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Automatic modal identification and variability in measured modal vectors of a cable-stayed bridge

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Abstract. An automatic modal identification program is developed for continuous extraction of modal parameters of three cable-supported bridges in Hong Kong which are instrumented with a long-term monitoring system. The program employs the Complex Modal Indication Function (CMIF) algorithm for identifying modal properties from continuous ambient vibration measurements in an on-line manner. By using the LabVIEW graphical programming language, the software realizes the algorithm in Virtual Instrument (VI) style. The applicability and implementation issues of the developed software are demonstrated by using one-year measurement data acquired from 67 channels of accelerometers permanently installed on the cable-stayed Ting Kau Bridge. With the continuously identified results, variability in modal vectors due to varying environmental conditions and measurement errors is observed. Such an observation is very helpful for selection of appropriate measured modal vectors for structural health monitoring use.

Key words: cable-stayed bridge; monitoring system; automatic modal identification; complex modal indication function; modal vector; modal variability.

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1. Introduction

Real-time or near real-time damage monitoring of civil infrastructure systems when subjected to natural or man-made disasters has widespread societal implications. Not only does this give infrastructure owners/managers knowledge of what and where damage may have occurred, but also whether immediate evacuation of the occupants/contents is necessary. In Hong Kong, a sophisticated on-structure instrumentation system, called Wind And Structural Health Monitoring System (WASHMS), has been devised and implemented by the Hong Kong SAR Government Highways Department for long-term monitoring of three cable-supported bridges (Lau *et al.* 1999, Wong *et al.* 2000). The objectives of devising this on-line monitoring system are: (i) to monitor the structural health (safety) conditions of the bridges; (ii) to provide information for facilitating the planning of inspection and maintenance activities; and (iii) to verify design assumptions and parameters for future construction of cable-supported bridges. The system consists of more than 800 sensors of various types and accomplishes continuous 24-hour monitoring per day. The large volume of collected data from a continuous monitoring system involving hundreds of sensors makes manual probing almost impossible. Especially, traditional human analysis leads to the breakdown of real-time or near real-time monitoring. Automatic and on-line data processing and analysis is therefore imperative for real-time or near real-time structural health monitoring.

This paper describes the development of an automatic modal identification program and its application to the cable-stayed Ting Kau Bridge which is one of the three bridges instrumented with WASHMS. Software that can automatically identify structural modal properties from continuous ambient vibration measurements is developed in the LabVIEW environment. The Complex Modal Indication Function (CMIF) algorithm is encoded into the software for modal parameter extraction, which is realized in Virtual Instrument (VI) style with the aid of a visualization program. The CMIF algorithm is especially efficient for output-only modal identification of large-scale structures with spatially distributed sensors. The developed software is then applied for continuous modal parameter identification, at one-hour intervals, of the cable-stayed Ting Kau Bridge by using one-year measurement data from 45 accelerometers (a total of 67 channels) permanently installed on the bridge.

From the point of view of vibration-based structural health monitoring, it is extremely important to discriminate abnormal changes in modal features caused by structural damage from normal changes due to varying environmental and operational conditions, so that the normal changes will not raise a false positive alarm in health monitoring. It has been accepted that monitoring of structures for at least one complete cycle of in-service/operating environment to include the whole range of environmental conditions is in need before reliably implementing damage identification methods. Considerable research efforts have been devoted to investigating the variability of modal frequencies caused by in-service environmental and operational conditions (Roberts and Pearson 1996, Abdel Wahab and De Roeck 1997, Farrar *et al.* 1997, Alampalli 1998, Sohn *et al.* 1999, Lloyd *et al.* 2000, Rohrmann *et al.* 2000, Peeters and Roeck 2001, Sohn *et al.* 2002, Ko *et al.* 2003, Lloyd *et al.* 2003), but no study addressing the normal variability of modal vectors was reported. In recognizing that the modal vector information is necessary for locating structural damage, we address the variability of measured modal vectors in the present study. It is worth mentioning that when the modal vectors are identified using ambient vibration data, the variation in their measured values is not only due to environmental variability, but also from measurement errors. The signal-to-noise ratio is different for a specific mode when it is fully motivated in an excitation event and weakly motivated in another excitation event. Because it is difficult to distinguish the environmental effects from the measurement causes, emphasis of the present study is laid to identify the modes with stationary measured modal vectors. For this purpose, modal properties of the Ting Kau Bridge are identified using one-year continuous ambient vibration measurements, and variability of the measured modal vectors is observed. Then stationary and non-stationary modes with respect to the environmental and measurement causes are classified and only the former will be used in vibration-based damage identification. Such an observation also provides information on appropriate sensor installation locations.

2. Algorithm and realization of automatic modal identification

2.1 Output-only modal identification

For the three cable-supported bridge instrumented with the long-term monitoring system WASHMS, a considerable number of accelerometers (106 channels in total) as part of the sensory system have been permanently deployed at the deck, towers, cables, anchorage and tower bases to continuously monitor structural dynamic response induced by wind loading, highway and railway traffic, and ground tremor (earthquake, wave and ship impact). Although anemometers and weigh-in-motion sensors are also installed on the bridges for the collection of wind data and traffic data, it is almost impossible to accurately measure the wind and traffic dynamic excitations in recognizing that they are distributed and moving loads, respectively. As a result, only the output-only identification methods are applicable for modal parameter estimation of the instrumented bridges.

The output-only modal identification methods can be classified into the frequency-domain methods and the time-domain methods. The most commonly adopted frequency-domain method is the peak-picking method in conjunction with the use of coherence and phase information. This method is simple to implement and user-friendly. However, when closely-spaced modes exist as in the case of cable-supported bridges, the peak-picking method fails to provide accurate identification of the modal vectors. The time-domain methods for output-only modal identification include the stochastic subspace algorithm, Ibrahim time decrement algorithm, and eigensystem realization algorithm. All these methods require *a priori* determination of the state-space dimension, which is relatively difficult for complex cable-supported bridge structures. In the present study, the Complex Modal Indication Function (CMIF) algorithm is adapted for output-only modal identification of the instrumented bridges in recognizing its following features: (i) as a frequency-domain approach the CMIF algorithm is user-friendly like the peak-picking method; (ii) the CMIF algorithm is able to accurately identify closely-spaced modes; and (iii) because of working in the spatial domain, the CMIF algorithm simultaneously uses data from spatially distributed sensors for reliable capture of modal properties. The CMIF algorithm was originally proposed to identify the proper order of system equation in the frequency domain and the modal parameters from frequency response functions (FRFs) (Shih *et al.* 1988a, 1988b). However, for output-only systems subjected to ambient excitations, only power spectral density matrix of system responses rather than FRFs is available. The refinement of the CMIF algorithm with respect to the response spectral density is briefed as follows.

