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Invited Review Paper

Dynamic analysis of structure/foundation systems

Joseph Penzien†

International Civil Engineering Consultants, Inc., U.S.A.

Abstract. A review of current procedures being used in engineering practice to analyze the response of structure/foundation systems subjected separately to different types of dynamic excitation, such as earthquake, sea-wave action, wind, or moving wheel loads, is presented. Separate formulations are given for analyzing systems in the time and frequency domains. Both deterministic and stochastic forms of excitation are treated. A distinction is made between demand and capacity analyses.

Key words: dynamic analysis; structure/foundation system; time-domain; frequency-domain; nonlinear analysis; performance evaluation.

1. Introduction

The procedures used in engineering practice to analyze the response of structures to dynamic excitations have changed rapidly in recent years due to an expanded understanding of the basic theories involved, continued improvement and implementation of effective computational procedures, more realistic modeling of elements and components, and advances in computer technology. The response of linear systems, which have constant mass, damping, and stiffness parameters, has traditionally been treated through normal-mode solutions; however, because of large improvements in computational speeds and storage capacities, the response of such systems can now be readily obtained by solving the initial coupled equations of motion directly. The response of nonlinear systems must, of course, be obtained through solutions of the coupled equations of motion.

In recent years, analytical modeling of nonlinear components and elements has improved considerably as a result of increasing experimental test results; and, improved modeling of foundations has also occurred. The commonly used “p-y”, “t-z”, “Q-d” method of modeling is now used effectively for slender-pile foundations; however, for massive foundations such as large mat foundations and caissons, improved modeling has become necessary to account for important soil-foundation inertia and radiation damping effects. Such modeling requires the use of complex frequency-dependent parameters; thus, the frequency domain method of dynamic analysis has become increasingly used in practice.

Since dynamic excitations produced by sea-wave action and wind turbulence are best characterized in the frequency domain as random processes, stochastic methods of dynamic analysis are being used more frequently and computer programs are being expanded to accommodate such solutions.

† Senior Principal and Chair

The dynamic analysis procedures discussed herein, focus mainly on evaluating structural demands (forces and deformations); however, to assess performance, structural capacities must also be evaluated; thus, a distinction has been made in this paper between demand and capacity evaluations.

2. Deterministic response of linear systems

2.1 Time domain analysis

The analysis of a general three-dimensional structure/foundation system, responding to dynamic excitation, e.g., earthquake, sea waves, wind, or moving wheel loads, is typically carried out in the time domain. When structure-foundation interaction effects are significant, it is convenient to divide the complete system into two substructures — one being the structure, the other being its complete foundation. This complete foundation system could be a single foundation or multiple foundations as in the case of a long bridge. For determining linear elastic response, the governing equations of motion used to evaluate internal forces and deformations can be expressed in the matrix form

$$\begin{bmatrix} \boldsymbol{M}_{ss} & \boldsymbol{M}_{sf} \\ \boldsymbol{M}_{sf}^{T} & \boldsymbol{M}_{ff}^{s} \end{bmatrix} \begin{Bmatrix} \ddot{\boldsymbol{U}}_s(t) \\ \ddot{\boldsymbol{U}}_f(t) \end{Bmatrix} + \begin{bmatrix} \boldsymbol{C}_{ss} & \boldsymbol{C}_{sf} \\ \boldsymbol{C}_{sf}^{T} & \boldsymbol{C}_{ff}^{s} \end{bmatrix} \begin{Bmatrix} \dot{\boldsymbol{U}}_s(t) \\ \dot{\boldsymbol{U}}_f(t) \end{Bmatrix} + \begin{bmatrix} \boldsymbol{K}_{ss} & \boldsymbol{K}_{sf} \\ \boldsymbol{K}_{sf}^{T} & \boldsymbol{K}_{ff}^{s} \end{bmatrix} \begin{Bmatrix} \boldsymbol{U}_s(t) \\ \boldsymbol{U}_f(t) \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{p}_s(t) \\ \boldsymbol{p}_f(t) \end{Bmatrix} \tag{1}$$

in which all of the letters $\boldsymbol{M}$, $\boldsymbol{C}$, and $\boldsymbol{K}$ denote mass, damping, and stiffness matrices, respectively; all $\boldsymbol{U}$ quantities denote time-dependent total-displacement vectors; subscript s denotes the number of degrees of freedom (DOF) in the structure, excluding its f DOF located at the structure/foundation interface; vector $\boldsymbol{p}_s(t)$ represents externally applied forces acting on the structure; and vector $\boldsymbol{p}_f(t)$ represents structure/foundation interaction forces. Force vectors $\boldsymbol{p}_s(t)$ and $\boldsymbol{p}_f(t)$ depend, of course, on the type of dynamic excitation present.

