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Damage detection of mono-coupled multistory buildings: Numerical and experimental investigations

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Abstract. This paper presents numerical and experimental investigations on damage detection of mono-coupled multistory buildings using natural frequency as only diagnostic parameter. Frequency equation of a mono-coupled multistory building is first derived using the transfer matrix method. Closed-form sensitivity equation is established to relate the relative change in the stiffness of each story to the relative changes in the natural frequencies of the building. Damage detection is then performed using the sensitivity equation with its special features and minimizing the norm of an objective function with an inequality constraint. Numerical and experimental investigations are finally conducted on a mono-coupled 3-story building model as an application of the proposed algorithm, in which the influence of modeling error on the degree of accuracy of damage detection is discussed. A mono-coupled 10-story building is further used to examine the capability of the proposed algorithm against measurement noise and incomplete measured natural frequencies. The results obtained demonstrate that changes in story stiffness can be satisfactorily detected, located, and quantified if all sensitive natural frequencies to damaged stories are available. The proposed damage detection algorithm is not sensitive to measurement noise and modeling error.

Key words: damage detection; mono-coupled multistory building; natural frequency.

1. Introduction

Vibration-based structural damage detection methods have attracted considerable attention in

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recent years for the assessment of health and safety of large civil structures. These methods are built on the idea that the measured modal parameters or the properties derived from these modal parameters are functions of the physical properties of the structure and, therefore, changes in the physical properties will cause detectable changes in the modal parameters (Doebling *et al.* 1998). Measurements of changes in the modal parameters can then be used not only to indicate overall trends in the integrity of a structure but also to help find the location and severity of the damage. Currently used vibration-based structural damage detection methods include those based on changes in modal properties (such as natural frequency, mode shape, mode shape curvature, strain mode shape, and response frequency function), the use of dynamically measured flexibility matrix to estimate changes in the static behavior of the structure, and the modification of structural model matrices to reproduce as closely as possible the measured dynamic response.

Since natural frequencies of a structure can be measured conveniently and accurately and the measurements of natural frequencies can be carried out with a few sensors only, the damage detection approach based on changes in natural frequencies can provide an inexpensive structural assessment technique. Cawley and Adams (1979) were among the first investigators to use an incomplete set of measured natural frequencies to identify damage location and provide a rough estimate of structural damage size. Hassiotis and Jeong (1995) used the first-order perturbation of the eigenvalue problem to yield a set of simultaneous equations that relate the local decrease in stiffness to the decreases in the eigenvalues, and then they solved the equations with the aid of an optimality criterion. Koh *et al.* (1995) formulated an improved condensation method for local damage detection of multistory frame buildings using measured natural frequencies only. Identification was executed recursively on a remedial model, yielding integrity indices for all stories. Morassi and Rovere (1997) presented an inverse technique to localize notch effects in steel frames using changes in natural frequency. Their study focused particularly on the accuracy of the assumed reference (undamaged) structural configuration and the practicality of making vibration measurements in the field. Zhu and Wu (2002) proposed a characteristic receptance method to identify the location and magnitude of damage in mono-coupled periodic structures using measured natural frequencies only. Significant efforts have been also devoted to the identification of crack-related damage in beam structures based on eigenfrequencies. Gudmundson (1983) determined theoretically and experimentally the eigenfrequencies of an edge-cracked cantilevered beam with different crack lengths and crack positions. Liang *et al.* (1992) developed an analytical relationship between the changes in eigenfrequencies and the location and magnitude of crack-related damage in both simply supported and cantilever beams with either uniform or segmented cross-sectional areas. Morassi (2001) identified a single crack using the knowledge of damage-induced shifts in a pair of natural frequencies of a vibrating rod and through an explicit expression of frequency sensitivity to damage. Excellent reviews on the use of natural frequency changes for damage diagnosis can be found in Salawu (1997) and Doebling *et al.* (1998). Recently, Capecchi and Vestroni (1999) also addressed the problem of understanding when it is sufficient to measure and use only natural frequencies to locate and quantify structural damage. They drew conclusions that a reliable estimate of the damage can be obtained using only few additional frequency data with respect to the number of damaged zones.

Though the aforementioned methods have demonstrated various degrees of success in damage detection, frequency-based damage detection methods have significant practical limitations for applications to some civil structures, such as offshore platforms and long span bridges because of low sensitivity of frequency shifts to structural damage and insufficient number of measured natural frequencies with significant changes. The presence of modeling and measurement errors may also considerably complicate the damage detection problem to reach the exact solution (Morteza 1998, Law and Shi 1998). By bearing all the points in mind, this paper presents numerical and experimental investigations on damage detection of mono-coupled multistory buildings, rather than other types of structures, using natural frequency as only diagnostic parameter. Frequency equation of a mono-coupled multistory building is first derived using the transfer matrix method. Closed-form sensitivity equation is then established to relate the relative change in the stiffness of each story to the relative changes in natural frequencies of the building. The sensitivity equation derived in such a way for the concerned building type is not sensitive to modeling error. Damage detection is finally carried out using the sensitive equation with its special features and minimizing an error objective function with an inequality constraint. To demonstrate the applicability of the proposed algorithm, numerical and experimental investigations are performed on a mono-coupled 3-story building model. A mono-coupled 10-story building is further used to examine the sensitivity of the proposed algorithm to measurement noise and incomplete measured frequencies.

2. Damage detection algorithm

Many multistory buildings can be modeled by a mono-coupled system consisting of N subsystems (S_1, S_2, S_3, ..., S_N) connected by end-to-end and terminating at the base of the building as a fixed end and at the top of the building as a free end (see Fig. 1). A subsystem can be composed of one or

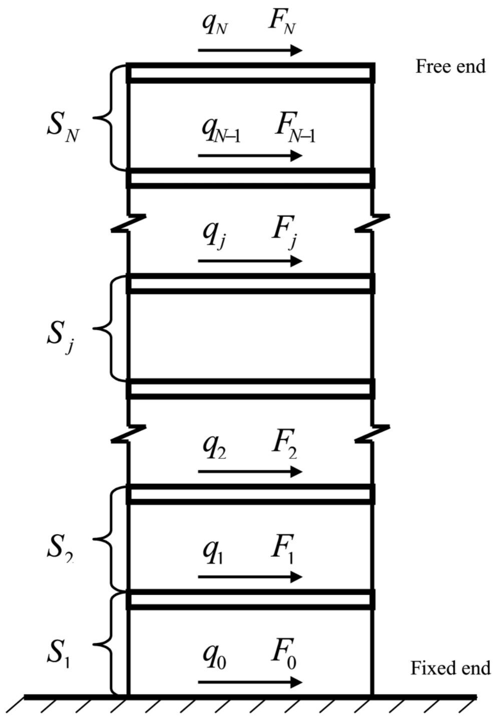


Fig. 1 Structural model of a mono-coupled building with N-stories

several stories, but in this paper a subsystem consists of one story only. The floor of each story is assumed to be rigid and the mass of each story is concentrated at the floor only. The axial deformation of the columns is neglected. As a result, the building becomes a mono-coupled multistory building with horizontal motion only.

