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Strength enhancement in confined concrete with consideration of flexural flexibilities of ties

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Abstract. The interaction between concrete core expansion and deformation of perimeter ties has been known to have a significant effect on the effective confinement of rectangular reinforced concrete (RC) tied columns. This interaction produces passive confining pressure to the concrete core. Most existing models for determining the response of RC tied columns do not directly account for the influence of flexural stiffness of the ties and the variation of confining stress along the column height. This study presents a procedure for determining the confined compressive strength of RC square columns confined by rectilinear ties with various tie configurations considering directly the influence of flexural flexibility of the ties and the variation of confining stress along the vertical direction. The concept of area compatibility is employed to ensure compatibility of the concrete core and steel hoop in a global sense. The proposed procedure yields satisfactory predictions of confined strengths compared with experimental results, and the influence of tie flexibility, tie configuration and degree of confinement can be well captured.

Key words: reinforced concrete columns; confinement; flexural flexibility; rectilinear ties; tie configurations.

1. Introduction

The characteristic of confined concrete is one of the most interesting research topics during the last two decades. Unlike the confinement provided by active hydrostatic pressure, it is well known that the confinement pressure in a reinforced concrete (RC) column is passive in nature and varies along the height of the column and over the cross-section of the concrete core. Under the applied axial strain, the level of stress in the transverse steel, which produces the passive confining pressure, depends on the amount of lateral expansion on the concrete core, and the flexibilities of the transverse steel. Therefore, in order to accurately predict the behavior of confined RC columns, the interaction between the deformation of the concrete core and transverse steel must be considered.

Due to the complexity of the problem, the majority of researchers have used approximate models to predict the effective confinement pressure exerted by rectilinear ties on the concrete core. Sheikh and Uzumeri (1982) and later Mander *et al*. (1988) made use of the concept of an effectively confined concrete area to determine the confinement effectiveness coefficient, K_e, defined as the

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ratio of the effective confinement pressure to the nominal value obtained from equilibrium consideration. Saatcioglu and Razvi (1992) have proposed an empirical formula, based on regression analyses of test results, to determine K_e. In both of these approaches, it is either explicitly or implicitly assumed that the perimeter ties and crossties reach the yield strength at the peak response. However, Ahmad and Shah (1982) and Madas and Elnashai (1992) showed that this assumption is not always valid. A rational method was introduced by Cusson and Paultre (1995) based on strain compatibility and transverse force equilibrium to obtain stress-strain relationship for high-strength square RC columns. The actual column is replaced by an equivalent circular column. Thus, only extension in the hoops is considered. An iterative procedure is needed for determining the steel stress at the peak confined strength. Modification of the Cusson and Paultre model has been made to extend its applicability to square and circular columns made of normal- or high-strength materials (Paultre and Légeron 1999, Légeron and Paultre 2003). A simpler approach based on regression analysis of test results was proposed by Razvi and Saaticioglu (1999) to determine the stress in transverse steel at peak confined strength. An empirical formula was given for this purpose, avoiding the need to perform lengthy iterations.

Although the influence of flexural stiffness of perimeter ties on the confining pressure has been investigated by experiments (Sun *et al*. 1996, Sato and Yamaguchi 2000), very few researchers studied this problem analytically. Assa *et al*. (2001a, 2001b) proposed analytical models based the expansion of concrete core and the transverse deformation of perimeter ties for determining the confining pressure at peak strength for RC columns. It should be noted that, for tied columns, the nonuniform confinement pressure along the height of the column is transformed to an equivalent uniform confining stress provided by an equivalent circular envelope, using the strain energy concept.

This study presents an alternative rational procedure for determining the strength enhancement of both normal- and high-strength RC square columns under uniaxial vertical loading, taking into account the interaction between the concrete core and transverse steel and the variation of confining stress along the height of the column. The effect of flexural flexibility of perimeter ties is directly included in this study. The proposed approach has the advantage that one can obtain the effective confining pressure directly from mechanics principles rather than from regression analyses of test results or from the concept of an effectively confined area which have more limitations in application. Buckling of longitudinal bars is not considered in the formulation.

2. The concept

Fig. 1 shows a RC column with hoops spaced vertically at spacing of *s*. Under a uniform axial strain, the core would expand uniformly [as shown in dashed lines in Fig. 1(b)] if there were no hoops confining the core. The lateral expansion of the core stretches and bends the hoop, resulting in interactive line forces at the hoop positions as depicted in Figs. 2(a) and 2(b). The interactive forces produce a non-linear confining pressure along the height as shown in Fig. 3. Results from three-dimensional finite element analyses of an elastic concrete core bound by elastic hoops are used to obtain simple relations for the variation of confining stress over the vertical hoop spacing and the confining force transferred by the perimeter ties. The equilibrium condition is invoked on the effective stress resultant in the concrete and the axial forces in the ties. The concept of area compatibility is employed to ensure compatibility of the concrete core and steel hoop in a global

