Structural Engineering and Mechanics, Vol. 19, No. 1 (2005) 97-106
DOI: http://dx.doi.org/10.12989/sem.2005.19.1.097

Seismic qualification using the updated finite element model of structures

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(Received March 12, 2004, Accepted July 14, 2004)

Abstract. The standard practice is to seismically qualify the safety related equipment and structural components used in the nuclear power plants. Among several qualification approaches the qualification by the analysis using finite element (FE) method is the most common approach used in practice. However the predictions by the FE model for a structure is known to show significant deviations from the dynamic behaviour of 'as installed' structure in many cases. Considering such limitation, few researchers have advocated re-qualification of such structures after installation at site to enhance the confidence in qualification vis-à-vis plant safety. For such an exercise the validation of FE model with experimental modal data is important. A validated FE model can be obtained by the Model Updating methods in conjugation with the in-situ experimental modal data. Such a model can then be used for qualification. Seismic analysis using the updated FE model and its advantage has been presented through an example of an in-core component – a perforated horizontal tube of a nuclear reactor.

Key words: modal test; FE modelling; model updating; seismic analysis.

1. Introduction

The potential of earthquake to cause severe damage to structures has been repeatedly observed. Hence from safety consideration it is mandatory to qualify the structures and equipment of Nuclear Power Plants (NPPs) to the design seismic loads. The qualification procedure is well known. It is generally carried out either by analysis or by test on shake-table or by comparison with the past experience. However the qualification purely by analysis or by shake table testing has some limitations. The seismic qualification by the shake-table testing is normally very exorbitant and besides such a facility may not be available in many places for testing large structures. Testing of scaled down model on available shake-table is another alternative but such a scale down model is

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known to show significant deviations from the dynamic behaviour of full scale model and so generally not recommended for the dynamic testing (IAEA 1992, Safety Series No. 50-SG-D15). The most important compromise often made during testing on shake-table is that the testing is carried out on single isolated structures/equipment without simulating structural connections to the secondary components which may change the dynamic behaviour compared to the 'as installed' structures. Another well accepted method for the seismic qualification is purely analytical (ASME 1981, Section-III) using finite element (FE) method (Cook *et al.* 1989, Zienkiewicz and Taylor 1994). The reliability of the method totally depends on the FE model. An FE model *a priori* is generally not found to be truly representing the dynamic behaviour of 'as installed' structures even for relatively simple structural components. This is because of a number of simplified assumptions made while FE modelling, such as for joints and boundary conditions, decoupling of the secondary components, etc. Few such case studies are discussed by Sinha and Friswell (2002, 2003a) and Sinha, Rao and Moorthy (2003b). Considering such limitation, few researchers have advocated re-qualification of such structures after installation at site to enhance the confidence in qualification vis-à-vis plant safety.

To overcome the above limitations, researchers have suggested few alternate approaches. Moorthy *et al.* (1994, 1996) suggested testing of components by mounting on a railway boggy as an alternative to the seismic qualification on shake table. The suggested approach uses the excitation induced by the unevenness in the railway track during the train journey on the track for the seismic qualification of structures. In fact the functional seismic qualification of a Diesel Generator (DG) set, which was too large for the shake table facility available in India and may be for many countries in the world, was successfully demonstrated by such an approach. Advantage of this approach is that even a large scale structure can be qualified during its transportation from manufacturer to the installation site. Other alternatives to the analytical method have also been suggested by many researchers (Sinha 2003c, Sinha and Moorthy 2000, Moorthy *et al.* 1994, Unruh *et al.* 1984, Asfusa 1985, Shye and Skreiner 1986). They have used in-situ modal test data directly for the seismic response estimation. Direct use of the modal test data obtained from the modal tests conducted on the 'as installed' structure has overcome the limitations when using FE model for seismic analysis. However these alternative methods may not always be practical for many structural components. The examples of such structures are the in-core components of the nuclear reactors and containments. The qualification of such structures by the railway track induced vibration using full scale model for the structure with the related auxiliary structures expected at site or conducting the modal tests on the structure at site would be difficult.

Hence for the structures like in-core components it is good to make a full scale laboratory model. It is not necessary that the full scale model need to be exact structural simulation expected after installation at site. From such laboratory model, the many required information like the parameters related to dynamic characteristics, damping, etc. can be extracted by the modal tests. This information can then be used for updating its initial FE model by the Model Updating methods (Friswell and Mottershead 1995, Sinha and Friswell 2002, 2003a). The Model Updating methods have now advanced to a matured area of research for reliable modelling of structures in the field of structural dynamics. Few commercially available FE codes have already implemented this method in their codes. Such updated FE model usually represents a true reflection of the structural dynamics predicted by modal tests. So the updated FE model completely reflecting the test setup can then be used to simulate the expected design conditions after installation at site and then the reliable estimation of the seismic responses due to design loads can be possible to qualify the component.