For an instrumented bridge with acceleration responses as observation variables, its state-space model can be represented as

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_c\boldsymbol{x}(t) + \boldsymbol{B}_c\boldsymbol{u}(t) \tag{1a}$$

$$\boldsymbol{y}(t) = \boldsymbol{C}_c\boldsymbol{x}(t) + \boldsymbol{D}_c\boldsymbol{u}(t) \tag{1b}$$

where $\boldsymbol{x} \in \mathbf{R}^{n\times 1}$ is an n-dimensional state vector; $\boldsymbol{y} \in \mathbf{R}^{m\times 1}$ is an m-dimensional output or measurement; $\boldsymbol{u} \in \mathbf{R}^{r\times 1}$ is a r-dimensional control input or excitation. $\boldsymbol{A}_c \in \mathbf{R}^{n\times n}$, $\boldsymbol{B}_c \in \mathbf{R}^{n\times r}$, $\boldsymbol{C}_c \in \mathbf{R}^{m\times n}$ and $\boldsymbol{D}_c \in \mathbf{R}^{m\times r}$ are the transition matrix, input coefficient matrix, output coefficient matrix and transfer matrix of the system, respectively.

If the singular value decomposition of the transition matrix is $\boldsymbol{A}_c = \boldsymbol{\Psi}\boldsymbol{\Lambda}_c\boldsymbol{\Psi}^{-1}$, Eq. (1) can be written as

$$\dot{\boldsymbol{x}}_m(t) = \boldsymbol{\Lambda}_c\boldsymbol{x}_m(t) + \boldsymbol{L}_c^T\boldsymbol{u}(t) \tag{2a}$$

$$\boldsymbol{y}(t) = \boldsymbol{V}_c\boldsymbol{x}_m(t) + \boldsymbol{D}_c\boldsymbol{u}(t) \tag{2b}$$

where $\boldsymbol{L}_c^T = \boldsymbol{\Psi}^{-1}\boldsymbol{B}_c$, $\boldsymbol{V}_c = \boldsymbol{C}_c\boldsymbol{\Psi}$, and the diagonal elements of $\boldsymbol{\Lambda}_c$ (i.e., the singular values of $\boldsymbol{A}_c$) are

$$\lambda_k = -\xi_k\omega_k + j\omega_k\sqrt{1-\xi_k^2} \tag{3}$$

in which ω_k and ξ_k are the modal frequency and damping ratio of the kth mode.

If the excitation is assumed to be a zero-mean white noise vector satisfying

$$\boldsymbol{R}_u(\tau) = E[\boldsymbol{u}(t+\tau)\boldsymbol{u}^T(t)] = \boldsymbol{R}_u\delta(\tau) \tag{4}$$

then the power spectral density matrix of the input can be expressed as

$$\boldsymbol{S}_u(s) = \int_{-\infty}^{\infty}\boldsymbol{R}_u(t)e^{-st}dt = \boldsymbol{R}_u \tag{5}$$

and the power spectral density matrix of the output is obtained as

$$\boldsymbol{S}_y(s) = \boldsymbol{H}_c(s)\boldsymbol{R}_u\boldsymbol{H}_c^T(s^*) \tag{6}$$

where $\boldsymbol{H}_c(s)$ is the transfer function of the system, which is obtained by the Laplace transform of Eq. (1) as

$$\boldsymbol{H}_c(s) = \frac{\boldsymbol{Y}(s)}{\boldsymbol{u}(s)} = \boldsymbol{C}_c(s\boldsymbol{I}-\boldsymbol{A}_c)^{-1}\boldsymbol{B}_c + \boldsymbol{D}_c \tag{7}$$

where $\boldsymbol{Y}(s)$ and $\boldsymbol{u}(s)$ are the Laplace transforms of the output and input, respectively. By expressing the transfer function in pole-residue form, we have

$$\boldsymbol{H}_c(s) = \sum_{k=1}^{n}\frac{s^2}{\lambda_k^2(s-\lambda_k)}\{\boldsymbol{v}_{ck}\}\{\boldsymbol{l}_{ck}\}^T \tag{8}$$

where $\{\boldsymbol{v}_{ck}\} \in \mathbf{R}^{m\times 1}$ is the kth modal vector; $\{\boldsymbol{l}_{ck}\} \in \mathbf{R}^{r\times 1}$ is the kth modal participation factor vector;

λ_k is the system pole value of the kth mode. Upon the substitution of Eq. (8) in Eq. (6), we obtain

$$\boldsymbol{S}_y(s) = \left\{ \sum_{k=1}^{n} \frac{s^2}{\lambda_k^2(s-\lambda_k)} \{\boldsymbol{v}_{ck}\}\{\boldsymbol{l}_{ck}\}^T \right\} \boldsymbol{R}_u \left\{ \sum_{k=1}^{n} \frac{(s^*)^2}{\lambda_k^2(s^*-\lambda_k)} \{\boldsymbol{l}_{ck}\}\{\boldsymbol{v}_{ck}\}^T \right\} \tag{9}$$

Taking the singular value decomposition of $\boldsymbol{S}_y(s)$ at $s = j\omega_k$ yields

$$\boldsymbol{S}_y(j\omega_k) = \boldsymbol{U}(j\omega_k)\boldsymbol{\Sigma}(j\omega_k)[\boldsymbol{U}^*(j\omega_k)]^T \tag{10}$$

where $\boldsymbol{U}(j\omega_k) = [\boldsymbol{u}_1\ \boldsymbol{u}_2\ \ldots\ \boldsymbol{u}_m]$ and $\boldsymbol{u}_k$ is the column vector of $\boldsymbol{U}(j\omega_k)$; $\Sigma(j\omega_k) = diag\ [s_1\ s_2\ \ldots\ s_m]$ and s_i is the singular value ordered in descending sort. The matrix $\boldsymbol{\Sigma}(j\omega_k)$ is herein referred to as the complex modal indication function (CMIF), which is the eigenvalues solved from the normal matrix formed by the transfer function matrix at each spectral line.

