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Orthotropic sandwich plates with interlayer slip and under edgewise loads

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Abstract. An elasticity solution for sandwich plates assembled with non-rigid bonding and subjected to edgewise loads is presented. The solution satisfies the equilibrium equations of the face and core elements, the compatibility equations of stresses and strains at the interfaces, and the boundary conditions. To investigate the effects of bonding stiffnesses on the responses of sandwich plates, numerical evaluations are conducted. The results obtained have shown that the bonding stiffness, up to a certain level, has a strong effect on the plate mechanical response. Beyond this level, the usual assumption of perfect bonding used in classical theories is quite acceptable. An answer to what constitutes perfect bonding is found in terms of the ratio of the core stiffness to the bonding stiffness.

Key words: composites; deformation; edge loads; interlayer shear; orthotropic; panels; plates; sandwich; stiffness; strain; stress.

1. Introduction

Significant advances in the development of lightweight materials have promoted the application of sandwich plates in structures (Haan 1996, Hussein and Cheremisinoff 1993, Hussein 1993, Karbhari 1997, Swanson 1997). The advantages of using sandwich plates are derived from judicious combinations of different materials for the skins and core to satisfy the structural and environmental performance requirements. Plate virtues include structural efficiency, weather seal, interior and exterior finish, thermal insulation, and durability.

Existing methods of analysis of sandwich plates have invariably assumed perfect bonding between layers (Allen 1969, Noor *et al.* 1996, Plantema 1966). Nevertheless, interlayer slips do occur because of the finite bonding stiffness; the bonding creep under sustained loads and environmental effects. The high local interlayer shear stress due to applied loads may contribute to an answer of the many delamination problems in structural sandwich plates.

Analysis of wood joist floor systems, taking into account interlayer shear stresses, was done by Goodman *et al*. (1968, 1969, 1974). In that study, the wood layer were assembled with nails or by gluing their ends, and although the interlayer slip in this system was accounted for in the analytical model, transverse shear deformations were neglected. The interlaminar shear in composites under plane stress was investigated analytically by Puppo and Evensen (1970), and with the finite element

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method by Isakson and Levy (1971).

Very few papers have been published which deal with the structural responses of sandwich plates with interlayer slip or orthotropic materials. In a series of analytic and experimental studies, the author has investigated the structural behaviors of sandwich plates with interlayer slips and under transverse and thermal loads (Hussein *et al.* 1989, 1986, 1984, 1982) and sandwich beam-columns with interlayer slip (Hussein *et al.* 1982). In those investigations, many common assumptions from the literature have been replaced with realistic ones such as the use of bonding having finite stiffness; the effects of core elastic properties and shear deformations on the plate deformations and stresses; the permission of faces to deform in their own planes. There remains the problem of orthotropic sandwich plates with interlayer slip and under edgewise loads.

This paper presents an analytical solution of orthotropic sandwich plates with interlayer slips and under edgewise loads. The solution satisfies the equilibrium equations of each layer and the compatibility of deformations at the interfaces. The objective is to ascertain the effects of interlayer slips on the performance of sandwich plates due to edgewise loads.

2. Description of problem

Consider a sandwich plate of span 2a and width 2b, subjected to in-plane biaxial loads as shown in Fig. 1. The plate is composed of three layers bonded together and made of orthotropic materials. The facings are thin, of equal thickness t_f. The core, of a thickness 2 t_c, has a modului of elasticity, E_{cx} and E_{cy} usually significantly less than those of the faces E_{fx} and E_{fy}. However, its shear modului G_{cxy}, G_{cxz} and G_{cyz} should be high enough to develop the interaction required between the layers. The bond between the facings and core has finite stiffness K_x and K_y. The extent of this composite action depends on the relative stiffness of the constituent materials as will be shown subsequently.

