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Rocking response of unanchored rectangular rigid bodies to simulated earthquakes

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Abstract. Rocking response of rigid bodies with rectangular footprint, freely standing on horizontal rigid plane is studied analytically. Bodies are subjected to simulated single component of horizontal earthquakes. The effect of baseline correction, applied to simulated excitations, on the rocking response is first examined. The sensitiveness of rocking motion to the details of earthquakes and geometric properties of rigid bodies is investigated. Due to the demonstrated sensitivity of rocking response to these factors, prediction of rocking stability must be made in the framework of probability theory. Therefore, using a large number of simulated earthquakes, the effects of duration and shape of intensity function of simulated earthquakes on overturning probability of rigid bodies are studied. In the case when a rigid body is placed on any floor of a building, the corresponding probability is compared to that of a body placed on the ground. For this purpose, several shear frames are employed. Finally, the viability of the energy balance equation, which was introduced by Housner in 1963 and widely used by nuclear power industry to estimate the rocking stability of bodies, is evaluated. It is found that the equation is robust. Examples are also given to show how this equation can be used.

Key words: rigid body; rocking response; simulated earthquakes; energy balance equation.

1. Introduction

There are many structures/bodies that are unanchored placed on the ground or on the floors of a building. These include oil or water tanks, mechanical or electrical equipment, art objects, computers, furniture and scaffoldings.

First studies toward understanding the behavior of rigid bodies date back to 1881. The researchers then tried to correlate the observed response of tombstones and monumental columns, some of which toppled over whilst some did not, to ground shaking intensity. Typical examples are the studies of Omori (1899, 1900), Ikegami and Kishinouye (1947, 1950). These researchers were, however, unable to find such a correlation. Ishiyama (1982) presents a summary of the other early studies of response of rigid bodies to earthquake excitations.

When the base on which an unanchored body is placed moves, the body may respond in different modes. Shenton (1995) identified these modes as rest, slide, rock and slide-rock. The body may leave the base momentarily if a vertical base excitation is involved. Depending on the system parameters, a rigid body responding to earthquake excitation in any of the four modes may also

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shift to a new mode of response during the earthquake. Such transitions between different modes have been studied in the past, see, for example, Yang *et al.* (2000).

Apparently, the rocking mode of response was first studied analytically by Housner in 1963. In addition to a limited amount of laboratory experimental work, the problem has since been studied analytically and numerically by solving the equation of rocking motion of a rigid body. This has been done either deterministically or probabilistically. When the problem is studied probabilistically, it is carried out either by using an ensemble of real or simulated earthquakes or by the theory of random vibration. The literature is too numerous to cite and the interested reader is referred to Aydin (2000).

In 1998, Shao, following the approach of Yim *et al.* (1980), used both simulated and real earthquakes to study the rocking behavior of unanchored bodies. There were, however, a number of issues not addressed by Shao. Using the simulated earthquakes in the study herein, the following are investigated:

1. the effect of baseline correction of simulated base motion on rocking response,
2. the sensitivity of rocking response to details of earthquake motion, the intensity of earthquake motion, the size of the body and the coefficient of restitution,
3. the effects of the duration of earthquake and the shape of the intensity function on the probability of overturning of an unanchored body,
4. the difference between the probabilities of overturning of an unanchored body when the body is placed on the ground and on the floors of a building.

A very important part of this study is to verify the validity of the energy balance equation proposed by Housner in 1963. The energy balance equation enables one to determine the level of excitation that would overturn a rocking rigid body with an approximately 50% probability. It is simple in form and has been used by engineers, especially those in nuclear industry, in dealing with unanchored bodies such as scaffoldings, ladders, trash cans, cabinets etc. The energy balance equation was derived using heuristic arguments and has not been verified either analytically or by experiment. With a large number of simulated earthquakes, this study is able to show that the energy balance equation is indeed very robust.

2. Analytical model

The body under consideration is a rigid homogenous rectangular block of mass M, height H and width B (see Fig. 1). The size of the body is measured by R and the parameter $\gamma = H/B$ is called the slenderness ratio. It has a rectangular footprint, and the base excitation is horizontal in the x direction-$a_x(t)$ such that its motion is confined in the x-y plane (Fig. 1b). In the figure, $Mg(=W)$ is the weight of the body. There is presumably a sufficient friction between the body and the surface on which it rests so that no slippage occurs under base excitation. The equation of rocking motion of the body when the motion is about the corner O is given by

$$I_O\ddot{\theta} + WR\sin(\theta_c - \theta) = -\frac{W}{g}R\cos(\theta_c - \theta)a_x(t) \tag{1}$$

where θ is the tilting of the body. If the body pivots about the corner O', the equation of motion becomes

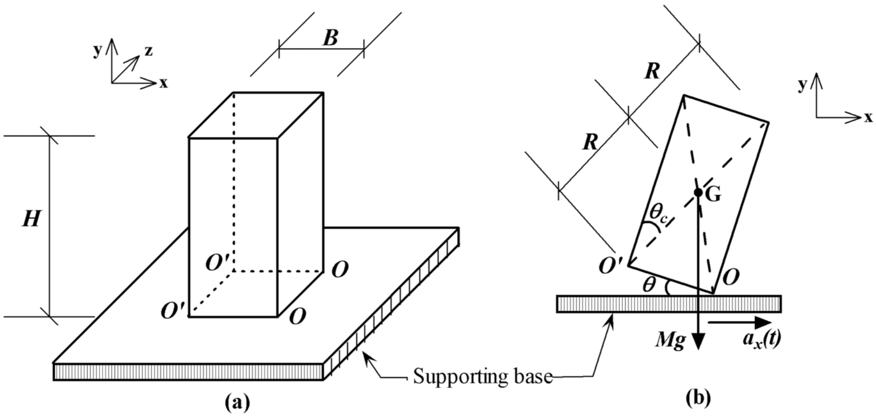

Fig. 1 Rectangular rigid body (a) at rest, (b) with a response θ from the horizontal plane

$$I_{O'}\ddot{\theta} - WR\sin(\theta_c + \theta) = \frac{W}{g}R\cos(\theta_c + \theta)a_x(t) \tag{2}$$

In the equations, $I_O = I_{O'} = (4/3)MR^2$ represents the mass moment of inertia of the body about O and O', $R = (\sqrt{H^2 + B^2})/2$ the size parameter, $\theta_c = \tan^{-1}(B/H)$ the critical angle-critical in the sense that it separates toppling and no-toppling of the body under the action of gravity alone and g the gravitational acceleration.

The equations are highly nonlinear and, under arbitrary earthquake-like base excitation, can only be solved numerically. The numerical solution is achieved by using a fourth-order Runge-Kutta integration scheme. For the accuracy of the solution, a very short time-step of 1/400 seconds is used. The rigid body is assumed to be initially at rest.

In solving the equation of motion, an additional quantity, the coefficient of restitution denoted by e, must be provided because, during the rocking motion of a rigid body, some energy is dissipated at each impact. The extent of energy dissipation depends on the magnitude of coefficient of restitution that is defined as $e = \dot{\theta}^+/\dot{\theta}^-$, in which $\dot{\theta}^-$ is the angular velocity before impact and $\dot{\theta}^+$ is the angular velocity after impact. The value of e consequently depends on the angular velocity of the body before and after impact, and the material properties of the base and the body. Aslam *et al.* (1980) determined the value of e by free rocking tests of a concrete body. Their study showed that e was effectively constant during the rocking motion and could be given a value of 0.925.

