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Local stress field for torsion of a penny-shaped crack in a transversely isotropic functionally graded strip

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Abstract. The torsion of a penny-shaped crack in a transversely isotropic strip is investigated in this paper. The shear moduli are functionally graded in such a way that the mathematics is tractable. Hankel transform is used to reduce the problem to solving a Fredholm integral equation. The crack tip stress field is obtained by taking the asymptotic behavior of Bessel function into account. The effects of material property parameters and geometry criterion on the stress intensity factor are investigated. Numerical results show that increasing the shear moduli's gradient and/or increasing the shear modulus in a direction perpendicular to the crack surface can suppress crack initiation and growth, and that the stress intensity factor varies little with the increasing of the strip's height.

Key words: torsion; penny-shaped crack; anisotropic media; functionally graded strip; stress intensity factor.

1. Introduction

The Functional Gradient Material (FGM) has received great interest in solving boundary value problems with crack-like discontinuity. The nonhomogeneity of the elastic body is assumed to depend on coordinates while the resulting equation could still be solved analytically. While such an approach has been used to solve nonhomogeneous elasticity problem in the past, it did not receive the same attention as in recent years because of the advent of composites such that FGMs could now be made and used in applications. Materials possessing functionally graded nonhomogeneity and containing cracks have been studied extensively for the isotropic case, and the solution to a class of problems for anti-plane shear and in-plane extension can be found in Kassir and Sih (1975). In fact, because of the techniques used to process the FGMs, they are seldom isotropic. For example, a plasma spray technique would usually lead to a lamellar structure while electron beam

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vapor deposition can be used to yield a highly columnar structure. It is therefore necessary to consider the anisotropic character of the FGMs. Ozturk and Erdogan (1997) analyzed the Mode I static crack problem, where an exponential form was used in their study. Li *et al*. (1999) and Li *et al*. (2001) respectively investigated the torsional problem of a penny-shaped crack and anti-plane shear problem of a Griffith crack in a FGM, where the generalized interface layer model (Wang *et al*. 1997) was used, and only the unbounded material in all directions was dealt with.

In what follows, the torsion of transversely isotropic strip with functionally graded shear moduli and a penny-shaped crack is considered. The objective is to obtain the local stress field and to examine the effects of material property parameters and geometry criterion on the fracture behaviors.

2. Material property model

Assumed in the FGM model are different variations of the shear modulus. Both types $\mu(z) = \mu_0|z|^m$ ($m > 0$) and $\mu(z) = \mu_0(c+|z|)^m$ ($c \neq 0$) have been assumed in Kassir and Sih (1975), where m can be both positive and negative in the latter case. The cases for $\mu(y) = \mu_0(1+c|y|)$ and $\mu(y) = \mu_0\exp(\gamma y)$ were considered by Gerasoulis and Srivastav (1980), Erdogan (1985) and Konda and Erdogan (1994), respectively. Recently, the material property model $\mu(z) = \mu_0(1+\alpha|z|)^2$ was used to study the crack tip behaviors of a penny-shaped crack or a Griffith crack (Li *et al*. 1998, 2001), respectively.

In this paper, we consider an orthotropic FGM as shown in Fig. 1. The coordinates r and z are assumed to be the principal axes of orthotropy. The shear moduli μ_r and μ_z are assumed to be functions of z only, and vary proportionately as

$$\mu_r(z) = (\mu_r)_0(1+\alpha|z|)^k \tag{1}$$

$$\mu_z(z) = (\mu_z)_0(1+\alpha|z|)^k \tag{2}$$

where α is a constant ($\alpha > 0$), $(\mu_r)_0$ and $(\mu_z)_0$ are the shear moduli at $z = 0$.

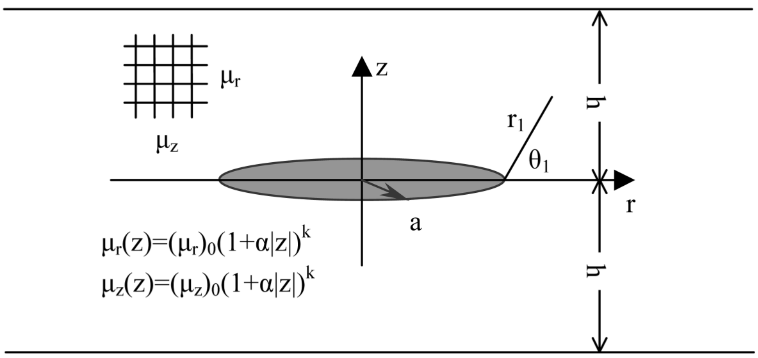


Fig. 1 Penny-shaped crack in transversely isotropic functionally graded strip

3. Formulation of the problem

Fig. 1 considers a penny-shaped crack of diameter $2a$. It is embedded in a FGM of height $2h$ and lies in the $z = 0$ plane. The solid extends to infinity in r direction. In cylindrical polar coordinates, the displacements are denoted as u_r, u_θ and u_z. For the present problem, we have