Suppose that the building is elastic and linear and subjected to a harmonic vibration. The dynamic displacement and the shear force at the base of the building (the fixed end) are denoted by $q_0 e^{i\omega t}$ and $F_0 e^{i\omega t}$, respectively, in which ω is the frequency; t is the time; i is the imaginary unit; and q_0 and F_0 are the amplitudes of the displacement and the shear force, respectively. The dynamic displacement and the horizontal force at the top of the building (the free end) are designated by $q_N e^{i\omega t}$ and $F_N e^{i\omega t}$, respectively. The dynamic displacement and the shear force at the jth floor are denoted by $q_j e^{i\omega t}$ and $F_j e^{i\omega t}$, respectively. Then, by employing the transfer matrix method (Yang and Lin 1981) and by assuming that there are no external dynamic forces acting on the floors, one may obtain the following relationship for the dynamic displacements and shear forces at the low and up boundaries of the jth story.

$$\begin{Bmatrix} q_j \\ F_j \end{Bmatrix} = \begin{bmatrix} t_{llj} & t_{luj} \\ t_{ulj} & t_{uuj} \end{bmatrix} \begin{Bmatrix} q_{j-1} \\ F_{j-1} \end{Bmatrix} \tag{1}$$

where $\begin{bmatrix} t_{llj} & t_{luj} \\ t_{ulj} & t_{uuj} \end{bmatrix}$ is called the transfer matrix of the jth story ($j = 1, 2, 3, \ldots, N$). Because the structural damping is not considered in this paper for damage detection, the elements in the transfer matrix of the jth story can be expressed as

$$t_{llj} = 1; \quad t_{luj} = \frac{1}{k_j}; \quad t_{ulj} = -m_j\omega^2; \quad t_{uuj} = -\frac{m_j\omega^2}{k_j} + 1 \tag{2}$$

in which k_j is the horizontal stiffness of the jth story; and m_j is the mass of the jth floor. Eq. (1) can be rewritten in the simple matrix form

$$Z_j = T_j Z_{j-1} \tag{3}$$

$$T_j = \begin{bmatrix} t_{llj} & t_{luj} \\ t_{ulj} & t_{uuj} \end{bmatrix}; \; Z_j = \begin{Bmatrix} q_j \\ F_j \end{Bmatrix} \;\; (j = 1, 2, \ldots, N) \tag{4}$$

Eq. (3) can be applied repeatedly to obtain the following relation Z_0 between Z_N.

$$Z_N = \begin{Bmatrix} q_N \\ F_N \end{Bmatrix} = T_N \ldots T_j \ldots T_1 \begin{Bmatrix} q_0 \\ F_0 \end{Bmatrix} = \begin{bmatrix} t_{LL} & t_{LU} \\ t_{UL} & t_{UU} \end{bmatrix} \begin{Bmatrix} q_0 \\ F_0 \end{Bmatrix} = T \cdot Z_0 \tag{5}$$

where T is the total transfer matrix for the whole building. The elements in the transfer matrix T can be obtained in terms of the transfer matrix of each story. Since the elements of the transfer matrix of each story are functions of floor mass, inter-story stiffness, and frequency, the total transfer matrix is also a function of structural physical parameters and frequency. The boundary conditions at the base and at the top floor of the building dictate that

$$q_0 = 0; \quad F_N = 0 \tag{6}$$

Applying Eq. (6) to Eq. (5) leads to

$$t_{UU}F_0 = 0 \tag{7}$$

Since F_0 is not zero, Eq. (7) yields the frequency equation for the building

$$t_{UU}(\omega(k_j, m_j), k_j, m_j) = 0 \qquad (j = 1, 2, \ldots, N) \tag{8}$$

N natural frequencies, $\omega_1, \omega_2, \ldots, \omega_N$, of the building can be found from the above frequency equation. In this study, the structural damage in the jth story is expressed directly by the change in the jth story stiffness k_j. The mass of the building is, however, assumed to remain unchanged. Thus, by knowing k_j and m_j and the change in k_j ($j = 1, 2, 3, \ldots, N$), one can calculate the natural frequencies of the building without and with damage and then the changes in natural frequency. Inversely, by knowing the changes in natural frequency, the inverse problem of determining changes in story stiffness can be formulated. Let us now consider the sensitivity of the nth natural frequency ω_n of the building to the jth story stiffness k_j. To this end, a partial differentiation of equation $t_{UU}(\omega_n(k_1, \ldots, k_N), k_1, \ldots, k_N) = 0$ with respect to k_j ($j = 1, 2, 3, \ldots, N$), when $\partial t_{UU}/\partial\omega \neq 0$, gives the sensitivity coefficient S_{nj} as

$$S_{nj} \equiv \frac{\partial\omega_n}{\partial k_j} = \frac{\dfrac{\partial t_{uu}}{\partial k_j}}{\dfrac{\partial t_{uu}}{\partial\omega_n}} \tag{9}$$

It is more reasonable to use the relative change in the jth story stiffness, $\Delta\bar{k}_j = \Delta k_j/k_j$, other than the absolute change in stiffness, Δk_j, to describe the severity of structural damage. Also, it is more proper to use the relative change in the nth natural frequency, $\Delta\bar{\omega}_n = \Delta\omega_n/\omega_n$, other than the absolute change in natural frequency, $\Delta\omega_n$, so as to reduce the effect of measurement noise on damage detection. Therefore, Eq. (9) is normalized to obtain the normalized sensitivity coefficient $\bar{S}_{nj}$.

$$\bar{S}_{nj} = \frac{\partial\bar{\omega}_n}{\partial\bar{k}_j} = \frac{\partial\omega_n}{\partial k_j}\frac{k_j}{\omega_n} = S_{nj}\cdot\frac{k_j}{\omega_n} \tag{10}$$

For any combination of size and location of structural damage at one or more stories, it is assumed that the decreases in the relative natural frequencies can be expressed as a linear combination of the decreases in the relative stiffness in terms of the normalized sensitivity coefficients

$$\Delta\bar{\omega}_n = \sum_{j=1}^{N}\frac{\partial\omega_n}{\partial k_j}\cdot\frac{k_j}{\omega_n}\cdot\Delta\bar{k}_j \equiv \sum_{j=1}^{N}\bar{S}_{nj}\cdot\Delta\bar{k}_j \tag{11}$$

Eq. (11) is called the sensitivity equation relating the relative changes in the stiffness of the N

stories to the relative changes in the measurable natural frequencies (n = 1, 2, …p). Eq. (11) can simply be expressed as

$$\{\Delta\overline{\omega}\} = [\overline{S}]\{\Delta\overline{k}\} \tag{12}$$

If the number of available natural frequency change, p, in $\{\Delta\overline{\omega}\}$ is equal to the number of building story, N, the solution of Eq. (12) may yield the relative change in the story stiffness $\{\Delta\overline{k}\}$ under certain circumstances. However, in practice the number of available natural frequency change is often less than the number of building story, and Eq. (12) accordingly becomes underdetermined. In such a case, the estimation of $[\Delta\overline{k}]$ may be obtained after the introduction of an optimality criterion (Hjelmstad 1996). In this study, the minimization of the norm of the following objective function is used as an optimality criterion.

$$g = \left\|[\overline{S}]\{\Delta\overline{k}\} - \{\Delta\overline{\omega}\}\right\| \tag{13}$$

Since a positive change in the stiffness can never be produced by structural damage, an inequality constraint is introduced to the optimality criterion.

$$\{\Delta\overline{k}\} \le 0 \tag{14}$$

Eqs. (13) and (14) are nonlinear when the number of available measured natural frequencies is less than the number of building story. Thus, multiple minima are possible. To identify the best one and discard the extraneous ones, a gradient-based search algorithm with the random starting point scheme suggested by Hjelmstad (1996) is adopted to minimize the objective function with the inequality constraint. Since the normalized sensitivity matrix depends on the estimated values of mass m_j and stiffness k_j (j = 1, 2, 3, …, N) in the undamaged building model (the reference model) which may deviate from the actual values, the other factor that may affect the degree of accuracy of the solution of damage detection is modeling uncertainties involved in the normalized sensitivity matrix $[\overline{S}]$. In this connection, the sensitivity of the matrix $[\overline{S}]$ to the stiffness k_j should be examined to assess the quality of the normalized sensitivity matrix defined in this study. The sensitivity of the (n, j)th element, $\overline{S}_{nj}$, in the matrix $[\overline{S}]$ to the relative stiffness $\overline{k}_m$ can be obtained by partially differentiating Eq. (10) with respect to $\overline{k}_m$

$$\overline{S}_{nj}^{m} = \frac{\partial\overline{S}_{nj}}{\partial\overline{k}_m} = \frac{\partial S_{nj}}{\partial k_m}\frac{k_j k_m}{\omega_n} \tag{15}$$

3. Application to a mono-coupled 3-story building model

3.1 3-story building model

To assess the accuracy of the proposed damage detection algorithm, both numerical and experimental studies are performed on a mono-coupled 3-story building model (see Fig. 2). The building model was constructed using 3 steel plates of 850 mm × 500 mm × 25 mm and 4 equally sized rectangular columns of 9.5 mm × 75 mm. The plates and columns were properly welded to