3. Simulated earthquakes

In order to specify design ground acceleration, researchers since the sixties have been trying to find the ways alternative to recorded real ground accelerations. Such a way is to produce simulated earthquakes. The reason that simulated earthquakes are used in this study is because, to obtain statistically meaningful results, a large number (≥200) of earthquakes must be employed. The idea underlying the simulation of an earthquake is based on the fact that a function can be represented by the sum of a series of sinusoids. That is,

$$x(t) = \sum_{j=1}^{n} A_j \sin(\omega_j t + \phi_j), \qquad 0 < t < \infty \tag{3}$$

Here n is the total number of harmonics to be superposed, $A_j = \sqrt{4S_g(\omega_j)\Delta\omega_j}$ is the amplitude of the j^{th} component, $\omega_j = \omega_{j-1} + \Delta\omega_j$ is the frequency of the j^{th} component, and $\Delta\omega_j$ is the incremental frequency. The quantity ϕ_j is a random variable (phase angle) uniformly distributed over the interval $[0, 2\pi]$. Thus, to obtain $x(t)$, n values of ϕ_j ($j = 1, \ldots, n$) are produced using the random number generator. The quantity $S_g(\cdot)$ is the power spectral density (PSD) function. Many forms of $S_g(\cdot)$ have been suggested (for example, Clough and Penzien 1993) but this study is not concerned with which $S_g(\cdot)$ to use since it is now common practice to first specify the response spectrum from which S_g is obtained. The manner in which this conversion is carried out is based on an approach developed by Park (1995).

The unknown PSD function of ground acceleration $S_g(\omega)$ is discretized at frequencies ω_j and considered as the sum of a series of m discretized power components. That is,

$$S_g(\omega) = \sum_{j=1}^{m} S(\omega_j)\Delta\omega_j \delta(\omega - \omega_j) = \sum_{j=1}^{m} p_j \delta(\omega - \omega_j) \tag{4}$$

in which $S(\cdot)$ is the discretized equivalent PSD function (per $\Delta\omega_j$), $\delta(\cdot)$ is the Dirac delta function, $p_j(\geq 0)$ is the discretized power components and $\omega_j = \omega_{j-1} + \Delta\omega_j$, which are equally spaced on a logarithmic scale. Approximating the target RS at frequencies ω_k, the discretized power components p_j may be obtained by solving a standard least squares problem as

$$\text{minimize} \sum_{k=1}^{n} \left\{ R_t^2(\omega_k, h) - \sum_{j=1}^{m} p_j R_{k,j}^2(\omega_k, \omega_j, h) \right\}^2, \qquad j = 1, \ldots, m \tag{5}$$

in which R_t is the target acceleration response spectrum. The term $R_{k,j}$ in Eq. (5) is the peak acceleration response of a single-degree-of-freedom (SDOF) system with a vibration frequency of ω_k and viscous damping ratio of h. The SDOF system is excited by a sinusoid of unit amplitude and vibration frequency ω_j. The $R_{k,j}$ is computed using the peak factor approximation given by Davenport (1964) as

$$R_{k,j}(\omega_k, \omega_j, h) = \left\{ \sqrt{2\ln(\nu_j T_e)} + \frac{5.772}{\sqrt{2\ln(\nu_j T_e)}} \right\} \sqrt{\frac{1 + 4h^2x^2}{(1 - x^2)^2 + 4h^2x^2}} \tag{6}$$

in which $\nu_j = \omega_j / 2\pi$, $x = \omega_j / \omega_k$ and T_e is the equivalent duration that is related to the intensity function $I(\cdot)$ as

$$T_e = \frac{\int_0^T I(\tau)d\tau}{\max\{I(\tau)\}} \tag{7}$$

Here, T stands for the total duration of the earthquake excitation. Eq. (5) is solved for p_j using the IMSL Mathematical Libraries and the equivalent PSD function is obtained as

$$S(\omega_j) = \frac{p_j}{\Delta\omega_j}, \qquad j = 1, \ldots, m \tag{8}$$

The target response spectrum (for 5% damping ratio) is chosen to be that given in Regulatory Guide 1.60 by US Atomic Energy Commission (1973). In the analysis here, n is set to be equal to m (see Eq. 5) and is selected to be 100. The lower and upper limits of frequency band are 0.15 cps and 50 cps. Therefore, the PSD function and response spectrum (RS) are discretized at 100 frequency points between 0.15 cps and 50 cps. The frequency of the j^{th} component is $\omega_j = \omega_{j-1} + \Delta\omega_j$ as mentioned before.

Upon determining the equivalent PSD function, the earthquake accelerations compatible with the target response spectrum can finally be simulated by using the following equation

$$a(t) = I(t)\sum_{j=1}^{n} A_j \sin(\omega_j t + \phi_j) \tag{9}$$

where $I(t)$ is the intensity function. This deterministic function, which rises from zero to unity and, after remaining constant for a period of time, decays to zero, is used to reflect the transient character of a real accelerogram on simulated earthquakes. In this study, the intensity functions considered are the boxcar, trapezoidal and compound type functions (see Fig. 2). Phase angles, ϕ_j, are produced by a random number generator in the IMSL mathematical libraries. Using n randomly selected value of ϕ_j, one obtains one time history $a(t)$. If it is wished to generate an ensemble of N $a(t)$'s, one randomly selects $N \times n$ number of ϕ_j's. The simulated time histories $a(t)$'s, thus, will be statistically similar but will be different in details.

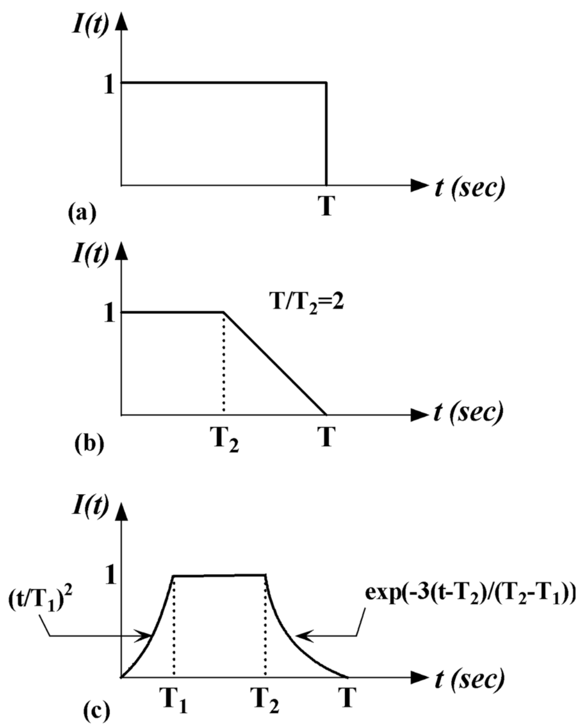

Fig. 2 Intensity functions type (a) boxcar (rectangular), (b) trapezoidal and (c) compound

A measure of the intensity of an earthquake time history is the peak acceleration (the maximum of the absolute value of the time function) denoted by $A_{gx}g$, g being the gravitational acceleration. When an ensemble of many earthquakes is dealt with, the peak acceleration, also denoted by $A_{gx}g$, is the average of that of the individual acceleration time histories.

4. Effect of baseline correction on the rocking response

It is well known that the baseline of a real or simulated earthquake acceleration time history must be corrected to compensate the base translation of the accelerograms caused by instrument error or by digitizing error (Jennings *et al.* 1968). This error firstly yields a linearly increasing velocity and secondly yields a parabolically increasing displacement curve. By baseline correction this type of error or unwanted trend insertion to simulated earthquakes can be removed such that non-zero values at the end of the earthquake time histories are obtained. A few baseline correction schemes are known to engineering community. In this study, a method which performs quadratic baseline correction is applied. In the method, original earthquake data points are subtracted by a quadratic calibration function which satisfies the condition of minimization of average square sum of integrated velocity history. The effect of this baseline correction on the acceleration time history itself is imperceptibly small. Thus, it is of interest to find out how this small difference between the acceleration time histories with and without baseline correction would affect the rocking response of a rigid body. Using the simulation program and Regulatory Guide 1.60 response spectrum, with a compound intensity function and a 20 sec duration, an earthquake is generated without correcting the baseline.