$$u_r = u_z = 0; \quad u_\theta = u_\theta(r, z) \tag{3}$$

The nonvanishing stress components $\tau_{\theta z}$ and $\tau_{r\theta}$ are

$$\tau_{\theta z} = \mu_z \frac{\partial u_\theta}{\partial z}; \quad \tau_{r\theta} = \mu_r\left(\frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}\right) \tag{4}$$

where the shear moduli μ_r and μ_z satisfy Eqs. (1) and (2), respectively.

Two of the motion equations are identically satisfied and the remaining one gives

$$\frac{\partial^2 u_\theta}{\partial r^2} + \frac{1}{r}\frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r^2} + \frac{\mu_z(z)}{\mu_r(z)}\frac{\partial^2 u_\theta}{\partial z^2} + \frac{\mu_z'(z)}{\mu_r(z)}\frac{\partial u_\theta}{\partial z} = 0 \tag{5}$$

Prime denotes derivative.

Suppose that a twisting action is applied to crack surfaces. Hence, the boundary conditions are

$$\tau_{\theta z}(r, 0) = -\tau_0 r/a; \quad 0 \le r < a \tag{6}$$

$$u_\theta(r, 0) = 0; \quad r \ge a \tag{7}$$

$$\tau_{\theta z}(r, \pm h) = 0; \quad r \ge 0 \tag{8}$$

4. Integral equation and solution

Considering the symmetry, it suffices to consider only the part $z > 0$. Introducing the pair of Hankel transform of the first order defined by

$$V(p, z) = \int_0^\infty u_\theta(r, z) J_1(pr) r dr \tag{9}$$

$$u_\theta(r, z) = \int_0^\infty V(p, z) J_1(pr) p dp \tag{10}$$

where $J_1(\ )$ is the Bessel function of the first kind, we obtain the transformed equation from Eq. (5)

$$\frac{\mu_z(z)}{\mu_r(z)}\frac{\partial^2 V(p, z)}{\partial z^2} + \frac{\mu_z'(z)}{\mu_r(z)}\frac{\partial V(p, z)}{\partial z} - p^2 V(p, z) = 0 \tag{11}$$

Substituting Eqs. (1) and (2) into Eq. (11) yields

$$\frac{\partial^2 V(p,z)}{\partial z^2} + \frac{k\alpha}{1+\alpha z}\frac{\partial V(p,z)}{\partial z} - P^2 V(p,z) = 0 \tag{12}$$

where $P = \gamma p$ with $\gamma = \sqrt{(\mu_r)_0/(\mu_z)_0}$. By defining

$$X = P(1+\alpha z); \quad Y = (1+\alpha z)^{\beta} V(p,z) \tag{13}$$

Eq. (12) can be rewritten as the modified Bessel differential equation as follows

$$\frac{d^2 Y}{dX^2} + \frac{1}{X}\frac{dY}{dX} - \left[\frac{1}{\alpha^2} + \frac{\beta^2}{X^2}\right] Y = 0 \tag{14}$$

in which $\beta = (k-1)/2$. By using the solution of Eq. (14), the solution of Eq. (12) can be easily expressed as

$$V(p,z) = A(p)(1+\alpha z)^{-\beta} I_\beta\left[(1+\alpha z)\frac{P}{\alpha}\right] + B(p)(1+\alpha z)^{-\beta} K_\beta\left[(1+\alpha z)\frac{P}{\alpha}\right] \tag{15}$$

where $I_\beta(\,)$ and $K_\beta(\,)$ are the modified Bessel functions of the first kind and the second, respectively.
Substituting Eq. (15) into Eq. (10) results in

$$u_\theta(r,z) = \int_0^\infty (1+\alpha z)^{-\beta}\left\{A(p) I_\beta\left[(1+\alpha z)\frac{P}{\alpha}\right] + B(p) K_\beta\left[(1+\alpha z)\frac{P}{\alpha}\right]\right\} J_1(pr)\, p\, dp \tag{16}$$