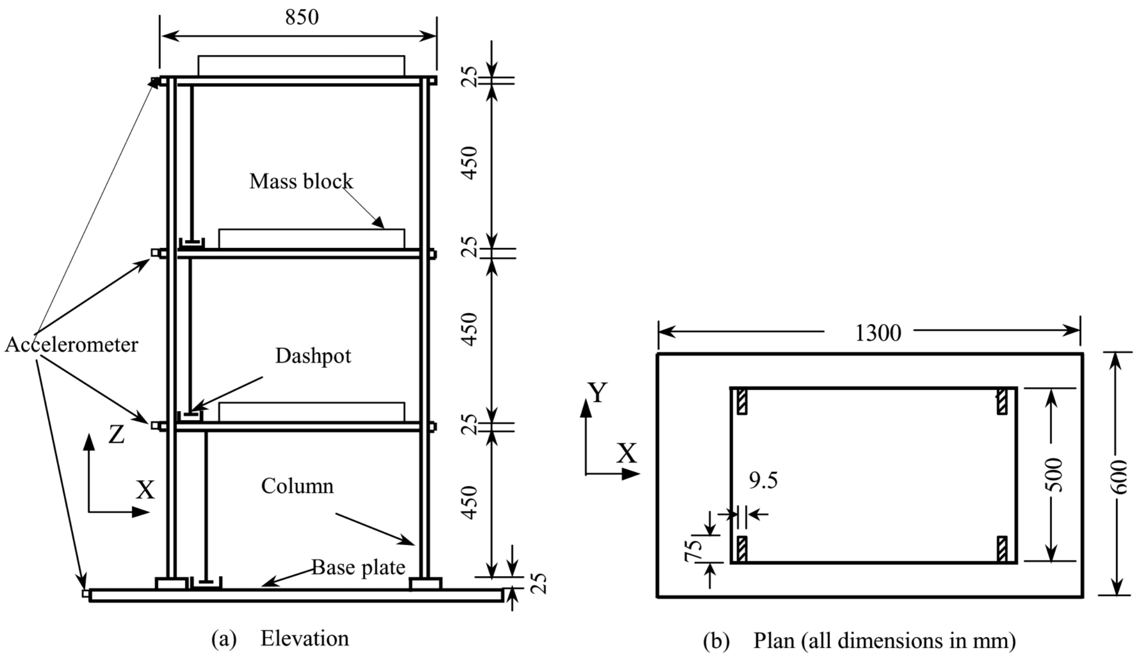


Fig. 2 Configuration of a mono-coupled 3-storey building model

form rigid connections. The building was then welded on a steel base plate of 20 mm thickness. The steel base plate was in turn bolted firmly on a shaking table using a total of 8 bolts of high tensile strength. The overall dimensions of the building were 1450 mm × 850 mm × 500 mm. All the columns were made of high strength steel of 435 MPa yield stress and 200 GPa modulus of elasticity. The 9.5 mm × 75 mm cross-section of the column was arranged in such a way that the first natural frequency of the building was much lower in the *x*-direction than in the *y*-direction. This arrangement restricted the building motion in the *x*-direction and thus the building was effectively reduced to planar building in the *x*-*z* plane. The thickness of each steel floor was 25 mm so that the floor can be regarded as a rigid plate in horizontal direction, leading to a shearing type of deformation. The geometric scale of the building model was assumed to be 1/5. To have a proper simulation, additional mass block of 135 kg was placed on each floor of the building model.

3.2 Instrumentation and measurement

The building model was subjected to a white noise random ground excitation generated by a 3 m × 3 m MTS shaking table at The Hong Kong Polytechnic University. The frequency range of the white noise random ground excitation was from 1.0 to 30 Hz. The peak acceleration and duration of the excitation was 0.05 g and 180 seconds, respectively, where g is the acceleration due to gravity in m/s^2, to ensure that vibration of the building model is linear and elastic. Each building floor was equipped with one B&K 4370 accelerometer in the *x*-direction (Fig. 2). The signals from the accelerometers were transferred to a personnel computer through B&K 2635 signal conditioners and sampled at 300 Hz by a data acquisition system. The digital data were subsequently analyzed by commercial computer software ARTeMIS, developed by Structural Vibration Solutions in

Denmark, to identify the natural frequencies and mode shapes using the method of Frequency Domain Decomposition (FDD). The three natural frequencies were identified as 3.369, 9.704, and 14.282 Hz for the undamaged building model.

3.3 Mathematical model

By measuring the weight of each plate, the mass of each floor (m_1, m_2, and m_3) was estimated at 231 kg. The horizontal stiffness of each story was computed first based on the geometric dimensions and material properties of the columns and then updated using the measured natural frequencies and mode shapes. The updated story stiffness was $k_1 = 4.8398 \times 10^5$ N/m, $k_2 = 5.7405 \times 10^5$ N/m, and $k_3 = 5.9520 \times 10^5$ N/m. The three natural frequencies computed from the updated model were 3.369, 9.704 and 14.282 Hz, which are the same as the measured natural frequencies. The lower value of the first story stiffness in the updated model is because the bottom support of the columns in the first story deviates from the fixed support to some extent. Based on the computed three modal shapes from the updated model and the measured three modal shapes, the Modal Assurance Criteria (MAC) values (Zhu 2000) of the three modal shapes were computed as 0.9989, 0.9969 and 0.9972, respectively, which indicate that the computed modal shapes matched well the measured ones. The updated mathematical model is thus considered as the undamaged building model (the reference building model).

3.4 Sensitivity study

For the mono-coupled 3-story building model, its frequency equation can be derived as

$$t_{UU}(\omega, k_1, k_2, k_3) \equiv m^3\omega^6 - (k_1 + 2k_2 + 2k_3)m^2\omega^4 + (k_1k_2 + 2k_1k_3 + 3k_2k_3)m\omega^2 - k_1k_2k_3 = 0 \qquad (16)$$

By using the above frequency equation, the partial differentiation of the nth natural frequency (n = 1, 2, and 3) with respect to k_1, k_2 and k_3, respectively, yields the following normalized sensitivity coefficients.

$$\begin{aligned}
\bar{S}_{n1} &= \frac{\partial\omega_n}{\partial k_1}\cdot\frac{k_1}{\omega_n} = \frac{m^2\omega_n^4 - 2(k_2 + k_3)m\omega_n^2 + 3k_2k_3}{6m^2\omega_n^4 - 4(k_1 + 2k_2 + 2k_3)m\omega_n^2 + 2(k_1k_2 + 2k_1k_3 + 3k_2k_3)} \\
\bar{S}_{n2} &= \frac{\partial\omega_n}{\partial k_2}\cdot\frac{k_2}{\omega_n} = \frac{m^2\omega_n^4 - (k_1 + 2k_3)m\omega_n^2 + 2k_1k_3}{6m^2\omega_n^4 - 4(k_1 + 2k_2 + 2k_3)m\omega_n^2 + 2(k_1k_2 + 2k_1k_3 + 3k_2k_3)} \\
\bar{S}_{n3} &= \frac{\partial\omega_n}{\partial k_3}\cdot\frac{k_3}{\omega_n} = \frac{m^2\omega_n^4 - (k_1 + 2k_2)m\omega_n^2 + k_1k_2}{6m^2\omega_n^4 - 4(k_1 + 2k_2 + 2k_3)m\omega_n^2 + 2(k_1k_2 + 2k_1k_3 + 3k_2k_3)}
\end{aligned} \qquad (17)$$