The advantage of such an approach has been presented in the paper through a typical example of a long horizontal perforated tube designed to distribute the flow all along its length in a nuclear reactor vessel. One of the gradient based sensitivity methods used for model updating in the present study has also been described in the paper for better understanding.

2. Gradient based sensitivity method for FE model updating

One of the gradient based model updating methods, namely the *Penalty Function method* (Friswell and Mottershead 1995), based on natural frequencies only, is used here to estimate the updating parameters and updating of the initial FE model. The vector of updating parameters, $\boldsymbol{\theta} = [\theta_1, \theta_2, \dots, \theta_p]^T$, where these parameters could be the boundary stiffness, material and geometrical properties of the structure. The selection of the updating parameters generally depends on the physical understanding of the structure and the assumptions made in the initial FE modelling. The first m eigenvalues (natural frequency squared) are measured and placed in the measurement vector, $\mathbf{z}_e = [\lambda_{e1}, \lambda_{e2}, \dots, \lambda_{em}]^T$. The corresponding eigenvalues computed from the FE model are $\mathbf{z}_c = [\lambda_{c1}, \lambda_{c2}, \dots, \lambda_{cm}]^T$. Mode shapes (or eigenvectors) may also be included in the measurement vector, although this is a minor extension to the approach, and is not considered further.

The eigenvalues may be written as a first order truncated Taylor series expansion in terms of the updating parameters, giving the error vector, ε, as (Friswell and Mottershead 1995),

$$\varepsilon = \delta\mathbf{z} - \mathrm{S}\delta\boldsymbol{\theta} \tag{1}$$

where $\delta\theta$ is the vector of perturbations in the updating parameters and $\delta\mathbf{z} = \mathbf{z}_e - \mathbf{z}_c$ is the eigenvalue error. Note that it is important to pair the correct modes, which is conveniently checked using the Modal Assurance Criteria (MAC) (Allemang and Brown 1982). The Sensitivity matrix, **S**, is the first derivative of eigenvalues with respect to the updating parameters. The element of the sensitivity matrix is computed (Fox and Kapoor 1968) as

$$S_{ij} = \frac{\partial\lambda_{ci}}{\partial\theta_j} = \phi_{ci}^T\left[\frac{\partial\mathbf{K}}{\partial\theta_j} - \lambda_{ci}\frac{\partial\mathbf{M}}{\partial\theta_j}\right]\phi_{ci} \tag{2}$$

where λ_{ci} and ϕ_{ci} are the ith eigenvalue and eigenvector. The penalty function, **J**, is formed as (Friswell and Mottershead 1995),

$$\mathbf{J}(\delta\theta) = \varepsilon^T\mathbf{W}_\varepsilon\varepsilon \tag{3}$$

where $\mathbf{W}_\varepsilon$ is the positive diagonal weighting matrix which reflects the confidence level in the frequency measurements. It is generally taken as the reciprocal of the variance (the square of the standard deviation) of the corresponding measurements (Friswell and Mottershead 1995).

The vector of desired updating parameters can be obtained by minimizing **J** with respect to $\delta\theta$ which involves the differentiation of **J** with respect to each parameter, and setting the result equal to zero. The perturbation in the parameter vector is then

$$\delta\theta = [\mathbf{S}^T\mathbf{W}_\varepsilon\mathbf{S}]^{-1}\mathbf{S}^T\mathbf{W}_\varepsilon\delta\mathbf{z} \tag{4}$$

