar{z}, (i = 1)$. In homogeneous and non-homogeneous cases, when the ratio E_1/E_2 increases, the values of S_{crd} decrease, but the values of K_d increase. In comparison with homogeneous cases, the effect of Young's moduli variation to S_{crd} values is 7% and to K_d values is 44%.

5. Conclusions

This study has considered torsional buckling of orthotropic cylindrical shells, with non-uniform Young's moduli and density, subjected to dynamic loading given by a power function of time was studied. For large torsional loading parameter values, the analytical solution for the critical parameters of orthotropic cylindrical shells with non-uniform Young's moduli and density in the thickness direction, have been found. Numerical computations were carried out for the Young's moduli and density vary in the form of periodic and power function and the power of time in the torsional loading expression.

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