The normalized sensitivity coefficients in Eq. (17) have the following relations:

$$\bar{S}_{n1} + \bar{S}_{n2} + \bar{S}_{n3} = \frac{\partial\omega_n}{\partial k_1}\cdot\frac{k_1}{\omega_n} + \frac{\partial\omega_n}{\partial k_2}\cdot\frac{k_2}{\omega_n} + \frac{\partial\omega_n}{\partial k_3}\cdot\frac{k_3}{\omega_n} = \frac{1}{2} \quad (n = 1, 2, 3) \qquad (18)$$

$$\bar{S}_{1j} + \bar{S}_{2j} + \bar{S}_{3j} = \frac{\partial\omega_1}{\partial k_j}\cdot\frac{k_j}{\omega_1} + \frac{\partial\omega_2}{\partial k_j}\cdot\frac{k_j}{\omega_2} + \frac{\partial\omega_3}{\partial k_j}\cdot\frac{k_j}{\omega_3} = \frac{1}{2} \quad (j = 1, 2, 3) \qquad (19)$$

Eq. (18) shows that the sum of the normalized sensitivity coefficients of a specified natural frequency ω_n with respect to the stiffness of each story is always equal to 0.5 and independent of the structural parameters no matter which natural frequency is designated. This indicates that for the specified natural frequency, if the sensitivity of the relative change in the specified frequency to the relative change in the stiffness of one story is higher, it will then be lower to the relative changes in the stiffness of the other two stories. On the other hand, Eq. (19) tells us that the sum of the normalized sensitivity coefficients of three natural frequencies with respect to the stiffness of a specified story k_j is always equal to 0.5 and independent of the structural parameters no matter which story is designated. This indicates that for a specified story, if the sensitivity of the relative change in one frequency to the relative change in the stiffness of the specified story is higher, it will then be lower for the other two frequencies.

The normalized sensitivity coefficients for the 3-story building model are calculated using Eq. (17) and plotted in Fig. 3. It is seen that the relative change in the first natural frequency is most sensitive to the relative change in the stiffness of the first story but least sensitive to the relative change in the stiffness of the third story. The relative change in the second natural frequency is less sensitive to the relative change in the stiffness of the second floor while the relative change in the third natural frequency is less sensitive to the relative change in the stiffness of the first story. All the computed normalized sensitivity coefficients comply with the rules stipulated by Eqs. (18) and (19).

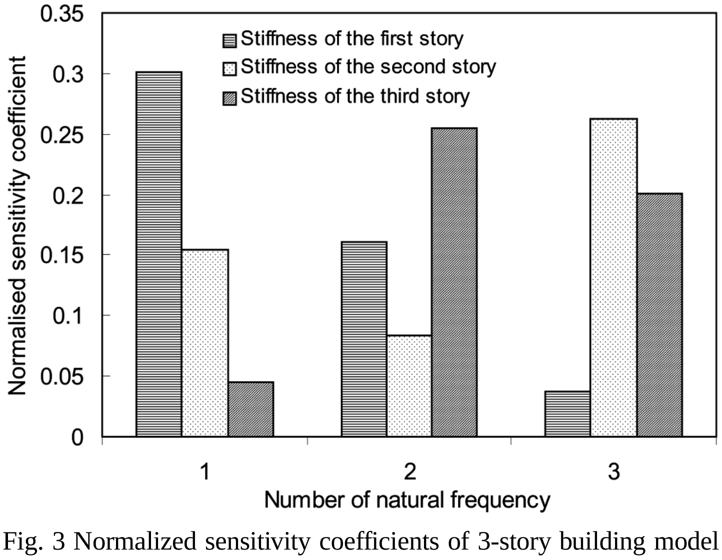


Fig. 3 Normalized sensitivity coefficients of 3-story building model

Table 1 The sensitivity of matrix $[\overline{S}]$ to relative stiffness of 3-storey building model

	$\overline{S}^m_{n1}$			$\overline{S}^m_{n2}$			$\overline{S}^m_{n3}$		
	m = 1	m = 2	m = 3	m = 1	m = 2	m = 3	m = 1	m = 2	m = 3
n = 1	–0.170	0.101	0.034	0.110	–0.124	0.01	0.060	0.023	–0.045
n = 2	0.005	–0.170	–0.099	–0.103	–0.074	0.248	0.098	0.244	–0.149
n = 3	0.020	–0.251	–0.276	0.048	0.441	–0.045	–0.068	–0.190	0.322

By using Eqs. (15) and (17), the sensitivity coefficient, $\overline{S}_{nj}^{m}$, of the (n, j)th element in the matrix $[\overline{S}]$ to the relative stiffness $\overline{k}_m$ can be obtained, and the results are listed in Table 1. The sensitivity coefficients listed in Table 1 indicate that $\sum_{j=1}^{3}\sum_{m=1}^{3}\overline{S}_{nj}^{m}=0$ (n = 1, 2, 3) and $\sum_{n=1}^{3}\sum_{j=1}^{3}\overline{S}_{nj}^{m}=0$ (m = 1, 2, 3), that is, the sum of all the sensitivity coefficients for either a given natural frequency or a given story is equal to zero. The maximum value in all the 27 sensitivity coefficients is 0.441, which corresponds to the sensitivity of the (3,2)th element in the matrix $[\overline{S}]$ to the relative stiffness $\overline{k}_2$. This indicates that if there is a stiffness modeling error of 1% in the second story, the value of the (3,2)th element in the matrix $[\overline{S}]$ will have a change of 0.00441, which is only about 1.5% of the largest value in the sensitivity matrix $[\overline{S}]$. This implies that the normalized sensitivity matrix $[\overline{S}]$ is not very sensitive to the modeling error in stiffness.

3.5 Damage scenarios

To apply the proposed algorithm for damage detection of the 3-story building model, four damage scenarios are considered in both numerical and experimental studies. In Scenario 1, the width b of each column in the first story of the building is reduced from the original 75 mm to 51.30 mm within a height of 60 mm from the bottom (see Fig. 4), leading to a theoretical value of 11.6% reduction in the horizontal stiffness of the first story. In Scenario 2, the width b of each column in the first story of the building is further reduced to 37.46 mm within the same height of 60 mm, yielding a theoretical value of 21.1% reduction in the horizontal stiffness of the first story. In Scenario 3, in addition to the damage simulated in Scenario 2, the width b of each column in the second story of the building is reduced from the original 75 mm to 51.30 mm within a height of 60 mm from the first floor, leading to 11.6% reduction in the horizontal stiffness of the second floor. In Scenario 4, in addition to the damage simulated in Scenario 2, the width b of each column in the second story of the building is further reduced to 37.46 mm within the height of 60 mm from the first floor, yielding a 21.1% reduction in the horizontal stiffness of the second story.