Usually, the mass of the structure is lumped at nodal points, leading to a diagonal mass matrix in Eq. (1); the damping matrix is assumed to be of the Raleigh form

$$\begin{bmatrix} \boldsymbol{C}_{ss} & \boldsymbol{C}_{sf} \\ \boldsymbol{C}_{sf}^{T} & \boldsymbol{C}_{ff}^{s} \end{bmatrix} = \alpha \begin{bmatrix} \boldsymbol{M}_{ss} & \boldsymbol{M}_{sf} \\ \boldsymbol{M}_{sf}^{T} & \boldsymbol{M}_{ff}^{s} \end{bmatrix} + \beta \begin{bmatrix} \boldsymbol{K}_{ss} & \boldsymbol{K}_{sf} \\ \boldsymbol{K}_{sf}^{T} & \boldsymbol{K}_{ff}^{s} \end{bmatrix}; \tag{2}$$

and, the stiffness matrix is assembled from the matrices of individual finite elements, including their geometric-stiffness matrices. Rayleigh damping as expressed by Eq. (2) leads to uncoupled normal modes of vibration. The two constants α and β are assigned numerical values to limit modal damping ratios to levels within acceptable bounds over the range of frequencies dominating dynamic response. It is this author's position that changes are needed to the Raleigh form of damping, and that structure analysis computer programs should be changed accordingly.

The letter s has been added as a superscript to matrices $\boldsymbol{M}_{ff}$, $\boldsymbol{C}_{ff}$, and $\boldsymbol{K}_{ff}$ in Eq. (1) to indicate that these matrices apply only to the structure when isolated from the foundation substructure. The structure/foundation interaction forces in vector $\boldsymbol{p}_f(t)$, which act on the structure in the f DOF, must be developed from an isolated soil/foundation model representing the foundation substructure (Tseng and Penzien 2000).

Often in engineering practice, simple quasi-static modeling is used to represent foundations, which ignores soil/foundation inertia forces and radiation damping. When the structure is subjected to direct dynamic forces represented by vector $\boldsymbol{p}_s(t)$, such as those produced by sea waves, wind, or moving wheel loads, vector $\boldsymbol{p}_f(t)$ is then expressed by the relation

$$\boldsymbol{p}_f(t) = -\boldsymbol{K}_{ff}^f \boldsymbol{U}_f(t) - \boldsymbol{C}_{ff}^f \dot{\boldsymbol{U}}_f(t) \tag{3}$$

in which $\boldsymbol{K}_{ff}^f$ is the static stiffness matrix of the isolated foundation representing its f DOF at the structure/foundation interface and $\boldsymbol{C}_{ff}^f$ is the corresponding viscous damping matrix.

In the above substructuring procedure, heavy masses such as pile caps and mat foundations should be included in the structure mass matrix $\boldsymbol{M}_{ff}^s$. Considering for example a bridge having multiple pile foundations, the structure/foundation interface should be specified as being at the lower surfaces of the footings, each having six degrees of freedom (three translations and three rotations). Thus, the footing masses would be included in the structure system; however, to satisfy pile-head boundary conditions, rigid massless footings should be included in the isolated foundation system. Substituting Eqs. (2) and (3) into Eq. (1), the resulting equation of motion can then be solved for displacement response vectors $\boldsymbol{U}_s(t)$ and $\boldsymbol{U}_f(t)$ and their corresponding velocity and acceleration vectors. This solution is performed after force vector $\boldsymbol{p}_s(t)$ has been fully defined and evaluated.

