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Non-linear rheology of tension structural element under single and variable loading history Part II: Creep of steel rope - examples and parametrical study

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Abstract. The substance of the use of the derived non-linear creep constitutive equations under variable stress levels (see first part of the paper, Kmet 2004) is explained and the strategy of their application is outlined using the results of one-step creep tests of the steel spiral strand rope as an example. In order to investigate the creep strain increments of cables an experimental set-up was originally designed and a series of tests were carried out. Attention is turned to the individual main steps in the production and application procedure, i.e., to the one-step creep tests, definition of loading history, determination of the kernel functions, selection and definition of constitutive equation and to the comparison of the resulting values considering the product and the additive forms of the approximation of the kernel functions. To this purpose, the parametrical study is performed and the results are presented. The constitutive equations of non-linear creep of cable under variable stress history offer a strong tool for the real simulation of stochastic variable load history and prediction of realistic time-dependent response (current deflection and stress configuration) of structures with cable elements. By means of suitable stress combination and its gradual repeating various loads and times effects can be modelled.

Key words: non-linear creep of steel rope; creep test; constitutive equation of creep under variable stress; kernel functions; parametrical study.

1. Introduction

The transition from various empirical expressions for creep and relaxation of engineering materials and primitive computational tools to the application of dramatically growing potential of the rheology and computer technology affects, among others, the development of the constitutive equations for non-linear time-dependent materials. It can be expected that simplistic rheological models will be gradually diminish and much more sophisticated methods will be developed.

Many analytical and numerical models have been developed to describe the mechanical behaviour of cables (Huang and Vinogradov 1996, Costello 1997, Raoof and Kraincanic 1998, Labrosse *et al.* 2000, Evans *et al.* 2001, Lefik and Schrefler 2002). Mathematical physical models of a cable as a system of interacting wires are presented. Roshan Fekr *et al.* (1999), Nawrocki and Labrosse (2000)

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and Jiang *et al.* (2000) proposed finite element models. More detailed description of the individual published works is presented in the first part of the paper (Kmet 2004). Many suggested methods do not involve the rheological properties.

To assess the structural reliability and serviceability performance of today's large span structures with the high strength tension cable elements, the general and accurate rheological models and constitutive equations of cable time-dependent behaviour must be available. Such models have to be considered for the following aspects:

a) Realistic consideration of cable properties, taking into account non-linear stress strain time relations.
b) The effect of imposed stress or strain originated during the previous time interval.
c) Variations of the imposed stress or strain.
d) The effect of the loading time, when the stress or strain is imposed.

More studies on the creep and relaxation of the steel wire and synthetic fibre cables have been carried out experimentally and theoretically (Nakai *et al.* 1975, Husiar and Switka 1986, Leech 1987, Guimaraes and Burgoyne 1992, Leech *et al.* 1993, Conway and Costello 1993, Banfield and Flory 1995, Kaci 1995, Leech 2002, Banfield *et al.* 2003), but only few theoretical approaches on the non-linear rheologic behaviour of the cables under varying loading history have been done (Kmet 1989, 1994, Kmet and Holickova 2000).

In the first part of the present paper (Kmet 2004), the mathematical derivations of the general constitutive equations for non-linear creep and relaxation of the tension elements such as the steel wire and synthetic fibre cables under one-step and the variable stress or strain inputs using the product and two types of additive approximations of the kernel functions were derived and presented. The time-dependent material constants – kernel functions determined from the results of one-step creep or relaxation tests were used for the description of constitutive equations for variable stress or strain history.

The potential of the derived non-linear constitutive equations using the approximation methods for determination of the kernel functions for variable loading history as a powerful tool for the practical simulations is described and emphasized in the present part of the paper. The illustrative examples based on the experimental and theoretical investigation of non-linear creep of steel spiral strand rope under single and variable stress history demonstrate the application of the presented theoretical expressions. Attention is turned to the individual main steps in the production and application procedure, i.e., to the one-step creep tests, definition of loading history, determination of the kernel functions, selection and definition of constitutive equation and to the comparison of the resulting values considering the product and the additive forms of the approximation of the kernel functions. To this purpose, the parametrical study is performed and the results are presented.

In order to investigate the creep strain increments of the steel cables and verify the derived equations, an experimental set-up was originally designed and a series of tests were carried out (Holickova 1997, Kmet and Holickova 2000).

2. Experimental program and theoretical background

To describe numerically and investigate the creep behaviour of the steel cables under arbitrary single and variable stress levels, the realization of constant input tests is inevitable. To this purpose, the creep tests of the cables were carried out under a constant load - constant uniaxial tension force.

Since the cables would be tested under a high tension, the load bearing capacity of the test equipment became a factor in setting the size of specimens.
Fulfilment of three basic decisions following requirements:

a) introduction of the real high tensile loading of the tested specimen,

b) placement of the cable specimen in the vertical position,

c) required length of the tested cable specimen,

influenced the design of the test equipment, the accuracy and the real response of measurement and test.

Even though the uniaxial tensile test is simple in its principle, in this case it turned out to be quite difficult to realize it technically taking into account all the above requirements.

The equipment that can keep a specimen with the length of 3000 mm under a constant tension load for the duration of the test (35 days) was designed and built. Fig. 1 shows the principle and the scheme of the experimental equipment. An appropriate loading system helps keep the specimen, vertical positioned between a passive anchorage and an active one, under constant tension load. Both anchorages are put inside a very stiff steel frame, which ensures that any stress is negligible enough to induce any displacement that could interfere with those of the specimen tested. In the certain place of one of the two columns of the frame, there is connected the arm of the lever mechanism by means of an unmovable hinged attachment. A passive anchorage creates a stiff transversal beam situated in the top part of the frame. The second movable support – active anchorage of a lever arm creates the tested specimen – a cable structural element with a cast-in