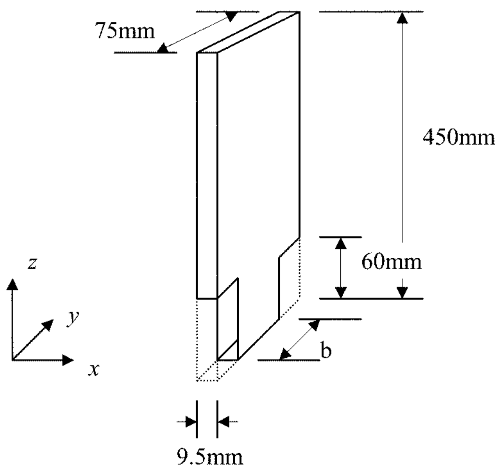


Fig. 4 Configuration of structural damage simulated in 3-story building model

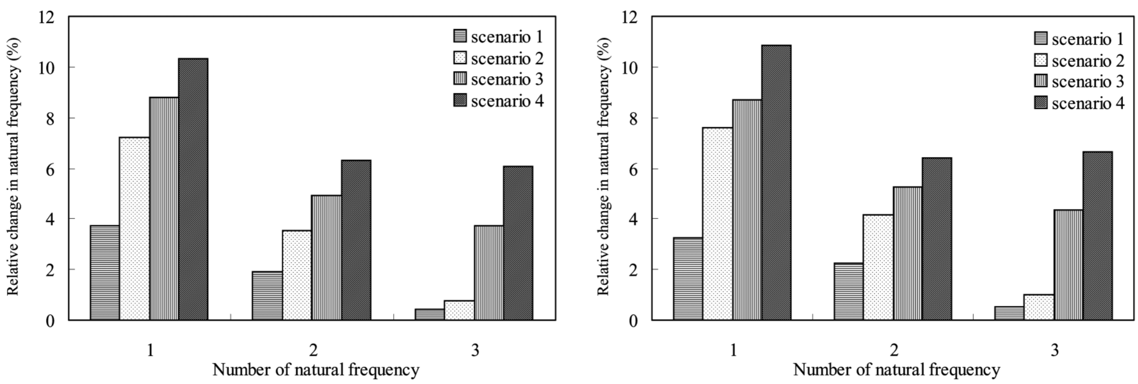
3.6 Numerical and experimental results

The natural frequencies of the 3-storey building model in intact and damage states are numerically computed and experimentally identified and listed in Table 2. The relative changes in natural frequency are plotted in Fig. 5(a) based on the numerical results and Fig. 5(b) using the experimental results. It is observed that the pattern of relative frequency change obtained from the measurement is similar to that from the numerical study. The structural damage in the first story (Scenarios 1 and 2) affects the first natural frequency most, which is consistent with the sensitivity result. Because of severe structural damage in Scenario 2, the relative changes in the three natural frequencies are larger in Scenario 2 than in Scenario 1. In Scenarios 3 and 4, though the relative change in the first natural frequency is the greatest among the three natural frequencies, the relative changes in the second and third natural frequencies are also significant. In particular for Scenario 4, the relative change in the third natural frequency is almost the same as that in the second natural frequency. However, one may also find that there exist some differences in the magnitude of relative frequency changes between the measurement and numerical computation, which should be attributed to both modeling errors and measurement errors. The modeling errors include mainly structural

Table 2 The natural frequencies of intact and damaged 3-storey building obtained from experiment and numerical computation (Hz)

Mode	Intact state		Damage scenario 1		Damage scenario 2		Damage scenario 3		Damage scenario 4	
	Ana.	Exp.	Ana.	Exp.	Ana.	Exp.	Ana.	Exp.	Ana.	Exp.
1	3.369	3.369 (0.00%)	3.243 (–3.74%)	3.259 (–3.26%)	3.126 (–7.21%)	3.113 (–7.60%)	3.073 (–8.79%)	3.076 (–8.70%)	3.021 (–10.33%)	3.003 (–10.86%)
2	9.704	9.704 (0.00%)	9.519 (–1.91%)	9.485 (–2.25%)	9.362 (–3.52%)	9.302 (–4.14%)	9.227 (–4.92%)	9.192 (–5.27%)	9.091 (–6.32%)	9.082 (–6.41%)
3	14.282	14.282 (0.00%)	14.220 (–0.43%)	14.209 (–0.51%)	14.173 (–0.76%)	14.136 (–1.02%)	13.752 (–3.71%)	13.660 (–4.35%)	13.415 (–6.07%)	13.330 (–6.66%)

Note: The value in the parenthesis represents the relative frequency change

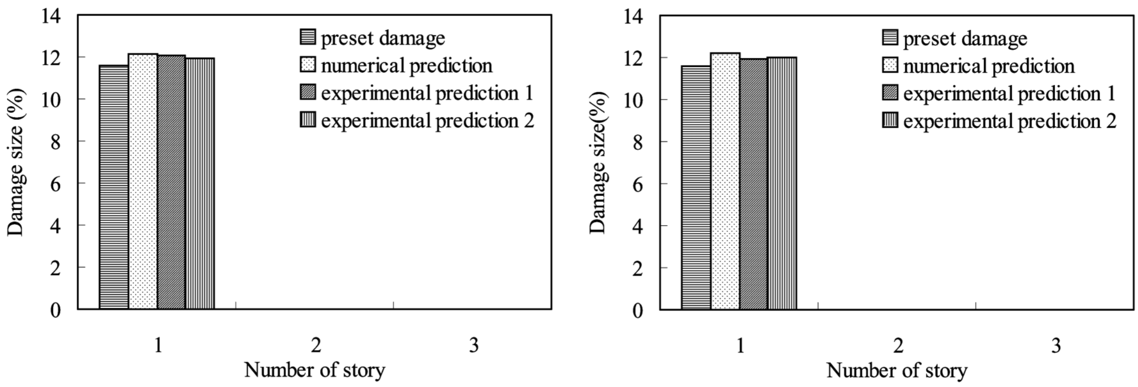


(a) Numerical results (b) Experimental results

Fig. 5 Relative changes in natural frequencies of 3-storey building model modeling error and damage modeling error while the measurement errors contain mainly measurement noise and the error in the frequency identification.

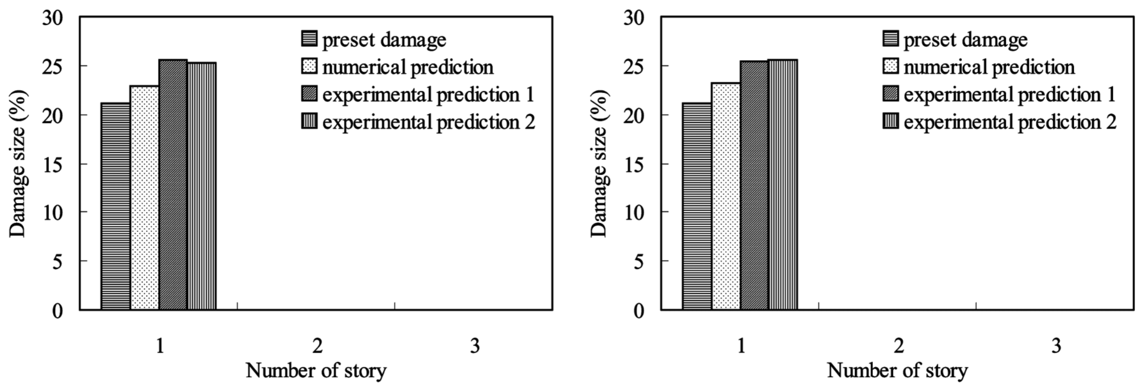
By using the calculated and test relative changes in the three natural frequencies, the detection of relative changes in structural stiffness can be quantitatively realized by solving the inverse problem of Eqs. (12) to (14). Since the number of available natural frequency is often less than that of the building story and lower natural frequencies are easier and more accurate to be measured than higher natural frequencies in practice, the relative changes in only the first two natural frequencies are also used to identify the damages to assess the influence of incomplete measured natural frequencies on damage detection. The identified structural damages (relative changes in stiffness) using the first two frequencies are then compared with those using all the three natural frequencies. Both the numerical results and experimental results (denoted experimental prediction 1) are depicted in Figs. 6, 7, 8 and 9 for Scenarios 1, 2, 3 and 4, respectively. In these figures, the preset damage means the relative changes in the stiffness set out in the scenarios.

Figs. 6 and 7 show the results of structural damage (relative change in stiffness) identified



(a) Three natural frequencies (b) First two natural frequencies

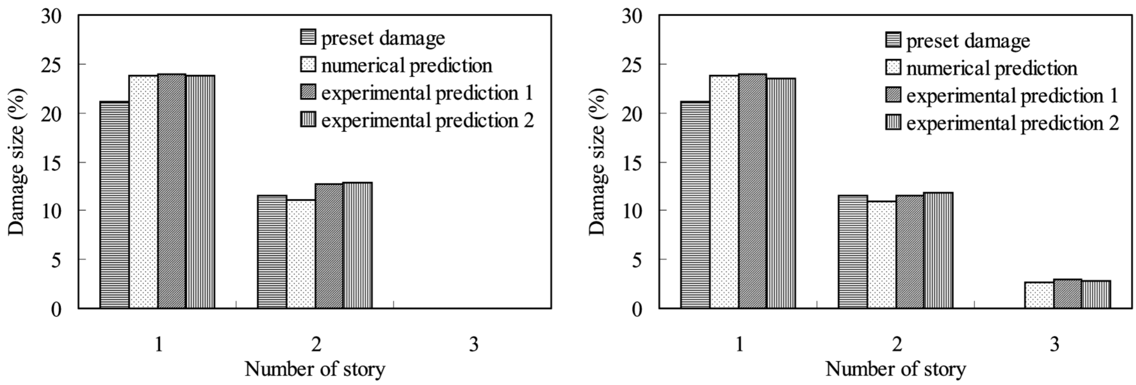
Fig. 6 Structural damage of 3-story building in scenario 1