It can be proved that if the value of $\boldsymbol{S}(j\omega)|_{\omega=\omega_k}$ is dominated by one mode, only the first singular value s_1 out of $\{s_1\ s_2\ \ldots\ s_m\}$ will reach its maximum. More generally, if the value of $\boldsymbol{S}(j\omega)|_{\omega=\omega_k}$ is dominated by i modes, there will be i singular values which reach their local maxima when ω approaches to ω_k. By setting the remaining $(m-i)$ singular values as zero, the rank of the diagonal matrix $\boldsymbol{\Sigma}(j\omega_k)$ will be equal to number of dominant modes at $\omega = \omega_k$, and consequently, the column vector $\boldsymbol{u}_i$ corresponding to the nonzero singular value s_i in $\boldsymbol{U}(j\omega_k)$ will represent the modal shape of the ith mode. The modal frequencies and modal shapes are thus identified. Because the CMIF algorithm easily detects the components of significantly contributing modes at any specific frequency, identification of closely-spaced modes becomes straightforward with fidelity. Applying the singular value decomposition to $\boldsymbol{S}_y(s)$ results in the spectral density function for each decomposed single-degree-of-freedom system, so the modal damping can be readily estimated by the half-power bandwidth method or the curve fitting technique.

When conventional spectral analysis methods are applied to a long-span cable-supported bridge, the spectrum of ambient vibration response at each measurement point contains many peaks in the low frequency range due to global vibration modes, local vibration modes, as well as, dominant frequency contents of excitation. As a result, it is considerably difficult to determine all true global modes without omission and adulteration by observation of the response spectra from individual measurement points. Instead, the CMIF algorithm simultaneously uses the signals from all measurement points for modal identification. Each of the CMIF functions is constructed based on the information from all accelerometers rather than one sensor. In the implementation of this algorithm, at least three CMIF curves are plotted which are defined by using the three maximum CMIF values among all measurement points at each frequency interval. So each CMIF curve is a synthesis of the CMIF values from different measurement points. Only the frequencies which achieve clear peaks at three or more CMIF curves are identified with fidelity as modal frequencies and the corresponding modal shapes and damping ratios are determined from Eq. (10) after accomplishing the singular value decomposition.

2.2 Visualization in virtual instrument style

The CMIF algorithm has been encoded as a program by using the LabVIEW graphical programming language. With the aid of a visualization program, the automatic modal parameter identification is performed in Virtual Instrument (VI) style. Typical user interfaces of the developed software are shown below. Fig. 1 illustrates the time history of acceleration records and the response power spectrum diagrams. Fig. 2 shows the extracted complex modal indication functions and the identified modal shape. The CMIF peaks within pre-determined ranges are picked automatically and the natural frequencies, modal shapes and damping ratios are identified accordingly.