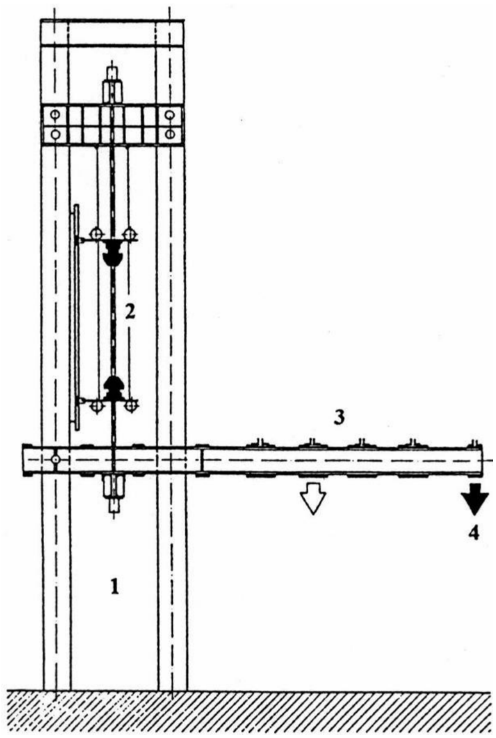

Fig. 1 Scheme of the test equipment. 1-test frame, 2-tested rope specimen, 3-lever arm, 4-weight

socket terminals in the both of the ends. An interdependent connection of a specimen with the active and passive anchorages is ensured by means of the stiff anchor transverse beams with special arrangement. A required constant load of the test specimens is obtained by means of a suitable placement of a dead weight on the lever of the creep tester mechanism.

The special clamping device - sockets for a placement of the measuring instruments of the extension of the specimens were designed. The sockets prevent sliding between a specimen and the measuring instruments and enable slewing of a cable specimen under a tension load. The sockets help eliminate any torsion displacement influences on the test results.

The values of the length increments – extension were simultaneously recorded and stored on a disk in the choice time intervals by the data-logging system driven by a microcomputer. During each creep test, contraction was measured and recorded temperature to evaluate its influence on the creep strain increments.

The specimen tested is a steel spiral strand rope with the open construction of type 1 + 6 + 12 (one core wire and 6 wires in the first and 12 wires in the second layer) with the rated diameter of D_r = 16 mm and the rated cross – sectional area of A_r = 148,1 mm^2. The rope has the mechanical characteristics as follows: the rated tensile load carrying capacity of N_r = 202,9 kN and the rated tensile strength of the wires of 1370 Nmm^{-2}. The cable specimens were cut of a new produced cable with the length of 100 m and were terminated with the cast-in socket terminals filled with zinc at the both ends. The cable specimens were initially stretched for a certain loading degree, where relatively small permanent strains occur after their unloading. This procedure is usually used before an introduction of a cable in a structure (European prestandard 2002). During the initial stretching, the cable was loaded within the time interval of 30 minutes by the force of 148 kN. This value represents the force of 17% bigger than is the force corresponding to the rope design strength.

The creep tests were carried out gradually under the constant uniaxial load levels that were equalled to 23%, 50% and 85% of the cable rated carrying capacity (Holickova 1997). The three tested stress levels σ_A, σ_B and σ_C were determined from the ratio $\sigma_r = N_r/A_r$ of the cable rated carrying capacity in tension N_r and the rated cross-sectional area A_r. For the experimentally investigated constant stress levels

$$\sigma_{A,\,B,\,C} = (23\% \,;\, 50\% \,;\, 85\%)\sigma_r = 315{,}1 \text{ Nmm}^{-2} \,;\, 685{,}0 \text{ Nmm}^{-2} \,;\, 1164{,}4 \text{ Nmm}^{-2} \tag{1}$$

the three corresponding average values of the creep strain increments $\varepsilon_A(t)$, $\varepsilon_B(t)$ and $\varepsilon_C(t)$ (for the number of the tests i = 1, 2, ..., N under each of the stress levels) are known

$$\varepsilon_\alpha(t) = \frac{\sum_{i=1}^{N} \varepsilon_{\alpha,\,i}(t)}{N} \quad \text{for} \quad \alpha = A, B, C \tag{2}$$

in a time t of the test time interval within <0 ; 35> days.

By the application of the constitutive equations in the form of polynomial of third order the courses of the creep strain increments can be formulated as follows

$$\varepsilon_\alpha(t) = F_1(t)\sigma_\alpha + F_2(t)\sigma_\alpha^2 + F_3(t)\sigma_\alpha^3 \quad \text{for} \quad \alpha = A, B, C \tag{3}$$

If the experimental creep strains from the three tests are described as functions of time by appropriate mathematical expressions (see Eq. (7) in the next chapter) and introduced on the left-hand side of Eqs. (3), these three equations may be solved simultaneously for the time functions $F_1(t)$, $F_2(t)$ and $F_3(t)$. Then from the known three stress levels σ_A, σ_B and σ_C and values of the creep strains $\varepsilon_A(t)$, $\varepsilon_B(t)$ and $\varepsilon_C(t)$ in the studied time t, the unknown values of constants $F_1(t)$, $F_2(t)$ and $F_3(t)$ – in the corresponding time t, after solution of system of Eqs. (4) are obtained, which can be written in the form

$$\begin{Bmatrix} F_1(t) \\ F_2(t) \\ F_3(t) \end{Bmatrix} = \begin{bmatrix} \sigma_A & \sigma_A^2 & \sigma_A^3 \\ \sigma_B & \sigma_B^2 & \sigma_B^3 \\ \sigma_C & \sigma_C^2 & \sigma_B^3 \end{bmatrix}^{-1} \cdot \begin{Bmatrix} \varepsilon_A(t) \\ \varepsilon_B(t) \\ \varepsilon_C(t) \end{Bmatrix} \tag{4}$$

Constants, i.e., values of the kernel functions $F_1(t)$, $F_2(t)$ and $F_3(t)$ are the time dependent characteristics of physical properties of the investigated cables.

If values of the kernel functions $F_1(t)$, $F_2(t)$ and $F_3(t)$ are introduced into the non-linear creep constitutive equation of the third order, as follows

$$\varepsilon(t) = F_1(t)\sigma + F_2(t)\sigma^2 + F_3(t)\sigma^3 \tag{5}$$

the expressions for a calculation of the cable creep strain increments for an arbitrary stress σ from the tested stress interval $<\sigma_A, \sigma_C>$ are obtained.

For the constitutive creep equation of the second order is necessary the system of two algebraic Eq. (3) as follows

$$\varepsilon_A(t) = \sum_{i=1}^{2} F_i(t)\sigma_A^i$$

$$\varepsilon_B(t) = \sum_{i=1}^{2} F_i(t)\sigma_B^i \tag{6}$$

and the other procedure is analogous.

The above approach is used for a numerical determination of the creep curves.

3. The cable creep curves calculated according to the constitutive equations and experimental data

By means of the constitutive equations of the third (Eq. (5)) and the second order (first two members in Eq. (5)), the creep curves – the creep strains in the corresponding time were calculated.