From Eqs. (16) and (4), the stress components are obtained as

$$\begin{aligned}\tau_{\theta z}(r,z) = \mu_z(z)\int_0^\infty \Bigg\{ & -\beta\alpha(1+\alpha z)^{-\beta-1}\left[A(p) I_\beta\left[(1+\alpha z)\frac{P}{\alpha}\right] + B(p) K_\beta\left[(1+\alpha z)\frac{P}{\alpha}\right]\right] \\ & + P(1+\alpha z)^{-\beta}\left[A(p) I'_\beta\left[(1+\alpha z)\frac{P}{\alpha}\right] + B(p) K'_\beta\left[(1+\alpha z)\frac{P}{\alpha}\right]\right]\Bigg\} J_1(pr)\, p\, dp\end{aligned} \tag{17}$$

$$\begin{aligned}\tau_{r\theta}(r,z) = \mu_r(z)\int_0^\infty & (1+\alpha z)^{-\beta}\left[A(p) I_\beta\left[(1+\alpha z)\frac{P}{\alpha}\right] + B(p) K_\beta\left[(1+\alpha z)\frac{P}{\alpha}\right]\right] \\ & \times\left[pJ'_1(pr) - \frac{J_1(pr)}{r}\right] p\, dp\end{aligned} \tag{18}$$

According to Eqs. (17) and (8), we have

$$A(p) = \frac{b(p)}{a(p)} B(p) \tag{19}$$

where

$$b(p) = \beta\alpha(1+\alpha h)^{-\beta-1} K_\beta\left[(1+\alpha h)\frac{P}{\alpha}\right] - P(1+\alpha h)^{-\beta} K'_\beta\left[(1+\alpha h)\frac{P}{\alpha}\right] \tag{20}$$

$$a(p) = -\beta\alpha(1+\alpha h)^{-\beta-1} I_\beta\left[(1+\alpha h)\frac{P}{\alpha}\right] + P(1+\alpha h)^{-\beta} I_\beta'\left[(1+\alpha h)\frac{P}{\alpha}\right] \tag{21}$$

From Eqs. (16), (17), (6), (7) and (19), a pair of dual integral equations are obtained as

$$\int_0^\infty E(p) J_1(pr)dp = 0; \quad r \ge a \tag{22}$$

$$\int_0^\infty pE(p)G(p)J_1(pr)dp = \frac{\tau_0 r}{\gamma(\mu_z)_0 a}; \quad 0 \le r < a \tag{23}$$

where

$$E(p) = pB(p)\left[K_\beta\left(\frac{P}{\alpha}\right) + \frac{b(p)}{a(p)} I_\beta\left(\frac{P}{\alpha}\right)\right] \tag{24}$$

$$G(p) = \frac{\beta\alpha\left[K_\beta\left(\frac{P}{\alpha}\right) + \frac{b(p)}{a(p)} I_\beta\left(\frac{P}{\alpha}\right)\right] - P\left[K_\beta'\left(\frac{P}{\alpha}\right) + \frac{b(p)}{a(p)} I_\beta'\left(\frac{P}{\alpha}\right)\right]}{P\left[K_\beta\left(\frac{P}{\alpha}\right) + \frac{b(p)}{a(p)} I_\beta\left(\frac{P}{\alpha}\right)\right]} \tag{25}$$

The dual integral Eqs. (22) and (23) can be solved by applying the method of Copson (1961), and the solution is

$$E(p) = \frac{4\tau_0 a^{5/2}}{3\sqrt{2\pi}\gamma(\mu_z)_0}\sqrt{p}\int_0^1 \sqrt{\xi}\Phi(\xi)J_{3/2}(pa\xi)d\xi \tag{26}$$

where $\Phi(\xi)$ should satisfy the Fredholm integral equation of the second kind

$$\Phi(\xi) + \int_0^1 \Phi(\eta)M(\xi, \eta)d\eta = \xi^2 \tag{27}$$

The kernel function $M(\xi, \eta)$ in Eq. (27) is

$$M(\xi, \eta) = \sqrt{\xi\eta}\int_0^\infty p\left[G\left(\frac{p}{a}\right) - 1\right]J_{3/2}(p\xi)J_{3/2}(p\eta)dp \tag{28}$$

The Fredholm integral Eq. (27) can be solved easily.

