Structural Engineering and Mechanics, Vol. 17, No. 1 (2004) 29-50

DOI: http://dx.doi.org/10.12989/sem.2004.17.1.029

Crack growth life model for fatigue susceptible structural components in aging aircraft

Karen C. Chou†

Department of Mechanical & Civil Engineering, 205 Trafton Science Center E., Minnesota State University, Mankato, MN 56001, U.S.A.

Glenn C. Cox‡

Lockwood Greene Engineers, Knoxville, TN, U.S.A.

Allison M. Lockwood‡†

Department of Civil & Environmental Engineering, University of Tennessee, Knoxville, U.S.A.

(Received January 8, 2003, Accepted August 18, 2003)

Abstract. A total life model was developed to assess the service life of aging aircraft. The primary focus of this paper is the development of crack growth life projection using the response surface method. Crack growth life projection is a necessary component of the total life model. The study showed that the number of load cycles N needed for a crack to propagate to a specified size can be linearly related to the geometric parameter, material, and stress level of the component considered when all the variables are transformed to logarithmic values. By the Central Limit theorem, the ln N was approximated by Gaussian distribution. This Gaussian model compared well with the histograms of the number of load cycles generated from simulated crack growth curves. The outcome of this study will aid engineers in designing their crack growth experiments to develop the stochastic crack growth models for service life assessments.

Key words: crack growth; fatigue; aging aircraft; response surface method; central limit theorem; structural reliability; Gaussian distribution; statistical analysis; Kolmogorov-Smirnov test; point estimate method.

1. Introduction

The central aging aircraft issue is the evaluation of airframe service lives. A realistic analysis must consider the randomness inherent in life prediction. In fatigue susceptible structural components, the path to “failure” (unacceptable condition) consists of 2 stages, crack initiation and crack growth. The physical model for a presumably perfect specimen to develop a detectable crack, taken as 0.01 inch in

† Associate Professor & Coordinator of Civil Engineering
‡ Assistant Engineer
‡† Former Undergraduate Research Assistant

this study, and the model for the crack to propagate to a critical size, taken as 0.3 inch, are quite different. Hence, the total life, L, of a component is defined as the sum of the crack initiation time, I, and the crack growth time, C. When the total life is less than the predetermined value, $L \le l_0$, the component is considered unserviceable or as failure. The corresponding probability, P_f, is given by

$$P_f = P[L \le l_0] = \int_0^{l_0} f_L(l)dl \tag{1}$$

in which $f_L(l)$ = probability density function of L. Eq. (1) can be rewritten in terms of crack initiation time and crack growth time,

$$P_f = \int_0^{l_0} \left[\int_0^{l_0 - i} f_C(c)dc \right] f_I(i)di \tag{2}$$

Eq. (2) assumes the crack initiation time, I, and the crack growth time, C, are statistically independent.

Chou (1998) showed that the total life model (Eq. 2) has the potential to be used in the reliability assessment of total service life expectancy. In order to further demonstrate the applicability of the total life model, a procedure to develop the probability density function for crack initiation time and crack growth time is desired. The focus of this paper is on the development of a probability model for crack growth time, $f_C(c)$ shown in Eq. (2). The information used in this study was provided by the engineers at the U.S. Navy when the first author was a visiting Summer Research Faculty Fellow and is for the potential cracks at the bolt holes of a cockpit longeron.

2. Theoretical development

Studies of crack growth are numerous. There have also been extensive studies on crack growth due to cyclic loads. To study the randomness inherent in the load-resistance behavior, researchers have developed stochastic models between crack size and load cycles (for example, Wirsching 1983, and Yang *et al*. 1985, a comprehensive list of studies in this area would be too numerous to list). These studies primarily focused on the crack size for a given number of load cycles. The interest here, for life prediction, is the crack growth time for a specified crack size. Fig. 1 shows the typical probability model developed by other researchers and the one needed for the total life model.

More recently, Yang and Manning (1996) presented a second order approximation for the crack growth distribution. The method can also compute the service time for a given crack size. The method yielded a cumulative distribution function (cdf) for service time. The parameters for the cdf were determined using a median crack growth curve. In the total life model proposed here, a probability density function (pdf) of the service time for a given crack size is needed. The pdf was developed based on the material, the geometry, and the stress level applied to the component.

Response surface method (RSM) was chosen for this study because of its simplicity in procedures and its flexibility of accepting limited experimental data. In the following sections, development of the RSM for the number of load cycles needed for a crack to propagate to a specified size is presented. The crack growth rate, as used by the Navy in this study, was assumed to follow Paris Law. And, the crack initiation size was defined as 0.01 inch and the critical size was defined as 0.3 inch.

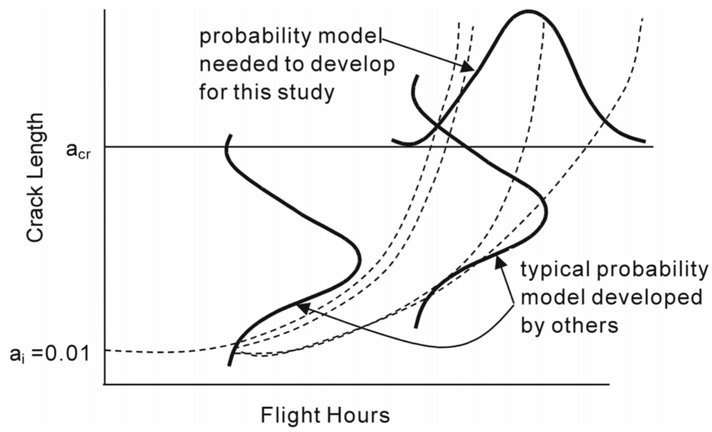

Fig. 1 Typical crack growth curves and probability models for crack size versus crack growth life

2.1 Fatigue crack growth model

As stated previously, the crack growth model used in this study was based on Paris Law (Bannantine *et al.* 1990) where the ratio of crack growth is defined as

$$\frac{da}{dN} = c(\Delta k^m) \tag{3}$$

in which a = crack size; N = number of load cycles and the duration of each load cycle was assumed to be constant; c = geometric parameter; m = material constant; and Δk = stress intensity factor and is defined as

$$\Delta k = \beta(\Delta\sigma)\sqrt{\pi a} \tag{4}$$

in which β = correction factor and $\Delta\sigma$ = stress intensity. For a complex geometry of the specimen, the integration of Eq. (3) does not yield a closed form solution. Numerical integration was used instead. In this study, the difference between crack initiation size and critical crack size was divided into 100 equal intervals. The number of load cycles ΔN needed to achieve each incremental crack length Δa was computed as

$$\Delta N_j = \frac{\Delta a}{c(\Delta k_j)^m} = \frac{\Delta a}{c[\beta_j(\Delta\sigma_j)\sqrt{\pi a_j}]^m} \tag{5}$$

in which

$$\Delta a = \frac{a_{critical} - a_{initiation}}{100} \tag{6}$$

and β_j, $\Delta\sigma_j$, and a_j are, respectively, the correction factor, the stress intensity, and crack size at the j-th increment. The total number of load cycles N equals to the summation of the incremental number of cycles computed by Eq. (5).

2.2 Response surface method

Response surface method (RSM) uses a limited amount of data to approximate a system in which a large number of variables influence the response of the system. The history of RSM can be found in a report written by Myers and his colleagues (Myers *et al.* 1989). The use of “response curves” has been dated back to the 1930’s, but it wasn’t introduced formally until an article by Box and Wilson (1951) on the notion of composite designs was published.