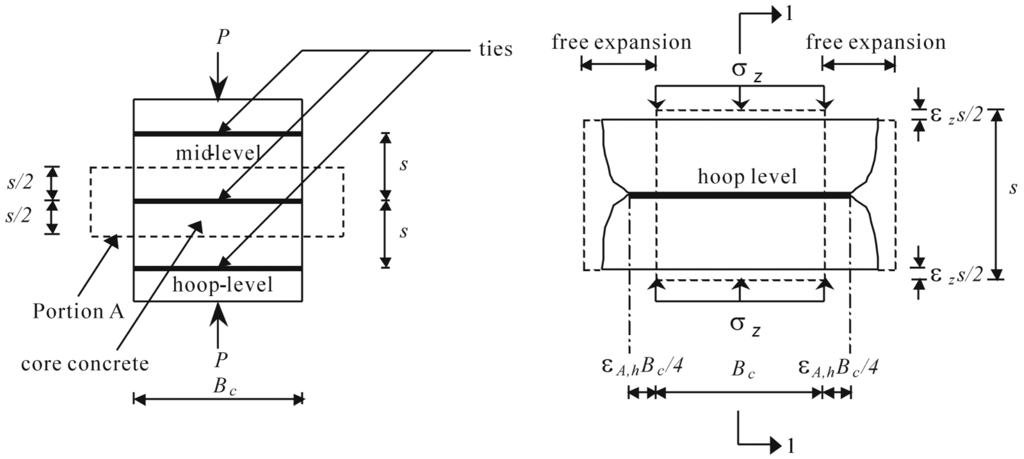

Fig. 1 (a) Column core under consideration, (b) Deformation of Portion A

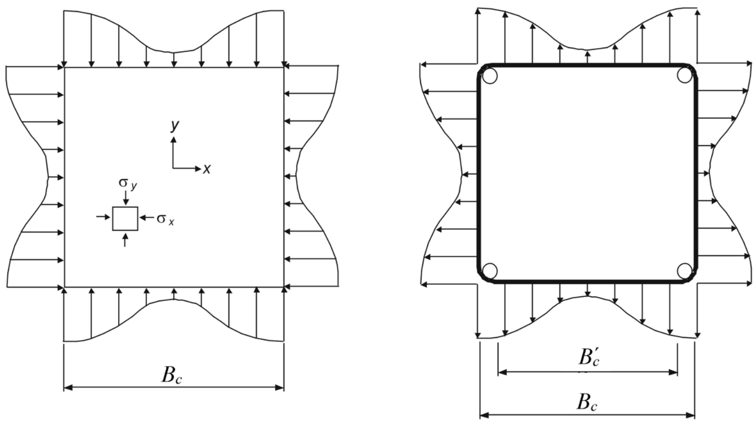

Fig. 2 Interactive confining forces at hoop level: (a) acting on concrete, (b) acting on perimeter tie

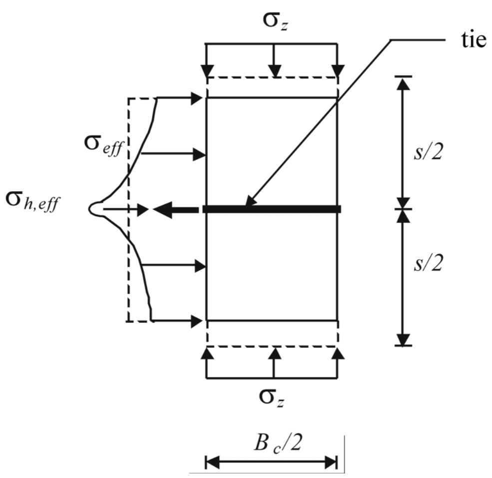

Fig. 3 Internal stress distribution in the concrete core at section 1-1

sense. The condition of zero volumetric strain is used to specify the state at which the concrete attains the peak compressive strength as suggested by Imran and Pantazopoulou (1996). Due to the non-linear nature of concrete behavior, an iterative procedure is needed in the computation. The resulting axial stress with the influence of confinement is then obtained using the well known Richart model (Richart *et al*. 1928).

3. Relationships between peak and average stresses and strains

As mentioned earlier, under the applied axial strain, the concrete core expands and induces interactive confining forces between the concrete core and the hoops. The interactive forces produce non-linear confining stresses over the cross-section and along the vertical direction. The determination of the strength of concrete under triaxial stress state involves a three-dimensional (3-D) problem, which is difficult to solve theoretically.

In order to transform the complicated 3-D problem to a 2-D one, a 3-D finite element model is utilized to determine the effective confining stress at the hoop level, $\sigma_{h,\,eff}$, the average effective confining stress, σ_{eff}, the interactive line forces acting on the hoop as well as the area strains averaged over the cross-sectional area at the hoop level, $\varepsilon_{A,\,h}$, and the average area strain over the column height, $\varepsilon_{A,\,ave}$. First, consider the effective confining stress at any level z defined as (Mau *et al*. 1998):

$$\sigma_{eff}(z) = \frac{1}{A}\int_A \frac{(\sigma_x + \sigma_y)}{2} dA \tag{1}$$

where σ_x and σ_y are the stresses in the x and y directions, respectively, and A is the area of the concrete core. In this study, compressive stresses and strains are assigned negative values. The average effective confining stress, σ_{eff}, is then determined by integrating $\sigma_{eff}(z)$ along the vertical direction and dividing the result by the spacing. It is thus uniform over the cross section and along the vertical spacing (see Fig. 3).

In the finite element modeling, the concrete core is modeled using 8-node brick elements, while the perimeter tie is discretized as beam elements. In addition, rigid truss elements are used to capture the interactive forces between the concrete core and the perimeter ties. Based on the results from 3-D finite element analyses shown in Figs. 4(a) and 4(b), the following relations are obtained by simple regression:

$$\frac{\sigma_{h,\,eff}}{\sigma_{eff}} = 1 + 3.1342\frac{s}{B_c'} \tag{2}$$

$$\varepsilon_{A,\,ave} - \varepsilon_{A,\,h} = \frac{2(1-\nu_e)(\sigma_{eff} - \sigma_{h,\,eff})}{E_c} \tag{3}$$

in which s and B_c' are, respectively, the hoop spacing and the clear distance between the two longitudinal bars at the corners of the hoop [as shown in Fig. 2(b)], and ν_e is the equivalent Poisson's ratio corresponding to the level of expansion of concrete which can be determined from the following relation:

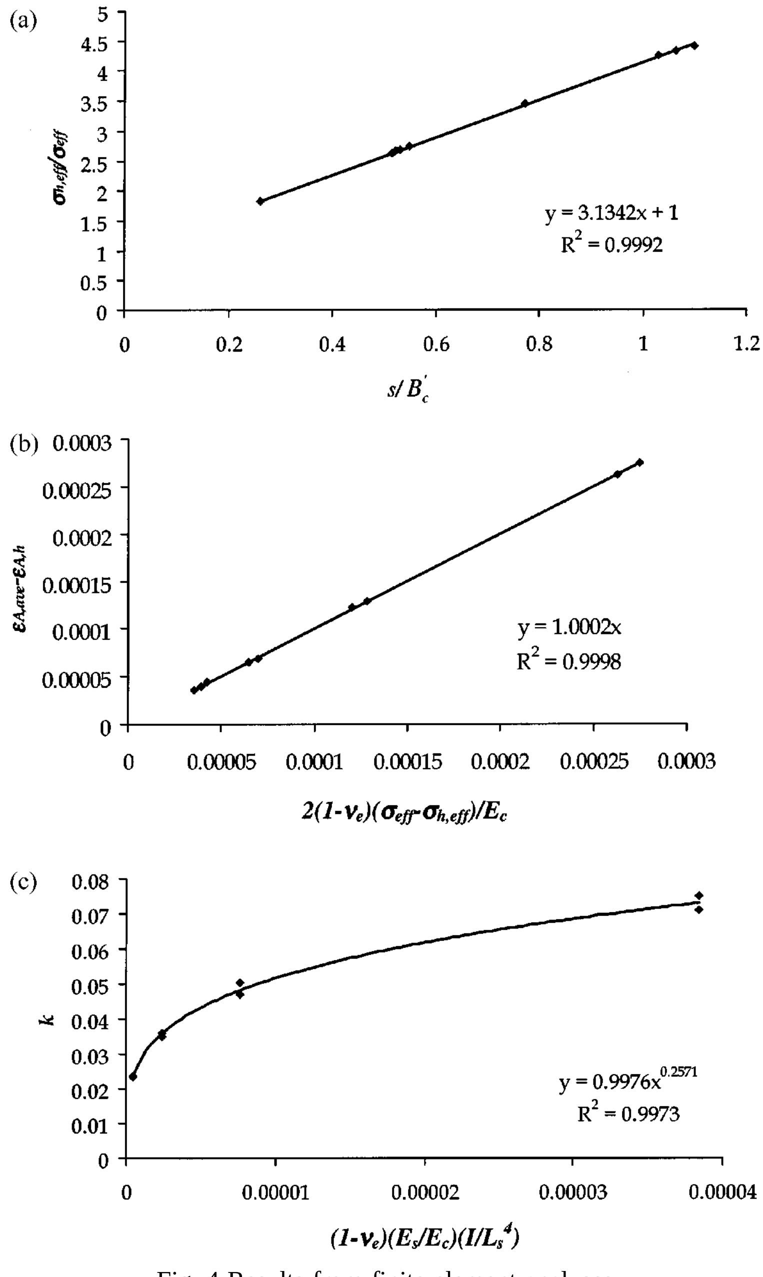

Fig. 4 Results from finite element analyses

$$\nu_e = \frac{\left(\frac{\Delta\sigma_{eff}}{E_c} - \frac{\Delta\varepsilon_{A,\,ave}}{2}\right)}{\left(\frac{\Delta\sigma_{eff}}{E_c} + \Delta\varepsilon_z\right)} \tag{4}$$

where ε_z is the applied axial strain, Δ denotes the incremental quantity, and E_c is the residual elastic stiffness in the concrete core which can be expressed in terms of the initial stiffness, $E_{c,\,i}$ and the average area strain as follows (Imran and Pantazopoulou 1996):

$$\frac{E_c}{E_{c,\,i}} = \frac{1}{1 + \dfrac{\varepsilon_{A,\,ave}}{0.05}} \tag{5}$$

4. Deformation of concrete under triaxial stress state

Pantazopoulou and Mills (1995) illustrated that damage in concrete due to microcracking was manifested by the volumetric expansion of the material. The rate of volume change (i.e., volume increment per unit of initial volume) represents the volumetric strain, ε_v. Partial or total restraint against expansion, usually imposed through the boundary conditions, has a profound influence on the internal stress state of the material.

Under a triaxial stress state, the volumetric strain, ε_v, of the concrete core is given by the following equations (Imran and Pantazopoulou 1996):

(a) Prior to cracking in the lateral direction, $\varepsilon_z > \varepsilon_z^{lim}$,

$$\varepsilon_v = (1 - 2\nu_i)\left(\frac{2\sigma_{eff}}{E_c} + \varepsilon_z\right) \tag{6}$$

where ε_z^{lim} and ν_i are the axial strain that induces cracking in the lateral direction and initial Poisson's ratio, respectively. The value of ε_z^{lim} is

$$\varepsilon_z^{lim} = \left(\frac{1 - \nu_i}{\nu_i E_c}\right)\sigma_{eff} - \frac{\varepsilon_{cr}}{\nu_i} \tag{7}$$

in which ε_{cr} is the cracking strain of concrete in direct tension.

(b) After cracking, $\varepsilon_z < \varepsilon_z^{lim}$,

$$\varepsilon_v = (1 - 2\nu_i)\left\{\frac{2\sigma_{eff}}{E_c} + \varepsilon'_{cc}\left[\frac{\varepsilon_z}{\varepsilon'_{cc}} - \left(\frac{\varepsilon_z - \varepsilon_z^{lim}}{\varepsilon'_{cc} - \varepsilon_z^{lim}}\right)^2\right]\right\} \tag{8}$$

where ε'_{cc} is the strain at peak confined strength. Subtracting Eq. (6) or (8) by the applied axial strain, ε_z, results in the average area strain of the concrete core, $\varepsilon_{A,\,ave}$. As suggested by Imran and Pantazopoulou (1996), ε'_{cc} is related to the peak confined compressive strength, f'_{cc}, by

$$\frac{\varepsilon'_{cc}}{\varepsilon_{co}} = 5\left[\frac{f'_{cc}}{f'_{co}} - 0.8\right] \tag{9}$$

$$f'_{cc} = f'_{co} + 4.1\sigma_{eff} \tag{10}$$

where f'_{co}, and ε_{co} are the unconfined compressive strength and the corresponding strain, respectively. The latter can be approximated from the following equation (Razvi and Saatcioglu 1999):

$$\varepsilon_{co} = 0.0028 - 0.0008\frac{40}{f'_{co}} \tag{11}$$

in which f'_{co} is in Megapascals.