In the case of earthquake excitation, the vector $\boldsymbol{p}_s(t)$ in Eq. (1) is a zero vector, since the structure is excited only through its foundation. Then, vector $\boldsymbol{p}_f(t)$ is expressed in the form

$$\boldsymbol{p}_f(t) = -\boldsymbol{K}_{ff}^f \{ \boldsymbol{U}_f(t) - \bar{\boldsymbol{U}}_f(t) \} - \boldsymbol{C}_{ff}^f \{ \dot{\boldsymbol{U}}_f(t) - \dot{\bar{\boldsymbol{U}}}_f(t) \} \tag{4}$$

in which vector $\bar{\boldsymbol{U}}_f(t)$ and its corresponding velocity vector $\dot{\bar{\boldsymbol{U}}}_f(t)$ represent seismically-induced motions in the f DOF of the isolated foundation system. These motions are referred to as "kinematic" foundation motions, due to the fact that foundation inertia effects have been ignored. Substituting Eqs. (2) and (4) into Eq. (1), the resulting equation of motion can then be solved for displacement response vectors $\boldsymbol{U}_s(t)$ and $\boldsymbol{U}_f(t)$ and their corresponding velocity and acceleration vectors.

2.2 Frequency domain analysis

The above-described time-domain form of solution of dynamic response is limited to cases of modeling using real constant parameters in the coefficient matrices. Often however complex frequency-dependent parameters are involved, e.g., in representing inertia and radiation damping effects in foundations (Tseng and Penzien 2000) and in representing hydrodynamic forces (Wang *et al.* 1991). In such cases, linear dynamic response must be evaluated in the frequency domain.

Because frequency domain analyses allow complex parameters in the coefficient matrices, it is recommended that complex-stiffness material damping be used rather than viscous damping as it is more realistic. In this case, the vector of damping forces in the equations of motion of the structure are expressed in the form

$$F_D(i\omega) = i\begin{bmatrix}\hat{K}_{ss} & \hat{K}_{sf}\\ \hat{K}_{sf}^T & \hat{K}_{ff}^s\end{bmatrix}\begin{Bmatrix}U_s(i\omega)\\ U_f(i\omega)\end{Bmatrix} \tag{5}$$

in which each stiffness matrix, $\hat{K}$, in this equation has been assembled from individual finite-element stiffness matrices $\hat{k}^{(m)}$ [superscript (m) denotes element m] of the form

$$\hat{k}^{(m)} = 2\xi^{(m)}k^{(m)} \tag{6}$$

in which $k^{(m)}$ denotes the individual elastic stiffness matrix for finite element m; and $\xi^{(m)}$ is a damping ratio selected to be appropriate for the material used in finite element m. If the same damping ratio is assumed throughout the system, so that $\xi^{(m)}$ is the same for all values of m, the stiffness matrix in Eq. (5) will posses the same orthogonality properties as does the elastic stiffness matrix in Eq. (1). However, because different materials are used, the $\xi^{(m)}$ values will differ with m; then, the stiffness matrix in Eq. (5) will not retain these orthogonality properties. These differing values of $\xi^{(m)}$ cause no difficulty however in solving the resulting equations of motion in the frequency domain.

These equations of motion are obtained by Fourier transforming Eq. (1) and substituting the above complex-stiffness damping for viscous damping. Doing so, one obtains

$$\left[\begin{bmatrix}K_{ss} & K_{sf}\\ K_{sf}^T & K_{ff}^s\end{bmatrix} - \omega^2\begin{bmatrix}M_{ss} & M_{sf}\\ M_{sf}^T & M_{ff}^s\end{bmatrix} + i\begin{bmatrix}\hat{K}_{ss} & \hat{K}_{sf}\\ \hat{K}_{sf}^T & \hat{K}_{ff}^s\end{bmatrix}\right]\begin{Bmatrix}U_s(i\omega)\\ U_f(i\omega)\end{Bmatrix} = \begin{Bmatrix}p_s(i\omega)\\ p_f(i\omega)\end{Bmatrix} \tag{7}$$

which can be expressed in the simplified form

$$\begin{bmatrix}I_{ss}(i\omega) & I_{sf}(i\omega)\\ I_{sf}^T(i\omega) & I_{ff}^s(i\omega)\end{bmatrix}\begin{Bmatrix}U_s(i\omega)\\ U_f(i\omega)\end{Bmatrix} = \begin{Bmatrix}p_s(i\omega)\\ p_f(i\omega)\end{Bmatrix} \tag{8}$$

The complete matrix $I(i\omega)$ in this equation is referred to as the impedance matrix of the structure.