Rajashekhar and Ellingwood (1993) gave a concise description of how to apply RSM to engineering problems. Assume a response variable Y which depends on the input parameters X_1, X_2, $X_{3,}$ ……., X_n, and each set of X_1, X_2, $X_{3,}$ ……., X_n and Y values is obtained experimentally or through simulations. Each set of X’s and Y values represents a point in an $(n + 1)$-dimensional space. Using sets of X’s and Y values (multiple points in the $(n + 1)$-dimensional space), one can develop a relationship between the response variable Y and the input parameters X’s through some form of regression analyses.

In most engineering systems, the relationship between the input parameters and the output value is very complex and is often difficult to derive mathematically or obtain a closed form expression. Response surface method offers a convenient approach to approximate such a relationship because only a limited number of input-response data points are required.

Laboratory testing on crack growth life can usually provide a small number of crack growth curves similar to those shown in Fig. 1. The limited number of data points is usually not sufficient to develop a quality statistical model. Moreover, as discussed in the previous section, except for some simple cases, the number of load cycles determined using Eq. (1) will not yield a closed form solution. Hence, the response surface method became a desirable choice for the modeling of the crack growth life.

To develop the response surface function, the number of load cycles, N, became the response variable and the geometric, material, and stress intensity parameters became the input variables. In this study, crack growth curves computed using Eq. (5) for various values of input variables were used to develop the response surface function. One of the objectives of this study was to determine the optimal number of crack growth curves needed to achieve an acceptable response surface function and its corresponding probability density function. The result will help engineers design the laboratory testing for new material, new geometry, and/or new stress level for future reliability studies.

Since the number of load cycles, N, is a function of three variables, a typical linear regression analysis (the simplest regression relationship) yields four coefficients. Hence, a minimum of 5 sets of (c, m, $\Delta\sigma$, and N) values are needed for the regression analysis. A computer software called Matlab and its statistical toolbox (by Mathwork) were used to perform the regression analysis which was based on the least-squared-error criteria. Both linear and nonlinear regression analyses were attempted. The most workable function was a linear regression with all the variables transformed to the natural logarithmic values,

$$ln\ N = b_1 lnc + b_2 lnm + b_3 ln(\Delta\sigma) + b_4 \tag{7}$$

in which b_i = coefficients obtained from the regression analysis.

2.3 Probability model and statistical verification

Since the response surface function for ln N (Eq. 7) is linear, ln N can be approximated by Gaussian distribution based on the Central Limit theorem. The statistics for ln N can be computed using either the point estimate method (Harr 1987, Rosenblueth 1975, Rosenblueth 1981) or the Taylor series approximation (Benjamin and Cornell 1970).

The point estimate method requires 8 points for the 3 variables c, m, and $\Delta\sigma$. These points are:

(c+, m+, $\Delta\sigma$+); (c+, m+, $\Delta\sigma$–); (c+, m–, $\Delta\sigma$+); (c+, m–, $\Delta\sigma$–); (c–, m+, $\Delta\sigma$+); (c–, m+, $\Delta\sigma$–); (c–, m–, $\Delta\sigma$+); (c–, m–, $\Delta\sigma$–);

in which "*+ " = mean of * plus (+) one standard deviation of *; and "*–" = mean of * minus (–) one standard deviation of *. The ln N values were computed for each of the points listed above using the response surface function given in Eq. (7). The mean and standard deviation of ln N becomes:

$$E[ln\ N] = \frac{1}{8}\sum_{i=1}^{8} ln\ n_i \tag{8}$$

$$\sigma_{lnN} = \sqrt{\frac{1}{8}\sum_{i=1}^{8}(ln\ n_i)^2 - (E[ln\ N])^2} \tag{9}$$

The other method used to estimate the mean and standard deviation of ln N from the response surface function was Taylor series approximation. Taylor series approximation requires that the RS function be continuous at least up to the second derivative. Second order Taylor series approximation was used to estimate the mean and first order approximation was used to estimate the variance. When the input parameters were assumed to be mutually independent, the resulting mean and standard deviatin were given as,

$$E[ln\ N] = b_1 ln(E[c]) + b_2 ln(E[m]) + b_3 ln(E[\Delta\sigma]) + b_4 + \frac{1}{2}[b_1\zeta_c^2 + b_2\zeta_m^2 + b_3\zeta_{\Delta\sigma}^2] \tag{10}$$

$$\sigma_{lnN} = \sqrt{b_1^2\zeta_c^2 + b_2^2\zeta_m^2 + b_3^2\zeta_{\Delta\sigma}^2} \tag{11}$$

in which

$$\zeta_*^2 = ln(V_*^2 + 1) \tag{12}$$

and V_* = the coefficient of variation of the variable *. When $V_* << 1$, $\zeta_*^2 \approx V_*^2$.

In order to verify the validity of the probability model proposed for ln N, the statistical characteristics of the proposed model should be compared with the actual data. Unfortunately, no actual data were available. Instead data from "pseudo population" were used. The "pseudo population" is a set of crack growth curves computed using Eq. (5). The input parametric values were simulated according to the statistical information of each random variable given in Table 1.

Table 1 Statistical data for input parameters

Parameter	Mean	Standard Deviation	Coefficient of Variation	Probability Distribution
c	1.0 E – 8	2.3 E – 9	0.23	lognormal
m	3.0	0.1	0.033	normal
$\Delta\sigma$ (ksi)	13.9	1.39	0.1	normal

The mean and standard deviation of ln N from these simulated crack growth curves were compared with the values computed using Eqs. (8) and (9) or Eqs. (10) and (11). The histogram from these simulated ln N was compared with the Gaussian distribution with the parameter computed using Eqs. (10) and (11). A goodness-of-fit test, Kologmorov-Smirnov (K-S) test, was used to verify the probability model. Detail discussion of this goodness-of-fit test is widely available in statistical text books (for example, Ang and Tang 1975, Benjamin and Cornell 1970). Additional statistical moments, the coefficient of skewness and the coefficient of Kurtosis, were also used to verify the probability model developed.

3. Analysis

During the study, a finite number of crack growth curves were determined as the basis for developing the response surface function. This is to emulate the condition when actual crack growth curves are available to develop the response surface function. The study was initiated for a specific model of aircraft, only one set of geometric and material parameters was considered. However, two types of stress level, $\Delta\sigma$, were considered. One study considered the stress level to be constant throughout the entire crack growth process. The second study considered the stress level to be a random value at each increment of ΔN (Eq. 5) computed. The rationale for the random stress study was that each pilot would induce a different level of stress to the structural component due to the way the pilot operates and maneuvers the aircraft.

3.1 Constant stress level

A range of 5 to 729 sets of (c, m, $\Delta\sigma$, N) values (“experimental” sample points) were used in this portion of the study. It was found that the number of samples was not the only factor influencing the response surface function. The range of input values of (c, m, $\Delta\sigma$) also influenced the “quality” of the RS function. Hence, various values of (c, m, $\Delta\sigma$) were considered as well. Table 1 summarizes the statistical data for the input parameters. These are the statistics for the material and component (bolt holes at cockpit longeron) of a specific model of aircraft whose monitoring record was also available.