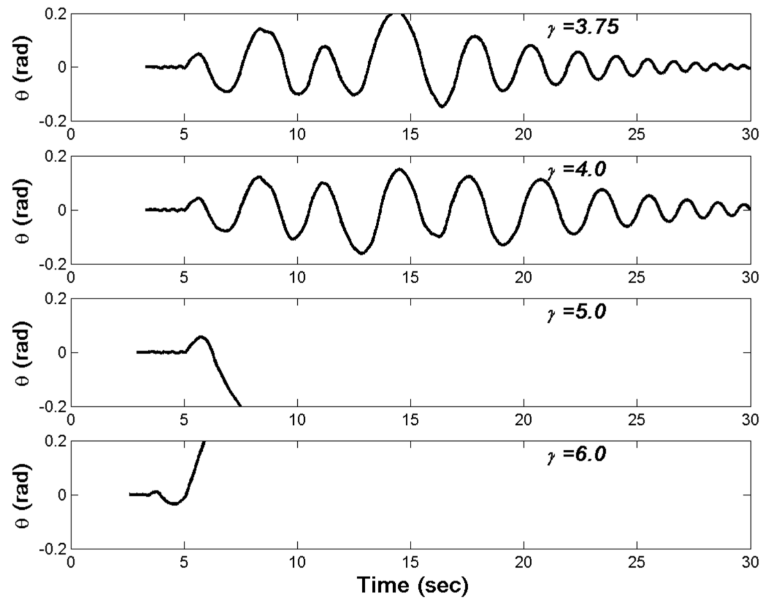

Fig. 3 Responses of bodies with R = 3.05 m, e = 0.925 subjected to baseline-uncorrected simulated earthquake (No.1) scaled to A_{gx} = 0.6

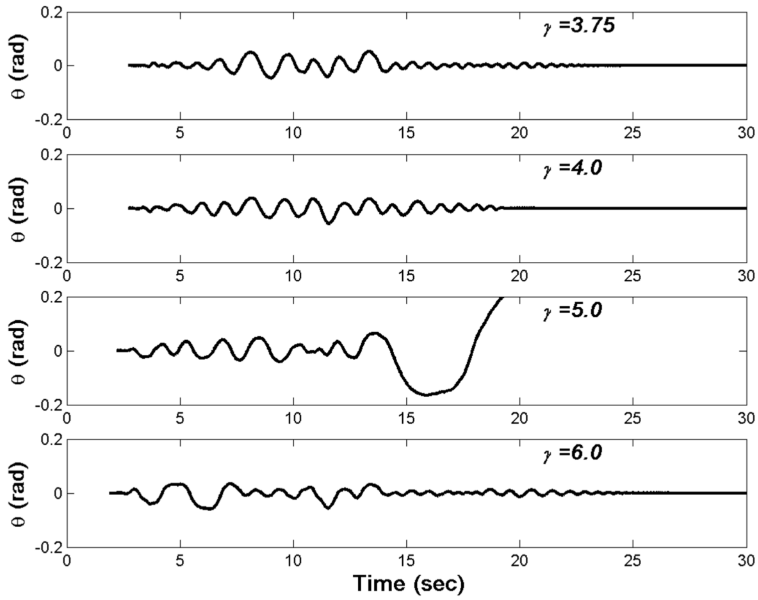

Fig. 4 Responses of bodies with $R = 3.05$ m, $e = 0.925$ subjected to baseline-corrected simulated earthquake (No.1) scaled to $A_{gx} = 0.6$

The intensity of this individual earthquake is specified by $A_{gx} = 0.6$. The rocking responses of rigid bodies with a size parameter $R = 3.05$ m and slenderness ratios $\gamma = 3.75$, 4.0, 5.0 and 6.0 are shown in Fig. 3. It is seen that the response θ of the body increases with an increase in the slenderness ratio. This observation contradicts those of previous researchers such as Yim *et al.* (1980) and Shao (1998). Fig. 4 shows the responses of the same rigid bodies as above subjected to the simulated earthquake with its baseline corrected. It is seen that, although the body with $\gamma = 6.0$ does not overturn, the one with $\gamma = 5.0$ does. In other words, the response of a rigid body does not necessarily increase with slenderness ratio. It is clear from this analysis that the rocking response of rigid bodies is affected by whether or not the baseline of the simulated earthquake is corrected. A paper by Shin (1999) also states that the rocking response is sensitive to baseline correction. Because of this, all the simulated acceleration time histories used in the remaining parts of this paper are baseline corrected.

5. Sensitivity analysis of the rocking response

The sensitivity of the rocking response to details of base excitation, intensity of earthquake motion α, size parameter R and coefficient of restitution e is investigated. It is already noted in the previous section that the slenderness ratio γ has no systematic effect on the rocking response of a rigid body.

To study how the details of base acceleration would affect the response of a body, 25 earthquakes are generated using the Regulatory Guide 1.60 response spectrum. Fig. 5 shows the influence of using four different members of the ensemble of 25 simulated earthquakes on the response θ. The rigid body is specified by $R = 3.66$ m and $\gamma = 5.0$. The peak acceleration A_{gx} of each the four members (No. 10, 12, 19 and 23) of the ensemble is set at 0.6. The body overturns under the

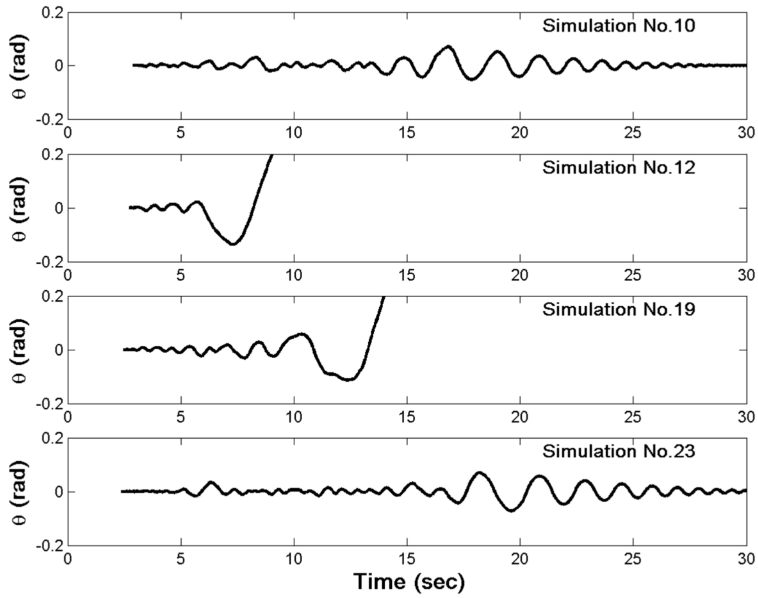

Fig. 5 Responses of a body with $R = 3.66$ m, $\gamma = 5.0$ and $e = 0.925$ subjected to four simulated earthquakes scaled to $A_{gx} = 0.6$