When the foundation is modeled rigorously, including mass and radiation damping, the structure-foundation interaction force vector $p_f(i\omega)$ is obtained directly from modeling and analysis of the soil/foundation substructure (Tseng and Penzien 2000) rather than indirectly by inverse Fourier transforming vector $p_f(t)$. Without attempting to explain the procedures involved, the resulting vector $p_f(i\omega)$ is obtained in the form

$$p_f(i\omega) = -I_{ff}^f(i\omega)U_f(i\omega) \tag{9}$$

in which the foundation impedance matrix, $I_{ff}^f(i\omega)$, is complex and frequency dependent. Substituting the right-hand side of Eq. (9) into Eq. (8) for vector $p_f(i\omega)$ and transferring it to the left hand side, allows $I_{ff}^f(i\omega)$ to be combined with $I_{ff}^s(i\omega)$. After having defined and evaluated vector $p_s(i\omega)$, the resulting Eq. (8) given by

$$\begin{bmatrix} I_{ss}(i\omega) & I_{sf}(i\omega) \\ I_{sf}^T(i\omega) & I_{ff}^s(i\omega) + I_{ff}^f(i\omega) \end{bmatrix} \begin{Bmatrix} U_s(i\omega) \\ U_f(i\omega) \end{Bmatrix} = \begin{Bmatrix} p_s(i\omega) \\ o \end{Bmatrix} \tag{10}$$

is then solved for response vectors $U_s(i\omega)$ and $U_f(i\omega)$.

In the case of earthquake excitation, there are no directly applied forces in the *s* DOF of the structure so that $p_s(i\omega)$ is a zero vector and $p_f(i\omega)$ is of the form

$$p_f(i\omega) = -I_{ff}^f(i\omega)\{U_f(i\omega) - \bar{U}_f(i\omega)\} \tag{11}$$

in which vector $\bar{U}_f(i\omega)$ represents motions in the *f* DOF of the isolated foundation when subjected to the earthquake excitation.

The motions represented by $\bar{U}_f(i\omega)$ are referred to in the literature as the foundation "scattered" motions (Tseng and Penzien 2000). They are evaluated through a separate analysis of the isolated foundation substructure subjected to its surrounding seismic free-field soil motions. Substituting Eq. (11) into Eq. (8) and setting $p_s(i\omega)$ equal to a zero vector, the resulting equation

$$\begin{bmatrix} I_{ss}(i\omega) & I_{sf}(i\omega) \\ I_{sf}^T(i\omega) & I_{ff}^s(i\omega) + I_{ff}^f(i\omega) \end{bmatrix} \begin{Bmatrix} U_s(i\omega) \\ U_f(i\omega) \end{Bmatrix} = \begin{Bmatrix} o \\ I_{ff}^f \bar{U}_f(i\omega) \end{Bmatrix} \tag{12}$$

can then be solved to obtain the seismic response vectors $U_s(i\omega)$ and $U_f(i\omega)$.

Having solved for response vectors $U_s(i\omega)$ and $U_f(i\omega)$ through the frequency-domain procedure described above, the resulting vector $U_f(i\omega)$ must be fed back into the *f* DOF of the isolated foundation model to evaluate forces and deformations in the foundation. Frequency domain modeling of heavy-mat and large-caisson foundations by the elastodynamic procedures in necessary in order to capture important soil/foundation inertia and radiation damping effects. Complex-stiffness material damping, as represented by Eq. (5) for the structure, can also be used to represent material damping in the complete soil/foundation system.

3. Deterministic response of nonlinear systems

When large nonlinearities are produced by dynamic response, time-domain procedures of analysis must be used. These nonlinearities usually occur in the form of hysteretic force-displacement relations of individual components in the structure, thus requiring that the linear forms represented in the third term on the left-hand side of Eq. (1) be changed to the appropriate nonlinear hysteretic forms. Special damping devices are now being used having nonlinear viscous properties which would require modifications to the second term on the left hand side of Eq. (1). Other forms of nonlinearities occur such as opening and closing of gaps, elements which carry tension or compression only, and Coulomb friction. Most common types of nonlinearities which occur in structures have been modeled and implemented into computer programs. Having implemented all nonlinear elements into the structure's model, the coupled equations of motion can then be solved by numerical procedures to obtain total displacements in vectors $U_s(t)$ and $U_f(t)$.