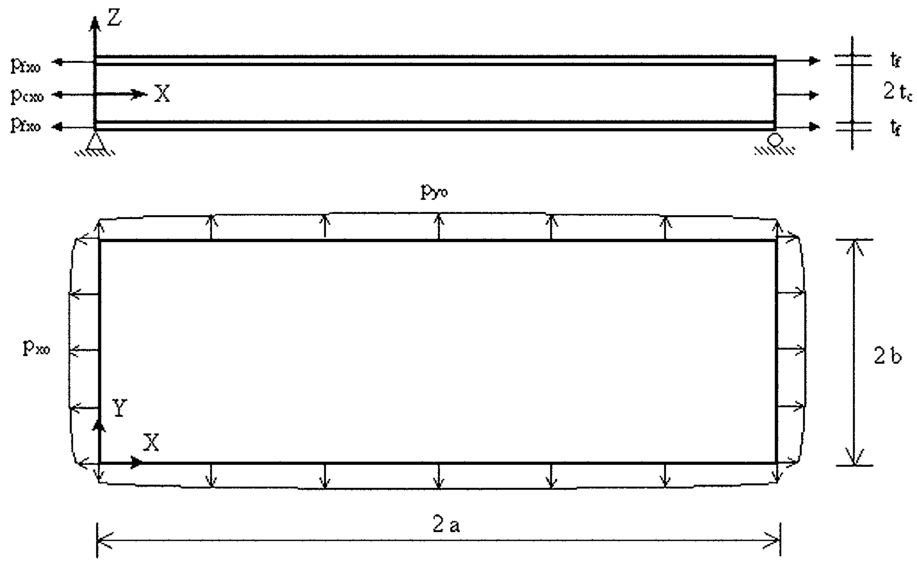

Fig. 1 Sandwich plate

3. Assumptions

The following assumptions underline the present development.

1. The materials are orthotropic and linear elastic.
2. Deformations are small, so first order strain-displacement relations are applicable.
3. Interlayer slip is proportional to the interlayer shears. The term "slip" is probably inaccurate; however, its use in this paper follows from its widespread acceptance in the literature.

4. Analytical development

4.1 General

This kind of problem has been attacked using the fundamentals of theory of elasticity (Hussein 2002a, 2002b, 1992, 1989, 1986, 1984a, 1984b, 1982). Generally, equations are set up to define the equilibrium of the separate faces and of the core and to prescribe the necessary continuity between the faces and the core. The result is a set of differential equations which may be solved in particular cases for the quantities of interest. In that kind of problems, the analytic investigation is sufficiently complex and differ from ordinary homogeneous plates in that the deformations are enhanced by the existence of non-zero shear strains in the core and bonding, and of the direct strains in the core.

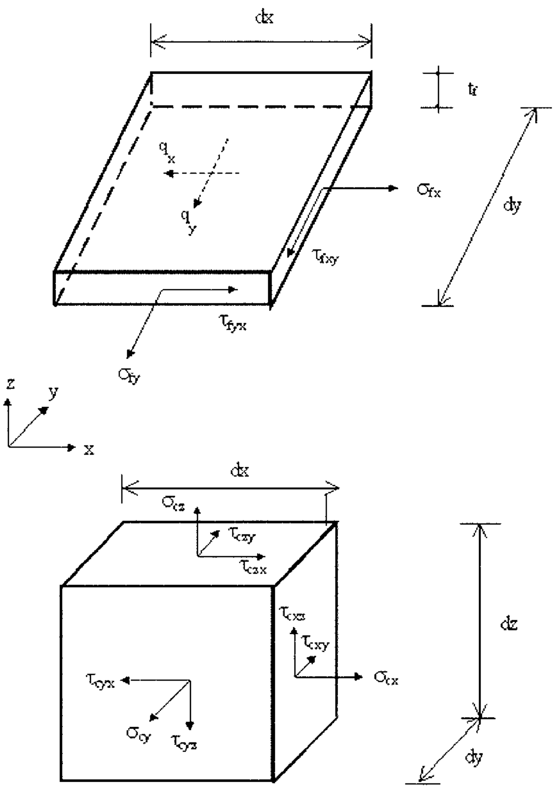

Fig. 2 Stress state in sandwich element

The stress state in the faces and core elements is shown in Fig. 2. The equilibrium of the face element requires that

$$\frac{\partial \sigma_{fx}}{\partial x} + \frac{\partial \tau_{fyx}}{\partial y} - \frac{q_x}{t_f} = 0 \tag{1}$$

$$\frac{\partial \sigma_{fy}}{\partial y} + \frac{\partial \tau_{fxy}}{\partial x} - \frac{q_y}{t_f} = 0 \tag{2}$$

in which

σ_{fx}, σ_{fy}	= Normal stress components in faces;
τ_{fxy}, τ_{fyx}	= Shear stress components in faces;
q_x and q_y	= Interlayer shear stress;
t_f	= The thickness of the face;
f	= Subscript denoting face;
x, y	= Coordinate axes.

The state of stress in the core must satisfy the following equilibrium equations.

$$\frac{\partial \sigma_{cx}}{\partial x} + \frac{\partial \tau_{cyx}}{\partial y} + \frac{\partial \tau_{czx}}{\partial z} = 0 \tag{3}$$

$$\frac{\partial \sigma_{cy}}{\partial y} + \frac{\partial \tau_{cxy}}{\partial x} + \frac{\partial \tau_{czy}}{\partial z} = 0 \tag{4}$$

in which

σ_{cx}, σ_{cy}, σ_{cz}	= Normal stress in the core;
τ_{cxy}, τ_{cyz}, τ_{czx}	= Shear stress in the core;
c	= Subscript denoting core.