5. Stress field around the crack tip

Integration of $E(p)$ in Eq. (26) by parts gives

$$E(p) = \frac{4\tau_0 a^{3/2}}{3\sqrt{2\pi}\gamma(\mu_z)_0}\frac{1}{\sqrt{p}}\left\{-\Phi(1)J_{1/2}(pa) + \int_0^1 \frac{1}{\sqrt{\xi}}J_{1/2}(pa\xi)\frac{d}{d\xi}[\xi\Phi(\xi)]d\xi\right\} \tag{29}$$

in which $\Phi(1)$ is the value of $\Phi(\xi)$ evaluated at the crack tip corresponding to $\xi = 1$. From Eqs. (29), (24), (19), (17) and (18), it is found that

$$\tau_{\theta z}(r,z) = \frac{4\tau_0 a^{3/2}\mu_z(z)}{3\sqrt{2\pi}(\mu_z)_0}\Phi(1)\int_0^\infty -\sqrt{p}Q(p)J_{1/2}(pa)J_1(pr)dp + \dots \tag{30}$$

$$\tau_{r\theta}(r,z) = \frac{4\tau_0 a^{3/2}\mu_r(z)}{3\sqrt{2\pi}\gamma(\mu_z)_0}\Phi(1)\int_0^\infty -\sqrt{p}R(p)J_{1/2}(pa)J_1'(pr)dp + \dots \tag{31}$$

with

$$Q(p) = \frac{(1+\alpha z)^{-\beta}\left[K_\beta'\left[(1+\alpha z)\frac{P}{\alpha}\right] + \frac{b(p)}{a(p)}I_\beta'\left[(1+\alpha z)\frac{P}{\alpha}\right]\right]}{K_\beta\left[\frac{P}{\alpha}\right] + \frac{b(p)}{a(p)}I_\beta\left[\frac{P}{\alpha}\right]} \tag{32}$$

$$R(p) = \frac{(1+\alpha z)^{-\beta}\left[K_\beta\left[(1+\alpha z)\frac{P}{\alpha}\right] + \frac{b(p)}{a(p)}I_\beta\left[(1+\alpha z)\frac{P}{\alpha}\right]\right]}{K_\beta\left[\frac{P}{\alpha}\right] + \frac{b(p)}{a(p)}I_\beta\left[\frac{P}{\alpha}\right]} \tag{33}$$

Because the integrands in Eqs. (30) and (31) are finite and continuous for any given values of p, the divergence of the integrals at the crack tips must be due to behavior as $p \to \infty$. By carrying out the expansion for large p and considering the asymptotic behavior of $K_\beta(x)$, $K_\beta'(x)$, $I_\beta(x)$ and $I_\beta'(x)$ as $x \to \infty$, we can obtain the lower-order terms of the stress components as follows

$$\begin{aligned}\tau_{\theta z}(r,z) &= \frac{4\tau_0 a\mu_z(z)}{3\pi(\mu_z)_0}\Phi(1)(1+\alpha z)^{-\beta-1/2}\int_0^\infty \sin(pa)\exp(-Pz)J_1(pr)dp \\ &= \frac{4\tau_0 a}{3\pi}\Phi(1)(1+\alpha z)^{k/2}\int_0^\infty \sin(pa)\exp(-Pz)J_1(pr)dp\end{aligned} \tag{34}$$

$$\begin{aligned}\tau_{r\theta}(r,z) &= -\frac{4\tau_0 a\mu_r(z)}{3\pi\gamma(\mu_z)_0}\Phi(1)(1+\alpha z)^{-\beta-1/2}\int_0^\infty \sin(pa)\exp(-Pz)J_0(pr)dp \\ &= -\frac{4\tau_0 a\gamma}{3\pi}\Phi(1)(1+\alpha z)^{k/2}\int_0^\infty \sin(pa)\exp(-Pz)J_0(pr)dp\end{aligned} \tag{35}$$