For each regression analysis, the values of c, m, and $\Delta\sigma$ were assumed to be at their mean values, or +/– some standard deviations from the mean. Column 3 in Table 2 presents the range of input parameters used to determine the sample size. For example, RS function number one in Table 2 has a sample size of 5 and a range of 0.01 standard deviations from the means of the input parameters. Thus, the values of c, m, and $\Delta\sigma$ for the five sets were:

Table 2 Sample size, input ranges, and coefficients from regression analysis for constant stress analysis

RS Function No.	Sample Size	No. of Std. Dev. from Mean	Coefficients			
			b1	b2	b3	b4
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	5	0.01	–0.8383	–0.3668	–2.3913	0.8531
2	5	0.05	–1.2163	–6.5087	–2.8365	1.8106
3	5	0.1	–1.0955	–7.1310	–2.9251	4.9527
4	5	0.5	–1.0085	–7.5415	–3.0187	7.2528
5	5	1	–1.0173	–7.6537	–3.0375	7.2654
6	5	1.5	–1.0266	–7.7639	–3.0562	7.2680
7	5	2	–1.0362	–7.8722	–3.0750	7.2632
8	5	2.5	–1.0461	–7.9790	–3.0937	7.2526
9	5	3	–1.0563	–8.0842	–3.1125	7.2354
10	8	1	–1.0000	–7.4098	–3.0000	7.2106
11	8	1.5	–1.0000	–7.3873	–3.0000	7.1810
12	8	2	–1.0000	–7.3554	–3.0000	7.1396
13	8	1.5	–1.0000	–7.3138	–3.0000	7.0852
14	8	3	–1.0000	–7.2616	–3.0000	7.0170
15	27	0.5, 0	–1.0000	–7.4242	–3.0000	7.2297
16	27	1, 0	–1.0000	–7.4141	–3.0000	7.2167
17	27	2, 0	–1.0000	–7.3732	–3.0000	7.1644
18	27	2.5, 0	–1.0000	–7.3419	–3.0000	7.1239
19	27	3, 0	–1.0000	–7.3029	–3.0000	7.0742
20	27	3.5, 0	–1.0000	–7.2557	–3.0000	7.0141
21	27	4, 0	–1.0000	–7.1996	–3.0000	7.0742
22	125	1, 0.5, 0	–1.0000	–7.4171	–3.0000	7.2211
23	125	2, 1, 0	–1.0000	–7.3850	–3.0000	7.1799
24	125	2.5, 1.5, 0	–1.0000	–7.3573	–3.0000	7.1443
25	125	2.5, 1, 0	–1.0000	–7.3625	–3.0000	7.1518
26	125	2.5, 2, 0	–1.0000	–7.3472	–3.0000	7.1303
27	125	3, 1.5, 0	–1.0000	–7.3302	–3.0000	7.1101
28	125	3.5, 2, 0	–1.0000	–7.3152	–3.0000	7.0930
29	343	1.5, 1, 0.5, 0	–1.0000	–7.4062	–3.0000	7.2072
30	343	2, 1, 0.5, 0	–1.0000	–7.3924	–3.0000	7.1900
31	343	2.5, 2, 1, 0	–1.0000	–7.3610	–3.0000	7.1489
32	343	3, 2, 1, 0	–1.0000	–7.3403	–3.0000	7.1231
33	343	3.5, 2, 1, 0	–1.0000	–7.3139	–3.0000	7.0905
34	729	2, 1.5, 1, 0.5, 0	–1.0000	–7.3916	–3.0000	7.1885
35	729	2.5, 2, 1, 0.5, 0	–1.0000	–7.3704	–3.0000	7.1617
36	729	3, 2, 1, 0.5, 0	–1.0000	–7.3522	–3.0000	7.1394
37	729	3.5, 2.5, 1, 0.5, 0	–1.0000	–7.3217	–3.0000	7.1011
38	729	3.5, 2, 1, 0.5, 0	–1.0000	–7.3289	–3.0000	7.1110
39	729	4, 3, 2, 1, 0	–1.0000	–7.2788	–3.0000	7.0453

Table 3 Summary of the sets of input values for crack growth computations (note: “0” = mean values; “+” = mean plus x standard deviations; “–” = mean minus x standard deviations; x given in third column of Table 2)

Sample Size	c	m	$\Delta\sigma$	c	m	$\Delta\sigma$	c	m	$\Delta\sigma$	c	m	$\Delta\sigma$
5	0	0	0	0	+	0	+	0	0	+	+	+
	0	0	+									
8	0	0	+	0	+	0	+	0	0	+	+	+
	0	0	–	0	–	0	–	0	0	–	–	–
27	0	0	+	0	+	–	+	+	+	–	+	0
	0	0	–	0	–	+	+	+	–	–	–	0
	0	0	0	+	0	+	+	–	+	–	+	+
	0	+	0	+	0	–	+	–	–	–	+	–
	0	–	0	+	0	0	–	0	+	–	–	+
	0	+	+	+	+	0	–	0	–	–	–	–
	0	–	–	+	–	0	–	0	0			

$$(\bar{c}, \bar{m}, \overline{\Delta\sigma});\ \ (\bar{c} + 0.01s_c, \bar{m}, \overline{\Delta\sigma});\ \ (\bar{c}, \bar{m} + 0.01s_m, \overline{\Delta\sigma});\ \ (\bar{c}, \bar{m}, \overline{\Delta\sigma} + 0.01s_{\Delta\sigma});$$

$$(\bar{c} + 0.01s_c, \bar{m} + 0.01s_m, \overline{\Delta\sigma} + 0.01s_{\Delta\sigma})$$

in which $\bar{c}, \bar{m}$ and $\overline{\Delta\sigma}$ represent the mean of c, m, and $\Delta\sigma$, respectively, and s_j represents the standard deviation of parameter j. Table 3 summarizes the combinations using sets of 5, 8, and 27 curves. For sample sizes of 125, 343, and 729, the permutation sequence is similar to that of sample size 27. Table 2 shows the coefficients of the RS function given by Eq. (7) for each sample size and range of input values.

3.1.1 Comparison of statistical moments

The mean and standard deviation of ln N for each RS function were calculated using the point estimation method (Eqs. 8 and 9) and the Taylor series approximation (Eqs. 10 and 11). Columns 3 and 4 in Tables 4 and 5 summarize the results from both methods. Using two significant figures, the mean of ln N is about 9.6 and the standard deviation is about 0.45. This corresponds to a mean number of load cycles of 16,340 and a standard deviation of 7,740 cycles.

One thousand sets of 10,000 crack growth curves were simulated to represent the pseudo population. The statistical data in Table 1 were used to randomly generate the c, m, and $\Delta\sigma$ values for each crack growth curve. Columns 5 and 6 in Tables 4 and 5 present the 95% confidence interval of the true population mean of ln N. For the point estimate method, all the pseudo population means fell within the 95% confidence interval determined from the RS functions except for RS functions 34 and 35 (Table 4, columns 7-9). However, more than 95% of the pseudo population means still fell within the confidence interval for these functions. The results for the Taylor series approximation (Table 5, columns 7-9) were similar. All the pseudo population means fell within 95% confidence intervals determined from the RS functions except for RS function number 39 where 98.8% of the pseudo population means fell within that confidence interval.

Table 4 Mean and standard deviations using the point estimate method and results of the pseudo population means at 95% confidence level for constant stress analysis