5. Equilibrium condition

The global equilibrium of the concrete-tie system can be readily determined from simple statics of the free body shown in Fig. 5 in which F_h, and F_c are the tensile forces in each perimeter tie and crosstie, respectively, and F_t is the tensile force in the concrete cover. The global equilibrium equation can be expressed as follows:

$$2F_h + \sum_{i=1}^{N} F_c + 2F_t = \sigma_{eff} s B_c \tag{12}$$

in which N and B_c are the number of crosstie legs in each direction and the width of the concrete core measured from center-to-center of perimeter tie, respectively. The tensile force in the concrete cover is given by

$$F_t = \sigma_t s t_{cov} \tag{13}$$

where σ_t is the tensile stress and t_{cov} is the thickness of the concrete cover. The tensile stress, σ_t, can be determined from the bilinear strain softening relation as a function of the crack width w (Rokugo *et al.* 1989) as shown in Fig. 6. The critical crack width, w_{cr}, can be obtained from the following equation:

$$w_{cr} = \frac{5G_F}{f_t} \tag{14}$$

where f_t is the tensile strength and G_F is the fracture energy of concrete.

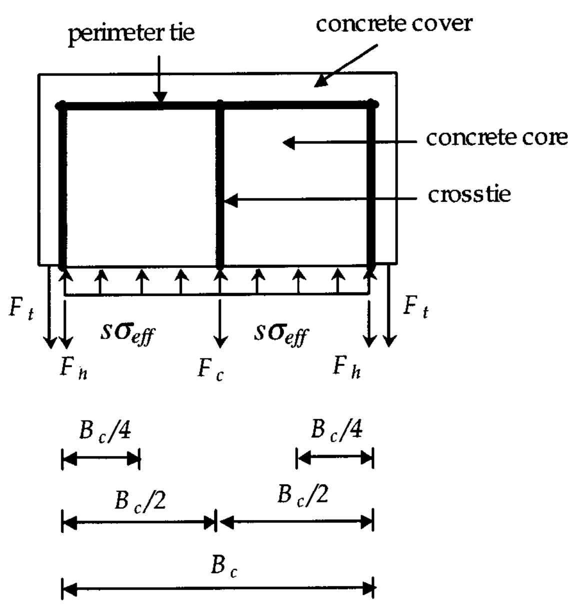

Fig. 5 Equilibrium condition

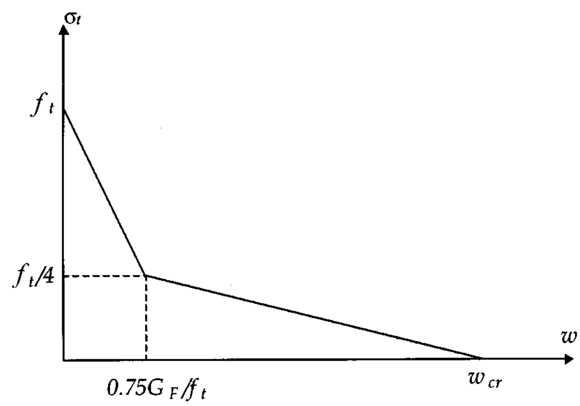

Fig. 6 Tensile strength-crack width relationship (Rokugo *et al.* 1989)

The equilibrium equation, Eq. (12), can only be solved rigorously by using finite element analyses. However, it can be further simplified if the relationship between F_h and F_c is known. To this end a number of finite element analyses of an elastic core confined by perimeter ties and center crossties having the same diameter were performed. With the volumetric ratio of the transverse steels varied from 0.5 to 6.7%, resulting in the tie leg clear span to tie diameter ratios of 8 to 33, respectively, the finite element results indicate that the F_c/F_h ratio varies from 1.3 to 2.1. In the computation procedure proposed, the tensile forces in perimeter ties and crossties are determined, for simplicity, by using the tributary area as shown in Fig. 5 resulting in the ratio of F_c/F_h equal to 2 for equally spaced ties. This leads to a maximum discrepancy of 6% in the computed average effective confining stress, σ_{eff}, compared with the finite element results. For the effective confinement index, σ_{eff}/f'_{co}, in the range of 0.01 to 0.27 covering low to high confinement levels, the resulting error in the confined compressive strength would not be greater than 3%. The simplified load distribution assumption of F_c/F_h equal to 2 is therefore acceptable. Substituting

$$F_c/F_h = 2 \tag{15}$$

into Eq. (12) leads to

$$F_c = \left(\sigma_{eff} - \frac{2F_t}{sB_c}\right)\left(\frac{B_c}{N+1}\right)s \tag{16}$$

After concrete cover spalling, the tension in the concrete cover vanishes and the axial forces in the perimeter ties and crossties can be determined from the above equations by setting F_t equal to zero.

6. Modeling of hoop

Due to the flexibility of the hoop under bending, the interactive transverse forces between the concrete core and the hoop vary non-linearly along the tie leg, with much concentration at the corners and at the crosstie intersections (if crossties are provided). Prediction of the transverse interactive force distribution in the tie leg is a very complicated problem due to the nonlinear behavior of concrete. Moreover, the distribution of the interactive transverse forces and the contact length between the core concrete and the perimeter hoop change significantly after spalling of the concrete cover. Therefore, the patterns of the transverse interactive force distribution can only be obtained by means of the nonlinear finite element method, which does not lend itself to practical computation intended for this study. However, the resultant of the transverse interactive forces can be easily obtained by using the global equilibrium condition given in Eq. (12). Thus, for simplicity, the deformation of the steel hoop is approximately computed from the resultant forces acting at distances kL_s from the supports as shown in Fig. 7. For ties with Young's modulus, E_s, moment of inertia, I, and unsupported length, L_s, the parameter k can be estimated from Fig. 4(c) which depicts the result of a regression analysis of 3-D finite element solutions of concrete core-tie systems described earlier.