The tested stress interval $<\sigma_A, \sigma_C>$ was divided into the partial stress subintervals with the increment of 5% of (N_r/A_r). In general, we can take into account the complete stress interval (application of the third order constitutive equation), or only the certain subintervals of stresses $<\sigma_A, \sigma_B>$, $<\sigma_B, \sigma_C>$ and $<\sigma_A, \sigma_C>$ (application of the second order constitutive equations) according

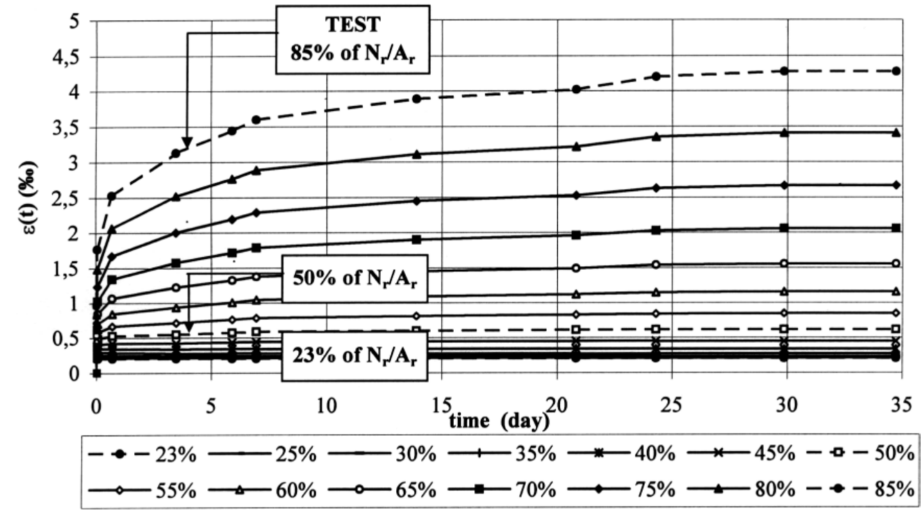

Fig. 2 Creep strain curves determined by use of the constitutive equation in the form of polynomial of third order with the stress interval within the boundaries of 23% to 85% of N_r/A_r

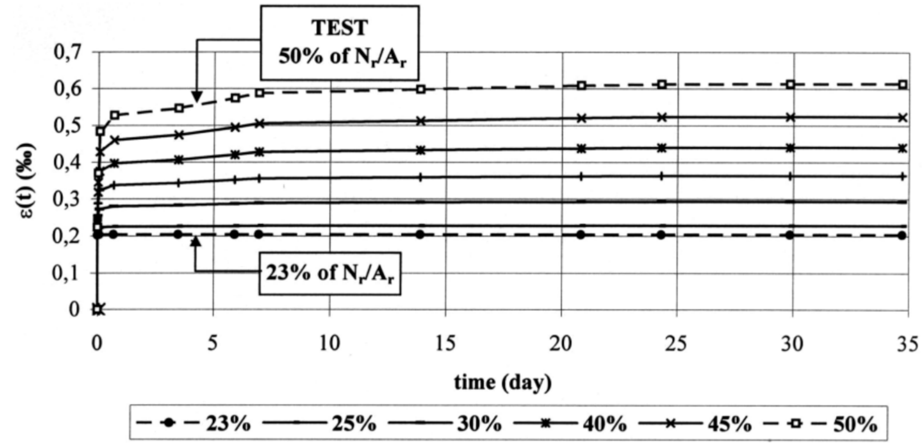

Fig. 3 Creep strain curves determined by use of the constitutive equation in the form of polynomial of second order with the stress interval within the boundaries of 23% to 50% of N_r/A_r

to the orientation on high or low loading levels. The creep strain increments determined for corresponding stress interval by the constitutive equations in the form of the polynomial of third and second order are shown in Fig. 2 to Fig. 5. The experimentally obtained curves are marked.

By a regression analysis of the experimentally and theoretically obtained data the optimal approximation creep function in logarithmic form was found, as follows

$$\varepsilon(t) = a + b \ln t \tag{7}$$

If one introduce in Eq. (7) time in days, the obtained strain is in ‰. The values of the coefficients of approximation functions and courses of approximation creep curves for the stress levels of 25

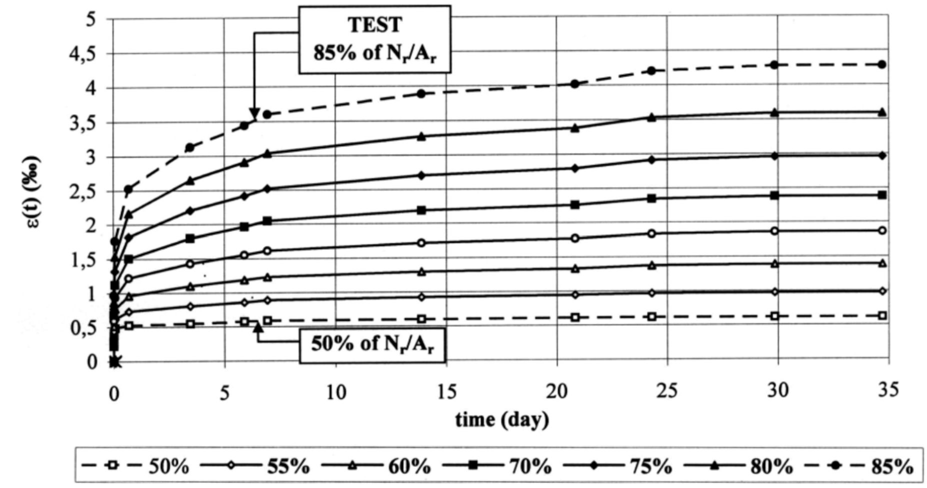

Fig. 4 Creep strain curves determined by use of the constitutive equation in the form of polynomial of second order with the stress interval within the boundaries of 50% to 85% of N_r/A_r