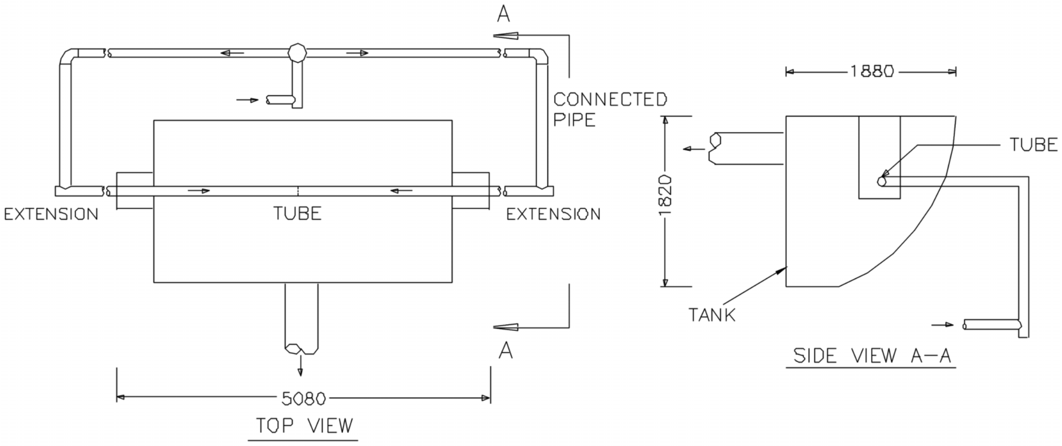
Since Eq. (4) is a linear approximation, the method is iterative. A new model with the updated values of the updating parameters is generated, and the revised computed eigenvalues and sensitivity matrix produced. The iteration process continues until the solution converges. The Eq. (4) may be ill-conditioned and the solution may require regularization. The various forms of regularization used in the process of model updatings are discussed by Friswell *et al.* (2001).

3. The example chosen

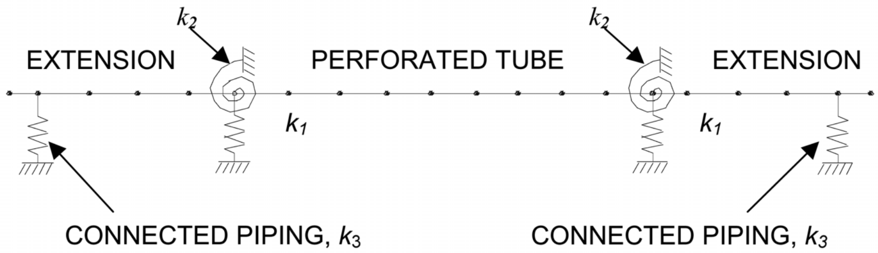
The reactor vessel in a nuclear power plant is usually a massive structure and is supported by rigid structure. Earlier experience has shown that such vessels have sufficient margin to withstand the safe shutdown level of seismic excitation (SSE), however many of its in-core components are required to be addressed adequately. One such example is a 5 m long perforated tube designed to carry the moderator through it and distribute all along its length in the vessel. The tube is perforated with different hole sizes at different pitches all along the length and is made of zircaloy (Sinha and Moorthy 1999). The tube would be rolled on to 50 mm thick side plates of the horizontal reactor vessel to realize a clamped-clamped condition in the vessel. The tube is always expected to be submerged in the moderator during the normal operation of plant. This is a typical example for which no practical experience of past earthquake is available and so it has to be qualified before being operational. Though the tube looks to be a simple structure but it has the following difficulties when any one of the known methods have to be used for qualification.

(a) Purely analytical qualification (ASME 1981, Section-III) may not be possible due to lack of knowledge about the interaction of tube and fluid, and its damping. So reliable dynamic characterization is difficult.
(b) It is difficult to conduct the modal tests at design stage and particularly for the tube, as it would be completely inaccessible to carry out the in-situ modal tests. Hence totally ruling out the possibility of directly using the modal test data in seismic analysis.
(c) A full scale model exactly simulating the plant condition to be used for the qualification by the shake-table testing or by the railway track induced vibration would also be a difficult task, if not impossible, it would be exorbitant. For the chosen example, the expected boundary stiffness in the plant condition would be about 8000 times the stiffness of the tube which is very difficult to achieve in a simple full scale laboratory model.

Considering these difficulties it was decided to tackle the problem by using a simple laboratory model. Hence a full scale model for the perforated tube with its extension on either sides has been made. The schematic of the setup is shown in the Fig. 1(a). As can be seen from Fig. 1(a) a tank has been used in the model which does not reflect the exact structural simulation of the reactor vessel, however the condition of submergence of the tube in moderator (water) can be satisfied. Hence the setup can then be used for estimating the known parameters – influence of the fluid mass on the tube dynamics and the damping – by conducting the modal tests on the tube. Obviously these modal data cannot be directly used for the seismic response estimation, as the boundary conditions of the tube in the setup are much different than the plant condition. But the experimentally obtained the added fluid mass data will remain same for the tube in the plant condition and the measured damping in setup can be conservatively assumed to be same for the plant condition. It is because the modal damping increases with increase in the amplitude of