For simplicity the perimeter tie is modeled as beam elements with the ends fixed, as shown in Fig. 7. In case the hoop is restrained internally by a crosstie, the intermediate crosstie support is also assumed to be restrained against rotation. The assumption of fixed end supports of the hoop is guided by 3-D finite element solutions of the core-tie system which indicate high bending moments

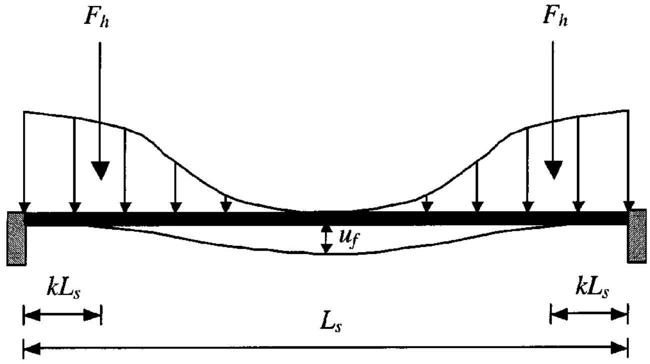

Fig. 7 Distribution of interactive line forces and resultant forces on part of perimeter tie at transverse restraints

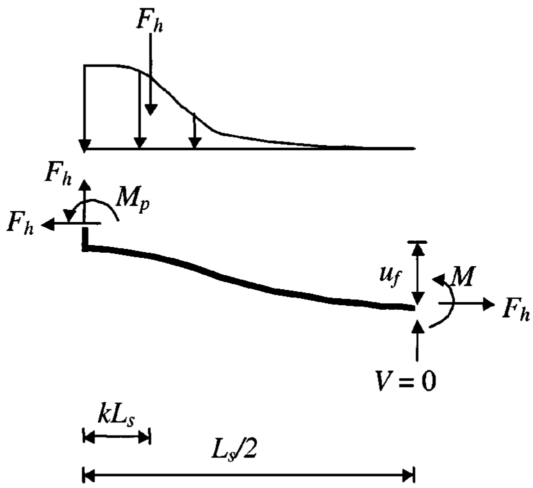

Fig. 8 Equilibrium condition of hoop leg with two plastic hinges formed at the end supports

developed at the joints of the hoop. Theoretically, such joints should be modeled as rotationally restrained supports. However, for simplicity, the fixed-end support conditions are acceptable in practice. Crossties (if provided) are modeled as axial bar elements.

Fig. 8 shows the free-body diagram of part of the perimeter tie. Neglecting bond between the concrete and hoop after the covering spalls off, and considering equilibrium and symmetry of the hoop under the action of the interactive transverse forces and hoop tension, one may readily show that the resultant of the interactive transverse forces must balance the axial force in the hoop leg in the direction considered. Furthermore, the first two plastic hinges are formed at the fixed ends. The value of the plastic moment, M_p, is influenced by the presence of the axial force, which can be approximated by the following equation (Chen and Sohal 1995):

$$M_p = \frac{d_h^3}{6} f_{yh} \cos\left(\frac{\pi F_h}{2 F_y}\right) \tag{17}$$

where d_h, f_{yh} and F_y are the hoop diameter, the yield strength and the yielding force of the perimeter hoop, respectively. The failure mechanism is attained when the final plastic hinge forms at mid-span.

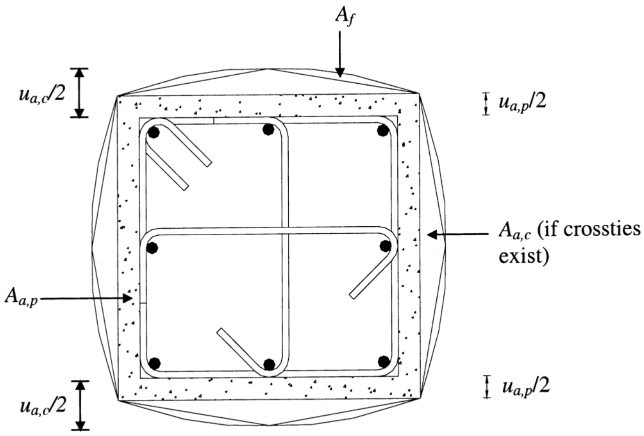

Fig. 9 Concept of area compatibility

7. Compatibility of concrete-tie system

The compatibility of the concrete core and hoop is treated in a global sense. Under bending and stretching, the hoop would expand with an increase in the enclosed area A_h. This can be obtained from the geometry of deformation of perimeter tie, shown in Fig. 9, as follows:

$$A_h = A_f + A_{a,p} + A_{a,c} \tag{18}$$

where A_f and $A_{a,p}$ are the increased areas due to bending deformation, u_f, and uniform expansion of the perimeter tie, $u_{a,p}$, respectively; and $A_{a,c}$ is the increase in area due to elongation of the crosstie, $u_{a,c}$, in excess of that of the hoop in the same direction. Dividing Eq. (18) by the undeformed core area yields the area strain of the steel hoop which has to be equal to that of the concrete core (at the hoop level) as the condition of compatibility.

8. Procedure for determining confined strength

For a given axial strain, we first assume a trial value of the average effective confining pressure, σ_{eff}. Then, the area strain of the concrete core at the level of the perimeter tie, $\varepsilon_{A,h}$, is determined from Eqs. (2)-(11). With the resultant force of the interactive transverse line load exerted by the core on the tie leg being determined from equilibrium, the resulting deformations of the perimeter tie and crosstie (if any) are computed. The area strain is then calculated from the deformation of the ties. If the difference between the area strain of the steel ties and the area strain of the concrete core is greater than the specified tolerance, which is set as 10^{-6} in this study, the new trial value of σ_{eff} is then assumed. The iteration is carried out until the desired tolerance is achieved. For each converged state of deformation, the value of the volumetric strain is then computed. These calculation procedures are repeated until the state of zero volumetric strain is reached when the confined concrete attains the peak confined strength. The associated confining pressure is computed and the confined strength of the core concrete can then be obtained from Eq. (10).