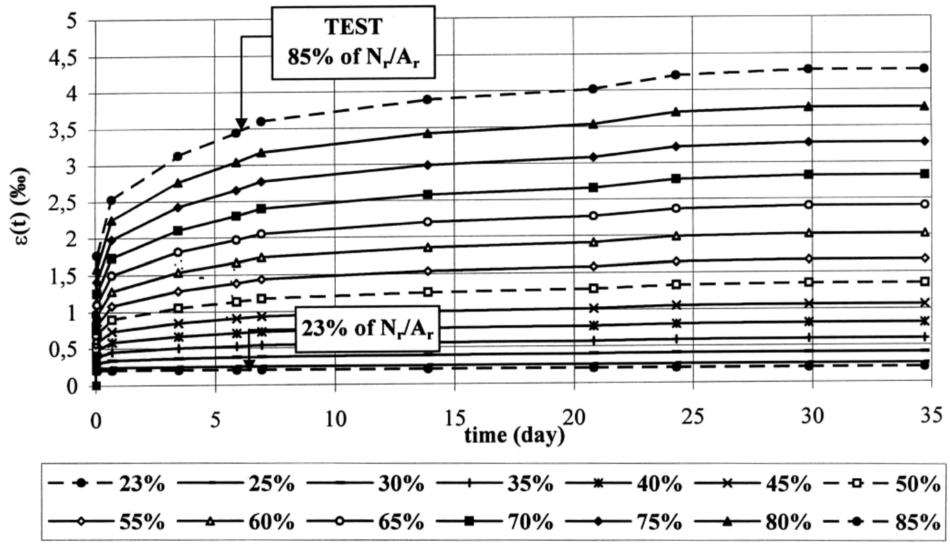

Fig. 5 Creep strain curves determined by use of the constitutive equation in the form of polynomial of second order with the stress interval within the boundaries of 23% to 85% of N_r/A_r

until 50% of (N_r/A_r) are presented in Table 1 and shown in Fig. 6.

The results can be summarized as follows:

a) The creep curves quantitatively correspond to the initial stress levels with a significant influence of non-linear behaviour. The creep strain increments increase gradually with the increased tensile loads.

b) The clearances between wire layers have an important influence on the creep strain values of the cable. By the use of the initial stretching of the cable structural elements, the creep strain

Table 1 Coefficients a and b of the approximation function for the corresponding stress levels

% of (N_r/A_r)	a	b
25	0,2257021	0,0008617
30	0,2796912	0,0041336
35	0,336852	0,0083302
40	0,3971127	0,0134981
45	0,4604037	0,0196521
50	0,5267743	0,0267673

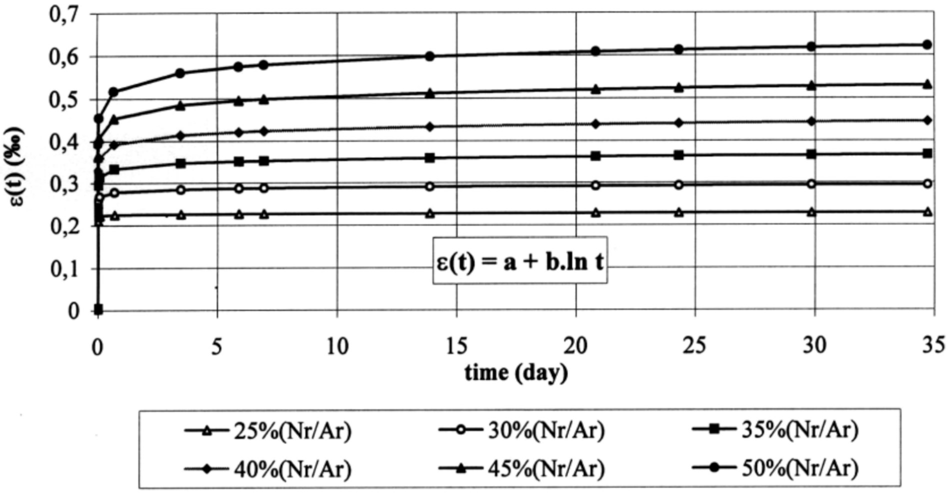

Fig. 6 Approximation functions of the creep curves in the stress interval within the boundaries of 25% to 50% of N_r/A_r

increments under the individual stress levels are reduced owing to a reduction of the clearances. In a case of the creep under the smaller stress levels the influence of the initial stretching of a cable is so significant, that the creep strain increments are already stabilized in the initial parts of the primary creep. For the stress levels of 50% of (N_r/A_r), there is a stabilizing process of the creep strain increments moved in the part of secondary creep. The creep strain increments under the stress levels of 85% of (N_r/A_r) (the stress level significantly exceeds the stress under the initial stretching) are considerable.

c) The experimentally obtained creep strains under the constant stress levels enable by means of the constitutive creep equations to determine the creep strain increments also for an arbitrary stress which lies inside of the tested stress interval.

4. Numerical illustration and verification of the constitutive equations of non-linear creep under variable stress

In this section, the results obtained from the constant one-step creep tests and derived constitutive equations are used to describe the non-linear rheologic behaviour of the cables under the variable input stress and to compare the non-linear constitutive equations with the product and additive forms of approximation of the kernel functions proposed in the first part of the paper (Kmet 2004). These equations are applied to predict the creep behaviour of a tested spiral strand rope under three and two-step stress.

4.1 Example 1

The example will show how to determine the creep strain in the required time t = 1000 minutes of the rope subjected to variable loading history, i.e., three stress increments $\Delta\sigma_0$ = 342,5 Nmm^{-2}, $\Delta\sigma_1$ = 68,5 Nmm^{-2} and $\Delta\sigma_2$ = 137 Nmm^{-2} (to demonstrate the applicability of derived equations deliberately are selected the stresses under which the creep were not tested) which gradually affect in the time subintervals $<t_0, t_1>$, $<t_1, t_2>$ and $<t_2, t_3>$ respectively, where t_0 = 0 minutes, t_1 = 900 minutes, t_2 = 990 minutes and the upper limitation t_3 = 10,000 minutes.

The input parameters of the rope such as stresses, time intervals and creep strain increments are selected or calculated from one-step creep tests as follows.

For the three constant stress levels σ_A = 342,5 Nmm^{-2}, σ_B = 465,8 Nmm^{-2} and σ_C = 698,7 Nmm^{-2} from the test interval can be experimentally and numerically obtained the creep strain increments $\varepsilon_A(t)$, $\varepsilon_B(t)$ and $\varepsilon_C(t)$ respectively in the investigated times, e.g., t = 10, 100, 1,000 and 10,000 minutes, which are inside the time of the test. The concrete input values of example are presented in Table 2. The corresponding kernel functions calculated according to Eq. (4) are presented in Table 3. If we introduce the values of obtained kernel functions in the non-linear constitutive equations of third order (Eq. 5), we can determine the cable creep strain for an arbitrary stress σ from the stress interval $<\sigma_A, \sigma_C>$.