The normal stress components in the facings and core must also satisfy the overall equilibrium equations, which are

$$2t_f \int_{y=0}^{y=2b} \sigma_{fx} dy + \int_{y=0}^{y=2b} \int_{z=-tc}^{z=tc} \sigma_{cx} dy\ dz + \int_{y=0}^{y=2b} p_x dy = 0 \tag{5}$$

$$2t_f \int_{y=0}^{y=2a} \sigma_{fy} dx + \int_{y=0}^{y=2a} \int_{z=-tc}^{z=tc} \sigma_{cy} dx\ dz + \int_{y=0}^{y=2a} p_y dx = 0 \tag{6}$$

where p_x and p_y are the applied edge loads.

At the interfaces between the core and the skins, the stresses and strains must be compatible. The compatibility equations in terms of stresses are

$$q_x = \tau_{czx}\big|_{z = \pm tc} \tag{7}$$

$$q_y = \tau_{czy}\big|_{z = \pm tc} \tag{8}$$

In terms of strains, the compatibility equations are written as

$$\frac{\partial \Delta_x}{\partial x} = \varepsilon_{fx} - \varepsilon_{cx}\big|_{z = \pm t_c} \tag{9}$$

$$\frac{\partial \Delta_y}{\partial y} = \varepsilon_{fy} - \varepsilon_{cy}\big|_{z = \pm t_c} \tag{10}$$

$$\gamma_{fxy} - \gamma_{cxy}\big|_{z = \pm t_c} = \frac{\dfrac{\partial q_x}{\partial y}}{K_x} + \frac{\dfrac{\partial q_y}{\partial x}}{K_y} \tag{11}$$

in which

ε and γ	= Normal and shear strain, respectively;
Δ_i	= Interlayer deformation in the i direction, where $i = x$ or y;
	$= \dfrac{q_i}{K_i}$;
K_i	= Stiffness of bonding in the i direction.

Solutions to the problem must also satisfy the prescribed displacement boundary conditions. With respect to a sandwich plate subjected to edgewise loads, the relevant boundary conditions are

1. At the plate edges, no normal or shear stresses should exist in the core and the face normal stress must equal the applied in-plane stress, thus

$$\text{at } x = 0, 2a \qquad \sigma_{fx} = \sigma_{fxo} \tag{12}$$

$$\text{at } y = 0, 2b \qquad \sigma_{fy} = \sigma_{fyo} \tag{13}$$

in which

$$\sigma_{fxo} = p_{fxo}/t_f$$

$$\sigma_{fyo} = p_{fyo}/t_f$$

2. For symmetrical loading about the plate middle plane and centerlines, the shear stresses vanish and no in-plane displacements occur. Thus

$$\text{at } x = a \qquad \tau_{fxy} = \tau_{cxy} = 0 \qquad u_c = u_f = 0 \tag{14}$$

$$\text{at } y = b \qquad \tau_{fyx} = \tau_{cyx} = 0 \qquad v_c = v_f = 0 \tag{15}$$

where u and v are displacements in the x and y directions, respectively.

4.2 Equilibrium of core

For the sandwich plate in Fig. 1, a solution for normal stress components in the core satisfying the boundary conditions in Eqs. (12) and (13) is considered as (Hussein *et al.* 1989, 1986, 1984a, 1984b, 1982)

$$\sigma_{cx} = \sum_{m=1,3,\dots}^{\infty} \sum_{n=1,3,\dots}^{\infty} A_{mn}\phi_x S_x S_y \tag{16}$$

$$\sigma_{cy} = \sum_{m=1,3,\dots}^{\infty} \sum_{n=1,3,\dots}^{\infty} C_{mn}\phi_y S_x S_y \tag{17}$$

in which

$$\phi_x = \theta_x(2\theta_x \cosh\theta_x z + z\theta_x^2 \sinh\theta_x z - t_c\theta_x^2 \coth\theta_x t_c \cosh\theta_x z) - \frac{\alpha_m^2}{2} k_{\phi x}\cos\alpha_{\phi x} z;$$

$$\phi_y = \theta_y(2\theta_y \cosh\theta_y z + z\theta_y^2 \sinh\theta_y z - t_c\theta_y^2 \coth\theta_y t_c \cosh\theta_y z) - \frac{\beta_n^2}{2} k_{\phi y}\cos\alpha_{\phi y} z;$$

$$k_{\phi x} = -\frac{\theta_x t_c}{\sinh\theta_x t_c} + \theta_x t_c \cosh\theta_x t_c;$$

$$k_{\phi y} = -\frac{\theta_y t_c}{\sinh\theta_y t_c} + \theta_y t_c \cosh\theta_y t_c;$$

$$\theta_x = \alpha_m \sqrt{\frac{E_{cx}}{G_{cxz}}};$$

$$\theta_y = \beta_n \sqrt{\frac{E_{cy}}{G_{cyz}}};$$

$$\alpha_{\phi x} = \frac{m\pi}{2t_c};$$

$$\beta_{\phi y} = \frac{n\pi}{2t_c};$$

S_x, S_y = sin $\alpha_m x$ and sin $\beta_n y$, respectively;

α_m, β_n = $\frac{m\pi}{2a}$ and $\frac{n\pi}{2b}$, respectively;

A_{mn}, C_{mn} = Unknown coefficients;
m, n = Integers.