RS Function No.	Sample Size	Average Value	Standard Deviation	95% Confidence Interval		Pseudo Population		Percent Acceptable
				Lower Limit	Upper Limit	Passed	Failed	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	5	9.63	0.310	9.36	9.91	1000	0	100%
2	5	9.65	0.457	9.25	10.1	1000	0	100%
3	5	9.65	0.457	9.25	10.0	1000	0	100%
4	5	9.65	0.459	9.24	10.0	1000	0	100%
5	5	9.65	0.463	9.24	10.1	1000	0	100%
6	5	9.65	0.468	9.24	10.1	1000	0	100%
7	5	9.66	0.472	9.24	10.1	1000	0	100%
8	5	9.66	0.477	9.25	10.1	1000	0	100%
9	5	9.67	0.481	9.25	10.1	1000	0	100%
10	8	9.64	0.454	9.33	9.96	1000	0	100%
11	8	9.64	0.454	9.32	9.95	1000	0	100%
12	8	9.63	0.453	9.32	9.94	1000	0	100%
13	8	9.62	0.453	9.31	9.94	1000	0	100%
14	8	9.61	0.452	9.30	9.92	1000	0	100%
15	27	9.64	0.455	9.47	9.82	1000	0	100%
16	27	9.64	0.455	9.47	9.81	1000	0	100%
17	27	9.64	0.454	9.46	9.81	1000	0	100%
18	27	9.63	0.453	9.46	9.80	1000	0	100%
19	27	9.62	0.452	9.45	9.79	1000	0	100%
20	27	9.61	0.452	9.44	9.78	1000	0	100%
21	27	9.74	0.451	9.57	9.91	1000	0	100%
22	125	9.64	0.455	9.56	9.72	1000	0	100%
23	125	9.64	0.454	9.56	9.72	1000	0	100%
24	125	9.63	0.453	9.55	9.71	1000	0	100%
25	125	9.63	0.454	9.56	9.71	1000	0	100%
26	125	9.63	0.453	9.55	9.71	1000	0	100%
27	125	9.63	0.453	9.55	9.71	1000	0	100%
28	125	9.63	0.453	9.55	9.71	1000	0	100%
29	343	9.64	0.454	9.59	9.69	1000	0	100%
30	343	9.64	0.454	9.59	9.69	1000	0	100%
31	343	9.63	0.454	9.59	9.68	1000	0	100%
32	343	9.63	0.453	9.58	9.68	1000	0	100%
33	343	9.63	0.453	9.58	9.67	1000	0	100%
34	729	9.64	0.454	9.61	9.67	964	36	96.4%
35	729	9.64	0.454	9.60	9.67	996	4	99.6%
36	729	9.63	0.453	9.60	9.67	1000	0	100%
37	729	9.63	0.453	9.60	9.66	1000	0	100%
38	729	9.63	0.453	9.60	9.66	1000	0	100%
39	729	9.62	0.452	9.59	9.65	1000	0	100%

Table 5 Mean and standard deviations using the Taylor series approximate method and results of the pseudo population means at 95% confidence level for constant stress analysis

RS Function No.	Sample Size	Average Value	Standard Deviation	95% Confidence Interval		Pseudo Population		Percent Acceptable
				Lower Limit	Upper Limit	Passed	Failed	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	5	9.61	0.309	9.34	9.88	1000	0	100%
2	5	9.62	0.456	9.22	10.0	1000	0	100%
3	5	9.62	0.455	9.22	10.0	1000	0	100%
4	5	9.62	0.458	9.22	10.0	1000	0	100%
5	5	9.62	0.462	9.22	10.0	1000	0	100%
6	5	9.63	0.467	9.22	10.0	1000	0	100%
7	5	9.63	0.471	9.22	10.0	1000	0	100%
8	5	9.63	0.475	9.22	10.1	1000	0	100%
9	5	9.64	0.480	9.22	10.1	1000	0	100%
10	8	9.62	0.453	9.30	9.93	1000	0	100%
11	8	9.61	0.453	9.30	9.92	1000	0	100%
12	8	9.60	0.452	9.29	9.92	1000	0	100%
13	8	9.60	0.451	9.28	9.91	1000	0	100%
14	8	9.58	0.451	9.27	9.90	1000	0	100%
15	27	9.62	0.453	9.45	9.79	1000	0	100%
16	27	9.62	0.453	9.45	9.79	1000	0	100%
17	27	9.61	0.453	9.44	9.78	1000	0	100%
18	27	9.60	0.452	9.43	9.77	1000	0	100%
19	27	9.60	0.451	9.43	9.77	1000	0	100%
20	27	9.59	0.450	9.42	9.76	1000	0	100%
21	27	9.58	0.449	9.41	9.75	1000	0	100%
22	125	9.62	0.453	9.54	9.70	1000	0	100%
23	125	9.61	0.453	9.53	9.69	1000	0	100%
24	125	9.61	0.452	9.53	9.69	1000	0	100%
25	125	9.61	0.452	9.53	9.69	1000	0	100%
26	125	9.60	0.452	9.52	9.68	1000	0	100%
27	125	9.60	0.452	9.52	9.68	1000	0	100%
28	125	9.60	0.451	9.52	9.68	1000	0	100%
29	343	9.62	0.453	9.57	9.66	1000	0	100%
30	343	9.61	0.453	9.57	9.66	1000	0	100%
31	343	9.61	0.452	9.56	9.65	1000	0	100%
32	343	9.60	0.452	9.56	9.65	1000	0	100%
33	343	9.60	0.451	9.55	9.65	1000	0	100%
34	729	9.61	0.453	9.58	9.65	1000	0	100%
35	729	9.61	0.452	9.58	9.64	1000	0	100%
36	729	9.61	0.452	9.57	9.64	1000	0	100%
37	729	9.60	0.452	9.57	9.64	1000	0	100%
38	729	9.60	0.452	9.57	9.64	1000	0	100%
39	729	9.59	0.451	9.56	9.63	988	12	98.8%

By the Central Limit theorem, the probability model of ln N can be approximated by Gaussian distribution. If a random variable is Gaussian distributed, the random variable's coefficient of skewness is zero (γ_1 = 0) due to symmetry and the coefficient of Kurtosis is 3.0 (γ_2 = 3.0). One hundred sets of 10,000 pseudo population crack growth curves were generated. The average γ_1 from these 100 sets of simulated crack growth curves was 0.048 and the standard deviation of γ_1 was 0.026. The average γ_2 from the same 100 sets of data was 3.03 and the standard deviation of γ_2 was 0.049. The average γ_1 indicates that the distribution of ln N skews slightly to the right. Although the pseudo population indicates that the probability distribution of ln N is not exactly normally distributed, both γ_1 and γ_2 were very close to the target values of zero and three, respectively.

3.1.2 Verification of probability model

The Kolmogorov-Smirnov (K-S) test was used to verify if Gaussian distribution is an acceptable model to describe the randomness of ln N. For comparison, three sets of pseudo population consisting of 10,000, 50,000, and 100,000 simulated crack growth curves were used. Columns 5, 7, and 9 in Table 6 present the maximum difference in cumulative distribution between the proposed model and the pseudo population. As can be seen from the Table, the maximum differences are significantly less than the critical value at 95% confidence level of a K-S test, D_{cr} (column 4 in Table 6). Note that D_{cr} depends on the number of sample points used to develop the theoretical probability model. In this study, the number of sample points for each RS function is given in column 2 of the Table.

To better visualize how the maximum difference in cumulative distribution (columns 5, 7, and 9) compared to D_{cr}, the fraction of D_{cr} (Columns 6, 8, and 10) was calculated by dividing columns 5, 7, and 9, respectively by D_{cr}. Most of the differences were within 10% of D_{cr} and the greatest difference was 36% of D_{cr}.

It was also observed that on the average, the K-S test results improved as the pseudo population size increased. Figs. 2 and 3 present a typical comparison between the theoretical probability model and the histogram based on the pseudo population. The comparison was for RS function number 17 and the pseudo populations size was 10,000 crack growth curves.