The following procedure is used for application of the constitutive equations with the product and additive forms of approximation of kernel functions to predict creep behaviour of cables under three step stress impulses:

The global time interval $<t_0, t_3>$ (see Fig. 1 in first part of the paper, Kmet 2004), in which one-step creep curves are known is divided into the three time subintervals $<t_0, t_1>$, $<t_1, t_2>$ and $<t_2, t_3>$. In the concrete case of the example e.g., t_0 = 0 minutes, t_1 = 900 minutes, t_2 = 990 minutes and the upper limitation t_3 = 10 000 minutes. In these intervals affect the constant stresses $\sigma_0 = \Delta\sigma_0$ in the time interval $<t_0, t_1>$, $\sigma_1 = \sigma_0 + \Delta\sigma_1$ in $<t_1, t_2>$ and $\sigma_2 = \sigma_1 + \Delta\sigma_2$ in the time interval $<t_2, t_3>$. The choice stress σ = 548 Nmm^{-2} from the defined interval we divide into the three stress increments $\Delta\sigma_0$ = 342,5 Nmm^{-2}, $\Delta\sigma_1$ = 68,5 Nmm^{-2} and $\Delta\sigma_2$ = 137 Nmm^{-2} which gradually affect in the time subintervals presented above (see second line in Table 4). Then, the constant stresses in the individual time intervals obtained the values as follows $\sigma_0 = \Delta\sigma_0$ = 342,5 Nmm^{-2} in the time interval $<t_0, t_1>$, $\sigma_1 = \sigma_0 + \Delta\sigma_1$ = 342,5 + 68,5 = 411 Nmm^{-2} in $<t_1, t_2>$ and $\sigma_2 = \sigma_1 + \Delta\sigma_2$ = 411 + 137 = 548 Nmm^{-2} in the time interval $<t_2, t_3>$.

Table 2 Creep strains of the rope for three constant stress levels in the studied times

Stress σ	Creep strains in time t				
(Nmm^{-2})	$\varepsilon(t)$ (‰)	10 (min)	100 (min)	1,000 (min)	10,000 (min)
σ_A = 342,5	$\varepsilon_A(t)$	0,02	0,05	0,07	0,1
σ_B = 465,8	$\varepsilon_B(t)$	0,06	0,09	0,12	0,16
σ_C = 698,7	$\varepsilon_C(t)$	0,4	0,6	0,77	0,84

Table 3 Values of the kernel functions in the investigated times

	Values of kernel functions in time t			
$F_K(t)$	10 (min)	100 (min)	1,000 (min)	10,000 (min)
$F_1(t)$ ($N^{-1}mm^2$)	$4{,}60239.10^{-4}$	$1{,}12307.10^{-3}$	$1{,}48698.10^{-3}$	$1{,}61311.10^{-3}$
$F_2(t)$ ($N^{-2}mm^4$)	$-2{,}45589.10^{-6}$	$5{,}23213.10^{-6}$	$-6{,}81587.10^{-6}$	$-7{,}00086.10^{-6}$
$F_3(t)$ ($N^{-3}mm^6$)	$3{,}74489.10^{-9}$	$6{,}94691.10^{-9}$	$8{,}96657.10^{-9}$	$9{,}17820.10^{-9}$

Using the non-linear creep constitutive equations with the product and additive forms of approximation of the kernel functions (see Eq. (14) for product and Eq. (22) for additive form in first part of the paper Kmet 2004) the creep strain increments of cable in the choice time t = 1000 minutes will be investigated (it means for time from the subinterval $<t_2, t_3>$). The necessary numerical values of the kernel functions in the corresponding times that occur in the constitutive equations we determine from Table 3, it means from the values obtained from one-step creep curves. The kernel functions have the values as follows

$$F_1(t) = F_1(1000) = 1{,}48698.10^{-3}\ N^{-1}mm^2, \text{ and similarly } F_2(t) = F_2(1000),\ F_3(t) = F_3(1000),$$

$$F_1(t-t_1) = F_1(1000-900) = F_1(100) = 1{,}12307.10^{-3}N^{-1}mm^2, \text{ and similarly}$$

$$F_2(t-t_1) = F_2(1000-900) = F_2(100),\ F_3(t-t_1) = F_3(1000-900) = F_3(100),$$

$$F_1(t-t_2) = F_1(1000-990) = F_2(10) = 4{,}60239.10^{-4}N^{-1}mm^2, \text{ and similarly}$$

$$F_2(t-t_2) = F_2(1000-990) = F_2(10),$$

$$F_3(t-t_2) = F_3(1000-990) = F_3(10). \tag{8}$$

After introducing the known values of the kernel functions (values in the required times are presented in Table 3) and stresses into the corresponding constitutive equations of non-linear creep of cable the creep strain $\varepsilon(t)$ in the investigated time t = 1000 minutes can be calculated. For the individual types of the constitutive equations valid as follows

- product form of the approximation of the kernel functions (Eq. (14) in first part of the paper, Kmet 2004)

$$\begin{aligned}\varepsilon(1000) &= F_1(1000)\Delta\sigma_0 + F_1(100)\Delta\sigma_1 + F_1(10)\Delta\sigma_2 + \\ &+ \{[F_2(1000)]^{1/2}\Delta\sigma_0 + [F_2(100)]^{1/2}\Delta\sigma_1 + [F_2(10)]^{1/2}\Delta\sigma_2\}^2 \\ &+ \{[F_3(1000)]^{1/3}\Delta\sigma_0 + [F_3(100)]^{1/3}\Delta\sigma_1 + [F_3(10)]^{1/3}\Delta\sigma_2\}^3\end{aligned} \tag{9}$$

for $t_2 < t \le t_3$.

- additive form of the approximation of the kernel functions (Eq. (22) in first part of the paper, Kmet 2004)

$$\begin{aligned}\varepsilon_c(1000) &= F_1(1000)\Delta\sigma_0 + F_1(100)\Delta\sigma_1 + F_1(10)\Delta\sigma_2 + \\ &+ [F_2(1000)\Delta\sigma_0 + F_2(100)\Delta\sigma_1 + F_2(10)\Delta\sigma_2](\Delta\sigma_0 + \Delta\sigma_1 + \Delta\sigma_2) + \\ &+ [F_3(1000)\Delta\sigma_0 + F_3(100)\Delta\sigma_1 + F_3(10)\Delta\sigma_2](\Delta\sigma_0 + \Delta\sigma_1 + \Delta\sigma_2)^2\end{aligned} \tag{10}$$

for $t_2 < t \le t_3$.