Note that the integrals in Eqs. (34) and (35) can be rewritten as

$$\int_0^\infty \sin(pa)\exp(-Pz)J_1(pr)dp = \mathrm{Im}\left[\frac{1}{r}\left(1 + \frac{i(a+i\gamma z)}{\sqrt{r^2-(a+i\gamma z)^2}}\right)\right] \tag{36}$$

$$\int_0^\infty \sin(pa)\exp(-Pz)J_0(pr)dp = \mathrm{Im}\left[\frac{1}{\sqrt{r^2-(a+i\gamma z)^2}}\right] \tag{37}$$

and that $r = a + r_1\cos\theta_1$, $z = r_1\sin\theta_1$ and $r_1 << a$ near the crack tip, we get

$$\int_0^\infty \sin(pa)\exp(-Pz)J_1(pr)dp = \frac{1}{\sqrt{2r_1 a}}\mathrm{Re}\left[\frac{1}{\sqrt{\cos\theta_1 - i\gamma\sin\theta_1}}\right] + O(r_1^0) \tag{38}$$

$$\int_0^\infty \sin(pa)\exp(-Pz)J_0(pr)dp = \frac{1}{\sqrt{2r_1 a}}\mathrm{Im}\left[\frac{1}{\sqrt{\cos\theta_1 - i\gamma\sin\theta_1}}\right] + O(r_1^0) \tag{39}$$

where the polar coordinates r_1 and θ_1 are defined in Fig. 1. Substituting Eqs. (38) and (39) into Eqs. (34) and (35), the local stress field is obtained as

$$\tau_{\theta z}(r_1, \theta_1) = \frac{K_{\mathrm{III}}}{\sqrt{2\pi r_1}}\mathrm{Re}\left[\frac{1}{\sqrt{\cos\theta_1 - i\gamma\sin\theta_1}}\right] + O(r_1^0) \tag{40}$$

$$\tau_{r\theta}(r_1, \theta_1) = \frac{K_{\mathrm{III}}}{\sqrt{2\pi r_1}}\mathrm{Re}\left[\frac{i\gamma}{\sqrt{\cos\theta_1 - i\gamma\sin\theta_1}}\right] + O(r_1^0) \tag{41}$$

where the stress intensity factor (SIF) K_{III} in Eqs. (40) and (41) is

$$K_{\mathrm{III}} = \frac{4}{3\pi}\tau_0\sqrt{\pi a}\Phi(1) \tag{42}$$

6. Results and discussion

The functional dependence of the stresses on r_1 and θ_1 as shown in Eqs. (40) and (41) reveals that the local stresses in orthotropic functionally graded strip also possess the inverse square root singularity in terms of r_1 and that the angular distribution in θ_1 is the same as the case in infinite homogeneous solids. Eq. (42) displays that the expressional form of the SIF is also the same as that in infinite homogeneous materials.

For comparison with the known results, as a special example, firstly examined is the normalized SIF of a penny-shaped crack for different h/a in the case of $\gamma\to 1$ and $k\to 0$ (and/or $\alpha a\to 0$). The numerical results are plotted in Fig. 2, where the SIF is normalized by $3\pi K_{\mathrm{III}}/4\tau_0\sqrt{\pi a}$. In fact, the corresponding results as $\gamma\to 1$ reflect the crack tip behaviors for isotropic materials, and the corresponding results as $k\to 0$ (and/or $\alpha a\to 0$) reveal the fracture properties for homogeneous