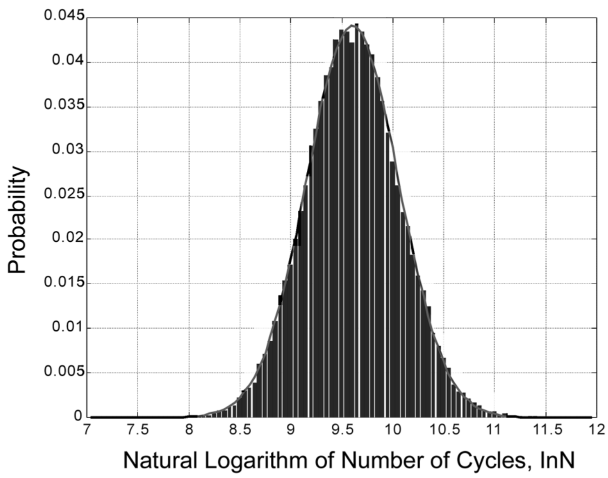

Fig. 2 Typical comparison between the proposed probability model based on response surface function and histogram from the pseudo population

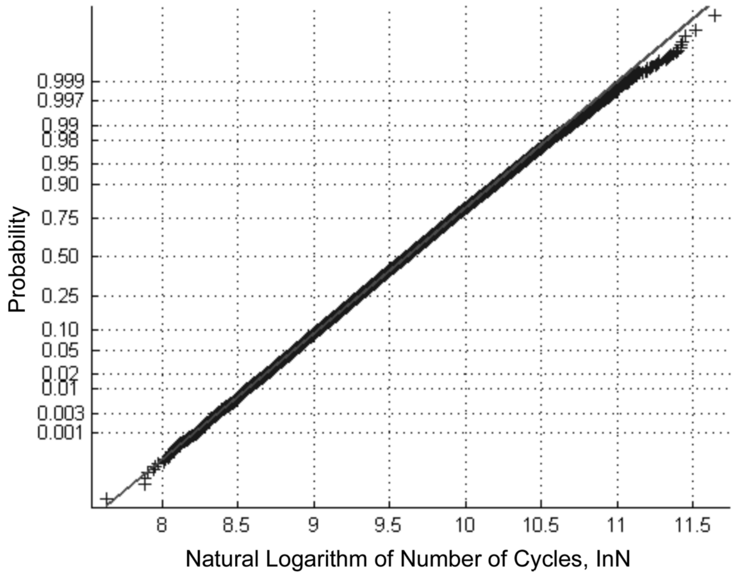

Fig. 3 Typical comparison in cumulative distribution between the proposed model based on response surface function and the pseudo population

3.1.3 Optimal sample size and range of input parameters

The response surface functions were developed using a limited number of crack growth curves that ranged from 5 to 729 as well as a wide range of input values (c, m, and $\Delta\sigma$) ranging from 0.01 standard deviations from the mean to 4 standard deviations from the mean. It is desirable to determine the optimal sample size and range of input values needed to achieve an acceptable RS function and its corresponding pdf. The result will help engineers design the laboratory testing for new material, new geometry, and/or new stress level in future reliability studies.

To determine the optimal sample size and range of input values, comparison was made between the maximum input range (maximum number of standard deviations from the mean given in column 3 of Table 6) and the fraction of D_{cr} (column 6, 8, or 10 of Table 6). Fig. 4 shows the comparison for a pseudo population consisting of 50,000 crack growth curves.

It was observed that as the sample size increases, the fraction of D_{cr} increases. However, this does not imply that the more samples one uses to determine a RS function, the worse the proposed model is. The reason for this observation is that D_{cr} decreases with increasing sample size. Despite this observation, for a given sample size, there was a fairly distinct trend that if the range of input values were significantly close (0 to 1 standard deviation) to or significantly far (3 to 4 standard deviations) from the mean, the fraction of D_{cr} would be larger. Based on this trend, it is recommended that the optimal choice is one with a sample size of 27 and an input range of 2 standard deviations from the mean. Hence, RS function number 17 would be the optimal RS function for modeling the number of load cycles needed for a crack to grow from the initiation size of 0.01 inches to 0.3 inches for the statistical information given in Table 1.

Table 6 Results of the Kolmogorov-Smirnov test for constant stress analysis

RS Function No.	Sample Size	No. of Std. Dev. from Mean	D_{cr}	Pseudo Population Size: 10,000		Pseudo Population Size: 50,000		Pseudo Population Size: 100,000	
				Difference	Fraction of D_{cr}	Difference	Fraction of D_{cr}	Difference	Fraction of D_{cr}
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1	5	0.01	0.560	0.096	0.171	0.094	0.168	0.093	0.166
2	5	0.05		0.020	0.035	0.011	0.020	0.008	0.014
3	5	0.1		0.020	0.037	0.012	0.021	0.008	0.015
4	5	0.5		0.022	0.039	0.013	0.023	0.010	0.018
5	5	1		0.024	0.042	0.016	0.029	0.013	0.022
6	5	1.5		0.027	0.049	0.020	0.036	0.016	0.029
7	5	2		0.032	0.057	0.025	0.044	0.021	0.037
8	5	2.5		0.037	0.066	0.030	0.054	0.026	0.046
9	5	3		0.043	0.077	0.036	0.064	0.032	0.057
10	8	1	0.470	0.017	0.036	0.009	0.019	0.005	0.010
11	8	1.5		0.013	0.027	0.005	0.010	0.003	0.006
12	8	1.5		0.012	0.025	0.012	0.026	0.016	0.033
13	8	2		0.009	0.020	0.007	0.016	0.008	0.017
14	8	3		0.019	0.039	0.021	0.045	0.025	0.053
15	27	0.5, 0	0.258	0.020	0.077	0.009	0.035	0.009	0.035
16	27	1, 0		0.018	0.071	0.007	0.028	0.007	0.029
17	27	2, 0		0.011	0.044	0.005	0.019	0.004	0.016
18	27	2.5, 0		0.010	0.037	0.010	0.038	0.007	0.027
19	27	3, 0		0.011	0.044	0.016	0.062	0.013	0.051
20	27	3.5, 0		0.016	0.061	0.023	0.090	0.021	0.080
21	27	4, 0		0.024	0.092	0.031	0.121	0.029	0.112
22	125	1, 0.5, 0	0.122	0.019	0.155	0.008	0.065	0.008	0.065
23	125	2, 1, 0		0.014	0.112	0.003	0.023	0.003	0.026
24	125	2.5, 1.5, 0		0.009	0.072	0.007	0.061	0.005	0.043
25	125	2.5, 1, 0		0.011	0.086	0.006	0.047	0.004	0.037
26	125	2.5, 2, 0		0.009	0.078	0.010	0.080	0.007	0.056
27	125	3, 1.5, 0		0.010	0.081	0.011	0.091	0.008	0.067
28	125	3.5, 2, 0		0.010	0.083	0.018	0.145	0.015	0.123
29	343	1.5, 1, 0.5, 0	0.073	0.017	0.233	0.006	0.083	0.006	0.085
30	343	2, 1, 0.5, 0		0.015	0.209	0.004	0.060	0.005	0.062
31	343	2.5, 2, 1, 0		0.009	0.128	0.007	0.092	0.005	0.067
32	343	3, 2, 1, 0		0.009	0.128	0.009	0.127	0.007	0.091
33	343	3.5, 2, 1, 0		0.010	0.141	0.012	0.169	0.010	0.132
34	729	2, 1.5, 1, 0.5, 0	0.050	0.015	0.293	0.004	0.077	0.004	0.079
35	729	2.5, 2, 1, 0.5, 0		0.012	0.232	0.005	0.091	0.004	0.077
36	729	3, 2, 1, 0.5, 0		0.010	0.190	0.007	0.131	0.005	0.098
37	729	3.5, 2.5, 1, 0.5, 0		0.010	0.196	0.011	0.211	0.008	0.156
38	729	3.5, 2, 1, 0.5, 0		0.009	0.187	0.009	0.178	0.007	0.130
39	729	4, 3, 2, 1, 0		0.012	0.241	0.018	0.361	0.016	0.308