Table 4 Creep strains of the rope in the studied time t = 1000 min under one and three stress levels. Times t_0, t_1, t_2 and t_3 of the individual time subintervals are considered as follows t_0 = 0 min, t_1 = 900 min, t_2 = 990 min and t_3 = 10,000 min

One stress level		Three stress levels			Product form	Additive form
$\sigma \in$ <t_0, t_3>	$\varepsilon(t)$	$\Delta\sigma_0 \in$ <t_0, t_1>	$\Delta\sigma_1 \in$ <t_1,t_2>	$\Delta\sigma_2 \in$ <t_2, t_3>	$\varepsilon(t)$	$\varepsilon(t)$
(Nmm^{-2})	(‰)	(Nmm^{-2})	(Nmm^{-2})	(Nmm^{-2})	(‰)	(‰)
479,5	0,1334	342,5	68,5	68,5	0,1294	0,1202
548	0,2436	342,5	68,5	137	0,2209	0,2082
548	0,2436	342,5	137	68,5	0,2221	0,2153
685	0,7024	342,5	68,5	274	0,5568	0,5520
685	0,7024	342,5	274	68,5	0,6052	0,6062

In the case of product form of approximation of the kernel functions, we obtain the creep strain of $\varepsilon(1000)$ = 0,2209‰ and in the case of additive form $\varepsilon(1000)$ = 0,2082‰ (see last two columns of second line presented in Table 4). In the second column and second line of presented table is state in order to compare the results also the value of creep strain under one-step creep (calculated according to Eq. (5)), it means under the constant one-stress level σ = 548 Nmm^{-2}, which is equal $\varepsilon(1000)$ = 0,2436‰.

On the base of a comparison of the results, it is evident that the cable affected under higher initial constant stress level during the whole time interval (creep under constant one-step stress history) shows higher creep strain than the cable which was affected with gradually increasing stress (creep under varying stress history). This stress reaches the value σ = 548 Nmm^{-2} only in the last time subinterval.

4.2 Example 2

The second example will show how to determine the creep strain in the required time t = 100 minutes of the rope subjected to variable loading history - two stress increments $\Delta\sigma_0$ = 342,5 Nmm^{-2} and $\Delta\sigma_1$ = 205,5 Nmm^{-2} which gradually affect in the time subintervals <t_0, t_1> and <t_1, t_2>, where t_0 = 0 minutes, t_1 = 90 minutes and the upper limitation t_2 = 200 minutes (see second line presented in Table 5).

Table 5 Creep strains of the rope in the studied time t = 100 min under one and two stress levels. Times t_0, t_1 and t_2 of the individual time subintervals are considered as follows t_0 = 0 min, t_1 = 90 min and t_2 = 200 min

One stress level		Two stress levels		Product form	Additive form
$\sigma \in$ <t_0, t_2>	$\varepsilon(t)$	$\Delta\sigma_0 \in$ <t_0, t_1>	$\Delta\sigma_1 \in$ <t_1, t_2>	$\varepsilon(t)$	$\varepsilon(t)$
(Nmm^{-2})	(‰)	(Nmm^{-2})	(Nmm^{-2})	(‰)	(‰)
466	0,009	342,5	123,5	0,09	0,08
548	0,1874	342,5	205,5	0,1773	0,1662
616,5	0,3115	342,5	274	0,296	0,285
685	0,5471	342,5	342,5	0,465	0,4568

The kernel functions have the values as follows

$$F_1(t) = F_1(100), \quad F_2(t) = F_2(100), \quad F_3(t) = F_3(100),$$
$$F_1(t-t_1) = F_1(100-90) = F_1(10), \quad F_2(t-t_1) = F_2(100-90) = F_2(10),$$
$$F_3(t-t_1) = F_3(100-90) = F_3(10) \tag{11}$$

Substituting the needed values of the kernel functions presented in Table 3 and values of stress increments into Eq. (13) and Eq. (21) (see first part of the paper, Kmet 2004), the resulting concrete forms of the constitutive equations for calculation the creep strain of the rope according to the type of the approximation of the kernel functions are as follows

- product form (Eq. (13) in first part of the paper, Kmet 2004)

$$\varepsilon(100) = F_1(100)\Delta\sigma_0 + F_1(10)\Delta\sigma_1 + \{[F_2(100)]^{1/2}\Delta\sigma_0 + [F_2(10)]^{1/2}\Delta\sigma_1\}^2 + \\ + \{[F_3(100)]^{1/3}\Delta\sigma_0 + [F_3(10)]^{1/3}\Delta\sigma_1\}^3 \tag{12}$$

for $t_1 < t \le t_2$,

- additive form (Eq. (21) in first part of the paper, Kmet 2004)

$$\varepsilon_c(100) = F_1(100)\Delta\sigma_0 + F_1(10)\Delta\sigma_1 + [F_2(100)\Delta\sigma_0 + F_2(10)\Delta\sigma_1](\Delta\sigma_0 + \Delta\sigma_1) + \\ + [F_3(100)\Delta\sigma_0 + F_3(10)\Delta\sigma_1](\Delta\sigma_0 + \Delta\sigma_1)^2 \tag{13}$$

for $t_1 < t \le t_2$.

In the case of the product form of approximation of the kernel functions, we obtain the creep strain of $\varepsilon(100)$ = 0,1773‰ and in the case of additive form $\varepsilon(100)$ = 0,1662‰ (see last two columns of second line presented in Table 5). In the second column and second line of presented table is state in order to compare the results also the value of creep strain increment under one-step creep, it means under the constant one-stress level σ = 548 Nmm^{-2}, which is equal $\varepsilon(100)$ = 0,1874‰. For the calculation, the non-linear creep constitutive equation of the second order was used (see first two members of Eq. (5)).

Presented numerical applications have a limited interest for the concrete type of investigated cable and loading, i.e., stress history. But on the other hand proposed constitutive equations of non-linear creep and approximation computational models for determination of the kernel functions have the general applicability for the arbitrary tested synthetic and steel cables under various stress history.