Since time-domain procedures of analysis do not permit the presence of frequency-dependent parameters in the coefficient matrices of the equations of motion, the foundation impedance matrix $\boldsymbol{I}^f_{ff}(\boldsymbol{i}\omega)$, accounting for inertia and radiation-damping, must be modified into a simpler form which is compatible with a time-domain solution. To accomplish this objective, the foundation impedance matrix is first separated into its real and imaginary parts in accordance with

$$\boldsymbol{I}^f_{ff}(\boldsymbol{i}\omega) = \boldsymbol{I}^{fR}_{ff}(\omega) + \boldsymbol{iI}^{fI}_{ff}(\omega) \tag{13}$$

in which $\boldsymbol{I}^{fR}_{ff}(\omega)$ and $\boldsymbol{I}^{fI}_{ff}(\omega)$ are real functions of ω. Then, these functions can be approximated using the relations

$$\boldsymbol{I}^{fR}_{ff}(\omega) \doteq \bar{\boldsymbol{K}}^f_{ff} - \omega^2 \bar{\boldsymbol{M}}^f_{ff} \; ; \quad \boldsymbol{I}^{fI}_{ff}(\omega) \doteq \omega \bar{\boldsymbol{C}}^f_{ff} \tag{14}$$

where the real constants in matrices $\bar{\boldsymbol{K}}^f_{ff}$, $\bar{\boldsymbol{M}}^f_{ff}$, and $\bar{\boldsymbol{C}}^f_{ff}$ are assigned numerical values to provide best fits to the individual functions in matrices $\boldsymbol{I}^{fR}_{ff}(\omega)$ and $\boldsymbol{I}^{fI}_{ff}(\omega)$ over the frequency range of major dynamic response. In this fitting process, it is sufficient to treat $\bar{\boldsymbol{M}}^f_{ff}$ as a diagonal matrix, thus affecting only the diagonal functions in matrix $\boldsymbol{I}^{fR}_{ff}(\omega)$. The particular forms of Eqs. (14) have been selected so that when they are substituted into Eq. (13), which in turn is substituted into Eqs. (9) and (11), the resulting expressions for $\boldsymbol{p}_f(\boldsymbol{i}\omega)$ can be inverse Fourier transformed to the time domain yielding the relations

$$\boldsymbol{p}_f(t) = -\bar{\boldsymbol{K}}^f_{ff}\boldsymbol{U}_f(t) - \bar{\boldsymbol{C}}^f_{ff}\dot{\boldsymbol{U}}_f(t) - \bar{\boldsymbol{M}}^f_{ff}\ddot{\boldsymbol{U}}_f(t) \tag{15}$$

and

$$\boldsymbol{p}_f(t) = -\bar{\boldsymbol{K}}^f_{ff}\{\boldsymbol{U}_f(t) - \bar{\boldsymbol{U}}_f(t)\} - \bar{\boldsymbol{C}}^f_{ff}\{\dot{\boldsymbol{U}}_f(t) - \dot{\bar{\boldsymbol{U}}}_f(t)\} - \bar{\boldsymbol{M}}^f_{ff}\{\ddot{\boldsymbol{U}}_f(t) - \ddot{\bar{\boldsymbol{U}}}_f(t)\} \tag{16}$$

respectively, which have no frequency dependent parameters.

After substituting Eq. (15) into Eq. (1) and transferring all three terms to the left-hand side of the equation, the resulting nonlinear equations of motion can be solved for response vectors $\boldsymbol{U}_s(t)$ and $\boldsymbol{U}_f(t)$ produced by the form of excitation represented by force vector $\boldsymbol{p}_s(t)$. Likewise, substituting Eq. (16) into Eq. (1) and transferring its three terms containing vectors $\boldsymbol{U}_f(t)$, $\dot{\boldsymbol{U}}_f(t)$, and $\ddot{\boldsymbol{U}}_f(t)$ to the left-hand side of the equation, but leaving those three terms containing vectors $\bar{\boldsymbol{U}}_f(t)$, $\dot{\bar{\boldsymbol{U}}}_f(t)$, and $\ddot{\bar{\boldsymbol{U}}}_f(t)$, the resulting nonlinear equations of motion can be solved for response vectors $\boldsymbol{U}_s(t)$ and $\boldsymbol{U}_f(t)$ produced by earthquake excitation.

As in the case of analyzing linear systems, the resulting vector $\boldsymbol{U}_f(t)$ must be fed back into the f DOF of the isolated foundation model to obtain its internal forces and deformations.