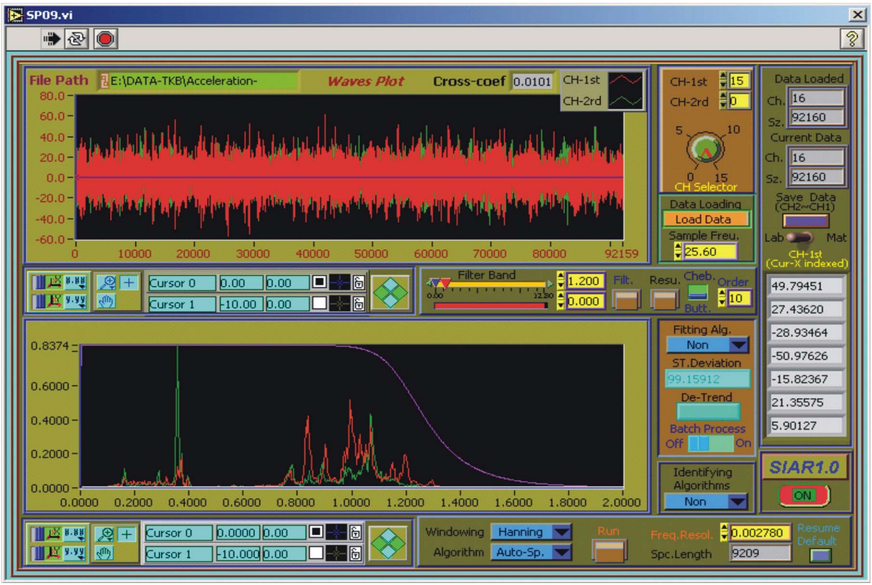

Fig. 1 Time-domain response and power spectral density

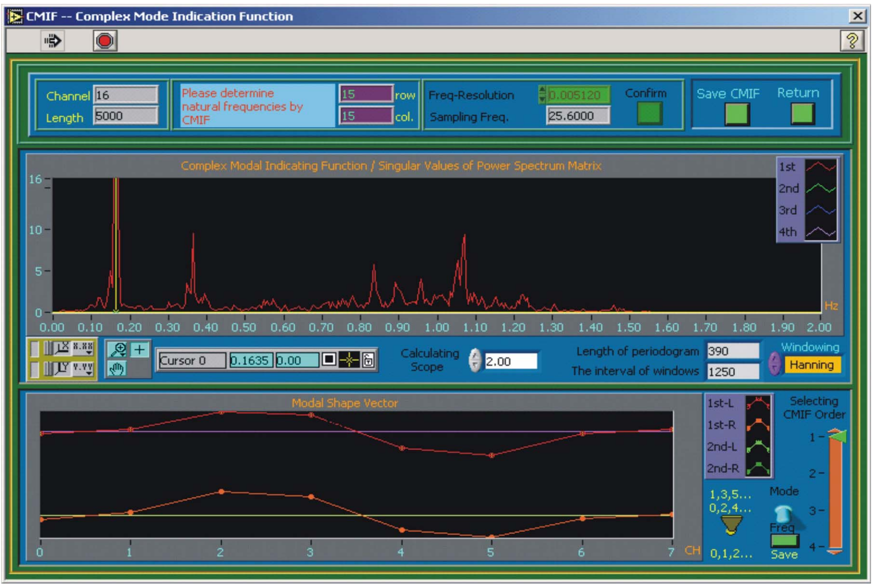

Fig. 2 Illustration of CMIF and modal shape

3. Application to Ting Kau Bridge

The Ting Kau Bridge, as shown in Fig. 3, is a three-tower cable-stayed bridge with two main spans of 448 m and 475 m respectively, and two side spans of 127 m each (Bergermann and Schlaich 1996). A sophisticated long-term monitoring system consisting of over 230 sensors has been installed on the bridge immediately after completion of its construction (Lau *et al.* 1999, Wong *et al.* 2000). The sensors include accelerometers, strain gauges, displacement transducers, anemometers, temperature sensors, weigh-in-motion sensors, and recently deployed global positioning system (Wong *et al.* 2001). A total of 24 uni-axial accelerometers, 20 bi-axial accelerometers and 1 tri-axial accelerometer (totally 67 accelerometer channels) are permanently deployed at the deck of the two main spans and the two side spans, the longitudinal stabilizing cables, the top of the three towers, and the base of central tower to monitor seismic excitation and dynamic response of the

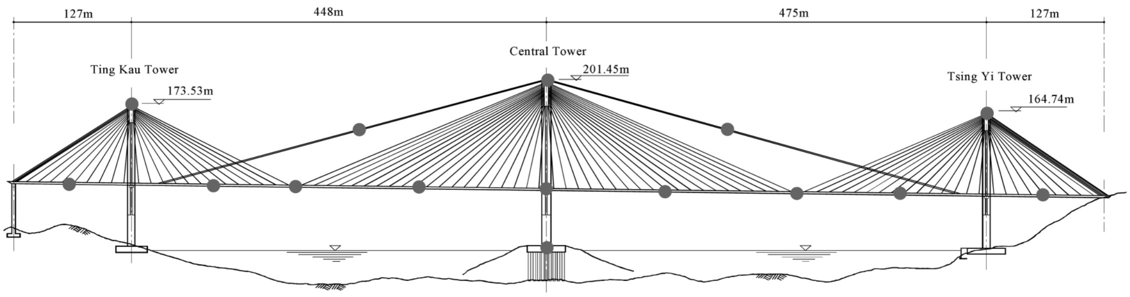

Fig. 3 Elevation of Ting Kau Bridge

Table 1 Information of accelerometers installed on Ting Kau Bridge

Cable_TK		Cable_TY		Deck_vertical		Deck_lateral		Tower	
Ch ID	Location	Ch ID	Location	Ch ID	Location	Ch ID	Location	Ch ID	Location
BGFC1Z	11718	BGKC1Z	12187	SGBE1Z	11429	SGBW2Y	11429	BGCT1X	11505
BGFC1Y	11718	BGKC1Y	12187	SGBW1Z	11429	SGDW2Y	11580	BGCT1Y	11505
BGFC2Z	11718	BGKC2Z	12187	SGDE1Z	11580	SGEW2Y	11688	BGHT1X	11953
BGFC2Y	11718	BGKC2Y	12187	SGDW1Z	11580	SGGW2Y	11823	BGHT1Y	11953
BGFC3Z	11718	BGKC3Z	12187	SGEE1Z	11688	SGJW2Y	12082	BGHT2X	11953
BGFC3Y	11718	BGKC3Y	12187	SGEW1Z	11688	SGLW2Y	12217	BGHT2Y	11953
BGFC4Z	11718	BGKC4Z	12187	SGGE1Z	11823	SGMW2Y	12352	TGHT1X	11953
BGFC4Y	11718	BGKC4Y	12187	SGGW1Z	11823	SGOW2Y	12503	TGHT1Y	11953
BGFC5Z	11718	BGKC5Z	12187	SGJE1Z	12082			TGHT1Z	11953
BGFC5Y	11718	BGKC5Y	12187	SGJW1Z	12082			BGNT1X	12428
BGFC6Z	11718	BGKC6Z	12187	SGLE1Z	12217			BGNT1Y	12428
BGFC6Y	11718	BGKC6Y	12187	SGLW1Z	12217				
BGFC7Z	11718	BGKC7Z	12187	SGME1Z	12352				
BGFC7Y	11718	BGKC7Y	12187	SGMW1Z	12352				
BGFC8Z	11718	BGKC8Z	12187	SGOE1Z	12503				
BGFC8Y	11718	BGKC8Y	12187	SGOW1Z	12503				

bridge. One-year continuously acquired acceleration data from all these 67 channels, covering one full cycle of in-service/operating conditions, are used herein for modal parameter identification and environmental variability observation.

Table 1 gives the accelerometer information. Notation of the senor locations (numbering of bridge cross-sections) refers to the references (Lau *et al.* 1999, Wong *et al.* 2000), and the sensor locations are also roughly marked in Fig. 3. The category ‘Cable-TK’ implies the sensors installed on the longitudinal stabilizing cables in the Ting Kau main span, while ‘Cable-TY’ indicates the sensors installed on the longitudinal stabilizing cables in the Tsing Yi main span. The indices ‘X’, ‘Y’ and ‘Z’ denote the longitudinal, lateral and vertical directions, respectively. The accelerometers on the cables were positioned in vertical and lateral directions. Also, the accelerometers on the deck were installed in vertical direction (at both deck edges) and lateral direction (at central girder). For the bridge towers, the accelerometers were mainly deployed in longitudinal (along the bridge axis) and lateral (sway) directions except for one accelerometer channel being oriented in vertical direction for seismic excitation measurement at the base of central tower. All signals from the 67 accelerometer channels were acquired with a sampling rate of 25.6 Hz through 24-hour continuous monitoring per day. One-year data have been used for this investigation and modal analysis was conducted using the developed software at one-hour intervals. Figs. 4 to 6 show the identified CMIFs and modal shapes for a typical predominantly vertical mode, a typical predominantly torsional mode and a typical predominantly lateral mode of the bridge. They are obtained by using one-hour response records during 4:00 to 5:00 pm of 1 March 1999. Only the vertical modal vectors of measurement points at two edges of the deck are plotted in Figs. 4 and 5, while only the lateral modal vectors of measurement points at central girder of the deck are plotted in Fig. 6.