5. Parametrical study

The analogous procedures are used for the parametrical study. The constitutive equations with product and additive form of the approximation of the kernel functions are applied to predict the creep behaviour of a tested spiral strand rope under one, two and three step stress. In this study, the influence of the following variable quantities is investigated:

a) size of the stress increment in the time subinterval,
b) length of the time subinterval, in which stress affects,
c) number of the stress levels – creep under one stress level, two and three stress levels,
d) form of the approximation of the kernel functions.

The individual stresses σ (constant stress levels for one-step creep) and the varying stress increments $\Delta\sigma_0$ and $\Delta\sigma_1$ affected in the time subintervals $<t_0, t_1>$ and $<t_1, t_2>$, respectively, are for the case of two stress levels presented in Table 5. For the time values of the individual subintervals valid as follows t_0 = 0 minutes, t_1 = 90 minutes and t_2 = 200 minutes. Thus the lengths of the time subintervals in which a stress with the corresponding level affects are predetermined. Gradually the maximum stress values and the values of the stress increments were changed.

The obtained results, i.e., the creep strains of cable in the studied time t = 100 minutes calculated according to Eq. (12) (product form of approximation) and Eq. (13) (additive form of approximation) are for the two stress levels presented in Table 5 and shown in Fig. 7. Also the results of one stress creep (stress σ affects in whole interval $<t_0, t_2>$) are presented in the table and shown in the figure. In Fig. 7, the global constant stresses σ_0 and σ_1 affected in the individual time subintervals as follows $\sigma_0 = \Delta\sigma_0$ in the time subinterval $<t_0, t_1>$ and $\sigma_1 = \sigma_0 + \Delta\sigma_1$ in the subinterval $<t_1, t_2>$ are presented. The sum of the varying stress increments in the individual time subintervals is equal to the stress level under one-step creep affected in the whole time interval. One line in the tables and/ or one column in the figures mean one independent example.

The parametrical study for varying three stress levels is followed. The individual stresses σ (constant stress levels for one-step creep) and the varying stress increments $\Delta\sigma_0$, $\Delta\sigma_1$ and $\Delta\sigma_2$ affected in the time subintervals $<t_0, t_1>$, $<t_1, t_2>$ and $<t_2, t_3>$, respectively are for the case of three stress levels presented in Table 4. For the time values of the individual subintervals valid as follows t_0 = 0 minutes, t_1 = 900 minutes, t_2 = 990 minutes and t_3 = 10,000 minutes.

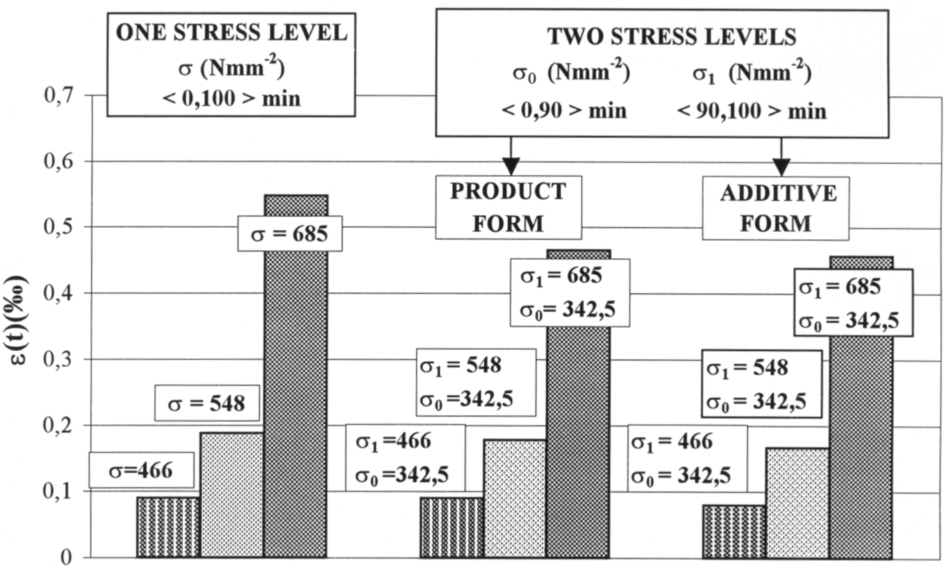

Fig. 7 Creep strains of the rope in the studied time t = 100 min under one and two stress levels (the values of the first, second and fourth line presented in Table 5). Times t_0, t_1 and t_2 of the individual time subintervals are considered as follows t_0 = 0 min, t_1 = 90 min and t_2 = 200 min

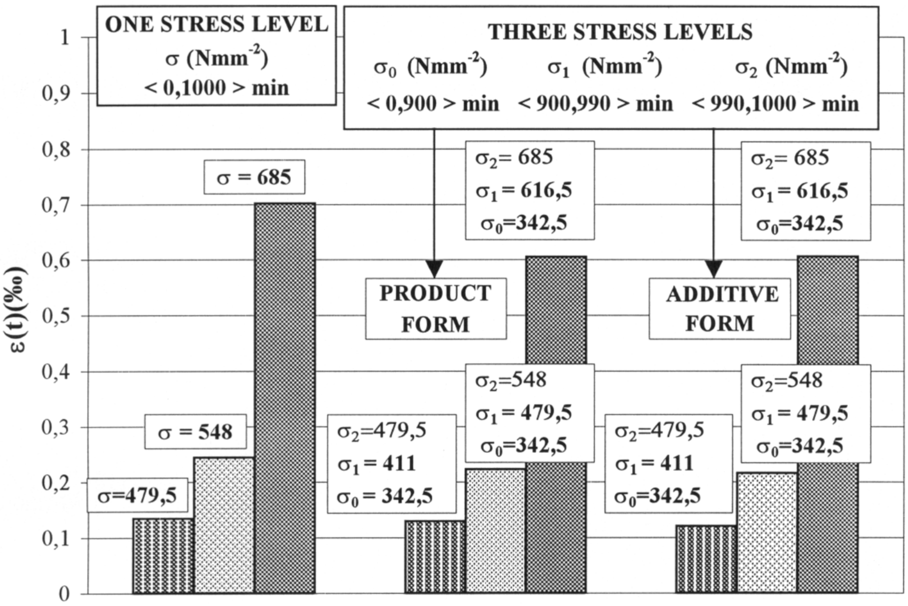

Fig. 8 Creep strains of the rope in the studied time t = 1000 min under one and three stress levels (the values of the first, third and fifth line presented in Table 4). Times t_0, t_1, t_2 and t_3 of the individual time subintervals are considered as follows t_0 = 0 min, t_1 = 900 min, t_2 = 990 min and t_3 = 10,000 min