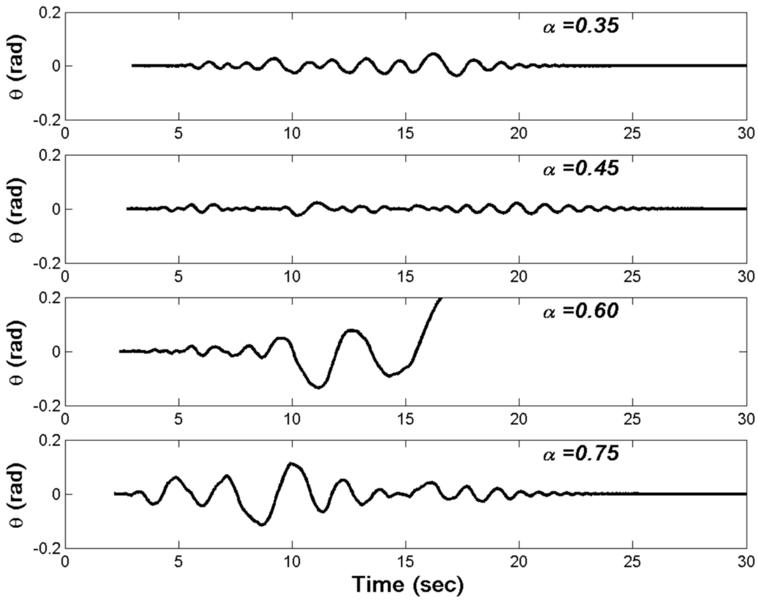

Fig. 6 Responses of a body with $R = 3.66$ m, $\gamma = 5.0$ and $e = 0.925$ subjected to simulated earthquake No.25

excitation of No. 12 and 19 accelerations but remains stable under that of No. 10 and 23 accelerations. It may be said, therefore, that the rocking response is sensitive to the details of base excitation.

Fig. 6 shows the effect of scaling factor α on rocking response. α is defined as the value of A_{gx} of individual acceleration time history. In other words, when dealing with a single earthquake, α is

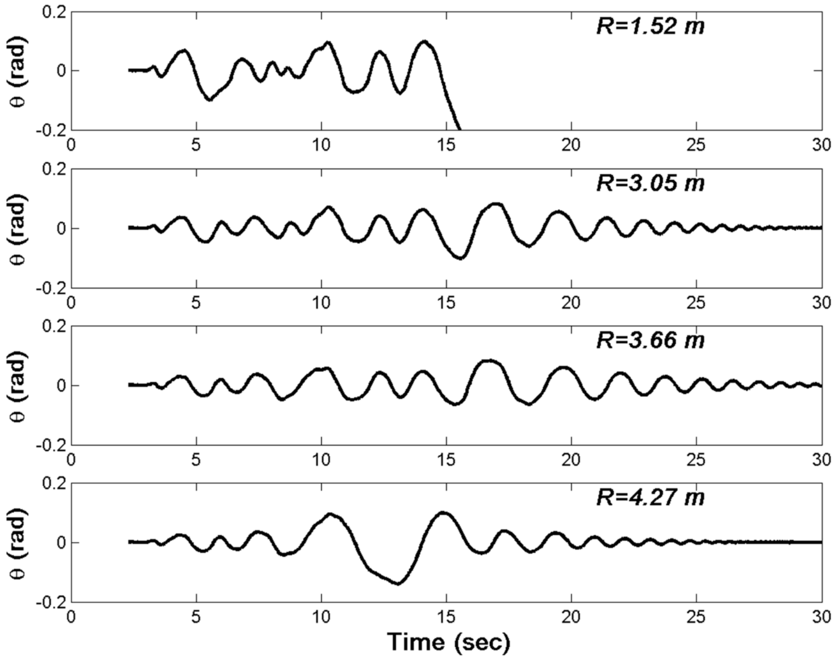

Fig. 7 Responses of bodies with $\gamma = 5.0$ and $e = 0.925$ subjected to simulated earthquake No.1 scaled to $A_{gx} = 0.5$

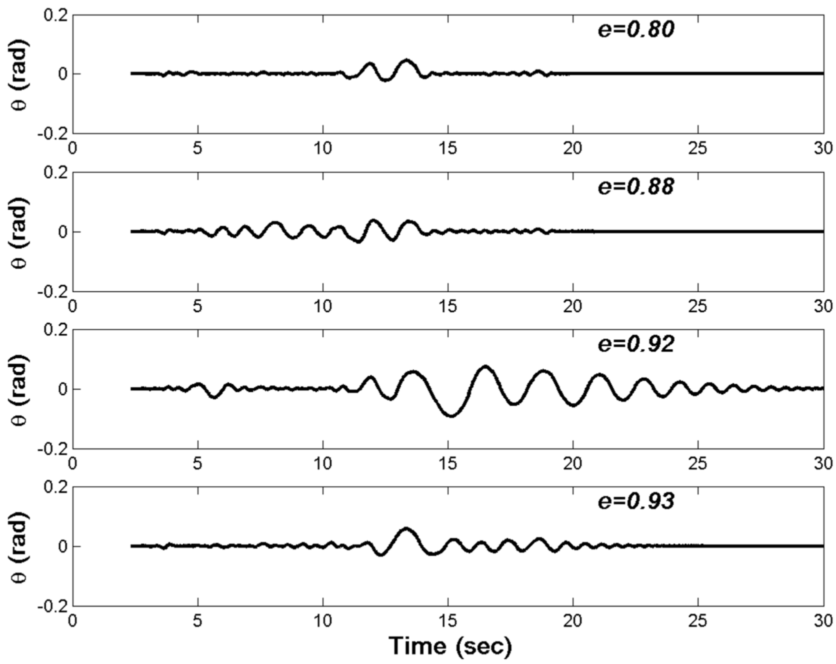

Fig. 8 Responses of a body with $R = 3.05$ m, $\gamma = 5.0$ and varying coefficient of restitution e subjected to simulated earthquake No.1 scaled to $A_{gx} = 0.5$

refers to intensity (absolute maximum acceleration) of that earthquake time history. However, in dealing with an ensemble of earthquakes, A_{gx} is used to define the average of the absolute maximum acceleration of the members of the ensemble. Therefore, it should be noted herein that the absolute maximum accelerations of the individual members (α's) are naturally different from A_{gx} value. Fig. 6 shows the responses of a body with $R = 3.66$ m and $\gamma = 5.0$ subjected to the No.25 simulated excitation with $\alpha = A_{gx}$ equal to 0.35, 0.45, 0.60 and 0.75. It is observed that the body overturns when $\alpha = 0.60$ but remains stable when $\alpha = 0.75$.

Fig. 7 shows the effect of size parameter R. The rigid bodies with $R = 1.52$, 3.05, 3.66, 4.27 m and $\gamma = 5.0$ are considered. The No. 1 simulated earthquake with $A_{gx} = 0.5$ is used as base excitation. The figure shows no systematic pattern on the response as R is varied.

The effect of coefficient of restitution e is studied using the rigid body with $R = 3.05$ m, $\gamma = 5.0$ and No.1 simulated earthquake with $A_{gx} = 0.5$. Fig. 8 gives the responses using $e = 0.80$, 0.88, 0.92 and 0.93. It is expected that the smaller the e value, the smaller the response because more energy is lost at each impact between the body and surface of the base. The response determined for $e = 0.93$ is, however, less than that determined for $e = 0.92$. Thus, the rocking response is not monotonically proportional to coefficient of restitution. Due to this observation an e value of 0.925, as assured by the study of Aslam *et al.* (1980), is used in this study.

It is due to this sensitivity of the rocking response that the following studies are carried out in the framework of theory of probability. The rocking response of a body is studied using a large number of simulated base excitations and attention is focused on the probability of the event of overturning of the body. The event of toppling is considered to occur if θ reaches $\pi/2$.