From Eqs. (16) and (17), expressions for the displacement components in the core satisfying the boundary conditions in Eqs. (14) and (15) are derived as

$$u_c = -\frac{1}{E_{cx}} \sum_{m=1,3,\ldots}^{\infty} \sum_{n=1,3,\ldots}^{\infty} \frac{A_{mn}\phi_x C_x S_y}{\alpha_m} + \frac{\nu_{cxy}}{E_{cy}} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{C_{mn}\phi_y C_x S_y}{\alpha_m} \tag{18}$$

$$v_c = -\frac{1}{E_{cy}} \sum_{m=1,3,\ldots}^{\infty} \sum_{n=1,3,\ldots}^{\infty} \frac{C_{mn}\phi_y S_x C_y}{\beta_n} + \frac{\nu_{cyx}}{E_{cx}} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{A_{mn}\phi_x S_x C_y}{\beta_n} \tag{19}$$

in which

ν = Poisson's ration;

C_x, C_y = cos α_m x and cos β_n y, respectively.

Eqs. (18) and (19) fulfill the boundary conditions in Eqs. (14) and (15). An expression for the shear strain in the core γ_{cxy} is obtained by properly differentiating Eqs. (18) and (19); thus

$$\gamma_{cxy} = \frac{1}{E_{cx}} \sum_{m=1,3,\ldots}^{\infty} \sum_{n=1,3,\ldots}^{\infty} A_{mn}\phi_x\left(-\frac{\beta_n}{\alpha_m} + \nu_{cyx}\frac{\alpha_m}{\beta_n}\right)C_x C_y + \frac{1}{E_{cy}} \sum_{m=1,3,\ldots}^{\infty} \sum_{n=1,3,\ldots}^{\infty} C_{mn}\phi_y\left(-\frac{\alpha_m}{\beta_n} + \nu_{cxy}\frac{\beta_n}{\alpha_m}\right)C_x C_y \tag{20}$$

Eq. (20) fulfills the boundary conditions in Eqs. (14) and (15). By substituting Eqs. (16), (17) and (20) in Eq. (3), an expression for the vertical shear stress τ_{cxz} in the core is obtained as

$$\tau_{cxz} = \sum_{m=1,3,\ldots}^{\infty} \sum_{n=1,3,\ldots}^{\infty} \int_{z=0}^{z} \phi_x dz A_{mn}\left[-\alpha_m + \frac{G_{cxy}}{E_{cx}}\beta_n\left(-\frac{\beta_n}{\alpha_m} + \nu_{cyx}\frac{\alpha_m}{\beta_n}\right)\right]C_x S_y + \frac{G_{cxy}}{E_{cy}} \sum_{m=1,3,\ldots}^{\infty} \sum_{n=1,3,\ldots}^{\infty} \int_{z=0}^{z} \phi_y dz C_{mn}\beta_n\left(-\frac{\alpha_m}{\beta_n} + \nu_{cxy}\frac{\beta_n}{\alpha_m}\right)C_x S_y \tag{21}$$

In a similar manner, the shear stress in the core τ_{cyz} is obtained from Eqs. (17), (20) and (4) as

$$\tau_{cyz} = \sum_{m=1,3,\ldots}^{\infty} \sum_{n=1,3,\ldots}^{\infty} \int_{z=0}^{z} \phi_y dz C_{mn}\left[-\beta_m + \frac{G_{cxy}}{E_{cy}}\alpha_m\left(-\frac{\alpha_m}{\beta_n} + \nu_{cxy}\frac{\beta_n}{\alpha_m}\right)\right]S_x C_y + \frac{G_{cxy}}{E_{cx}} \sum_{m=1,3,\ldots}^{\infty} \sum_{n=1,3,\ldots}^{\infty} \int_{z=0}^{z} \phi_x dz A_{mn}\alpha_m\left(-\frac{\beta_n}{\alpha_m} + \nu_{cyx}\frac{\alpha_m}{\beta_n}\right)S_x C_y \tag{22}$$