(a) The test setup



(b) FE model

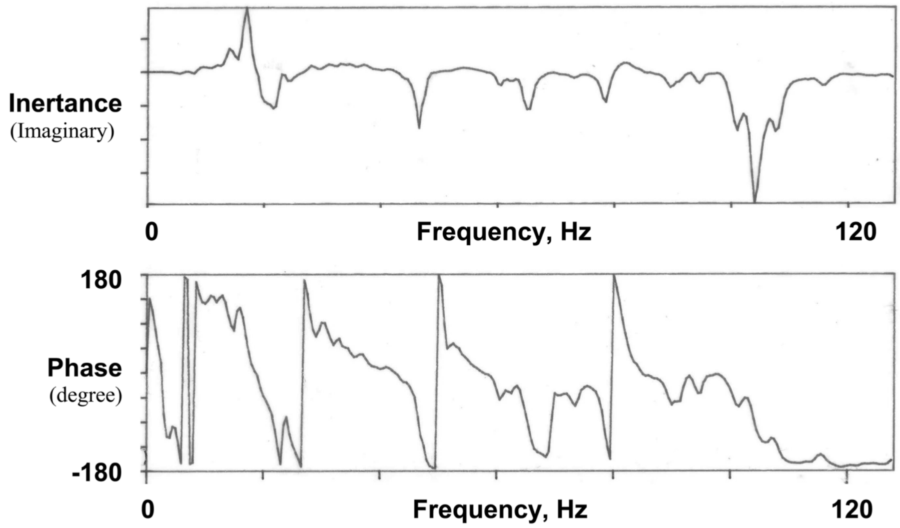
Fig. 1 The test setup of the perforated tube and its FE model

excitation (Stevenson 1980, Ibaner *et al.* 1981) and a very low level excitation generally used in modal test compared to the excitation level expected during the event of the earthquake. So the damping expected during the seismic event would be more than the measured modal damping.

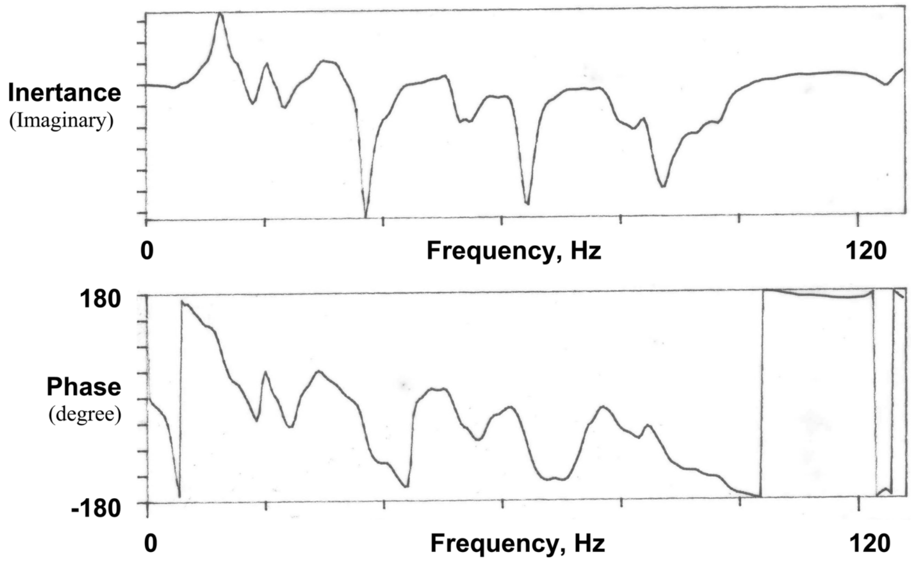
Hence in the present study, the experimental modal data generated for the full length tube in the setup and its FE model was developed. The deviation in the initial FE model was then updated by the gradient based sensitivity method (Friswell and Mottershead 1995, Sinha and Friswell 2002) using the experimental modal data so that the updated FE model fully reflect the dynamic behaviour of the tube in the setup. Once this model was achieved, the model was then used to simulate the expected boundary conditions in plant and then the seismic response was estimation by the response spectrum method. Such an approach would certainly give a much closer estimation of the actual behaviour.

4. Modal tests

The modal tests were conducted on the tube when the tank was empty (non-submerged condition) as well as when the tank was filled with water (submerged condition) (Sinha and Moorthy 1999).