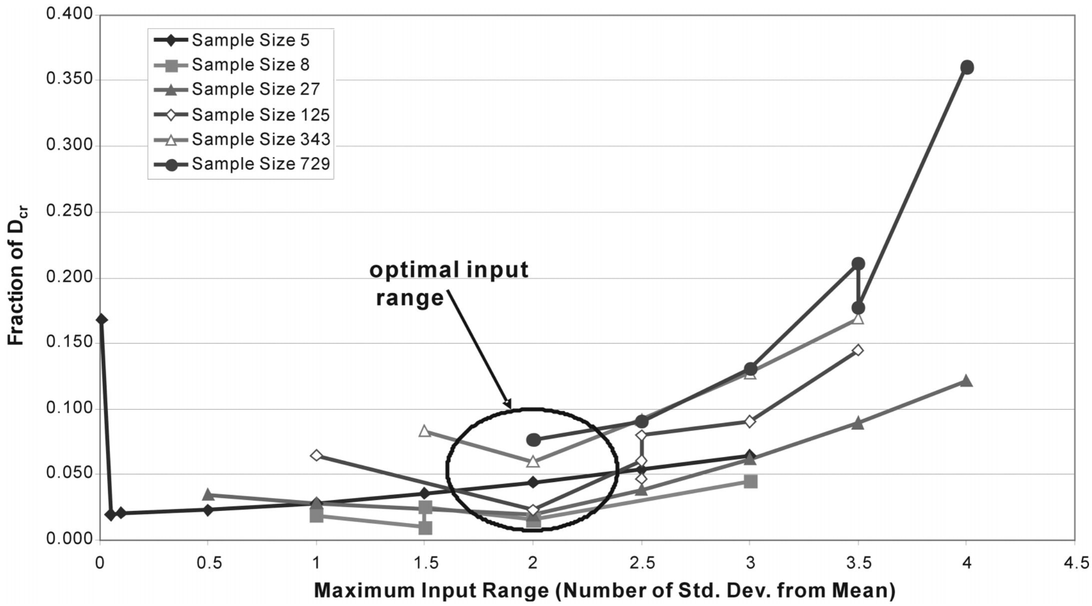

Fig. 4 Comparison of fraction of D_{cr} and maximum input range of (c, m, and $\Delta\sigma$) for a pseudo population size of 50,000 crack growth curves

3.2 Random stress block

The constant stress level analysis assumed the load induced on the structural component was constant. The random stress block analysis assumed the load induced on the structural component changed either continuously or within each flight exercise. For simplicity, during the study, the stress level was assumed to be constant for each increment of load cycles, ΔN, calculation (Eq. 5), and the stress level was different between any two ΔN calculations. These stress levels were randomly selected based on the statistical information given in Table 1.

A range of 9 to 81 sets of (c and m) values as shown in column 3 of Table 7 were used to develop the RS function. Four sets of 25 RS functions were developed. More than one set of RS functions was analyzed to ensure consistency among the RS functions because the stress level $\Delta\sigma$ was a random variable generated for each incremental load cycles calculation. All four sets of RS functions used the same sample sizes and range of input values for c and m. Again, Eq. (5) was used to develop the crack growth curve for each set of (c, m, simulated $\Delta\sigma$) values. Similar to the constant stress analysis, a linear regression was performed on each set of (c, m, $\overline{\Delta\sigma}$, and N) to determine the coefficients of the RS function. The mean value of $\Delta\sigma$, $\overline{\Delta\sigma}$, was used instead of a specified value. The reason was that a random $\Delta\sigma$ value was used among crack growth curves and within each crack growth curve calculations. Overall, the values of $\Delta\sigma$ were centralized towards the mean value. The coefficients of the linear RS functions (Eq. 7) for one set (set 1) of input values (25 RS functions) are shown in Table 7. The coefficients given in Table 7 are quite different from those shown in Table 2 for constant stress level. The coefficients for the random stress block vary more among the RS functions than those coefficient for the constant stress level.

Table 7 Sample sizes, input ranges, and coefficients from regression analysis for RS functions set 1 of random stress block analysis

Set 1						
RS Function No.	Sample Size	No. of Std. Dev. from Mean	Coefficients			
			b1	b2	b3	b4
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	9	0.5, 0	–1.0764	–7.6857	–2.9294	5.9954
2	9	1, 0	–0.9860	–7.7167	–1.8748	4.8982
3	9	2, 0	–0.9960	–7.1420	–1.7821	3.8418
4	9	2.5, 0	–1.0048	–7.1392	–3.6392	8.5623
5	9	3, 0	–0.9873	–7.0882	–2.5506	5.9580
6	9	3.5, 0	–0.9923	–7.1602	–2.5929	6.0526
7	9	4, 0	–0.9988	–7.2555	–3.7718	9.1179
8	25	1, 0.5, 0	–1.0274	–7.1966	–2.5699	5.4003
9	25	2, 1, 0	–0.9873	–7.3006	–2.8595	7.0110
10	25	2.5, 1.5, 0	–1.0049	–7.4089	–3.1474	7.5614
11	25	2.5, 1, 0	–0.9994	–7.3434	–2.6179	6.1931
12	25	2.5, 2, 0	–1.0141	–7.3210	–3.5680	8.3889
13	25	3, 1.5, 0	–1.0124	–7.1745	–3.7681	8.7967
14	25	3.5, 2, 0	–1.0068	–7.3509	–4.2062	10.2243
15	49	1.5, 1, 0.5, 0	–1.0249	–7.2475	–3.1146	6.9448
16	49	2, 1, 0.5, 0	–1.0033	–7.4570	–2.8443	6.8514
17	49	2.5, 2, 1, 0	–0.9954	–7.3256	–2.5492	6.0685
18	49	3, 2, 1, 0	–1.0058	–7.3341	–3.0202	7.1276
19	49	3.5, 2, 1, 0	–1.0043	–7.2638	–2.7447	6.3533
20	81	2, 1.5, 1, 0.5, 0	–1.0026	–7.1761	–2.8431	6.5500
21	81	2.5, 2, 1, 0.5, 0	–0.9945	–7.3736	–3.3822	8.3313
22	81	3, 2, 1, 0.5, 0	–0.9989	–7.2768	–3.1634	7.5694
23	81	3.5, 2.5, 1, 0.5, 0	–0.9989	–7.2129	–3.2588	7.7499
24	81	3.5, 2, 1, 0.5, 0	–0.9996	–7.3273	–2.5532	5.9963
25	81	4, 3, 2, 1, 0	–1.0030	–7.2492	–2.6345	6.0579

3.2.1 Comparison of statistical moments

The point estimate method and the Taylor series approximation were used to calculate the mean and standard deviation of ln N for each RS function. Columns 3 and 4 in Tables 8 and 9 summarize the results from both methods for set 1 of RS functions. Using two significant figures, the mean of ln N was 9.7 and the standard deviation was about 0.34. This corresponds to a mean number of load cycles of 17,290 and a standard deviation of 6,050 cycles. It is of interest to note that the standard deviation using random stress blocks was smaller than that using constant stresses. The reason may be due to the less variation in stress level among crack growth curves. The stress levels within each crack growth curve are random but have a central tendency towards the mean stress level. Regardless of the input range of c and m, and the sample size, the average stress level for each

Table 8 Mean and standard deviations using the point estimate method and results of the pseudo population means at 95% confidence level for RS functions set 1 of random stress block analysis