6. The effect of duration and shape of intensity function on the rocking response

An intensity function is needed in the earthquake simulation process to reflect the transient characteristics of a real earthquake. A variety of intensity functions have been proposed in the past. It is, therefore, useful to examine whether different intensity functions affect the rocking response of rigid bodies. Three commonly used intensity functions: boxcar (rectangular), trapezoidal and compound functions are investigated here (see Fig. 2). The duration of earthquakes is varied from 10 to 30 sec. To obtain statistically meaningful results, 500 earthquakes for each of the case are simulated. Figs. 9(a) and 9(b) give the overturning probabilities versus A_{gx} computed using the boxcar intensity function with earthquake durations of 10, 15, 20, 25 and 30 sec for a bodies with $R = 1.22$ m, $\gamma = 4.0$ and $R = 3.66$ m, $\gamma = 4.0$ respectively. The figure shows that as duration increases, overturning probability also increases as expected but the difference between the overturning probability curves for the cases of 25 and 30 sec earthquake duration is rather small. The same conclusion is reached when computation is carried out for other body configurations using the other two intensity functions. The respective figures are not shown for terseness.

Fig. 10 shows the overturning probabilities for the body (with $R = 1.22$ m, $\gamma = 4.0$) subjected to simulated earthquakes with durations of 15, 20, 25, and 30 sec using the intensity functions of rectangular, trapezoidal and compound type. This figure and those produced using the other body configurations show that the overturning probability of a rigid body is not appreciably affected by the choice of the intensity function.

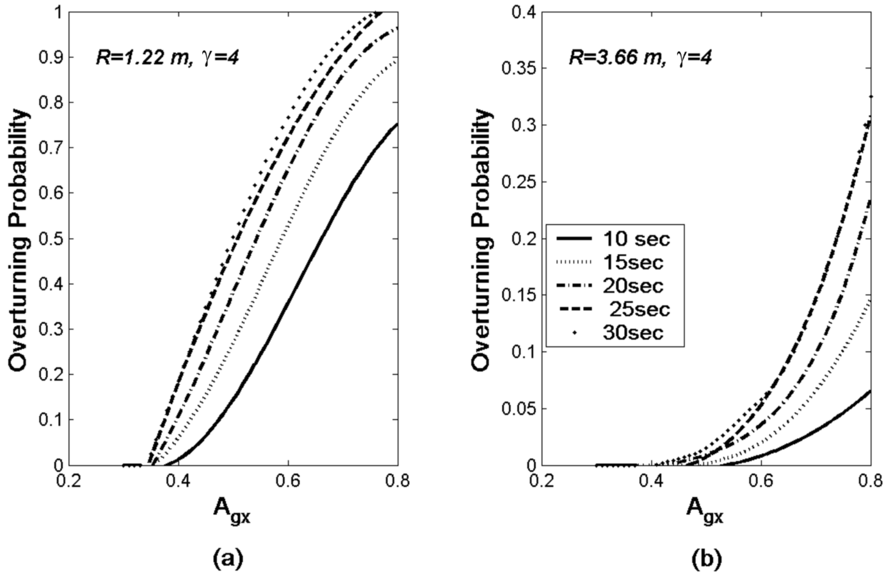

Fig. 9 Overturning probabilities of bodies with (a) $R = 1.22$ m and $\gamma = 4.0$, and (b) $R = 3.66$ m and $\gamma = 4.0$ ($e = 0.925$), using the boxcar intensity function

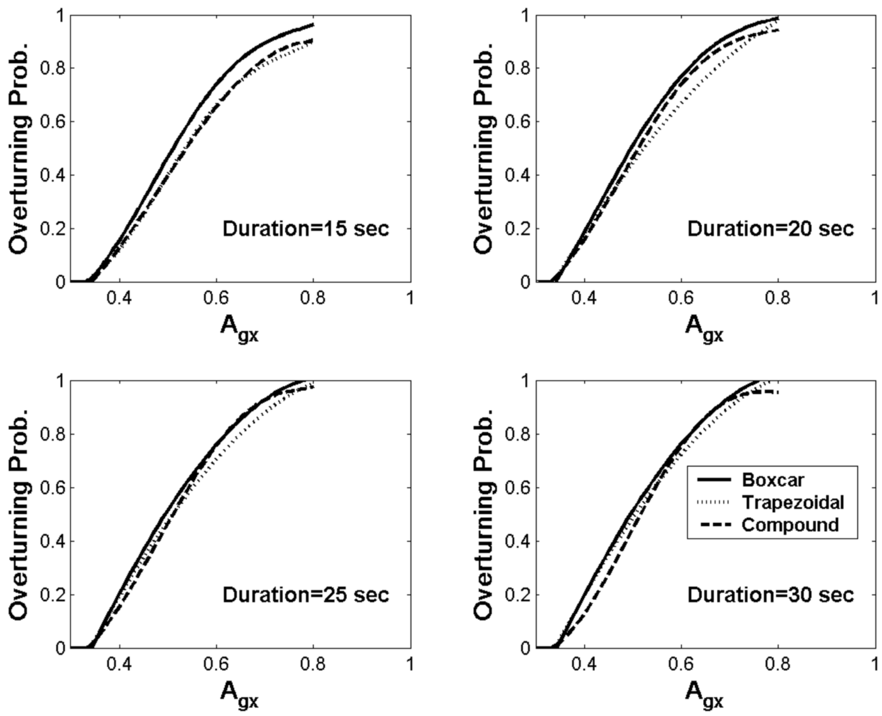

Fig. 10 Overturning probabilities of a body with $R = 4.0$ ft and $\gamma = 4.0$ ($e = 0.925$), using a duration values of 15, 20, 25, and 30 sec

7. The effect of floor accelerations on the rocking response

A study is performed to determine how an unanchored body responds differently when placed on the ground and on the floors of a building. For this purpose, a 5-story shear frame of uniform floor mass and story stiffness (with a 5% modal damping) is used. It is expected that the accelerations on the floors would be amplified and the energy contained in various frequency bands would be altered. The second factor is investigated here, since it is already known, probabilistically, that the amplified earthquakes (i.e., greater intensity) increase the overturning probability of a rigid body. To compare the overturning probability of a rigid body on the ground and on the floors, the intensity (A_{gx}) of the floor acceleration time histories is scaled to be the same as that of the ground.

The uniform 5-story shear frame with a fundamental period of vibration $T_n = 2.0$ sec is first considered. It was assumed that the structural masses are lumped at each floor level with a value of 444.82 kN/g and the total story stiffness for each floor is 55.23 kN/cm. The structure with a fundamental natural period $T_n = 2$ sec is much more flexible than typical five story frames. It is chosen such that the low frequency content of the ground excitations can be retained and reflected in floor responses. It establishes a margin in the analysis because the rocking response of rigid bodies of interest is sensitive to the low frequency components of input forcing functions. The damping ratio is assumed to be 5% for all the modes.

It is assumed that the mass of a rigid body is considerably small compared to that of the floors of a shear frame. This ensures that the mass of a rigid body does not affect the vibrational periods and modal shapes.

500 input accelerations to the base of the structure are simulated based on Regulatory Guide 1.60 (1973) response spectrum and using the compound intensity function with 20 sec duration. For each floor as well as the ground floor, the average peak acceleration, A_{gx}, ranges from 0.3 to 0.8.