(a) Three natural frequencies (b) First two natural frequencies

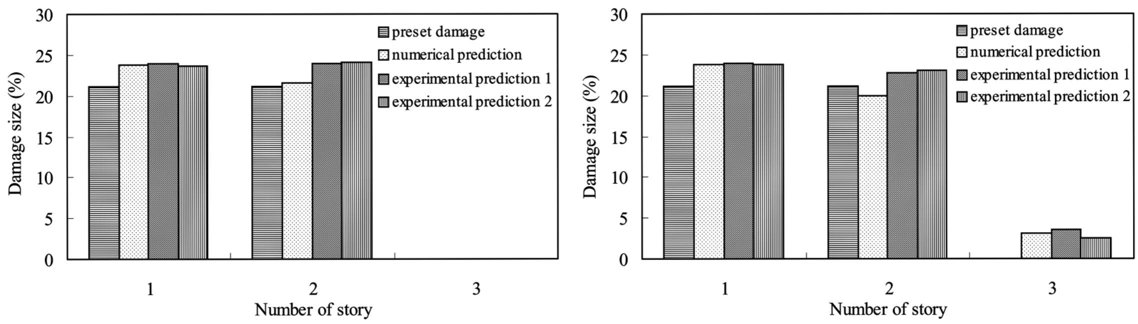
Fig. 7 Structural damage of 3-story building in scenario 2 numerically and experimentally together with the preset structural damage for Scenarios 1 and 2, respectively, where only single damage occurs in the first building story. In Scenario 1, the damage occurring in the first story is numerically identified as 12.13% and 12.21% using the three and first two natural frequencies, respectively. It is experimentally identified as 12.04% and 11.92% using the three and first two natural frequencies, respectively, which are slightly larger than the preset damage of 11.6%. In Scenario 2, the first story of 22.69% and 23.10% damage size is numerically identified using the three and first two natural frequencies, respectively. It is experimentally identified as 25.52% and 25.40% using the three and first two natural frequencies, respectively, as compared with the preset damage of 21.2%. It is clearly seen from the above results that not only the location of damage can be identified accurately in the first story, but also the size of damage in the first story can be predicted with only small overestimation. The similar results are obtained by using the first two natural frequencies only. This indicates that if the damage occurs only in the first story, the changes in the first two natural frequencies can be used to adequately detect the location and size of the damage.

Figs. 8 and 9 display the results of structural damage identified numerically and experimentally together with the preset structural damage for Scenarios 3 and 4, respectively, where multiple



(a) Three natural frequencies (b) First two natural frequencies

Fig. 8 Structural damage of 3-story building in scenario 3



(a) Three natural frequencies (b) First two natural frequencies

Fig. 9 Structural damage of 3-story building in scenario 4 damages occur in the first and second stories. Similarly, if the relative changes in the three natural frequencies are used to detect damages, the locations of damage can be accurately identified in the first and second stories. When the three computed natural frequencies are used, the identified damage sizes of the first and second stories are about 23.8% and 10.3% in Scenario 3, respectively, and about 23.8% and 21.6% in Scenario 4, respectively. When the three measured natural frequencies are used, the identified damage sizes of the first and second stories are about 24.0% and 12.64% in Scenario 3, respectively, and about 23.9% and 23.96% in Scenario 4, respectively. These damage sizes slightly deviate the preset damage sizes of 21.2% in the first story and 11.6% in the second story in Scenario 3, and 21.2% in both the first and the second stories in Scenario 4. When only the first two computed natural frequencies are used, the identified damage sizes of the first and second stories are about 23.8% and 10.9% in Scenario 3, respectively, and about 23.8% and 20.1% in Scenario 4, respectively. When only the first two measured natural frequencies are used, the identified damage sizes of the first and second stories are about 24.0% and 11.5% in Scenario 3, respectively, and about 24.0% and 22.7% in Scenario 4, respectively. The differences between the predicted and preset ones are not significant. However, when only the first two natural frequencies are used, the locations and sizes of damage in the first and second stories can be identified, but the third story is falsely predicted as a lightly damaged story of about 3% damage size. This implies that if the damage occurs in both the first and second stories, the changes in the three natural frequencies other than in the first two natural frequencies only should be considered in order to adequately detect both locations and sizes of damage. This is because the third natural frequency is most sensitive to the stiffness of the second story, as demonstrated in the sensitivity study. In summary, the satisfactory comparison between the experimental and numerical results clearly demonstrates that the proposed algorithm for detecting both damage location and size of the mono-coupled 3-story buildings is workable and practical.

As mentioned in the sensitivity study, the normalized sensitivity matrix $[\bar{S}]$ of the mono-coupled 3-story building is not very sensitive to modeling error in stiffness. To verify this point, the random modeling error of a normal distribution of zero mean and 10% standard deviation is used to change the building stiffness in the first, second and third stories by 2.44%, –0.49% and –1.95%, respectively. The damage detection results using the changed building stiffness and the measured relative changes in the natural frequencies are also plotted in Figs. 6 to 9 and denoted experimental prediction 2. It is seen that the identified damage locations and sizes in experimental prediction 2 are almost the same as those in experimental prediction 1, which implies that small modeling error in stiffness will hardly affect damage detection results.

4. Application to a mono-coupled 10-storey building

4.1 Damage scenarios

To further examine the applicability of the proposed algorithm to multistory buildings with incomplete measured frequencies and its sensitivity to measurement noise, a 10-story building reported by Topole and Stubbs (1995) is used in this investigation. The building is modeled as a mono-coupled shear building of 10 degrees of freedom (Fig. 10). Two damage scenarios are arranged. In Scenario 1, the horizontal stiffness in the third, fourth and fifth stories is reduced by 10%, whereas in Scenario 2, the horizontal stiffness in the first and sixth stories is reduced by 10%

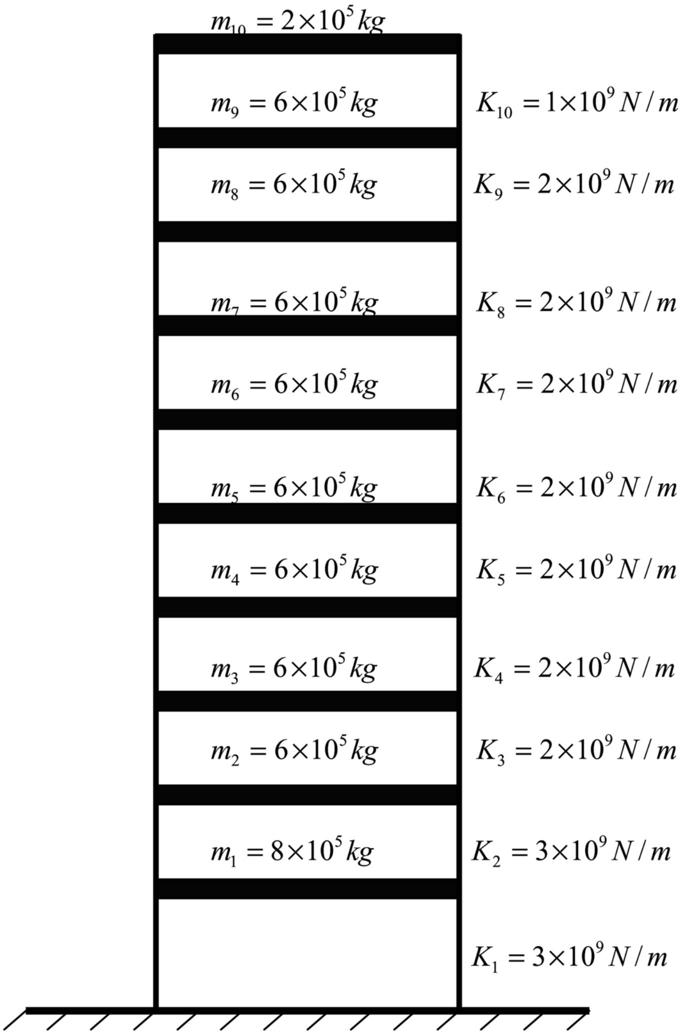


Fig. 10 Structural model of 10-story shear building

and the stiffness in the third and tenth stories is reduced by 25% and 15%, respectively. The natural frequencies of the building with and without damage and their relative changes corresponding to each damage scenario are computed and listed in Table 3. To examine the sensitivity of the proposed algorithm to measurement noise, the theoretical natural frequencies are polluted by a random noise of a normal distribution with zero mean and 5% standard deviation. The polluted natural frequencies and the corresponding relative frequency changes are also given in Table 3. To assess the applicability of the proposed algorithm to the building with incomplete measured frequencies, only the first 5, 4 and 3 natural frequencies before and after damage are used to detect the damage.