4.3 Interlayer shear stresses

Expressions for the interlayer shear stresses q_x and q_y are obtained from Eqs. (21) and (22) in accordance with the compatibility Eqs. (7) and (8); thus

$$q_x = \sum_{m=1,3,\ldots}^{\infty} \sum_{n=1,3,\ldots}^{\infty} (A_{mn}\lambda_{gn1} + C_{mn}\lambda_{gn2})C_x S_y \tag{23}$$

$$q_y = \sum_{m=1,3,\dots}^{\infty} \sum_{n=1,3,\dots}^{\infty} (C_{mn}\lambda_{gk1} + A_{mn}\lambda_{gk2})S_x C_y \quad (24)$$

in which

$$\lambda_{gn1} = \int_{z=0}^{z} \phi_x dz|_{z=t_c}\left[-\alpha_m + \frac{G_{cxy}}{E_{cx}}\beta_n\left(-\frac{\beta_n}{\alpha_m} + \nu_{cyx}\frac{\alpha_m}{\beta_n}\right)\right]$$

$$\lambda_{gn2} = \frac{G_{cxy}}{E_{cy}}\int_{z=0}^{z} \phi_y dz|_{z=t_c}\beta_n\left(-\frac{\alpha_m}{\beta_n} + \nu_{cxy}\frac{\beta_n}{\alpha_m}\right)$$

$$\lambda_{gk1} = \int_{z=0}^{z} \phi_y dz|_{z=t_c}\left[-\beta_n + \frac{G_{cxy}}{E_{cy}}\alpha_m\left(-\frac{\alpha_m}{\beta_n} + \nu_{cxy}\frac{\beta_n}{\alpha_m}\right)\right]$$

$$\lambda_{gk2} = \frac{G_{cxy}}{E_{cx}}\int_{z=0}^{z} \phi_x dz|_{z=t_c}\alpha_m\left(-\frac{\beta_n}{\alpha_m} + \nu_{cyx}\frac{\alpha_m}{\beta_n}\right)$$

4.4 Equilibrium of face

An expression for the in-plane shear stress τ_{fxy} in the facings is obtained from Eqs. (11) and (20) as

$$\begin{aligned}\tau_{fxy} = {} & \sum_{m=1,3,\dots}^{\infty} \sum_{n=1,3,\dots}^{\infty} A_{mn}\left[\frac{G_{fxy}}{E_{cx}}\phi_x|_{z=t_c}\left(-\frac{\beta_n}{\alpha_m} + \nu_{cyx}\frac{\alpha_m}{\beta_n}\right) + \right. \\ & \left. \frac{G_{fxy}\beta_n}{K_x}\lambda_{gn1} + \frac{G_{fxy}\alpha_m}{K_y}\lambda_{gk2}\right]C_x C_y + \\ & \sum_{m=1,3,\dots}^{\infty} \sum_{n=1,3,\dots}^{\infty} C_{mn}\left[\frac{G_{fxy}}{E_{cy}}\phi_y|_{z=t_c}\left(-\frac{\alpha_m}{\beta_n} + \nu_{cxy}\frac{\beta_n}{\alpha_m}\right) + \right. \\ & \left. \frac{G_{fxy}\beta_n}{K_x}\lambda_{gn2} + \frac{G_{fxy}\alpha_m}{K_y}\lambda_{gk1}\right]C_x C_y \end{aligned} \quad (25)$$

An expression for the normal stress σ_{fx} in the facings is obtained by substituting Eqs. (23) and (25) in Eq. (1); Thus

$$\sigma_{fx} = \sum_{m=1,3,\dots}^{\infty} \sum_{n=1,3,\dots}^{\infty} (A_{mn}\lambda_{z1} + C_{mn}\lambda_{z2})S_x S_y + f(x, y) \quad (26)$$

in which

$$\lambda_{z1} = \frac{G_{fxy}}{E_{cx}}\phi_x|_{z=t_c}\frac{\beta_n}{\alpha_m}\left(-\frac{\beta_n}{\alpha_m}+\nu_{cyx}\frac{\alpha_m}{\beta_n}\right)+\frac{\lambda_{gn1}}{\alpha_m}\left(\frac{G_{fxy}\beta_n^2}{K_x}+\frac{1}{t_f}\right)+\frac{G_{fxy}\lambda_{gk2}\beta_n}{K_y}$$

$$\lambda_{z2} = \frac{G_{fxy}}{E_{cy}}\phi_y|_{z=t_c}\frac{\beta_n}{\alpha_m}\left(-\frac{\alpha_m}{\beta_n}+\nu_{cxy}\frac{\beta_n}{\alpha_m}\right)+\frac{\lambda_{gn2}}{\alpha_m}\left(\frac{G_{fxy}\beta_n^2}{K_x}+\frac{1}{t_f}\right)+\frac{G_{fxy}\lambda_{gk1}\beta_n}{K_y}$$

$f(x, y)$ = A function representing the constant of integration.