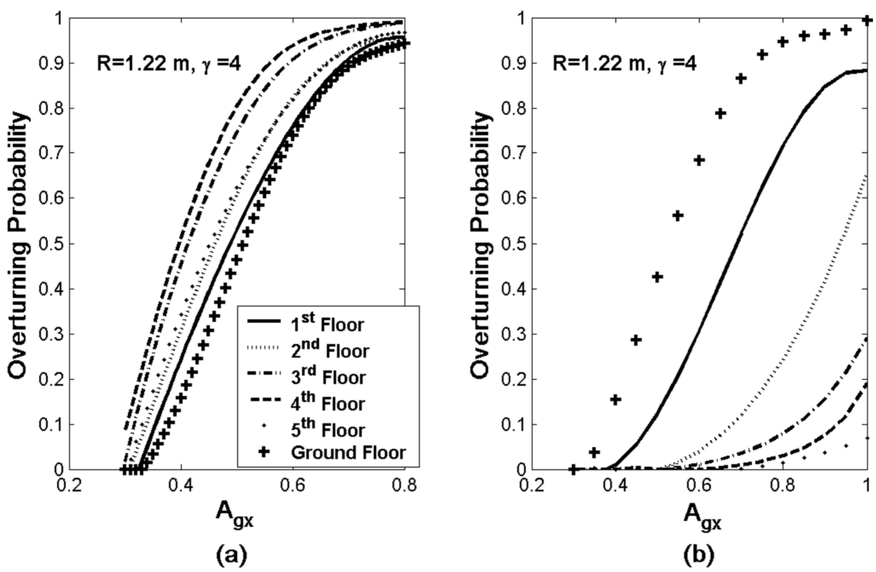

Fig. 11 Overturning probabilities of body with $R = 1.22$ m and $\gamma = 4.0$ ($e = 0.925$) placed on the shear frame with (a) $T_n = 2.0$ sec, and (b) $T_n = 0.5$ sec

Six rigid bodies with $R = 1.22$, 2.44, 3.66 m and $\gamma = 3.0$, 4.0 are analyzed. Fig. 11(a) shows the overturning probabilities for the body with $R = 1.22$ m and $\gamma = 4.0$. It is observed from the figure that the largest and the least overturning probabilities are obtained respectively when the body is placed on the fourth floor and on the ground. This trend also exists for relatively weak bodies but it starts to change as the body parameters vary such that the bodies become more stable. However, in all the cases the maximum probability of toppling is obtained for the fourth floor.

In order to understand the reason for the observed trend of toppling probabilities, average pseudo-velocity floor response spectra for 5% damping ratio for each story and ground floor are computed. The spectra are determined using an A_{gx} value of 0.6 (assuming that the average peak acceleration for a specific site is 0.6) and the results are given in Fig. 12(a). This figure shows the response quantities over a range of vibrational cyclic frequencies. The plots are presented so as to give an idea about the kinetic energy of the earthquake excitations that may be fed into the body. Note, however, that the responses of rigid bodies are not linear and do not have a unique frequency of rotation. It is naturally recognized that response spectrum gives the maximum response of a linear elastic single-degree-of-freedom system. That is, it is not the intention of this study to estimate the S_v value that corresponds to the oscillation of the rigid body of interest.

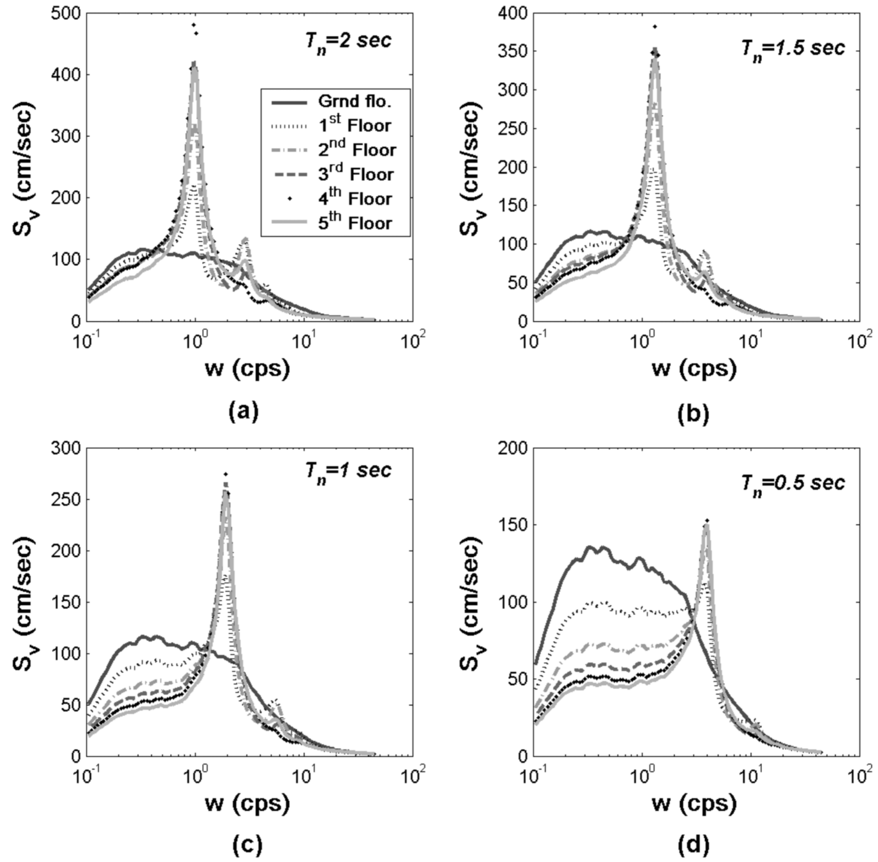

Fig. 12 Pseudo-velocity response spectra of the floors of a shear frame with (a) $T_n = 2.0$, (b) $T_n = 1.5$, (c) $T_n = 1.0$, and (d) $T_n = 0.5$ sec

It is seen from the figure that around the frequency value of 1 cps, the S_v values increase in the following order: ground, 1st floor, 2nd floor, 5th floor, 3rd floor and 4th floor. The probabilities of toppling of the bodies with $R = 1.22$ m, $\gamma = 3.0$ and $R = 1.22$ m and $\gamma = 4.0$ also increase in the same order. However, for the bodies that are relatively more stable with the frequency of oscillation in the range below 1 cps, such correspondence does not exist. This may be due to the fact that the shear frame currently employed is highly flexible. The flexibility of the shear frame accentuates the contributions of higher modes of vibration of the structure to the total response of the system (or floor accelerations in this case).

For this reason, it was decided to investigate the behavior bodies situated in a second shear frame. The total story stiffness of the shear frame is increased to 175.11 kN/cm and the floor mass is reduced to 87.59 kN/g. The fundamental natural period T_n is calculated to be 0.5 sec.

Fig. 11(b) gives the curve of the probability of toppling for the rigid body considered previously with $R = 1.22$ m and $\gamma = 4.0$. It is observed that the ground floor gives the maximum probability of toppling, and the probability of toppling of the body decreases as the floor level on which it is placed increases. This trend exists for all the rigid bodies analyzed here. The average pseudo-velocity response spectrum for each floor is again computed ($A_{gx} = 0.6$) and given in Fig. 12(d). It is seen that the ground floor produces the maximum values of pseudo-velocity up to a frequency value of 2.6 cps. The first, second, third, fourth and fifth floor give the pseudo-velocity values in a decreasing fashion. This trend is identical to the one corresponding to the probability of toppling. This suggests that the pseudo-velocity spectra may be used to give an indication of the relative magnitude of the probability of toppling of a body placed on the floors of a structural system whose fundamental period is around 0.5 sec.

Two more shear frames with $T_n = 1$ sec and $T_n = 1.5$ sec are also analyzed. Figs. 12(b) and 12(c) (again using $A_{gx} = 0.6$) show the response spectra for the shear frame with $T_n = 1$ sec and $T_n = 1.5$ sec. It is seen that there is a gradual transition pattern of the response spectra from a flexible ($T_n = 2$ sec) to a stiff structure ($T_n = 0.5$ sec). The plots of probability of toppling of the bodies placed on the ground and on the floors of these shear frames versus the ensemble average peak acceleration values are not shown here for brevity.

Comparing the four S_v figures (Figs. 12a, 12b, 12c and 12d); it may be observed that as the shear frame gets more flexible, the pseudo-velocity curves in the lower frequency range are more closely-spaced. Therefore, it may be inferred that it is hard to distinguish the severity of the response of a body located on any floor of a flexible structural system. This is especially true, when the rigid body under consideration is relatively stable against rocking motion.