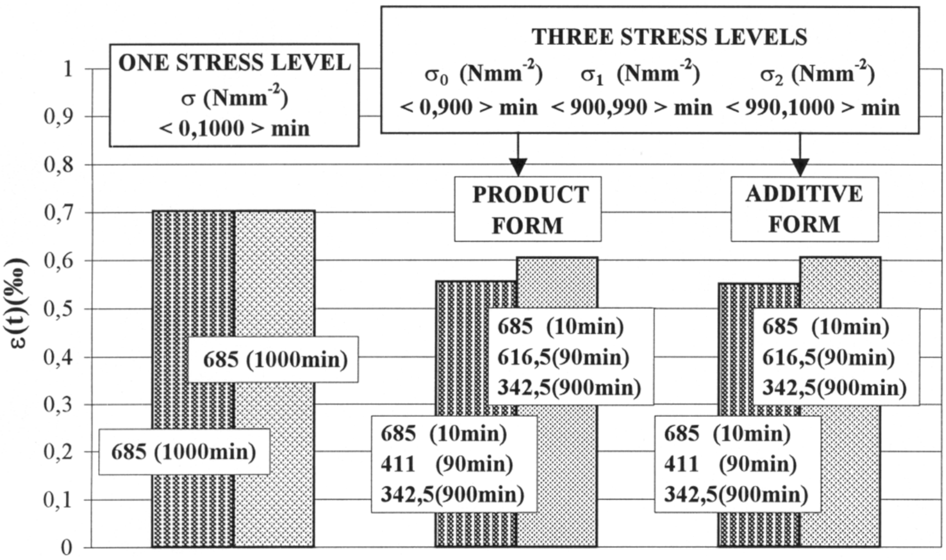

Fig. 9 Comparison of the creep strains under the varying three stress levels, gradually affected in the time subintervals with the variable lengths

The obtained results, i.e., the creep strains of cable in the studied time t = 1000 minutes calculated according to Eq. (9) (product form of approximation) and Eq. (10) (additive form of approximation) are for the three stress levels presented in Table 4 and shown in Fig. 8 (the values of the first, third and fifth line presented in Table 4 are used) and Fig. 9. Also the results of one stress creep (stress σ affects in whole interval <t_0, t_3>) are presented in the table and shown in the figures. In Fig. 8 and Fig. 9, the global constant stresses σ_0, σ_1 and σ_2 affected in the individual time subintervals as follows $\sigma_0 = \Delta\sigma_0$ in the time interval <t_0, t_1>, $\sigma_1 = \sigma_0 + \Delta\sigma_1$ in <t_1, t_2> and $\sigma_2 = \sigma_1 + \Delta\sigma_2$ in the time interval <t_2, t_3> are introduced.

The influence of the varying three stress levels on the resulting creep strain, with the lengths of the time subintervals in which a stress with the corresponding level affects is illustrated in Fig.9. Product and additive form of the approximation of the kernel functions are considered. Fourth and fifth line of the values presented in Table 4 is used.

From comparison of numerical results presented in Table 4, Table 5 and shown in Fig. 7, Fig. 8 and Fig. 9, which are obtained by use of the constitutive equations with the product and additive form of an approximation of the kernel functions under two (Eq. (12) for product and Eq. (13) for additive form of approximation) and three-stress levels (Eq. (9) for product and Eq. (10) for additive form of approximation) as well as by use of the constitutive equation of non-linear creep under one-step stress (Eq. (5)) follows, that proposed rheological constitutive equations, which serve to calculation and prediction of the creep behaviour of cable under one-step and variable stress history, are sufficiently accurate. The obtained values indicate and confirm the influence of the stress increment size and time subinterval length, in which the stress affects, on the resulting quantity of creep strain. The analysis of the results confirms logical and physical correctness of the obtained constitutive equations. By reason of this are these constitutive equations suitable for practical using in a design of the cable structural elements and tension structures with the time dependent properties. The results obtained by the creep constitutive equations with the product and additive forms of approximation of the kernel functions do not differ significantly, it means that for rheological analysis arbitrary from them can be used. The creep strains obtained by the use of the constitutive equation with product form of the approximation of the kernel functions are a little larger than that obtained by the additive form of approximation. Therefore, the results obtained by the use of the constitutive equations with product form of the approximation of the kernel functions lie on the more safety side. From point of view of the number of an algebraic operation constitutive equation with additive form approximation of the kernel functions is simpler.

6. Conclusions

The substance of the use of the derived non-linear creep constitutive equations under variable stress levels (see first part of the paper, Kmet 2004) is explained and the strategy of their application is outlined using the results of one-step creep tests of the steel one-strand cable as an example. In order to investigate the creep strain increments of cables, an experimental set-up was originally designed and a series of tests were carried out.

On the base of the experimental results and theoretical approaching the approximation values of the time-dependent kernel functions in the product and additive forms and the concrete types of the creep constitutive equations for the investigated cable were obtained and verification of the derived constitutive equations were performed.

The results from one-step creep tests and from the numerical experiments have been implemented in the non-linear creep constitutive equations of the steel cables under a variable stress history. The product and additive forms of approximation of the kernel functions were used in the constitutive equations to simulate variable stress history from the single step creep tests. Resulting response determined according to these forms of approximation are compared. Both results are of very good agreement. The useful information about the creep behaviour of the cables under variable stress levels was obtained from the analysis.

The constitutive equations of non-linear creep of cable under variable stress history offer a strong tool for the real simulation of stochastic variable load history in comparison to the one step creep constitutive expressions. The derived equations can be implemented in a suitable way into the discrete non-linear computational transformation models and existing software. It enables to make a quick rheological structural analysis and obtain real response characteristics of the constructions with cable structural elements (current deflection and stress configuration). By means of suitable stress combination and its gradual repeating various times and loads effects can be modelled.

The derived concrete forms of the constitutive equations enable to predict and assess non-linear creep rheologic behaviour of the cables for variable stress history by using the kernel functions determined only from one-step constant creep tests, what is the main advantage. The analogous methodology can be used for the synthetic fibre cables with non-linear rheological properties as well as for the other types of the uniaxial tension structural elements affected under single and variable loading history.

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