4. Stochastic response of linear systems

When the dynamic excitation of a three-dimensional structure/foundation system is expressed in the form of a stationary random process, such as the wave-height spectrum representing random sea waves (Malhotra and Penzien 1970) or the power spectral density function representing the dynamic contribution to wind velocity caused by turbulence, the corresponding response is evaluated as a stochastic process (Clough and Penzien 1993). In this case, the force vector shown on the right-hand side of Eq. (10) represents a stationary random process which is characterized by its $N \times N$ spectral density matrix of the form

$$\boldsymbol{S}_p(i\omega) = \left[\begin{array}{cccc|c} S_{p_1p_1}(\omega) & S_{p_1p_2}(i\omega) & \dots & S_{p_1p_s}(i\omega) & \\ S_{p_2p_1}(i\omega) & S_{p_2p_2}(\omega) & \dots & S_{p_2p_s}(i\omega) & \\ \dots & \dots & \dots & \dots & \boldsymbol{o} \\ S_{p_sp_1}(i\omega) & S_{p_sp_2}(i\omega) & \dots & S_{p_sp_s}(\omega) & (s \times f) \\ \hline & \boldsymbol{o} & & & \boldsymbol{o} \\ & (f \times s) & & & (f \times f) \end{array}\right] \tag{17}$$

in which the individual density functions are defined as

$$S_{p_jp_k}(i\omega) = \lim_{d \to \infty} \frac{\left[\int_{-d/2}^{d/2} p_j(t)\exp(-i\omega t)dt\right]\left[\int_{-d/2}^{d/2} p_k(t)\exp(+i\omega t)dt\right]}{2\pi d} \qquad j, k = 1, 2, ..., s \tag{18}$$

and the zero matrices are of the size indicated.

Consider now Eq. (10) written in the condensed form

$$\boldsymbol{U}(i\omega) = \boldsymbol{H}(i\omega)\boldsymbol{p}(i\omega) \tag{19}$$

in which

$$\boldsymbol{U}(\boldsymbol{i}\omega) = \begin{Bmatrix} \boldsymbol{U}_s(\boldsymbol{i}\omega) \\ \boldsymbol{U}_f(\boldsymbol{i}\omega) \end{Bmatrix}; \quad \boldsymbol{p}(\boldsymbol{i}\omega) = \begin{Bmatrix} \boldsymbol{p}_s(\boldsymbol{i}\omega) \\ \boldsymbol{o} \end{Bmatrix} \tag{20}$$

and

$$\boldsymbol{H}(\boldsymbol{i}\omega) = \begin{bmatrix} \boldsymbol{I}_{ss}(\boldsymbol{i}\omega) & \boldsymbol{I}_{sf}(\boldsymbol{i}\omega) \\ \boldsymbol{I}_{sf}^T(\boldsymbol{i}\omega) & \boldsymbol{I}_{ff}^s(\boldsymbol{i}\omega) + \boldsymbol{I}_{ff}^f(\boldsymbol{i}\omega) \end{bmatrix}^{-1} \tag{21}$$

Consistent with the condition of stationary response, (1) postmultiply each side of Eq. (19) by the transpose of its own complex conjugate, (2) divide both sides of the resulting equation by $2\pi d$ where d represents duration of the process, and (3) take the limit as $d \to \infty$. Doing so, one obtains

$$\lim_{d \to \infty} \frac{\boldsymbol{U}(\boldsymbol{i}\omega)\boldsymbol{U}(-\boldsymbol{i}\omega)^{\boldsymbol{T}}}{2\pi d} = \lim_{d \to \infty} \frac{\boldsymbol{H}(\boldsymbol{i}\omega)\boldsymbol{p}(\boldsymbol{i}\omega)\boldsymbol{p}(-\boldsymbol{i}\omega)^{\boldsymbol{T}}\boldsymbol{H}(-\boldsymbol{i}\omega)^{\boldsymbol{T}}}{2\pi d} \tag{22}$$

Using the definition of cross-spectral density given by Eq. (18), Eq. (22) becomes

$$\boldsymbol{S_U}(\boldsymbol{i}\omega) = \boldsymbol{H}(\boldsymbol{i}\omega)\boldsymbol{S_p}(\boldsymbol{i}\omega)\boldsymbol{H}(-\boldsymbol{i}\omega)^{\boldsymbol{T}} \tag{23}$$

in which $\boldsymbol{S_U}(\boldsymbol{i}\omega)$ is the $N \times N$ spectral density matrix for displacement response as expressed by

$$\boldsymbol{S}_U(i\omega) = \begin{bmatrix} S_{U_1U_1}(\omega) & S_{U_1U_2}(i\omega) & \dots & S_{U_1U_N}(i\omega) \\ S_{U_2U_1}(i\omega) & S_{U_2U_2}(\omega) & \dots & S_{U_2U_N}(i\omega) \\ \dots\dots & \dots\dots & \dots\dots & \dots\dots \\ S_{U_NU_1}(i\omega) & S_{U_NU_2}(i\omega) & \dots & S_{U_NU_N}(\omega) \end{bmatrix} \tag{24}$$