(a) Non-submerged (in air) condition



(b) Submerged (in water) condition

Fig. 2 Experimental frequency responses functions

The impulse response method was used for modal tests (Ewins 2000). Typical experimental frequency response functions of the tube for both non-submerged and submerged conditions are shown in Fig. 2. A single degree of freedom circle curve fit method was used on the experimental frequency response functions to extract the modal properties. The identified natural frequencies for first three flexural modes are listed in Table 1. The measured modal damping at the first three modes under submerged conditions is 3.66%, 1.96% and 1.20% respectively.

Table 1 Measured and computed natural frequencies of the tube and the estimated boundary stiffnesses of the test setup

Updating parameters		Non-submerged (in Air)			Submerged (in Water)			
		Modal Tests Data (a)	Updated FE model (b)	Error (%) (a & b)	Modal Tests Data (c)	Updated FE model (d)	Error (%) (c & d)	Plant condition
k_1, MN/m			21.63	---		21.63	----	Clamped-Clamped Codition
k_2, kNm/rad		UNKNOWN	0.00	---	UNKNOWN	0.00	-----	
k_3, MN/m			22.08	---		22.08	----	22.08
Modes (Hz)	f_1	17.24	18.24	+5.800	12.65	12.23	–3.32	13.90
	f_2	46.56	49.95	+7.281	37.12	32.76	–11.74	37.92
	f_3	103.9	103.8	–0.096	64.06	65.120	+1.65	72.72

5. FE modelling and model updating

An FE model of the tube in the setup was developed. Two node Euler-Bernoulli beam element (each node with two DoFs – one translational and other rotational) was used to model the perforated tube and the extended portion of the pipe on either side of the tube. Since the object for the study was the tube, the tank and its supporting structure were not included in the FE modelling, however their effects on the dynamics of the tube were considered by providing the translational and rotational springs of stiffnesses k_1 & k_2 at the location of the tube support at the side plate of the tank. A translational spring element of stiffness (k_3) has also been used on either side extended pipe to simulate the stiffness effect of the connected piping. The FE model is shown in Fig. 1(b). To account for the inertial and stiffness effects due to the large number of holes distributed all over, thickness equivalent to the volume of holes was removed uniformly from the inner diameter of the tube in each element of the FE model. Since the material and geometrical properties of the tube and the extended pipes were known exactly, the boundary stiffnesses, k_1, k_2 and k_3 in the FE model were then updated by the gradient based sensitivity approach discussed in the paper using the experimental modal data of the tube in air. Sinha *et al.* (2004) gave the details of the updating of the tube FE Model in the setup.

The computed natural frequencies, listed in Table 1, for the updated FE model are much closer to the measured frequencies for the non-submerged tube, hence the updated model becomes a true representative of the tube behaviour in the setup. On this updated model the effect of water on the dynamics of the tube under submergence was included as the added mass. The added mass of the water was estimated according to the formula suggested by Sinha and Moorthy (1999). The computed natural frequencies are listed in Table 1 for the submerged tube which are close to the modal test frequencies, and hence highlighting the potential of the updated FE model. The updated model can be confidently used for design qualification and design modification of the tube.

6. Plant conditions

The estimated values of the boundary stiffnesses during the process of the model updating are also listed in Table 1. As can be seen in Table 1 the rotational boundary stiffness (k_2) tends close to zero indicating that the tube supported on the side plates of the tank is not sufficiently thick to provide any bending rotational stiffness. The small value of the translational stiffness (k_1) indicates that the tank supports are flexible and allowing some deflection. However the tube boundary conditions would be much different in the plant. The thickness of the side plates of the vessel is sufficiently thick (50 mm) to restrict the rotation of the tube and the tube will have zero deflection as the vessel is supported on the rigid structure. Hence to simulate the plant conditions, the values of boundary stiffnesses - k_1 and k_2 are replaced by sufficient high values to realize the clamped-clamped conditions of the tube in the vessel. The computed natural frequencies for the tube in the plant condition are listed in Table 1. The stiffness k_3 would not influence the dynamics of the perforated tube in the plant, hence the value of k_3 was assumed to be same as setup.