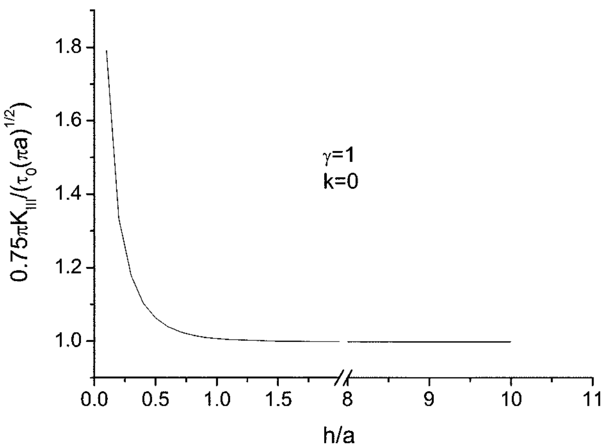
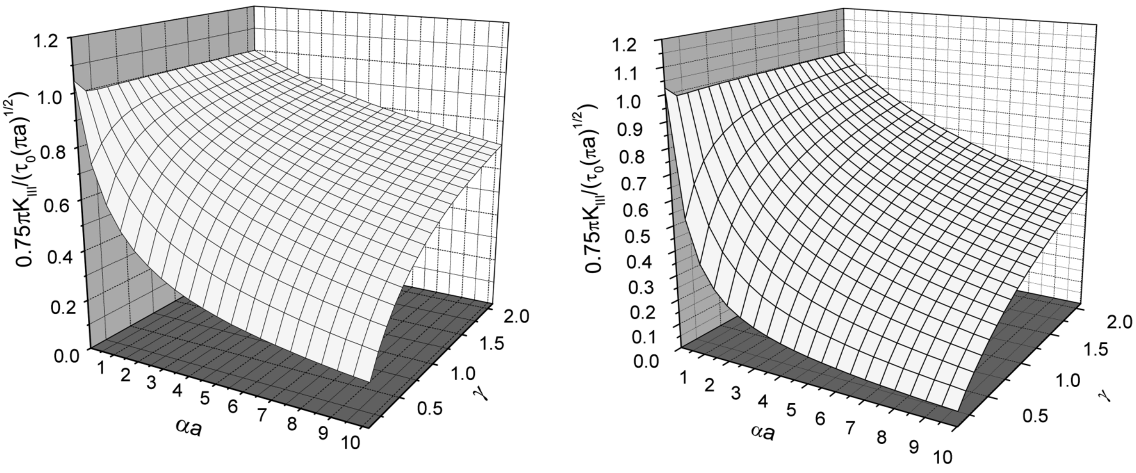
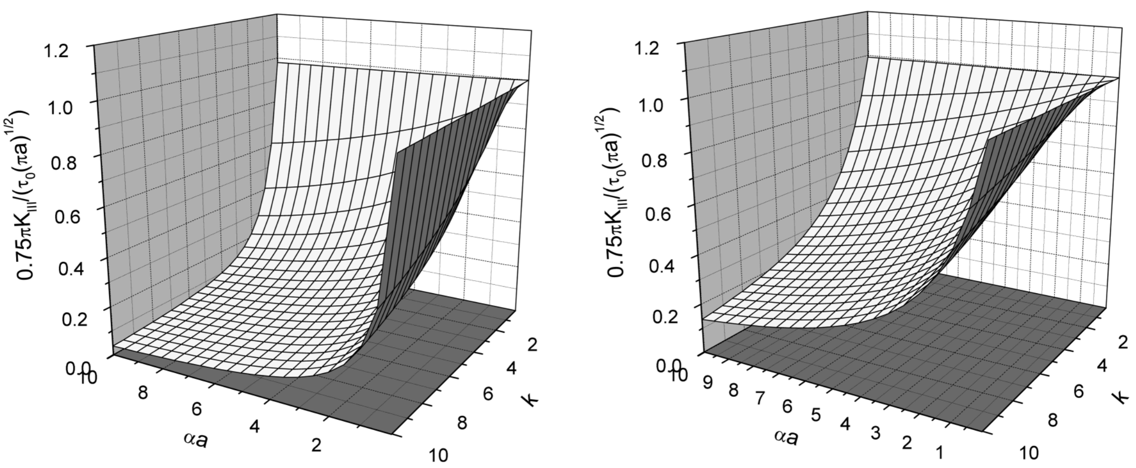


Fig. 2 Normalized stress intensity factor $0.75\pi K_{\mathrm{III}}/\tau_0(\pi a)^{1/2}$ with h/a for isotropic homogeneous strip



(a) k=1, h/a=5 (b) k=2, h/a=5

Fig. 3 The relation of normalized stress intensity factor $0.75\pi K_{\mathrm{III}}/\tau_0(\pi a)^{1/2}$ with γ and αa