Table 2 gives the identified modal frequencies, damping ratios and description of the first ten global modes of the Ting Kau Bridge. Figs. 7 to 11 show the identified modal shapes of the first five global modes. They are obtained by using one-hour response records during 7:00 to 8:00 am of

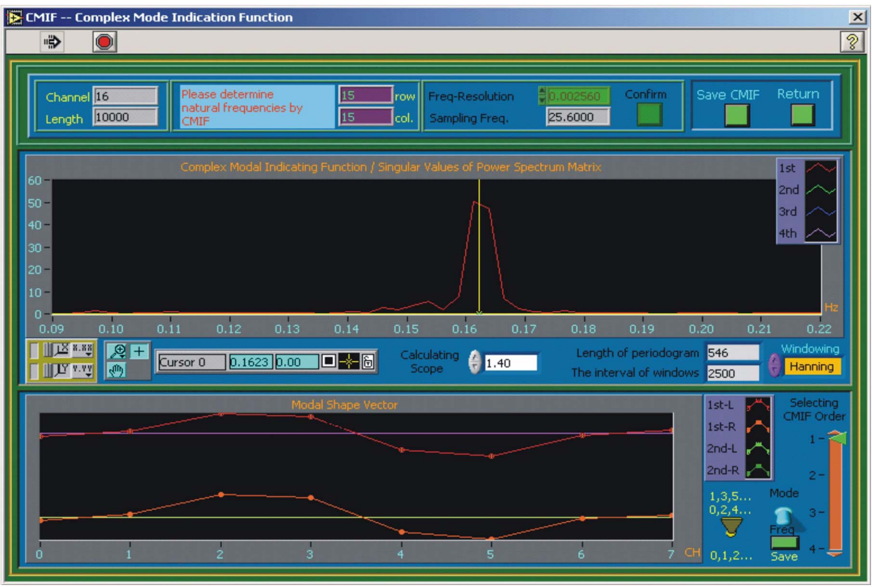

Fig. 4 Identified CMIF and modal shape for a predominantly vertical mode

Fig. 5 Identified CMIF and modal shape for a predominantly torsional mode

Fig. 6 Identified CMIF and modal shape for a predominantly lateral mode

23 March 1999. In the figures, the legend ‘U’ denotes the undeflected bridge deck configuration; the legends ‘V1’ and ‘V2’ denote the deck vertical modal components at western and eastern edges, respectively; and the legend ‘L’ denotes the deck lateral modal components at central girder. It is found that the first mode of the Ting Kau Bridge is a predominantly vertical mode with its natural frequency f = 0.165 Hz, which is less than the first modal frequency (0.199 Hz) of the world’s longest cable-stayed Tatara Bridge (Endo *et al.* 1991). It implies that the Ting Kau Bridge is one of

Table 2 Identified modal frequencies, damping ratios and description of first ten modes

Mode No.	Modal frequency (Hz)	Damping ratio	Mode description
1	0.165	1.304%	Predominantly vertical
2	0.228	0.489%	Coupled torsional and lateral
3	0.264	0.701%	Predominantly lateral
4	0.291	0.637%	Coupled lateral and torsional
5	0.300	0.362%	Predominantly vertical
6	0.323	0.839%	Coupled torsional and lateral
7	0.361	1.172%	Coupled lateral and torsional
8	0.373	0.761%	Predominantly vertical
9	0.386	0.320%	Predominantly vertical
10	0.395	0.338%	Predominantly vertical

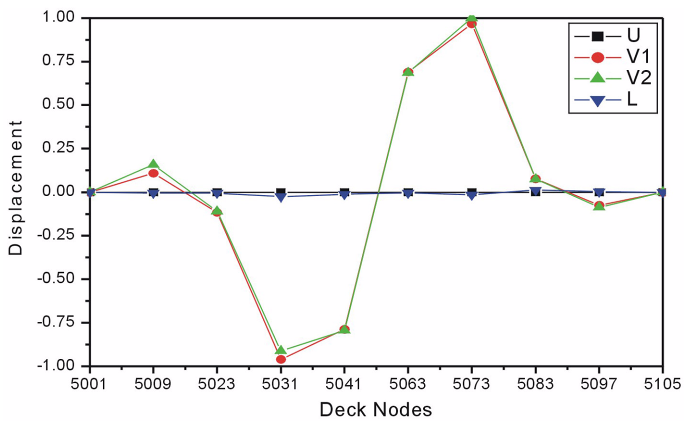

Fig. 7 Identified shape of the first mode (f_1 = 0.165 Hz)

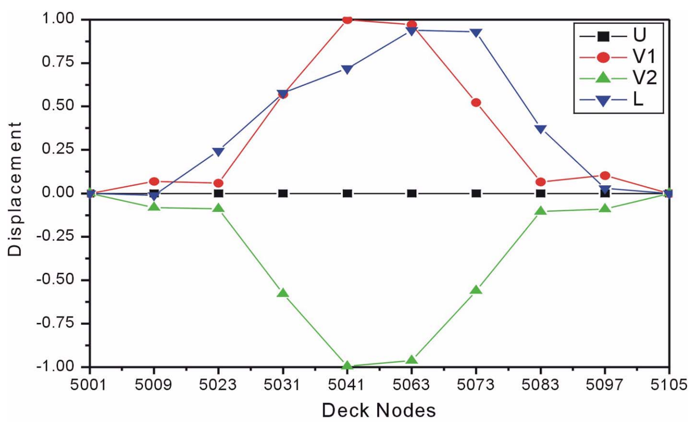

Fig. 8 Identified shape of the second mode (f_2 = 0.228 Hz)