This study shows that it is practically impossible to find any simple rule connecting the rocking response of a rigid body placed on a floor and on the ground of a frame. This is true for the simple uniform shear frame and is certainly true for a frame of irregular configuration and mass and stiffness distributions.

8. Energy balance equation

In 1963, Housner provided an equation for estimating the rocking stability of a rigid body. Roughly speaking, he equates the energy required to overturn a body with the energy input from the base to the body that is computed from the pseudo-velocity response spectrum S_v.

The energy balance equation takes the form

$$\frac{1}{2}\frac{W}{g}\frac{MR^2}{I_O}S_v^2\cos\theta_c^2 = W\Delta \quad (10)$$

in which $\Delta = R(1 - \cos\theta_c)$ is the vertical displacement of the center of mass of the body from the position $\theta = 0$ to that when $\theta = \theta_c$. Upon substituting I_O and Δ into Eq. (10)

$$S_v = \frac{1}{\cos\theta_c}\sqrt{\frac{8}{3}gR(1-\cos\theta_c)} \quad (11)$$

is obtained. When S_v satisfies Eq. (11), there is approximately 50% chance that the base motion will cause the body to overturn. He also noted that Eq. (11) was derived assuming that the ground acceleration is such that the average velocity response spectrum (undamped) is constant.

To verify the validity of this equation, 200 earthquakes with a compound intensity function and duration of 20 sec are produced from the response spectrum of Regulatory Guide 1.60. For a given body, say, with $R = 1.22$ m and $\gamma = 2.0$, a plot of overturning probabilities versus A_{gx} is constructed and is given in Fig. 13. From this figure, the A_{gx} value corresponding to a given level of probability of overturning can be obtained. Housner (1963) suggested that the energy balance equation is satisfied when there is approximately 50% probability of overturning. To verify the equation, the level of probability of overturning is therefore selected to be 50%. From Fig. 13, the A_{gx} value corresponding to 50% probability of overturning is 1.02. The time histories of the 200 base accelerations are adjusted accordingly and the pseudo-velocity response spectra are obtained. The average pseudo-velocity response spectrum is shown in Fig. 14. This average pseudo-velocity response spectrum is to be distinguished from the average of the pseudo-velocity response spectra generated from the 200 base accelerations. The former, referred to as the calculated response

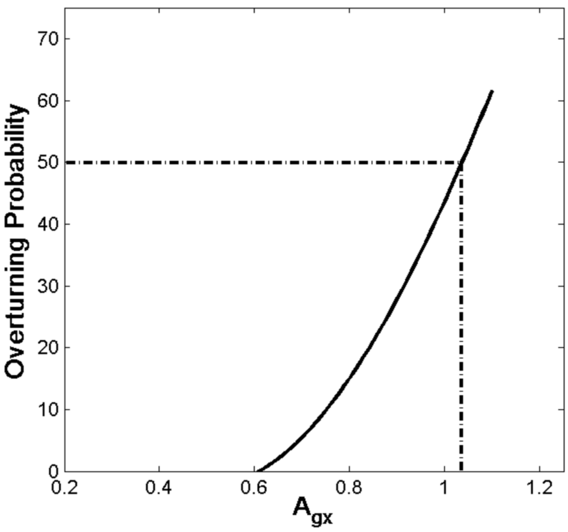

Fig. 13 Overturning probability of body with R=1.22 m and γ = 2.0

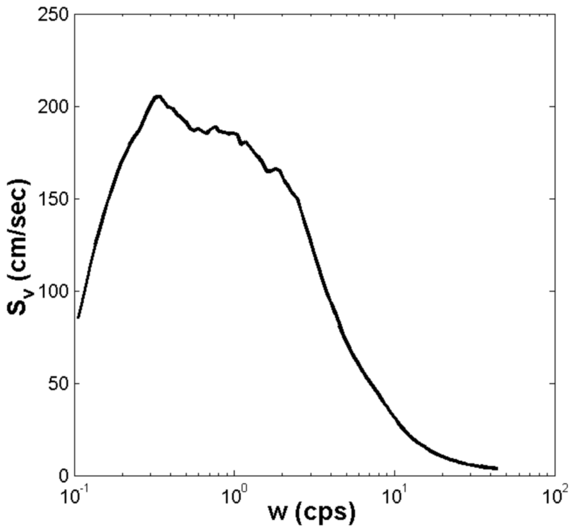

Fig. 14 The average S_v spectrum corresponding to 50% overturning probability of a body with R = 1.22 m and γ = 2.0 (e = 0.925)

spectrum and denoted S_{v1}, is dependent on the specific body whose stability against overturning is being examined and the latter, denoted S_{v2}, is independent of the body under consideration. Since the pseudo-velocity response spectrum is not constant but varies with frequency, it is not possible to specify which value of S_v to use to compare it with the required S_v value given by Eq. (11). If we

Table 1 Required S_v (cm/sec) to overturn a body with 50% probability (Eq. 11)

		γ								
		2.0	**2.5**	**3.0**	**3.5**	**4.0**	**4.5**	**5.0**	**5.5**	**6.0**
***R* (m)**	**1.22**	205.11	163.40	134.82	115.19	100.56	89.26	80.24	72.90	66.78
	1.52	229.34	181.84	150.75	128.78	112.45	99.80	89.71	81.48	74.65
	1.83	251.23	199.19	165.13	141.07	123.16	109.30	98.27	89.28	81.79
	2.13	271.35	215.16	178.36	152.37	133.05	118.08	106.15	96.42	88.34
	2.44	290.09	230.02	190.68	162.92	142.24	126.24	113.46	103.07	94.44
	2.74	307.70	243.97	202.23	172.80	150.85	133.88	120.37	109.32	100.15
	3.05	316.71	257.15	213.18	182.14	159.03	141.12	126.87	115.24	105.59
	3.35	340.16	269.72	223.57	191.03	166.78	148.03	133.07	120.88	110.72

Table 2 Calculated S_{v1} (cm/sec) to overturn a body with 50% probability

		γ								
		2.0	**2.5**	**3.0**	**3.5**	**4.0**	**4.5**	**5.0**	**5.5**	**6.0**
***R* (m)**	**1.22**	205.11	166.90	138.73	116.64	102.54	90.47	84.46	76.40	70.38
	1.52	229.24	184.99	152.83	132.72	114.60	102.54	90.47	86.46	78.41
	1.83	255.37	201.07	166.90	144.78	124.66	110.59	102.54	90.47	82.45
	2.13	275.49	221.18	180.98	158.85	134.72	120.65	110.59	100.53	93.50
	2.44	293.57	233.25	197.05	170.92	146.79	126.67	116.64	106.58	98.53
	2.74	301.63	245.31	211.12	180.98	154.84	140.77	124.66	114.60	106.58
	3.05	335.81	269.44	225.20	191.03	164.90	150.80	134.72	120.65	110.59
	3.35	359.94	285.52	241.30	209.12	176.94	160.86	144.78	130.71	120.65