9. Verification of proposed procedure

To verify the validity of the method proposed, we first examine the accuracy in predicting the axial strains in perimeter ties of the column specimens tested by Sun *et al.* (1996). Table 1 lists the details of the test specimens. It is to be noted that the specimens were reinforced with high-strength steel ties. The predicted values of the axial strains normalized to the yield strain of the hoop are plotted as functions of the volumetric ratio, ρ_h, in Fig. 10, together with the experimental results. It is seen that, for columns confined with 10-mm diameter ties, the numerical and experimental values agree reasonably well, with a discrepancy around 10-30% in general. For columns with 6-mm ties, the agreements are poor. It should be observed, however, that while the experimental method may be susceptible to error due to difficulty in measuring strains in hoops, the analytical approach can be more consistent in predicting the trend of influencing parameters. This can be seen by comparing the test results of Specimens HB6-70 and HB10-80, both having the same column sections and tie configurations. The former was confined by 6 mm diameter transverse steel at 70 mm spacing with a volumetric ratio of 1.70%, while the latter specimen was reinforced with 10-mm diameter ties at 80 mm spacing, resulting in a volumetric ratio about twice that of Specimen HB6-70. The proposed analytical method predicts strains in perimeter ties of Specimen HB10-80 higher than those of Specimen HB6-70 by 38%, consistent with previous findings that suggest a higher stress in the transverse reinforcement in a column with more confinement steel (Cusson and Paultre 1995, Razvi and Saatcioglu 1999). On the other hand, the experimental results for these two specimens failed to reflect this fact (refer to Fig. 10). Furthermore, at a given volumetric ratio, a tie configuration which results in a higher hoop stiffness, as indicated by a higher value of the ratio of the hoop diameter d_h to the unsupported length L_s, should give rise to a larger axial strain in the hoop. Again, this is nicely reflected in the analytical results shown in Fig. 10.

Table 1 Strength enhancement in square columns tested by Sun *et al.* (1996)

Column label	ρ_h (%)	Tie configuration	B_c (mm)	d_h (mm)	s (mm)	f_{yh} (MPa)	f_{co}' (MPa)	f_{cc}' (MPa) Experimental	f_{cc}' (MPa) Analytical	Analytical/ experimental
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
HA6-20	3.48	Type A	162	6.4	20	1025	43.8	62.7	68.7	1.10
HA6-30	2.32	Type A	162	6.4	30	1025	43.8	58.4	60.2	1.03
HA6-40	1.74	Type A	162	6.4	40	1025	43.8	55.1	53.9	0.98
HB6-35	3.39	Type B	162	6.4	35	1025	43.8	72.4	77.3	1.07
HB6-50	2.38	Type B	162	6.4	50	1025	43.8	67.9	65.0	0.96
HB6-70	1.70	Type B	162	6.4	70	1025	43.8	58.7	58.0	0.99
HA10-35	4.40	Type A	158	9.6	35	872	45.6	69.8	73.7	1.06
HA10-47	3.28	Type A	158	9.6	47	872	45.6	66.4	65.9	0.99
HA10-60	2.57	Type A	158	9.6	60	872	45.6	65.8	61.3	0.93
HB10-60	4.40	Type B	158	9.6	60	872	44.2	84.5	93.4	1.10
HB10-80	3.30	Type B	158	9.6	80	872	44.2	73.1	76.0	1.04
HB10-100	2.64	Type B	158	9.6	100	872	44.2	67.3	66.7	0.99

Note : Type A consists of perimeter ties only; Type B consists of perimeter and inner ties.

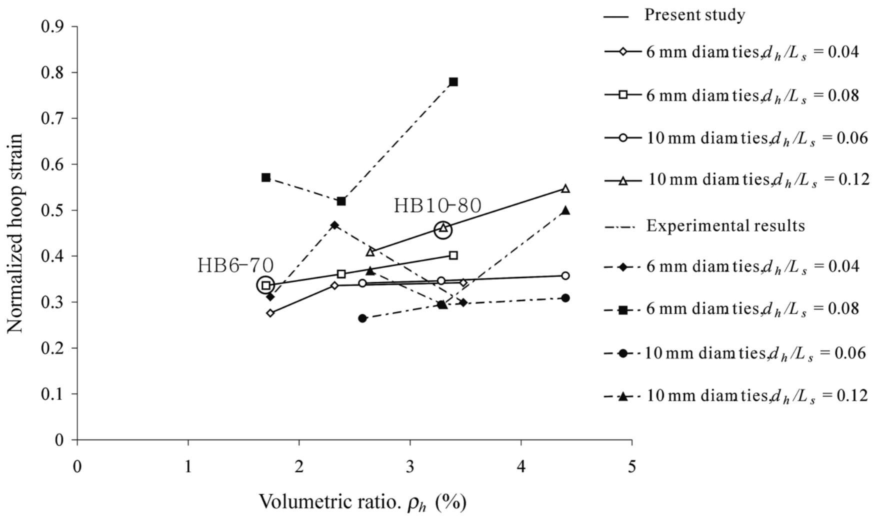

Fig. 10 Variation of normalized hoop strains with volumetric ratios

Table 2 Strength enhancement in square columns tested by Razvi and Saatcioglu (1999)