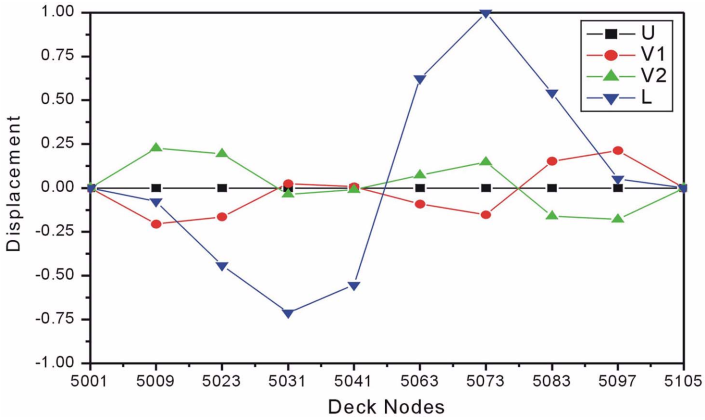

Fig. 9 Identified shape of the third mode (f_3 = 0.264 Hz)

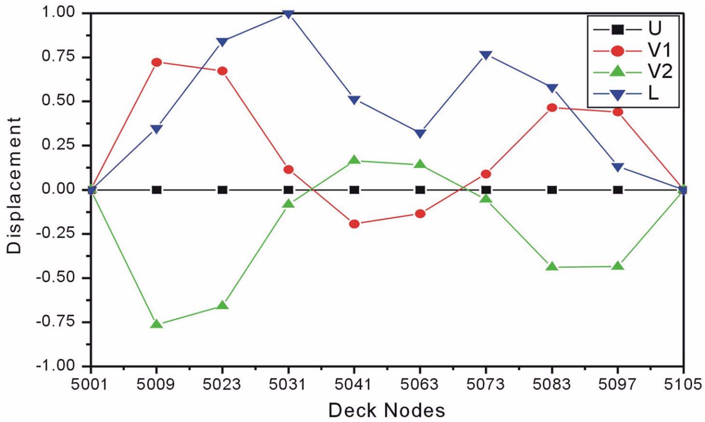

Fig. 10 Identified shape of the fourth mode (f_4 = 0.291 Hz)

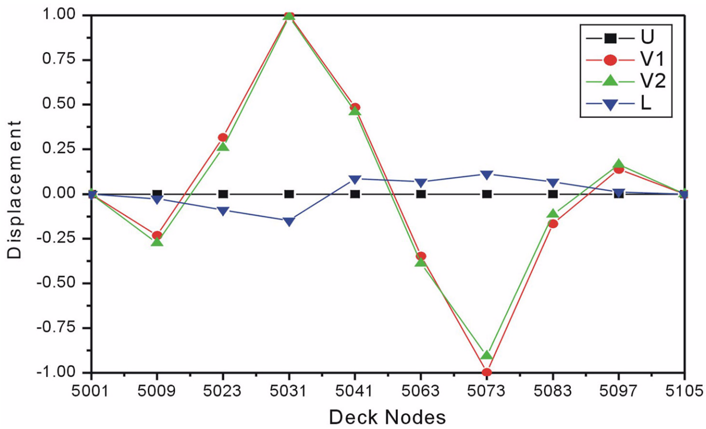

Fig. 11 Identified shape of the fifth mode (f_5 = 0.300 Hz)

the most flexible cable-stayed bridges in the world. The bridge has the first predominantly torsional mode at the natural frequency f = 0.228 Hz and the first predominantly lateral mode at the natural frequency f = 0.264 Hz. Due to the extremely long longitudinal stabilizing cables (up to 465 m), slender monoleg towers and separated deck system, the bridge exhibits strong modal coupling and interaction. For example, the first predominantly torsional mode is coupled with significant lateral modal components as shown in Fig. 8, and the first predominantly lateral mode incorporates torsional modal components as shown in Fig. 9. Significant modal interaction between the deck, towers and cables is observed. In the first predominantly vertical mode, for instance, besides both the deck and central tower participating greatly in the modal motion, the longitudinal stabilizing cables vibrate with very large amplitude.

4. Variability of measured modal vectors

The developed automatic modal identification software has been applied for continuous modal parameter extraction of the Ting Kau Bridge with the use of one-year ambient vibration data. It aims to observe variability of the measured modal parameters caused by varying environmental conditions and measurement errors and to identify stationary modes of which modal parameters will be employed to construct appropriate modal indices for vibration-based damage detection (Ni *et al.* 2001, Sun *et al.* 2001). Following the vibration-based structural health monitoring approach, damage in a structure is identified from changes in selected modal features extracted from vibration measurements. However, most civil structures are directly exposed to the environment and are thus subjected to variations in temperature, humidity, wind, traffic, insolation as well as other influences. These varying environmental and operational conditions also cause changes in modal features which may mask subtler structural changes caused by damage. Environmental variability in modal parameters must be considered before reliable use of vibration-based damage detection methods. A thorough understanding of this variability is necessary so that changes in modal features resulting from damage can be discriminated from changes resulting from such variability. Because the modal vector information is necessary for locating structural damage, variability of the measured modal vectors of the Ting Kau Bridge is observed and presented in the present study. Observation results on the modal frequency variability refer to Ko *et al.* (2003). Variations in measured modal frequencies are mainly attributed to varying environmental and operational conditions. However, when using ambient vibration data, measurement errors may contribute significantly to variations in measured modal vectors due to different modal response levels (therefore different signal-to-noise ratios) for a specific mode under different ambient excitation conditions.

Figs. 12 to 16 show 100 samples of measured modal vectors corresponding to the modes shown in Figs. 7 to 11. For clarity only modal vectors of the measurement points at the deck are plotted. The 100 samples cover the measurements in February, March, July, August, October and December 1999. For each sample, one-hour acceleration data are used to identify a set of modal frequencies and modal vectors by means of the developed software. The mean and standard deviation of the measured modal vectors for the 100 samples are listed in Table 3.