The function $f(x, y)$ is obtained by using the overall equilibrium Eq. (5); Eqs. (16) and (26), and by expanding the applied load in double trigonometric series. In the case of a uniform load of intensities p_{fxo} and p_{cxo}, σ_{fx} is found as

$$\sigma_{fx} = \sum_{m=1,3,\dots}^{\infty}\sum_{n=1,3,\dots}^{\infty}(A_{mn}\lambda'_{z1}+C_{mn}\lambda'_{z2})S_xS_y+\sigma_{fxo}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{2}{a\alpha_m}\frac{2}{b\beta_n}S_xS_y \tag{27}$$

in which

$$\lambda'_{z1} = \lambda_{z1}\left(1-\frac{2}{b^2\beta_n^2}\right)-\frac{2\int_0^{tc}\phi_x dz}{t_f b^2\beta_n^2}$$

$$\lambda'_{z2} = \lambda_{z2}\left(1-\frac{2}{b^2\beta_n^2}\right)$$

In a similar manner, an expression for the normal stress σ_{fy} is obtained from Eqs. (2), (6), (17) and (24) as

$$\sigma_{fy} = \sum_{m=1,3,\dots}^{\infty}\sum_{n=1,3,\dots}^{\infty}(A_{mn}\lambda'_{z3}+C_{mn}\lambda'_{z4})S_xS_y+\sigma_{fyo}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{2}{a\alpha_m}\frac{2}{b\beta_n}S_xS_y \tag{28}$$

in which

$$\lambda'_{z3} = \lambda_{z3}\left(1-\frac{2}{a^2\alpha_m^2}\right)$$

$$\lambda'_{z4} = \lambda_{z4}\left(1-\frac{2}{a^2\alpha_n^2}\right)-\frac{2\int_0^{t_c}\phi_y dz}{t_f a^2\alpha_m^2}$$

$$\lambda_{z3} = \frac{G_{fxy}}{E_{cx}}\phi_x|_{z=t_c}\frac{\alpha_m}{\beta_n}\left(-\frac{\beta_n}{\alpha_m}+\nu_{cyx}\frac{\alpha_m}{\beta_n}\right)+\frac{\lambda_{gk2}}{\beta_n}\left(\frac{G_{fxy}\alpha_m^2}{K_y}+\frac{1}{t_f}\right)+\frac{G_{fxy}\lambda_{gn1}\alpha_m}{K_x}$$

$$\lambda_{z4} = \frac{G_{fxy}}{E_{cy}}\phi_y|_{z=t_c}\frac{\alpha_m}{\beta_n}\left(-\frac{\alpha_m}{\beta_n}+\nu_{cxy}\frac{\beta_n}{\alpha_m}\right)+\frac{\lambda_{gk1}}{\beta_n}\left(\frac{G_{fxy}\alpha_n^2}{K_y}+\frac{1}{t_f}\right)+\frac{G_{fxy}\lambda_{gn2}\alpha_m}{K_x}$$

4.5 Solutions for A_{mn} and C_{mn}

At this stage, the only unknowns are the coefficients A_{mn} and C_{mn}. These can be determined by using the compatibility Eqs. (9) and (10). By substituting Eqs. (16), (17), (23), (24), (27) and (28) in Eqs. (9) and (10); A_{mn} and C_{mn} are obtained as

$$A_{mn} = \frac{\dfrac{\lambda_{y3}}{\lambda_{y2}} - \dfrac{\lambda_{y6}}{\lambda_{y5}}}{\dfrac{\lambda_{y1}}{\lambda_{y2}} - \dfrac{\lambda_{y4}}{\lambda_{y5}}} \tag{29}$$

$$C_{mn} = \frac{\dfrac{\lambda_{y3}}{\lambda_{y1}} - \dfrac{\lambda_{y6}}{\lambda_{y4}}}{\dfrac{\lambda_{y2}}{\lambda_{y1}} - \dfrac{\lambda_{y5}}{\lambda_{y4}}} \tag{30}$$

in which

$$\lambda_{y1} = \frac{\lambda'_{z1}}{E_{fx}} - \frac{\nu_{fxy}\lambda'_{z3}}{E_{fy}} - \frac{\phi_x|_{z=tc}}{E_{cx}} + \frac{\alpha_m \lambda_{gn1}}{K_x}$$

$$\lambda_{y2} = \frac{\lambda'_{z2}}{E_{fx}} - \frac{\nu_{fxy}\lambda'_{z4}}{E_{fy}} + \frac{\nu_{cxy}\phi_y|_{z=tc}}{E_{cy}} + \frac{\alpha_m \lambda_{gn2}}{K_x}$$

$$\lambda_{y3} = \frac{2}{a\alpha_m}\frac{2}{b\beta_n}\left[\frac{\sigma_{fxo}}{E_{fx}} - \nu_{fxy}\frac{\sigma_{fyo}}{E_{fy}}\right]$$

$$\lambda_{y4} = \frac{\lambda'_{z3}}{E_{fy}} - \frac{\nu_{fyx}\lambda'_{z1}}{E_{fx}} + \frac{\nu_{cyx}\phi_x|_{z=tc}}{E_{cx}} + \frac{\beta_n \lambda_{gk2}}{K_y}$$

$$\lambda_{y5} = \frac{\lambda'_{z4}}{E_{fy}} - \frac{\nu_{fyx}\lambda'_{z2}}{E_{fx}} - \frac{\phi_y|_{z=tc}}{E_{cy}} + \frac{\beta_n \lambda_{gk1}}{K_y}$$

$$\lambda_{y6} = \frac{2}{a\alpha_m}\frac{2}{b\beta_n}\left[\frac{\sigma_{fyo}}{E_{fy}} - \nu_{fyx}\frac{\sigma_{fxo}}{E_{fx}}\right]$$