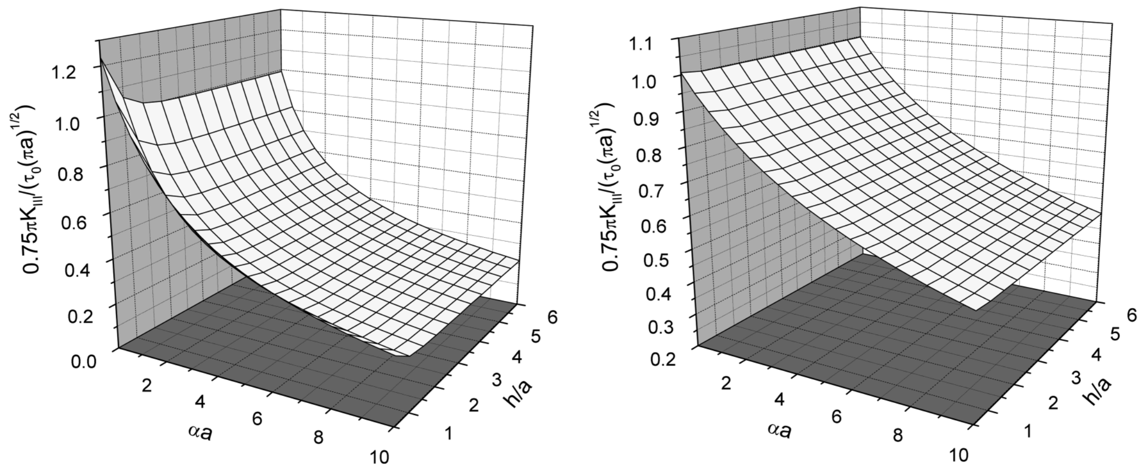


(a) γ=0.5, h/a=5 (b) γ=2.0, h/a=5

Fig. 4 The relation of normalized stress intensity factor $0.75\pi K_{\mathrm{III}}/\tau_0(\pi a)^{1/2}$ with k and αa

materials. In addition, the corresponding results as $h/a\rightarrow\infty$ reflect the crack tip behaviors for infinite materials. It is seen that the SIF decreases with increasing h/a and tends to one, i.e., approaches the SIF of infinite medium, rapidly.

The effects of γ and αa, k and αa, h/a and αa on the normalized SIF are respectively plotted in Figs. 3, 4 and 5. A table containing some typical data (Table 1) is listed as well. As shown in these figures and Table 1, for definite values of γ, k and h/a, the values of the SIF are less for larger values of αa, and k and h/a have the similar influences on the SIF as αa. However, for definite



(a) γ=0.5, k=2 (b) γ=2.0, k=2

Fig. 5 The relation of normalized stress intensity factor $0.75\pi K_{III}/\tau_0(\pi a)^{1/2}$ with h/a and αa

Table 1 Normalized SIFs of four kinds of typical cases

Case 1		$k = 2.0$, $h/a = 5.0$, $\alpha a = 2.0$										
		$\gamma = 0.1$	0.3	0.5	0.7	0.9	1.0	1.1	1.3	1.5	1.7	1.9
	$3\pi K_{III}/4\tau_0(\pi a)^{1/2}$	0.2274	0.4340	**0.5510**	0.6267	0.6801	0.7013	0.7199	0.7508	0.7755	0.7957	0.8125
Case 2		$k = 2.0$, $h/a = 5.0$, $\gamma = 0.5$										
		$\alpha a \to 0.0$	0.5	1.0	1.5	2.0	3.0	4.0	5.0	6.5	8.0	10.0
	$3\pi K_{III}/4\tau_0(\pi a)^{1/2}$	0.9912	0.8160	0.6996	0.6154	**0.5510**	0.4580	0.3935	0.3456	0.2930	0.2547	0.2171
Case 3		$h/a = 5.0$, $\alpha a = 2.0$, $\gamma = 0.5$										
		$k = 0.0$	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
	$3\pi K_{III}/4\tau_0(\pi a)^{1/2}$	0.9911	0.7263	**0.5510**	0.4304	0.3514	0.2960	0.2555	0.2246	0.2003	0.1807	0.1645
Case 4		$k = 2.0$, $\alpha a = 2.0$, $\gamma = 0.5$										
		$h/a = 0.5$	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	6.0
	$3\pi K_{III}/4\tau_0(\pi a)^{1/2}$	0.7525	0.6020	0.5675	0.5572	0.5536	0.5522	0.5515	0.5512	0.5511	**0.5510**	0.5509

values of k, αa and h/a, the values of the SIF are less for less values of γ. From Fig. 5, we can also know that the SIF nearly has no variations for larger values of h/a. In fact, the phenomenon has been pointed out before for isotropic homogeneous medium.

7. Conclusions

The local stress field at the crack tip is investigated for an orthotropic functionally graded strip with a penny-shaped crack under torsion. The numerical results of stress intensity factor show that the nonhomogeneity, orthotropy and gradient index of the strip have more significant effects on the fracture behavior than the strip's height. Both increasing the shear moduli's gradient and increasing the shear modulus in direction perpendicular to crack surface can restrain the stress intensity factor.

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