Set 1								
RS Function No.	Sample Size	Average Value	Standard Deviation	95% Confidence Interval		Pseudo Population		Percent Acceptable
				Lower Limit	Upper Limit	Passed	Failed	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	9	9.70	0.359	9.47	9.94	100	0	100%
2	9	9.68	0.346	9.45	9.91	100	0	100%
3	9	9.68	0.333	9.47	9.90	100	0	100%
4	9	9.68	0.335	9.46	9.90	100	0	100%
5	9	9.68	0.331	9.46	9.89	100	0	100%
6	9	9.67	0.333	9.45	9.89	100	0	100%
7	9	9.65	0.337	9.43	9.87	100	0	100%
8	25	9.69	0.340	9.55	9.82	100	0	100%
9	25	9.68	0.336	9.55	9.81	100	0	100%
10	25	9.68	0.341	9.55	9.81	100	0	100%
11	25	9.68	0.339	9.54	9.81	100	0	100%
12	25	9.67	0.341	9.53	9.80	100	0	100%
13	25	9.68	0.337	9.55	9.81	100	0	100%
14	25	9.66	0.340	9.52	9.79	100	0	100%
15	49	9.70	0.341	9.60	9.79	100	0	100%
16	49	9.69	0.342	9.59	9.78	100	0	100%
17	49	9.68	0.338	9.58	9.77	100	0	100%
18	49	9.68	0.340	9.59	9.78	100	0	100%
19	49	9.68	0.338	9.59	9.78	100	0	100%
20	81	9.68	0.335	9.61	9.76	100	0	100%
21	81	9.68	0.339	9.61	9.75	100	0	100%
22	81	9.68	0.337	9.61	9.75	100	0	100%
23	81	9.68	0.336	9.61	9.75	100	0	100%
24	81	9.67	0.338	9.60	9.74	100	0	100%
25	81	9.67	0.337	9.59	9.74	100	0	100%

crack growth curve was very close to that of another crack growth curve. Hence, the random stress level had less impact on the standard deviation of ln N than the constant stress level had.

One hundred sets of 10,000 crack growth curves were simulated to represent the pseudo population. The parameters c and m were simulated according to the statistics given in Table 1 for each crack growth curve. The stress level $\Delta\sigma$ was simulated for each increment of ΔN calculation. Columns 5 and 6 of Tables 8 and 9 show the 95% confidence interval of the mean computed using, respectively, the point estimate method and the Taylor series approximation of the RS functions. As can be seen from columns 7-9 in these two Tables, all the means from the pseudo population fell within the 95% confidence intervals. Similar results were observed for the other 3 sets of RS functions (Cox 2000).

Table 9 Mean and standard deviations using the Taylor series approximation method and results of the pseudo population means at 95% confidence level for RS functions set 1 of random stress block analysis

Set 1								
RS Function No.	Sample Size	Average Value	Standard Deviation	95% Confidence Interval		Pseudo Population		Percent Acceptable
				Lower Limit	Upper Limit	Passed	Failed	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	9	9.69	0.359	9.45	9.92	100	0	100%
2	9	9.66	0.345	9.44	9.89	100	0	100%
3	9	9.67	0.333	9.45	9.88	100	0	100%
4	9	9.67	0.334	9.45	9.89	100	0	100%
5	9	9.66	0.330	9.45	9.88	100	0	100%
6	9	9.66	0.332	9.44	9.88	100	0	100%
7	9	9.64	0.336	9.42	9.86	100	0	100%
8	25	9.67	0.339	9.54	9.81	100	0	100%
9	25	9.67	0.335	9.54	9.80	100	0	100%
10	25	9.67	0.340	9.54	9.80	100	0	100%
11	25	9.66	0.338	9.53	9.80	100	0	100%
12	25	9.66	0.340	9.52	9.79	100	0	100%
13	25	9.67	0.336	9.54	9.80	100	0	100%
14	25	9.65	0.339	9.52	9.78	100	0	100%
15	49	9.68	0.340	9.59	9.78	100	0	100%
16	49	9.67	0.341	9.58	9.77	100	0	100%
17	49	9.66	0.337	9.57	9.76	100	0	100%
18	49	9.67	0.339	9.57	9.76	100	0	100%
19	49	9.66	0.337	9.57	9.76	100	0	100%
20	81	9.67	0.334	9.60	9.74	100	0	100%
21	81	9.67	0.338	9.60	9.74	100	0	100%
22	81	9.67	0.336	9.60	9.74	100	0	100%
23	81	9.67	0.335	9.60	9.74	100	0	100%
24	81	9.66	0.338	9.58	9.73	100	0	100%
25	81	9.65	0.336	9.58	9.73	100	0	100%

For the pseudo population created above, the average coefficient of skewness, γ_1, was 0.046 and the standard deviation of γ_1 was 0.026. The average coefficient of Kurtosis, γ_2, was 3.03 and the standard deviation of γ_2 was 0.054. Similar to the results observed for the constant stress analysis, ln N values were skewed slightly to the right. The probability distribution of ln N is not exactly Gaussian distributed. However, both γ_1 and γ_2 were close to the target values of zero and three, respectively.

3.2.2 Verification of probability model

For the goodness-of-fit test, Kolmogorov-Smirnov (K-S) test was used. One set of 10,000 simulated growth curves was used as pseudo population. Table 10 presents the results for all four

Table 10 Results of the Kolmogorov-Smirnov test for RS functions sets 1 to 4 of random stress block analysis

RS Function No.	Sample Size	No. of Std. Dev. from Mean	D_{cr}	Pseudo Population Size Used for all Comparisons: 10,000							
				Set 1		Set 2		Set 3		Set 4	
				Difference	Fraction of D_{cr}	Difference	Fraction of D_{cr}	Difference	Fraction of D_{cr}	Difference	Fraction of D_{cr}
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1	9	0.5, 0	0.440	0.038	0.086	0.028	0.063	0.037	0.084	0.034	0.077
2	9	1, 0		0.007	0.016	0.034	0.076	0.031	0.071	0.024	0.055
3	9	2, 0		0.008	0.019	0.006	0.013	0.005	0.011	0.006	0.014
4	9	2.5, 0		0.015	0.034	0.003	0.008	0.016	0.036	0.019	0.043
5	9	3, 0		0.007	0.015	0.019	0.044	0.010	0.022	0.004	0.008
6	9	3.5, 0		0.008	0.019	0.043	0.099	0.021	0.048	0.025	0.057
7	9	4, 0		0.026	0.058	0.003	0.006	0.011	0.026	0.015	0.033
8	25	1, 0.5, 0	0.270	0.014	0.052	0.021	0.077	0.032	0.118	0.025	0.093
9	25	2, 1, 0		0.012	0.044	0.024	0.087	0.013	0.049	0.014	0.053
10	25	2.5, 1.5, 0		0.010	0.035	0.019	0.071	0.016	0.058	0.005	0.018
11	25	2.5, 1, 0		0.003	0.013	0.014	0.051	0.017	0.061	0.018	0.068
12	25	2.5, 2, 0		0.007	0.026	0.017	0.061	0.004	0.014	0.006	0.021
13	25	3, 1.5, 0		0.010	0.038	0.009	0.033	0.005	0.018	0.003	0.013
14	25	3.5, 2, 0		0.016	0.060	0.021	0.078	0.014	0.053	0.017	0.064
15	49	1.5, 1, 0.5, 0	0.192	0.028	0.144	0.023	0.119	0.024	0.124	0.020	0.106
16	49	2, 1, 0.5, 0		0.016	0.084	0.019	0.100	0.013	0.070	0.022	0.116
17	49	2.5, 2, 1, 0		0.004	0.021	0.009	0.045	0.011	0.058	0.005	0.028
18	49	3, 2, 1, 0		0.010	0.051	0.009	0.046	0.012	0.060	0.005	0.026
19	49	3.5, 2, 1, 0		0.003	0.016	0.003	0.017	0.010	0.054	0.003	0.016
20	81	2, 1.5, 1, 0.5, 0	0.151	0.012	0.081	0.019	0.128	0.017	0.112	0.021	0.136
21	81	2.5, 2, 1, 0.5, 0		0.011	0.070	0.006	0.042	0.008	0.053	0.005	0.034
22	81	3, 2, 1, 0.5, 0		0.012	0.080	0.008	0.055	0.009	0.060	0.010	0.068
23	81	3.5, 2.5, 1, 0.5, 0		0.012	0.079	0.008	0.054	0.007	0.048	0.003	0.019
24	81	3.5, 2, 1, 0.5, 0		0.007	0.048	0.005	0.034	0.012	0.079	0.004	0.024
25	81	4, 3, 2, 1, 0		0.011	0.074	0.009	0.061	0.010	0.069	0.009	0.061

sets of RS functions. The maximum differences in cumulative distribution between the proposed model and the pseudo population are shown in columns 5, 7, 9, and 11 of the Table. Like the constant stress analysis, the maximum differences are significantly less than the critical value at 95% confidence level of the K-S test, D_{cr} (column 4 in Table 10). The fraction of D_{cr} shows in columns 6, 8, 10, and 12 are mostly within 10% of D_{cr} with the maximum being 14% of D_{cr}.