5. Numeric evaluation of effects of bonding on behavior of sandwich plates

The complexity of the preceding solution makes it difficult to see the effect of bonding on the sandwich plate responses. To demonstrate these effects, a square plate is considered. The plate is made of two aluminum faces, a plastic foam core, and assembled with a non-rigid bonding. The plate, facings and core properties are:

For plate:
$a = b$ = 20 in. (1219.2 mm)

For facings:
t_f = 0.04 in. (1.016 mm);
$E_{fx} = E_{fy}$ = 10^7 psi (68.9 GPa);
$\nu_{fxy} = \nu_{fyx}$ = 0.33

For core:
t_c = 1.0 in. (50.8 mm);
$E_{cx} = E_{cy}$ = 2×10^6 psi (137.8 MPa)
$G_{cxy} = G_{cxz} = G_{cyz}$ = 10^4 psi (68.9 MPa)
$\nu_{cxy} = \nu_{cyx}$ = 0.20

Two loading cases are considered. In the first case a biaxial uniformly distributed stress of intensity $\sigma_{fxo} = \sigma_{fyo}$ = 208.3 psi is used. In the second case a uniaxial uniformly distributed stress of intensity σ_{fxo} = 208.3 psi is applied. In each loading case, the load is applied independently first to the face and core, and then concurrently to face and core as shown in Fig. 3. The face normal and shear stresses are calculated for a chosen range of bonding stiffness from $K_x = K_y = 10^3$-10^4 psi/in. The normal stress in the facings at the plate center and the shear stress in the facings at the plate corner are shown graphically in Figs. 4 and 5.

It is seen that the face normal stress shows greater sensitivity to the variation of bond stiffness value when the latter is in the lower range; and beyond a certain level of stiffness, the bonding can be practically considered as rigid. A change in K_x (or K_y) value for example from 10^3 to 2×10^3 psi/in