It is observed from Figs. 12 to 16 that the variability has different levels for different modes. The modal vectors for the first and fourth modes exhibit very good stationarity for all the measurement points under varying environmental and ambient excitation conditions. It is therefore concluded that using modal information of these two modes will provide most reliable damage identification

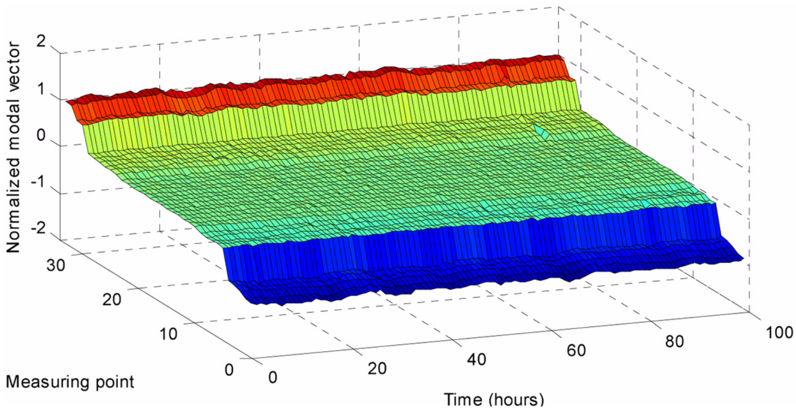

Fig. 12 Variability of measured modal vectors for the first mode

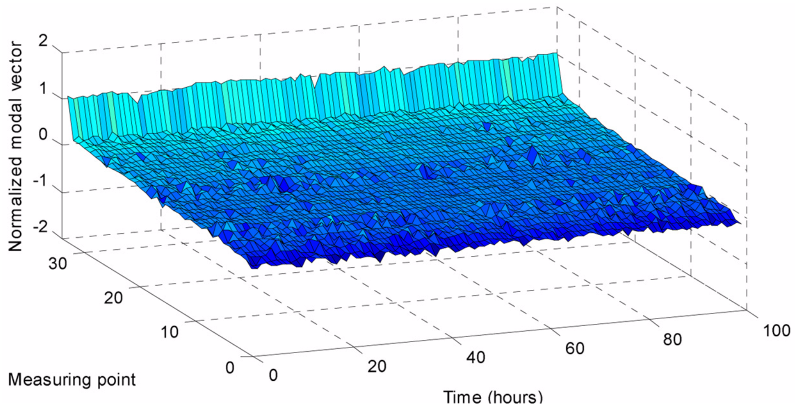

Fig. 13 Variability of measured modal vectors for the second mode

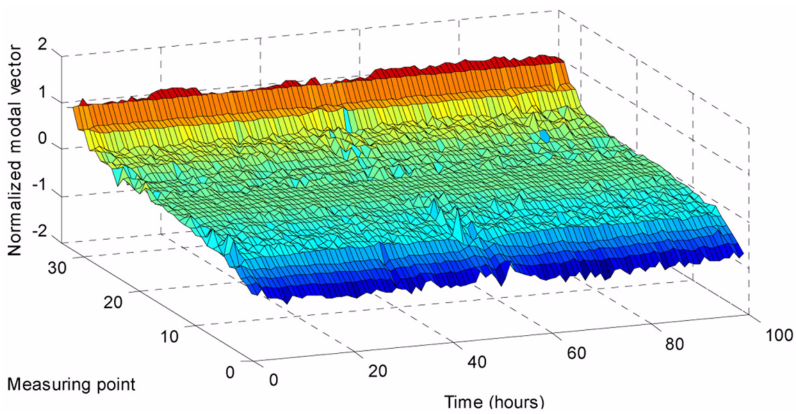

Fig. 14 Variability of measured modal vectors for the third mode

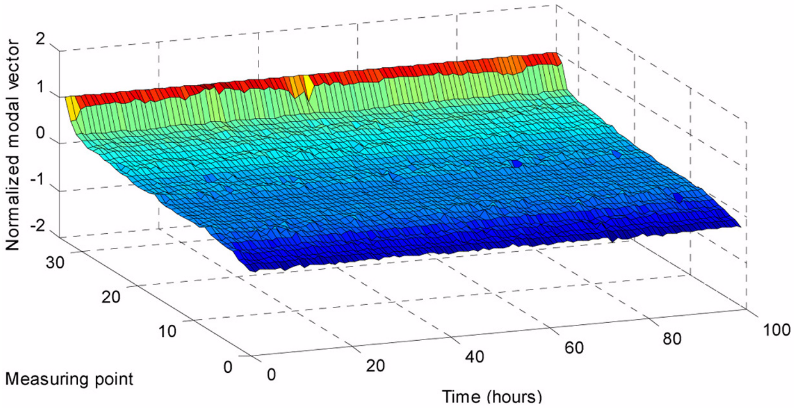

Fig. 15 Variability of measured modal vectors for the fourth mode

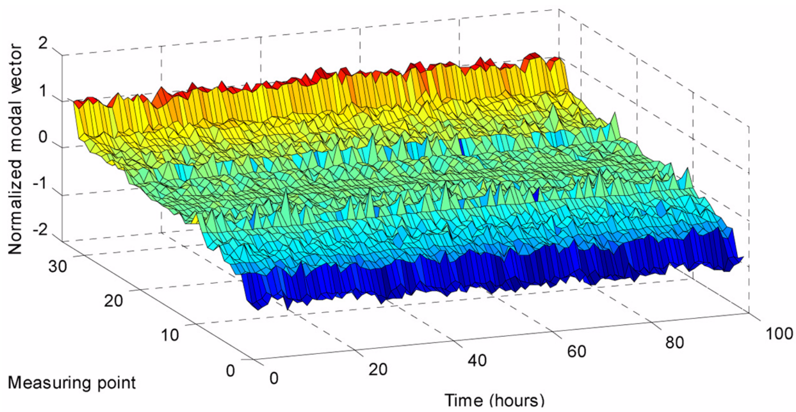

Fig. 16 Variability of measured modal vectors for the fifth mode

results. For the second and third modes, the modal vectors at some locations are found to be fairly stationary but the others exhibit noticeable non-stationarity. Variability level of the measured modal vectors for the fifth mode is considerably large. Because changes in modal parameters caused by structural damage are usually insignificant, such a variability level may be greater than the modal shape change caused by structural damage. Therefore, these measured modal parameters can be used for damage detection without false identification only after the non-stationarity of the modal parameters is quantified and the change arising from environmental and measurement causes is discriminated from that arising from structural damage. In summary, an observation of variability of the modal vectors can select the most appropriate modes employed for vibration-based damage identification and determine the sensor positions which are tolerant of environmental and measurement variability.