Column label	ρ_h (%)	Tie configuration	B_c (mm)	d_h (mm)	s (mm)	f_{yh} (MPa)	f'_{co} (MPa)	f'_{cc} (MPa) Experimental	f'_{cc} (MPa) Analytical	Analytical/ experimental
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
CS-1	3.33	Type 1	218.7	11.3	55	400	105.4	120.8	116.3	0.96
CS-2	1.62	Type 2	223.5	6.5	55	570	105.4	121.6	114.9	0.94
CS-3	2.16	Type 3	223.5	6.5	55	570	105.4	129.1	121.4	0.94
CS-4	2.17	Type 2	222.5	7.5	55	1000	105.4	123.4	121.7	0.99
CS-5	1.32	Type 3	222.5	7.5	120	1000	105.4	122.5	114.8	0.94
CS-6	1.05	Type 2	223.5	6.5	85	400	105.4	115.7	110.6	0.96
CS-7	0.99	Type 3	223.5	6.5	120	400	105.4	115.0	111.5	0.97
CS-8	3.23	Type 2	218.7	11.3	85	400	105.4	117.8	122.5	1.04
CS-9	3.06	Type 3	218.7	11.3	120	400	105.4	134.2	124.7	0.93
CS-11	4.58	Type 1	218.7	11.3	40	400	68.9	93.9	83.4	0.89
CS-12	3.33	Type 1	218.7	11.3	55	400	68.9	82.1	79.1	0.96
CS-13	1.62	Type 2	223.5	6.5	55	570	78.2	85.9	87.6	1.02
CS-14	2.16	Type 3	223.5	6.5	55	570	78.2	94.3	94.9	1.01
CS-15	2.17	Type 2	222.5	7.5	55	1000	68.9	95.5	89.3	0.94
CS-16	1.87	Type 3	222.5	7.5	85	1000	68.9	95.2	87.4	0.92
CS-17	1.05	Type 2	223.5	6.5	85	400	68.9	75.2	73.5	0.98
CS-18	1.4	Type 3	223.5	6.5	85	400	68.9	76.4	77.2	1.01
CS-19	3.23	Type 2	218.7	11.3	85	400	78.2	104.2	96.1	0.92
CS-20	4.32	Type 3	218.7	11.3	85	400	78.2	106.3	105.4	0.99
CS-22	1.4	Type 2	222.5	7.5	85	1000	51.0	68.0	61.0	0.90
CS-23	1.32	Type 3	222.5	7.5	120	1000	51.0	71.3	61.5	0.86
CS-24	3.23	Type 2	218.7	11.3	85	400	51.0	69.7	69.1	0.99
CS-25	3.06	Type 3	218.7	11.3	120	400	51.0	72.6	70.1	0.97
CS-26	2.16	Type 3	223.5	6.5	55	570	51.0	76.7	70.2	0.92

Table 3 Strength enhancement in square columns tested by Nishiyama *et al.* (1993)

Column label	ρ_h (%)	Tie configuration	B_c (mm)	d_h (mm)	s (mm)	f_{yh} (MPa)	f_{co}' (MPa)	f_{cc}' (MPa) Experimental	f_{cc}' (MPa) Analytical	Analytical/ experimental
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Unit-1	4.06	Type 3	214	6	31	813	92.4	145.0	135.5	0.93
Unit-2	4.06	Type 3	214	6	31	813	92.4	137.0	135.5	0.99
Unit-3	4.06	Type 3	214	6	31	813	92.4	145.0	135.5	0.93
Unit-4	2.78	Type 3	214	6	45	813	92.4	122.0	117.1	0.96
Unit-5	2.10	Type 3	214	6	60	813	92.4	120.0	109.1	0.91
Unit-6	2.10	Type 3	214	6	60	813	92.4	110.0	109.1	0.99
Unit-7	2.10	Type 3	214	6	60	813	92.4	120.0	109.1	0.91
Unit-8	1.78	Type 3	216	4	31	840	92.4	120.0	105.9	0.88
Unit-9	4.06	Type 3	214	6	31	462	96.2	134.0	120.9	0.90
Unit-10	4.06	Type 3	214	6	31	462	96.2	133.0	120.9	0.91
Unit-11	2.78	Type 3	214	6	45	462	96.2	117.0	113.4	0.97
Unit-12	2.10	Type 3	214	6	60	462	96.2	115.0	109.6	0.95
Unit-13	2.10	Type 3	214	6	60	462	96.2	115.0	109.6	0.95
Unit-14	1.78	Type 3	216	4	31	481	96.2	115.0	107.5	0.93

Table 4 Strength enhancement in square columns tested by Nagashima *et al.* (1992)

Column label	ρ_h (%)	Tie configuration	B_c (mm)	d_h (mm)	s (mm)	f_{yh} (MPa)	f_{co}' (MPa)	f_{cc}' (MPa) Experimental	f_{cc}' (MPa) Analytical	Analytical/ experimental
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
HH08LA	1.73	Type 3	199.9	5.1	55	1387	98.8	122.8	112.0	0.91
HH10LA	2.12	Type 3	199.9	5.1	45	1387	98.8	122.5	116.5	0.95
HH13LA	2.73	Type 3	199.9	5.1	35	1387	98.8	131.5	123.6	0.94
HL06LA	2.03	Type 3	200	5	45	807	100.4	118.2	117.0	0.99
HL08LA	2.62	Type 3	200	5	35	807	100.4	133.2	122.6	0.92
LL05LA	1.67	Type 3	200	5	55	807	51.3	68.9	65.4	0.95
LL08LA	2.62	Type 3	200	5	35	807	51.3	79.4	77.3	0.97
LH08LA	1.73	Type 3	199.9	5.1	55	1387	51.3	70.9	65.5	0.92
LH13LA	2.73	Type 3	199.9	5.1	35	1387	51.3	85.7	88.6	1.03
HH13MA	2.73	Type 3	199.9	5.1	35	1387	100.4	131.8	124.5	0.94
HH13HA	2.73	Type 3	199.9	5.1	35	1387	100.4	129.2	124.5	0.96
LL08MA	2.62	Type 3	200	5	35	807	51.3	79.6	74.9	0.94
LL08HA	2.62	Type 3	200	5	35	807	51.3	78.0	74.9	0.96
HH13LD	2.45	Type 2	199.9	5.1	25	1387	100.4	128.2	121.9	0.95
LL08LB	3.39	Type 3	200	5	27	807	52.4	82.4	84.1	1.02
LL08LD	2.36	Type 2	200	5	25	807	52.4	77.3	71.1	0.92
HH13MSA	2.73	Type 3	199.9	5.1	35	1387	100.4	129.7	123.8	0.95
HH13HSA	2.73	Type 3	199.9	5.1	35	1387	100.4	134.8	123.8	0.92
LL08MSA	2.62	Type 3	200	5	35	807	52.4	79.0	75.5	0.96
LL08HSA	2.62	Type 3	200	5	35	807	52.4	80.5	75.5	0.94