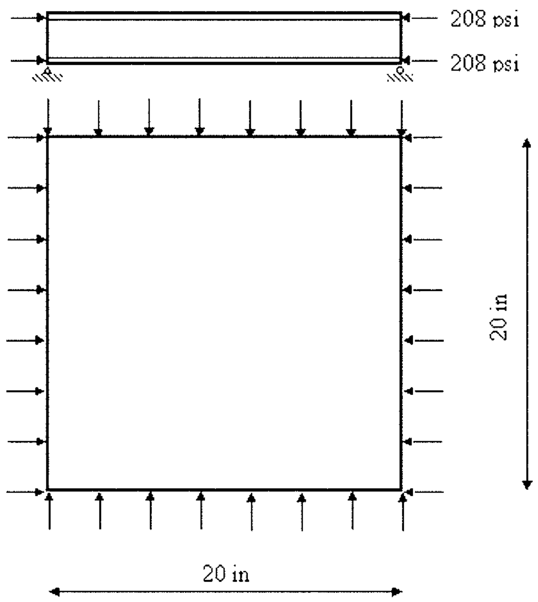

Fig. 3 A sandwich plate under biaxial edge load applied to skin

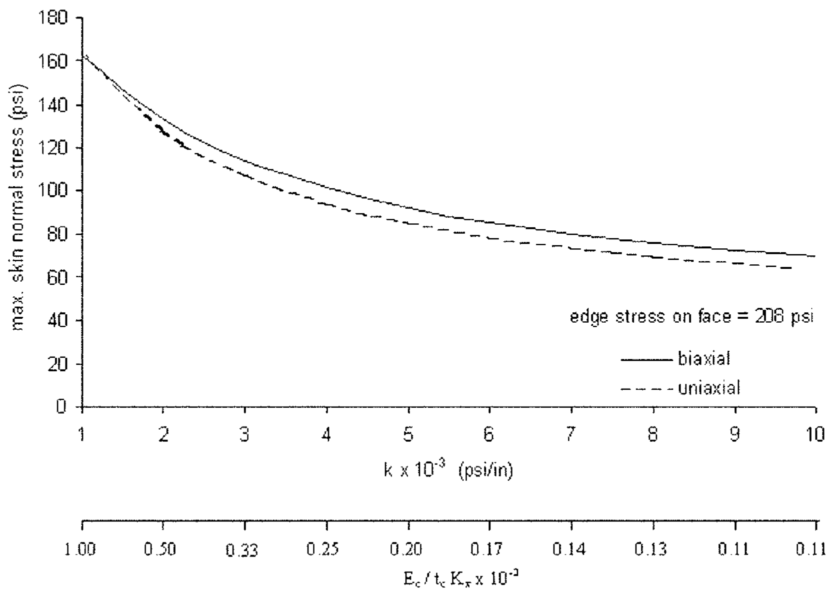

Fig. 4 Effects of bond stiffness on face normal stress due to face edge load

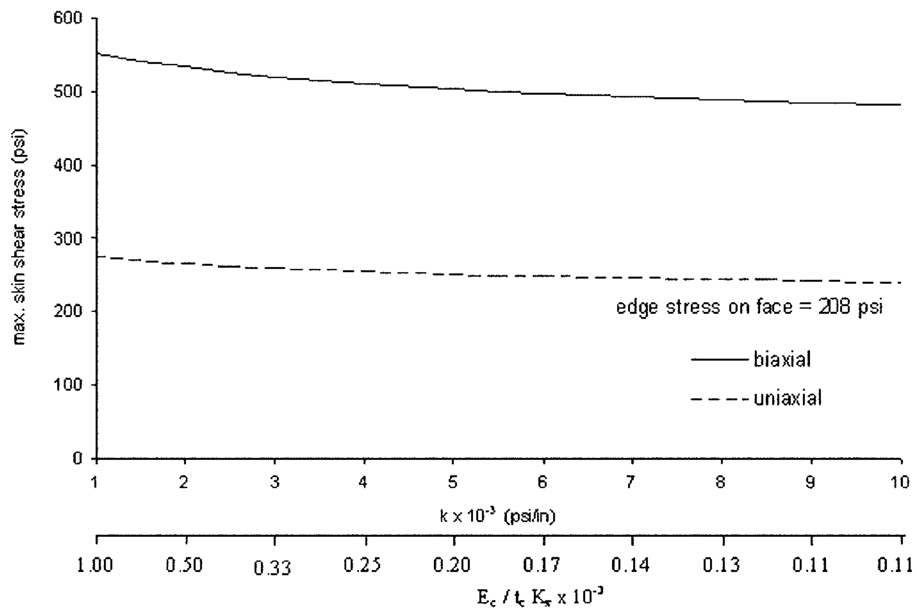

Fig. 5 Effects of bond stiffness on face shear stress due to face edge load

induces a stress decrease almost 6 times in the uniaxial case and 5 times in the biaxial case greater than when K_x (or K_y) changes from 9×10^3-10^4 psi/in. The face shear stress is practically independent of bonding stiffness. Unlike the mechanical behavior of other sandwich plates with non-rigid bonding and under transverse and thermal loads (Hussein *et al*. 2002a, 2002b, 1993, 1992, 1989, 1986, 1984a, 1982), this study reveals that interlayer shears are insignificant. This is due to the absence of transverse loads which induce high transverse shear forces.

This analysis has yet to bring up an important point. By using existing theories (Allen 1969, Plantema 1966), stress components in sandwich plates may be determined only at high values of bond stiffness with a small margin of error, otherwise the K values must be included in the analysis. The results presented here are virtually identical to those by existing theories for the case of perfect; i.e. very rigid, bonding.

Another important point has yet to come out of this analysis. By common sense, it can be felt that a very stiff bonding would be unnecessary if the core were too soft, and the converse would be unwise. This is quantitatively shown in Figs. 4 and 5 which show that the ratio of core stiffness to bond stiffness is one of the main parameters influencing the behavior of sandwich plates.

Finally, it is worth while to mention that the literature has no record of elasticity based analytic investigations of sandwich plates with bonds having finite stiffness and under edgewise loads. In this regard, this paper has advanced the state-of-the art.

6. Conclusions

In the literature, very few papers have been published which deal with the effects of bonding on the structural response of sandwich plates. Realistically, the core and bond in sandwich plates are rigid enough to make a significant contribution to the overall structural integrity of the plate, yet flexible enough to permit shear deformation.

An analysis of orthotropic sandwich plates taking into account the effects of the finite bonding stiffness has been presented in this paper. The edgewise load can be uniaxial or biaxial. The solution satisfies the equilibrium equations of the face and core elements, the compatibility equations of stresses and strains at the interfaces, and the boundary conditions.

The numerical results have shown that the bonding stiffness, up to a certain level, has a strong effect on the plate response. Beyond this level, the usual assumption of perfect bonding in the literature is quite acceptable. The answer to what constitute perfect bonding may be best answered in terms of the ratio of core stiffness to the bond stiffness, rather than on the individual constituent material.

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