An effort has been made to quantitatively understand the effects of various environmental factors (temperature, wind, etc.) on the modal properties. Based on simultaneous measurements of acceleration, temperature and wind velocity on the Ting Kau Bridge, a correlation analysis of modal properties (modal frequencies and modal vectors) versus temperature and wind velocity has been

Table 3 Mean and standard deviation of measured modal vectors

Channel ID	1st mode		2nd mode		3rd mode		4th mode		5th mode	
	Mean	St. dev.	Mean	St. dev.	Mean	St. dev.	Mean	St. dev.	Mean	St. dev.
SGBE1Z	0.138	0.034	0.077	0.088	0.231	0.077	–0.824	0.051	0.392	0.086
SGBW1Z	0.104	0.008	–0.078	0.073	–0.183	0.071	0.678	0.053	0.196	0.111
SGDE1Z	–0.103	0.013	–0.077	0.068	0.182	0.082	–0.647	0.046	–0.196	0.112
SGDW1Z	–0.103	0.006	–0.150	0.090	–0.153	0.072	0.676	0.049	–0.194	0.109
SGEE1Z	–0.896	0.103	0.308	0.101	–0.052	0.076	–0.059	0.051	–0.985	0.338
SGEW1Z	–0.965	0.104	–0.614	0.134	0.034	0.069	0.146	0.053	–0.990	0.345
SGGE1Z	–0.759	0.101	1.000	0.135	0.003	0.235	0.176	0.110	–0.392	0.178
SGGW1Z	–0.793	0.069	–0.923	0.102	–0.013	0.052	–0.205	0.081	–0.394	0.108
SGJE1Z	0.690	0.067	0.769	0.098	0.074	0.073	0.147	0.071	0.391	0.105
SGJW1Z	0.690	0.068	–0.845	0.088	–0.088	0.048	–0.145	0.076	0.389	0.176
SGLE1Z	1.000	0.103	0.538	0.105	0.159	0.053	0.000	0.077	1.000	0.446
SGLW1Z	0.966	0.104	–0.385	0.101	–0.167	0.071	0.088	0.065	0.998	0.389
SGME1Z	0.103	0.032	0.154	0.112	–0.148	0.064	–0.441	0.069	0.195	0.084
SGMW1Z	0.069	0.028	–0.077	0.082	0.159	0.057	0.471	0.077	0.193	0.109
SGOE1Z	–0.069	0.008	0.076	0.083	–0.181	0.066	–0.440	0.071	–0.194	0.122
SGOW1Z	–0.068	0.007	–0.153	0.131	0.208	0.064	0.469	0.075	–0.190	0.110
SGBW2Y	0.000	0.000	0.000	0.015	–0.067	0.022	0.323	0.028	0.000	0.000
SGDW2Y	0.000	0.000	–0.231	0.033	–0.406	0.051	0.853	0.025	0.000	0.000
SGEW2Y	–0.034	0.005	–0.461	0.032	–0.657	0.067	1.000	0.051	0.000	0.188
SGGW2Y	0.000	0.000	–0.615	0.028	–0.518	0.072	0.501	0.056	0.000	0.000
SGJW2Y	0.000	0.000	–0.769	0.021	0.609	0.055	0.265	0.020	0.000	0.000
SGLW2Y	0.000	0.000	–0.846	0.027	1.000	0.069	0.734	0.077	0.000	0.000
SGMW2Y	0.000	0.000	–0.384	0.017	0.551	0.041	0.588	0.055	0.000	0.000
SGOW2Y	0.000	0.000	0.000	0.019	0.032	0.038	0.147	0.024	0.000	0.000

conducted. It is found that the maximum variance of modal frequencies for the first five modes due to temperature change (from 5^{o}C to 55^{o}C) is 1.43%. The variations show an inversely proportional trend in modal frequencies with temperature for all the five modes. However, variations in the measured modal vectors are found out of proportion to temperature for all the five modes. In addition, the maximum variation of modal frequencies for the first five modes identified under low wind speed conditions and under typhoon conditions is 0.51%. The frequency for the first mode decreases with increasing wind speed while the frequencies for other modes slightly increase as wind speed becomes higher. Also, there is no proportional trend of variations in the measured modal vectors with wind speed. These analyses show that it is possible to quantify environmental effects (temperature and wind) on the modal frequencies and to isolate these effects in damage detection process through introducing appropriate correlation functions. However, the majority of variations in the modal vectors may arise from measurement errors and a quantitative representation of environmental effects on the modal vectors is very difficult. We can only identify stationary modes and use these stationary modal vectors to achieve reliable damage identification.

5. Conclusions

An automatic modal identification program employing the Complex Modal Indication Function (CMIF) algorithm has been developed for modal parameter extraction of the cable-stayed Ting Kau Bridge from continuously acquired ambient vibration response signals. The program accomplishes the data analysis in Virtual Instrument style. With one-year measurement data from 45 accelerometers (a total of 67 channels) which are permanently installed on the bridge as part of a long-term monitoring system, modal parameters of the bridge under a full cycle of in-service/ operating conditions are identified and the variability of modal properties due to environmental and measurement causes is observed.

The modal analysis results show that the Ting Kau Bridge exhibits strong modal coupling with simultaneous modal components in the three dimensions and significant modal interaction among the deck, towers and cables. Observation on the variability of measured modal properties indicates that variations in the modal frequencies are mainly attributed to the temperature effect. There is an inversely proportional trend of the modal frequencies with the temperature. The modal frequencies of the bridge may increase or decrease with the wind speed, depending on specific modes. Correlations between the modal frequencies and temperature and between the modal frequencies and wind speed can be quantitatively modeled, and therefore environmental effects on the modal frequencies are able to be isolated in damage detection process. Variations in the modal vectors arise considerably from measurement errors and there is no proportional trend of variations in the modal vectors with temperature or wind speed. As a result, it is difficult to eliminate environmental effects on the modal vectors in damage detection. A feasible alternative is to identify stationary modes and to use these stationary modal vectors for reliable damage identification.

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