Table 3 The natural frequencies of intact and damaged 10-storey building without and with 5% noise pollution (Hz)

Mode	Intact state		Scenario 1		Scenario 2	
	Without noise	5% noise	Without noise	5% noise	Without noise	5% noise
1	1.5698	1.5835	1.5278 (–2.68%)	1.5508 (–2.07%)	1.5012 (–4.37%)	1.4994 (–5.31%)
2	4.5951	4.5886	4.5696 (–0.55%)	4.5433 (–0.99%)	4.4699 (–2.73%)	4.4778 (–2.41%)
3	7.3057	7.3240	7.1804 (–1.72%)	7.2080 (–1.58%)	7.1645 (–1.93%)	7.0715 (–3.45%)
4	9.6620	9.6544	9.5253 (–1.42%)	9.5588 (–0.99%)	9.1652 (–5.14%)	9.1610 (–3.60%)
5	11.7333	12.7529	11.5197 (–1.82%)	11.5407 (–1.81%)	11.1374 (–5.08%)	11.0954 (–5.59%)
6	13.5063	13.5374	13.3340 (–1.28%)	13.3713 (–1.23%)	12.8839 (–4.61%)	12.8918 (–4.77%)
7	15.1871	15.1813	14.9426 (–1.61%)	14.9328 (–1.64%)	14.8433 (–2.26%)	14.8081 (–2.46%)
8	16.7283	16.7256	16.4278 (–1.80%)	16.4157 (–2.00%)	16.5746 (–0.92%)	16.5947 (–0.94%)
9	17.8677	17.8521	17.7811 (–0.48%)	17.7961 (–0.31%)	17.5598 (–1.72%)	17.5556 (–1.66%)
10	18.6398	18.6217	18.2098 (–2.31%)	18.1823 (–2.36%)	17.9627 (–3.63%)	17.9025 (–3.86%)

Note: The value in the parenthesis represents the relative frequency change

4.2 Sensitivity study and damage detection

Before damage detection, the normalized sensitivity matrix $[\overline{S}]$ of the 10-story building and its sensitivity to the relative stiffness in the *m*th story, $\overline{S}_{nj}^{m}$, are calculated using Eqs. (10) and (15). The maximum value in the matrix $[\overline{S}]$ is about 0.095 while the minimum value is zero. Similar to the 3-story building model, there also exist $\sum_{j=1}^{10}\overline{S}_{nj}=\frac{1}{2}$, $\sum_{n=1}^{10}\overline{S}_{nj}=\frac{1}{2}$, $\sum_{j=1}^{10}\sum_{m=1}^{10}\overline{S}_{nj}^{m}=0$ and $\sum_{n=1}^{10}\sum_{j=1}^{10}\overline{S}_{nj}^{m}=0$ for the 10-story building. The maximum value of the sensitivity coefficient $\overline{S}_{nj}^{m}$ is 0.803038, which means that the maximum error of the matrix $[\overline{S}]$ is 0.00803038 if there is 1% modeling error in stiffness. Furthermore, if only the first five natural frequencies are concerned, the maximum value of the sensitivity coefficient $\overline{S}_{nj}^{m}$ (n = 1, ..., 5; j = 1, ..., 10; m = 1, ...,10) is 0.04865 only while the maximum value of the matrix $[\overline{S}]$ for n = 1, ..., 5 and j = 1, ..., 10 is 0.095. This implies that the maximum error of the matrix $[\overline{S}]$ for n = 1, ...,5 and j = 1, ..., 10 is 0.0004865 only if there is 1% modeling error in stiffness. The value of 0.0004865 is about 0.51% of the maximum value of 0.095 in $[\overline{S}]$ for n = 1, ..., 5 and j = 1, ...,10. This means that the normalized sensitivity matrix $[\overline{S}]$ is not sensitive to modeling error in stiffness of the mono-coupled 10-story building.

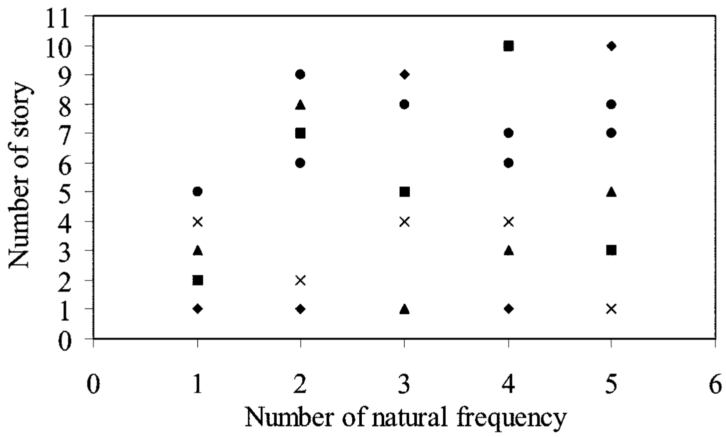
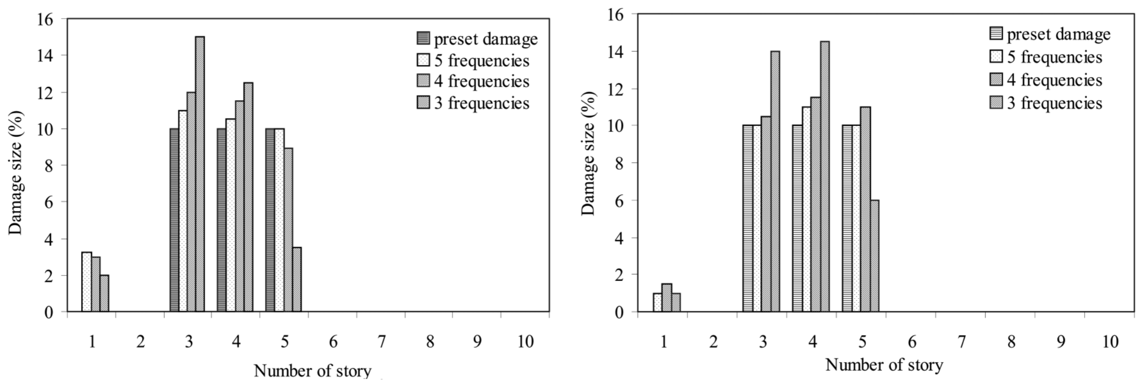


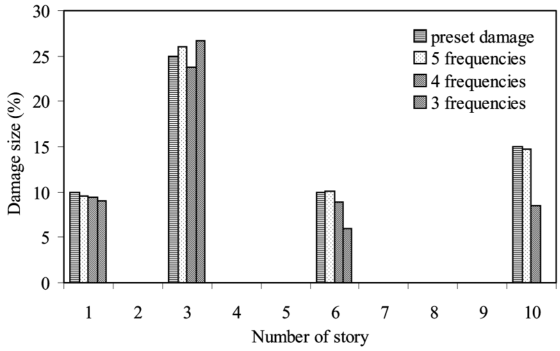
Fig. 11 Distribution of the five largest sensitivity coefficients of 10-story building (Largest→Smallest: ◆, □, △, ×, ●)