If one is interested in an r-component response vector $\boldsymbol{Z}(t)$ as given in the time and frequency domains, respectively, by

$$\boldsymbol{Z}(t) = \boldsymbol{AU}(t); \qquad \boldsymbol{Z}(i\omega) = \boldsymbol{AU}(i\omega) \tag{25}$$

where $\boldsymbol{A}$ is a known $r \times N$ coefficient matrix, then the $r \times r$ spectral density matrix for vector $\boldsymbol{Z}(t)$ is given by

$$\boldsymbol{S_z}(\boldsymbol{i}\omega) = \boldsymbol{AH}(\boldsymbol{i}\omega)\boldsymbol{S_p}(\boldsymbol{i}\omega)\boldsymbol{H}(-\boldsymbol{i}\omega)^{\boldsymbol{T}}\boldsymbol{A}^{\boldsymbol{T}} \tag{26}$$

Having this spectral density matrix, stochastic response $\boldsymbol{Z}(t)$ is fully characterized (Clough and Penzien 1993).

Integrating each power spectral density function along the diagonal of Eq. (24), over the infinite frequency range, yields the variance, σ^2, of the corresponding response. Likewise, integrating each cross-spectral density function appearing as an off-diagonal term yields the covariance of the corresponding pair of responses. Extreme-value theory can then be used to obtain mean extreme values of response and the corresponding standard deviations of the extreme values about their mean values.

5. Performance evaluations

The analysis procedures presented above focus on evaluating internal forces and deformations in structure/foundation systems subjected to specified dynamic excitations. Of primary interest is obtaining maximum values of such forces and deformations; thus, such analyses are commonly referred to as "demand" analyses. However, to assess the performance of such systems, the dynamic demands (maximum values) must be compared with corresponding capacities. It is therefore standard practice, especially when evaluating seismically-induced demands, to evaluate capacities by conducting inelastic static "pushover" analyses under controlled monotonic displacement and/or force conditions, noting the formation of plastic hinges, etc. as they take place up to the point of impending collapse. If a structural system can be modeled adequately with only one independent DOF, e.g., a transverse frame supporting a single bridge deck, then the pushover analysis is straightforward. In this case, one controls incrementally the single displacement at deck level and, as it increases monotonically, one follows the local member deformations and corresponding forces. If the frame is supporting decks at two levels, one has essentially two independent degrees of freedom. The proper control of increasing monotonically the displacements and/or forces in these two DOF cannot be rigorous specified; thus, considerable guidance must be provided to the designer. The more independent degrees-of-freedom contributing to dynamic response, the more difficult it is to perform a meaningful pushover analysis.

A meaningful pushover analysis requires very refined modeling to capture localized member capacities, i.e., refinements not normally used in modeling for demand analyses. Due to the above-mentioned difficulties in conducting a meaningful pushover analysis when multiple independent DOF are present, pushover analyses for local elements or subsystems may be more meaningful. Consider, for example, a strongly coupled three-dimensional subassemblage of members removed from a large structural system. It is suggested that time-dependent displacements, corresponding to those determined from a global time-history demand analysis of the complete structural system, be used as boundary inputs to the subassemblage in verifying performance or in determining capacity. In conducting a capacity analysis of the subassemblage subjected to these boundary inputs, the modeling of individual members must be more rigorous than required in conducting the global time-history demand analysis. The number of members in, or size of, the subassemblage should be limited so that inertia and viscous damping forces can be ignored in conducting the capacity analysis; thus allowing a quasi-static solution. This approach should be investigated in a research mode to determine its feasibility.

6. Conclusions

It is the intent of this paper to present some of the more advanced and less-frequently used methods of dynamic analysis of structures, hoping to stimulate their broader use in engineering practice. Their effective use will require further development of general-purpose structural analysis computer programs. All of the analysis procedures presented have been previously published in the literature.

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