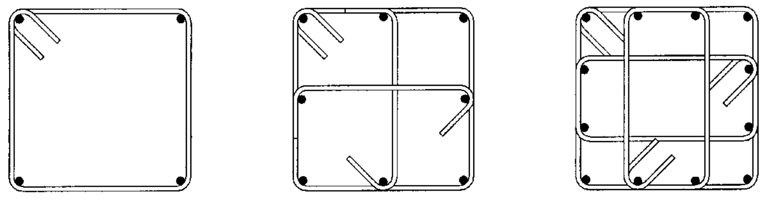

Fig. 11 Typical arrangements of transverse steel

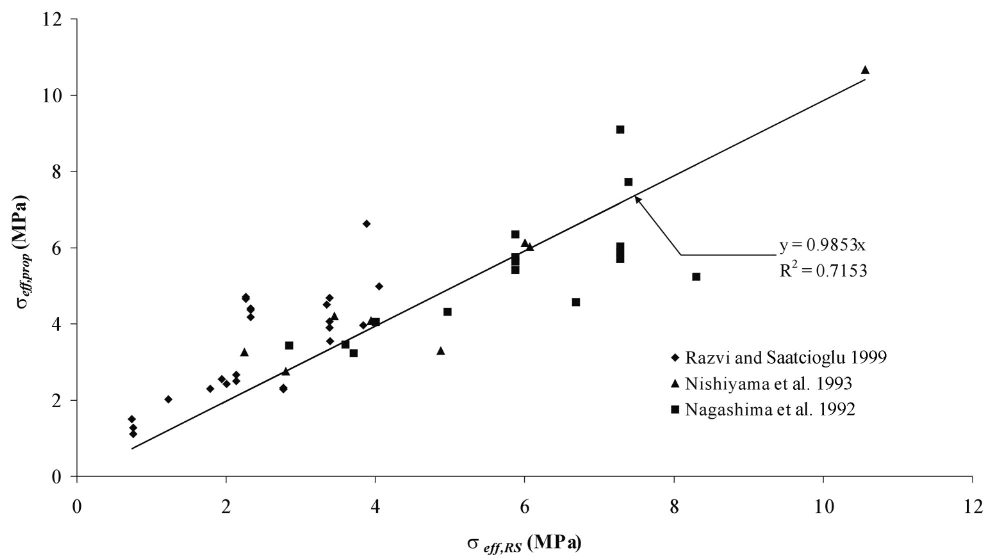

Fig. 12 Comparison of $\sigma_{eff,\,prop}$ with $\sigma_{eff,\,RS}$

The axial strains in the ties are related to the effective confinement pressure. Therefore, it is interesting to compare the effective confinement pressures predicted by Razvi and Saatcioglu (1999), $\sigma_{eff,\,RS}$, and the present study, $\sigma_{eff,\,prop}$. Tables 2-4 list the relevant details of the specimens considered for comparison, which cover both normal-and high-strength concrete and various tie configurations. Typical arrangements of the transverse steel are shown in Fig. 11. The unconfined concrete strength and the yield strength of transverse steel ranged from 50 to 105 Mpa and 400 to 1400 Mpa, respectively. The values of the effective confinement pressure are plotted in Fig. 12 together with the best fit curve which is given by

$$\sigma_{eff,\,prop} = 0.9853\sigma_{eff,\,RS} \tag{19}$$

with the goodness-of-fit index R^2 of 0.72.

The predicted peak confined strengths of the above specimens are compared with the experimental results in Fig. 13. Good agreement is observed with the maximum difference between the experimental and analytical results being about 14%.

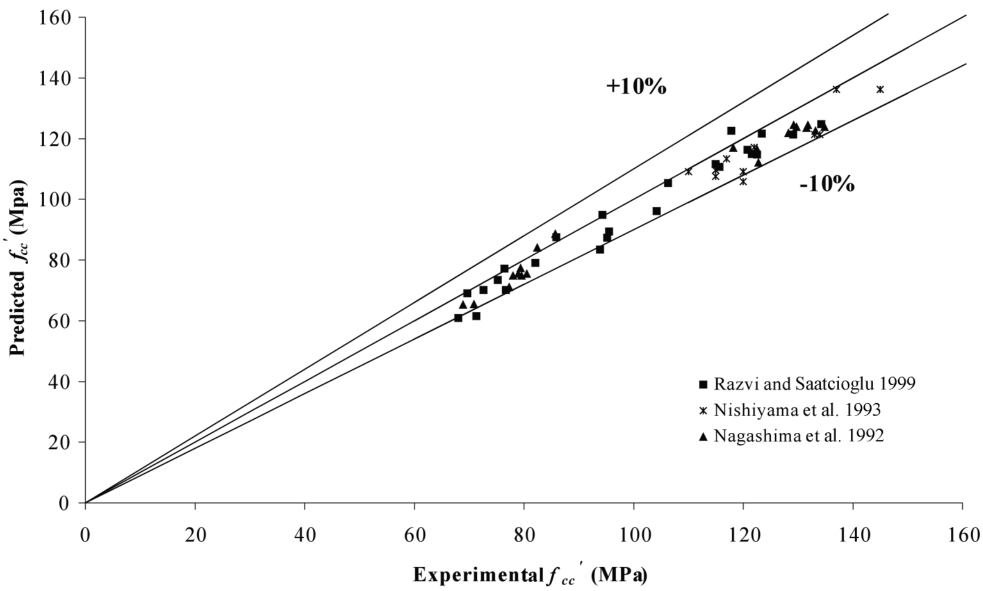

Fig. 13 Comparison of predicted confined compressive strength with experimental results

10. Conclusions

A confinement model is proposed which is applicable to both normal-strength and high-strength concrete columns confined with normal- or high-strength transverse steel. The interaction between the concrete core-transverse steel system and the variation of the confining stress along the column height were realistically accounted for, considering flexural flexibility of the rectilinear ties. Compatibility of the concrete core and steel hoop is imposed at the hoop level using the area compatibility concept. An iterative procedure is used to obtain the effective confinement pressure at the peak compressive strength.

Application of the method proposed to columns with various tie configurations and confinement reinforcement under uniaxial loading yields satisfactory predictions of confined strengths compared with experimental results. In general, the discrepancy is within 10%. The proposed procedure can capture the influence of tie flexibility, tie configuration and degree of confinement which govern the peak strength of the confined concrete.

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