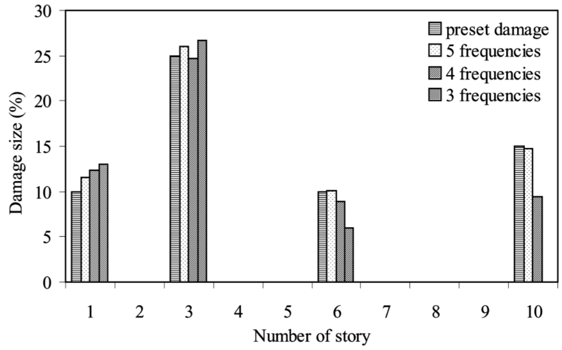
(a) Without noise pollution (b) With noise pollution

Fig. 12 Structural damage of 10-story building in scenario 1

Fig. 11 shows the distribution of the five largest sensitivity coefficients of the first five natural frequencies to the stiffness of the building. For the first natural frequency, the five largest sensitivity coefficients are distributed in the order of the first, second, third, fourth, and fifth stories. For the second frequency, the five largest sensitivity coefficients are distributed close to the two ends of the building. For further higher natural frequency, the distribution of the five largest sensitivity coefficients becomes more and more uniform along the height of the building. Figs. 12(a) and 12(b) display the damage predicted from Scenario 1 using both noise-free and noise-polluted natural frequencies, respectively, together with the preset damage location and size. It is seen that the preset damage locations (the third, fourth, and fifth stories) can be identified by using either the first 3 or 4 or 5 natural frequencies no matter whether the measurement noise is included or not. The first story is, however, falsely identified as a slightly damaged location. The accuracy of damage size detection is acceptable only when the relative changes in the first four or more natural frequencies are included in the damage detection. If the relative changes in only the first three natural frequencies are used for damage size detection, the identified relative changes in the stiffness of the third and



(a) Without noise pollution



(b) With noise pollution

Fig. 13 Structural damage of 10-storey building in scenario 2

fourth stories are much larger than the preset damage size while that in the stiffness of the fifth story is much less than the preset one. This is because the fourth and fifth natural frequencies, which are excluded in the damage detection, are sensitive to the damage occurring in the third to fifth stories. By comparing the identified results between Fig. 12(a) and Fig. 12(b), one may conclude that the 5% random noise in the natural frequencies does not significantly affect the damage detection results only when the first four or more natural frequencies are included in the damage detection.

Displayed in Figs. 13(a) and 13(b) are the damage location and size detected by the proposed algorithm for the 10-story building with damage scenario 2 with and without the random noise, respectively. It is noted that all the damaged stories (the first, third, sixth, tenth stories) can be correctly located when the relative changes in the first four or more natural frequencies are included. The damage in the tenth story cannot be correctly identified if only the first three natural frequencies are considered. This is because the fourth and fifth frequencies are very sensitive to the damage in the tenth story, as shown in Fig. 11. When the relative changes in the first five natural frequencies are included in the damage detection, the damage size in the four damaged stories can be satisfactorily detected. With the first four natural frequencies, the accuracy of the damage size detected is reduced. Again, the comparison of the identified results between Fig. 13(a) and Fig. 13(b) shows that the 5% random noise in the natural frequencies does not significantly affect the damage detection results.

4.3 Further discussion

Although the damage detection results presented above demonstrate that damage sizes can be satisfactorily predicted using the proposed algorithm with a sufficient number of natural frequencies, there still exist some differences between the preset and predicted damage sizes. Such differences are actually unavoidable because the sensitivity matrix derived is based on the linear assumption while the actual relationship between the relative changes in the stiffness and the natural frequencies are nonlinear.

Let us take the two buildings investigated in this paper to explain this point of view. Because the sum of the normalized sensitivity coefficients in the matrix $[\bar{S}]$ for a specified story is equal to

$\sum_{n=1}^{N} \bar{S}_{nj} = \frac{1}{2}$ (j = 1, ..., N), the summation of all the equations in Eq. (12) yields

$$\sum_{n=1}^{N} \Delta\bar{\omega}_n = \sum_{n=1}^{N} \bar{S}_{n1} \cdot \Delta\bar{k}_1 + \ldots + \sum_{n=1}^{N} \bar{S}_{nj} \cdot \Delta\bar{k}_j + \ldots + \sum_{n=1}^{N} \bar{S}_{nN} \cdot \Delta\bar{k}_N = 0.5\sum_{j=1}^{N} \Delta\bar{k}_j \tag{20}$$

In consideration that the number of measured natural frequencies is often less than the degrees of freedom N, Eq. (20) can be written as

$$\left|\sum_{j=1}^{N} \Delta\bar{k}_j\right| = 2\left|\sum_{n=1}^{N} \Delta\bar{\omega}_n\right| \geq 2\left|\sum_{n=1}^{p} \Delta\bar{\omega}_n\right| \tag{21}$$

where p is the number of available measured natural frequencies.

Eq. (21) shows that the sum of the relative changes in stiffness for all the stories identified by Eq. (12) is twice the sum of the relative changes in all the natural frequencies. If only limited measured natural frequencies are available, the sum of the identified relative changes in stiffness for all the stories will not be less than twice the sum of the relative changes in all the available natural frequencies. For the 3-story building model with damage scenario 1, the preset total damage size is 11.6% in the first story. Twice the sum of relative changes in the three natural frequencies is computed as 12.16% using the exact frequency Eq. (8). The sum of the damage sizes identified for all the stories based on the computed changes in the three natural frequencies is 12.16% other than 11.6%. If the relative changes in only the first two natural frequencies are used to predict the damage, the sum of the damage sizes identified for all the stories based on the changes in the first two natural frequencies is 12.22% while twice the sum of the relative changes in the first two natural frequencies is 11.3%. These results comply with Eq. (21). The difference between the preset total damage size of 11.6% and the predicted total damage size of 12.16% by using all the 3 computed frequency changes is because the linear assumption is used to derive the sensitivity equation. The actual sensitivity equation is nonlinear as shown in the following equation.

$$\{\Delta\bar{\omega}\} = [\bar{S}]\{\Delta\bar{k}\} + \frac{1}{2}\{\Delta\bar{k}\}^T\frac{\partial[\bar{S}]}{\partial\bar{k}}\{\Delta\bar{k}\} + \ldots \tag{22}$$

The same results are obtained for the 3-story building model with other damage scenarios and for the 10-story building with two damage scenarios. The same observation is also made for the 3-story building using the experimental data. Based on this understanding, one may conclude that the error in identified damage size will increase as actual damage level increases.

5. Conclusions

Damage detection of mono-coupled multistory buildings using natural frequency as only diagnostic parameter has been discussed in this paper. Closed-form sensitivity equation has been presented to relate the relative change in the stiffness of each story to the relative changes in the natural frequencies of the building. The special features of such a sensitivity equation and its low sensitivity to modeling errors have been demonstrated. Damage detection has been performed through the minimization of the error objective function with an inequality constraint. The proposed algorithm has finally been applied to a mono-coupled 3-story building model and a mono-coupled 10-story building and verified by the experiments on the 3-story building model. The damage detection results from the 3-story building model showed that the preset damage locations of the 3-story building model could be accurately identified when the relative changes in all the three natural frequencies were included. The preset damage locations could also be correctly located for the 3-story building model when the relative changes in the first two natural frequencies were included and for the 10-story building when the relative changes in the first four or more natural frequencies were included, but a false damage location with very small damage size may be predicted because of incomplete information on input natural frequencies. The damage detection results also demonstrated that the damage size at each damage location could be satisfactorily predicted for the 3-story building model when the first two or more natural frequencies are considered and for the 10-story building when the first five natural frequencies are considered. Small differences between the preset and predicted damage sizes are due to the linear assumption in deriving the sensitivity matrix. The investigation on the 10-story building further showed that the 5% random noise in the natural frequencies did not significantly affect the damage detection results, and the investigation on the 3-story building model further manifested that the 10% random error in the stiffness of the reference model did not significantly affect the damage detection results. The experimental investigation on the 3-story building model confirmed the numerical damage detection results.

Acknowledgements

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