Table 3 Ratio of S_{v1} to S_v

		γ								
		2.0	**2.5**	**3.0**	**3.5**	**4.0**	**4.5**	**5.0**	**5.5**	**6.0**
***R* (m)**	**1.22**	1.000	1.021	1.029	1.013	1.020	1.014	1.053	1.048	1.054
	1.52	1.000	1.017	1.014	1.031	1.019	1.027	1.008	1.061	1.050
	1.83	1.016	1.009	1.011	1.026	1.012	1.012	1.043	1.013	1.008
	2.13	1.015	1.028	1.015	1.043	1.013	1.022	1.042	1.043	1.058
	2.44	1.012	1.014	1.033	1.049	1.032	1.003	1.028	1.034	1.043
	2.74	0.980	1.006	1.044	1.047	1.026	1.051	1.036	1.048	1.064
	3.05	1.060	1.048	1.056	1.049	1.037	1.069	1.062	1.047	1.047
	3.35	1.058	1.059	1.079	1.095	1.061	1.087	1.088	1.081	1.090

choose the maximum value of the average pseudo-velocity (= 205.11 cm/sec) in Fig. 14 and compare it with required S_v value (= 205.11 cm/sec) from Eq. (11), we see that these two values are exactly the same. Encouraged by this result, the analysis is then extended to 72 rigid bodies with eight size parameter R values of 1.22 to 3.35 m (with 0.30 m increment) and nine slenderness ratio γ values of 2.0 to 6.0 (with 0.5 increment). The required S_v and the calculated S_{v1} are given separately in Tables 1 and 2 and the ratio of S_{v1} to S_v in Table 3. The agreement between S_v and S_{v1} is seen to be extremely good. In other words, the energy balance equation does what Housner claimed it would do. The goodness of the energy balance equation may also be shown in another way. For each given body, the required S_v value is given by Eq. (11) and the A_{gx} value corresponding to 50% probability of overturning is determined as described earlier. Fig. 15 gives the relation between the required S_v value and A_{gx} for the 72 bodies considered. The solid line is obtained by curve fitting. The relation between the required S_v and A_{gx} is linear and the 72 points all lie very close to the solid line indicating that the energy balance equation predicts accurately the energy level required to overturn a body with 50% probability.

To see if the energy balance equation works equally well for a body placed on the floors of a building, the five-story uniform shear frame described earlier with fundamental natural period T_n = 2.0 sec is considered. The absolute accelerations of the floors due to the 200 ground excitations are obtained. For each floor, the average A_{gx} is adjusted to be the same as that of the ground and the acceleration time histories are adjusted accordingly. The 72 bodies are placed on the floors and the probabilities of overturning are obtained for a range of values of A_{gx}. The floor A_{gx} values corresponding to 50% probability of overturning are determined and plotted against the required S_v. The points in Fig. 16, and in the other respective figures not presented here, are somewhat more scattered than those in Fig. 15 but the scatter is small so that the energy balance equation may still be considered quite satisfactory.

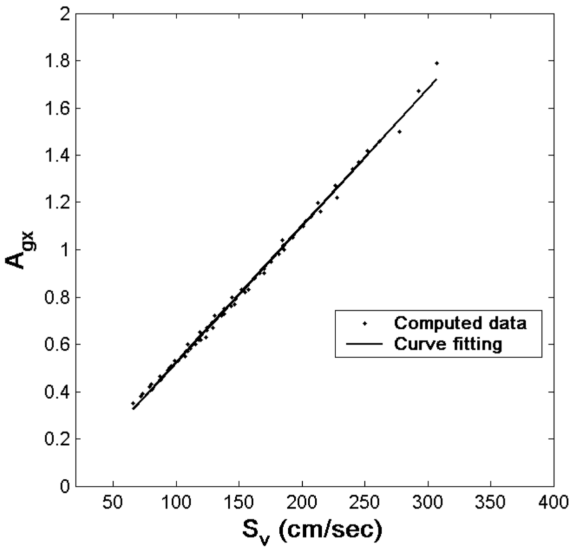

Fig. 15 S_v and A_{gx} values required to overturn the 72 bodies placed on the ground

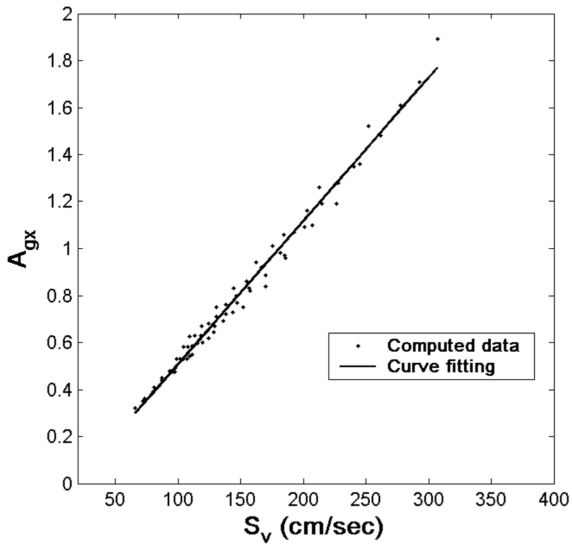

Fig. 16 S_v and A_{gx} values required to overturn the 72 bodies placed on the first floor of a shear frame with T_n = 2.0 sec

It should be mentioned that the average floor-velocity response spectra almost invariably have several spikes at certain frequencies. Using the maximum value of the spectrum may lead to erroneous or ambiguous conclusions. One should, however, realize that the frequency of oscillation of a rigid body tends to be rather small, especially when overturning is impending. This being the case, in this study, we have used the maximum values of S_v in the frequency range between 0.04 and 0.6 cps.

Having shown the robustness of the energy balance equation, the following examples are given to explain how it can be used. The pseudo-velocity response spectra for the ground and for the floors (scaled to have the same A_{gx} value as that of the ground) of a building are first obtained. Using the Regulatory Guide 1.60 response spectrum with $A_{gx} = 0.6$, such response spectra are given for the uniform 5-story shear frame with $T_n = 2.0$ sec and 5% modal damping ratio in Fig. 12(a). Consider a body with $R = 1.22$ m and $\gamma = 2.0$ placed on the first floor of the shear frame. This figure shows that in the low frequency range (<0.6 cps), the maximum S_v value is approximately 114.30 cm/sec. Eq. (11) or Table 2 shows that the required S_v is 205.11 cm/sec, which is greater than 114.30 cm/sec. Therefore, the body is safe against overturning. As a second example, consider a less stable body with $R = 1.22$ m and $\gamma = 4.0$ placed on the first floor. Eq. (11) gives 100.56 cm/sec for the value of the required S_v, which is smaller than 114.30 cm/sec. The body will, therefore, overturn with a probability of more than 50%.

9. Conclusions

By examining the rocking response of a body subjected to a single base excitation, it is found that

1. the rocking response is sensitive to baseline correction of the simulated acceleration time history,
2. the details of excitation time history, the intensity of an earthquake, the size of the body, its slenderness ratio as well as the coefficient of restitution, all affect the rocking response of a rigid body in a manner that no systematic pattern can be discerned,

   However, when studied statistically, a clear trend of behavior emerges. It is found that,
3. as the peak acceleration increases, the overturning probability also increases,
4. as the duration of earthquake increases, the overturning probability increases,
5. as the duration of earthquake increases beyond 25 sec, the overturning probability does not increase appreciably,
6. the shape of intensity function has a little effect on the overturning probability,
7. in general, there is no clear correlation between the overturning probability of a body placed on the ground and on the floors of a building,
8. if the structure is stiff with natural fundamental period less than 0.5 sec, say, the overturning probability of a body placed on the ground tends to be larger than that placed on the floors of the structure,
9. Housner's energy balance equation has been verified: it is robust, easy to use and can be applied to assess the overturning potential of a body placed on the ground and on the floors of a building.

In closing, it is emphasized that the work reported in this paper necessarily has its limitations. This stems from the fact that the model of the body considered is rather simple: it is rigid, homogeneous, rectangular and has a rectangular footprint. Also, the base excitations are horizontal and one-dimensional and are generated by simulation using a specific intensity function (and duration) and based on a specific response spectrum. The building model considered is a simple shear frame with uniform story mass and stiffness.

Using the method adopted in this study, the models of the body, the ground motion and the structure can all be modified.

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