7. Seismic response estimation

The FE model simulating plant condition was used for the seismic response estimation. As can be seen from Table 1, the first flexural mode of the perforated tube is at 13.90 Hz and is the only mode below 33 Hz. It will be same in horizontal and vertical directions due to symmetry feature of the tube. The tube lower mode in axial direction was found to be much above 33 Hz and hence not considered. The 3.66% of modal damping measured at first flexural mode in the test setup can be used conservatively for the plant condition. It is because the excitation used in the modal tests was much smaller compared to the design seismic loads. Hence the measured modal damping is considered to be smaller than the damping expected during the seismic event. However for the present case a most conservative design response spectrum at 2% damping were used (Soni *et al.* 1986). The response was estimated by the response spectrum method using the maximum design

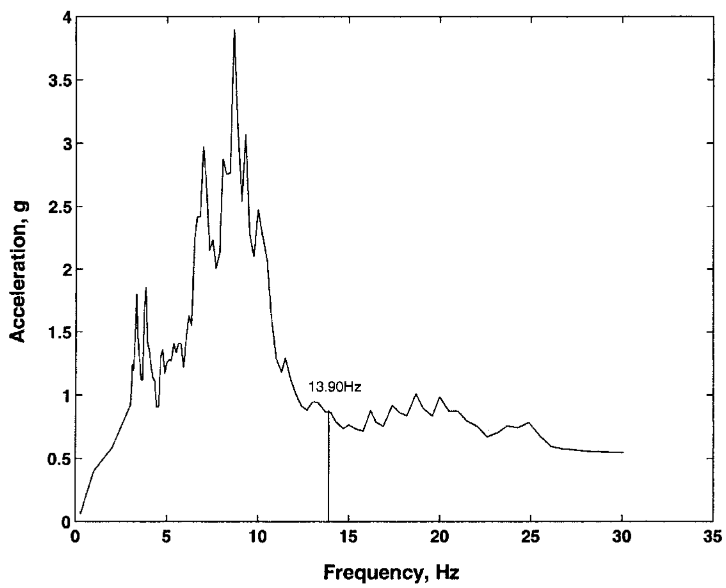


Fig. 3 The design response spectrum in the horizontal direction

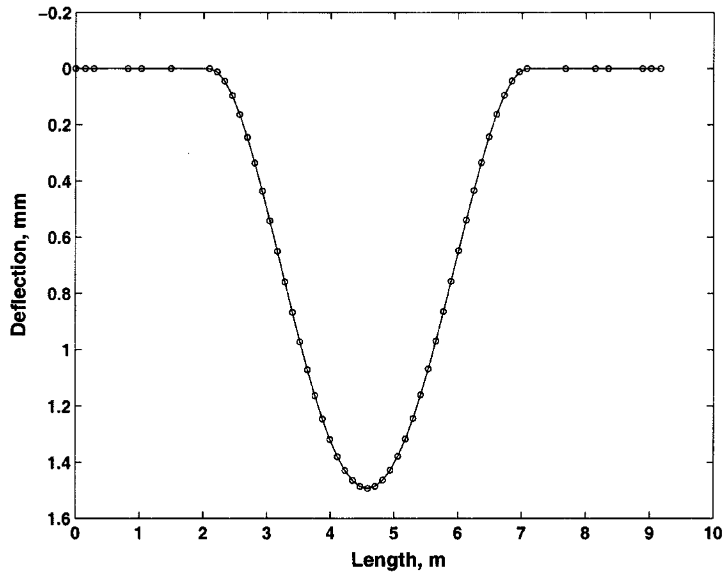


Fig. 4 The maximum deflection of the tube

response in horizontal direction as shown in Fig. 3. The estimated deflection of the tube is shown in Fig. 4. The estimated maximum stress due to this deflection is found to be at the clamped ends. The pipe is perforated and hence the stress concentration factor (SCF) has also been conservatively accounted for, though it is not necessary as the maximum stress is at clamped location where there are no perforation. The SCF is generally available for the perforated plate type structures under tension. However during bending, some portion – upper or lower depending on curvature of the tube – would be under tension so considering the developed portion of the tensile region as a perforated plate, a conservative estimate of SCF was made (Slot 1972). The maximum stress including SCF was found to be well within the allowable stress limit of zircaloy-2 material. Hence the designed in-core component can be considered to be seismically qualified.

8. Conclusions

The presented study has exploited the strength of the experimental and analytical tools for the seismic qualification of the in-core components of nuclear reactors in a realistic manner. In the experimental phase the modal tests were conducted on a full scale test setup which may not be an exact structural simulation of 'as installed' structure. The experimental modal data were then used to update the initial FE model by Model updating method. Once the updated FE model was obtained for the test setup, the model was then modified to extrapolate the 'as installed' condition. The modified FE model reflecting the 'as installed' behaviour was then used for the qualification. This has been shown through a typical in-core component of the perforated tube of nuclear reactor. In fact the strength of the recently developed model updating methods for reliable FE modelling should now be always utilized to enhance the confidence in the qualification.

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