3.2.3 Optimal sample size and range of input parameter

Fig. 5 shows the graphical comparison between the maximum number of standard deviations from the mean (for c and m only given in column 3 of Table 10) and the fraction of D_{cr} for each RS

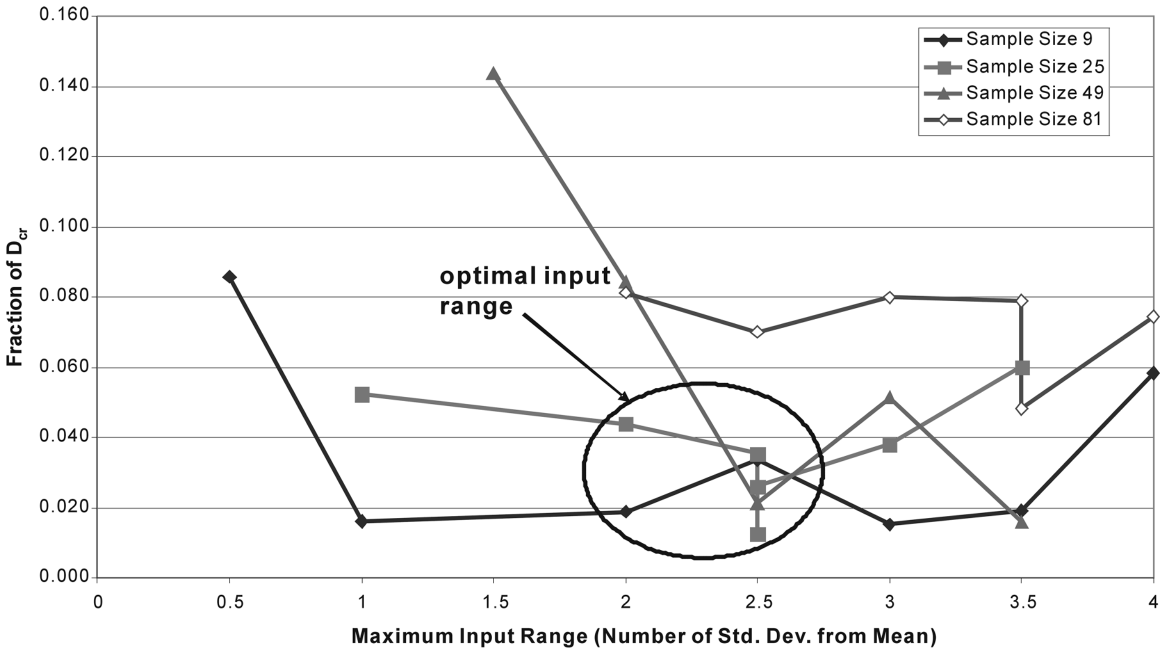

Fig. 5 Comparison of fraction of D_{cr} and maximum input range of (c and m) for a pseudo population size of 10,000 crack growth curves (RS function set 1)

function in set 1 (Table 7). Unlike the constant stress analysis where the fraction of D_{cr} shows a distinct trend with respect to sample size and range of input values, the trend is not quite as clear for the random stress block. Another interesting observation was that almost all sample sizes showed a high fraction of D_{cr} when the input values are close to the mean (0.5 to 1 standard deviation from the mean). Examination of the comparison shown in Fig. 5 and similar comparisons for the other 3 sets of RS functions suggested that the optimal choice is the one with a sample size of 9 to 25 and an input range of 2.0 to 2.5 standard deviation from the mean for c and m.

4. Applications of crack growth model

Once the probability model for ln N is known, at a specified risk level, a decision can be made on the number of flight hours an aircraft can continue to fly after a crack has just been initiated. For example, assume response surface function 17 in Table 2 was used to determine the maximum flight hours the aircraft should fly. For simplicity, assume each load cycle, N = 1, is equal to 0.5 flight hour. If a risk level of 1 in a million (probability of $N < n$ is 10^{-6}) is specified, one can compute the maximum flight hours as,

$$P[N < n] = 10^{-6} \Leftrightarrow P[lnN < ln\ n] = 10^{-6}$$

since, ln N is Gaussian distributed, the above equation can be rewritten as

$$\Phi\left(\frac{ln\ n - \overline{ln\ N}}{s_{lnN}}\right) = 10^{-6}$$

in which $\Phi(*)$ is the cumulative probability of the standardize normal distribution; $\overline{ln\ N}$ and s_{lnN} are, respectively, the mean and standard deviation of ln *N*. For RS function 17, the mean and standard deviation can be found in Table 5 as 9.61 and 0.453, respectively. The maximum flight hours or maximum number of load cycles becomes,

$$\begin{aligned} ln\ n &= \overline{ln\ N} - \Phi^{-1}(1 - 10^{-6})s_{lnN} \\ &= 9.61 - 4.75(0.453) \\ ln\ n &= 7.458 \\ n &= 1734\ \textit{cycles}\ \text{or}\ 867\ \textit{flight hours} \end{aligned}$$

5. Conclusions

A study was performed to assess aging aircraft by examining crack growth at the cockpit longeron of an aircraft. A total life model was shown to have the potential to be used in the reliability assessment of total service life expectancy (Chou 1998). Two probability models are required by the total life model. One is the probability density function (pdf) for the crack initiation time. The other is the pdf for the crack growth life. This paper presents the development of the pdf for the crack growth life projection using the response surface method (RSM).

Through regression analysis on a limited sample size, a linear relationship between the number of load cycles *N* (response) and the input geometric parameter, material, and stress level (c, m, and $\Delta\sigma$) was found when all the variables were transformed to the natural logarithmic values. By the central limit theorem, the natural log of *N*, ln *N*, computed using the response surface (RS) function was approximated by the Gaussian Distribution. The RSM presented here was applied to two types of stress, constant stress level through the entire crack growth curve and random stress level for each incremental crack size developed along the crack growth curve.

Comparison of the means of the ln *N* from the pseudo populations to the 95% confidence interval computed from the RS functions indicated that the RS function provide an acceptable mean value for the pseudo population. The coefficient of skewness and coefficient of Kurtosis were slightly higher than the target values of zero and three, respectively, for a true Gaussian distribution. The results of the Kolmogorov-Smirnov test indicated that Gaussian distribution was an acceptable pdf for ln *N* at a 95% confidence level. The above characteristics were observed in both the constant stress level case and the random stress block study.

During the study, it was found that the RS functions did not only depend on the sample size, it also depended on the range of input values. From the spectrum of sample sizes and ranges of input values examined, it was found that a sample size of 27 and a maximum of 2 standard deviations from the means of input variables (c, m, and $\Delta\sigma$) would be optimal for the constant stress case. For the random stress block situation, the optimal choice was 9 to 25 samples and a maximum of 2 to 2.5 standard deviations from the means of input variables (c and m). This information will be valuable in designing experiments for crack growth curves used to determine the RS functions for service life assessments.

While the probability model presented here for crack growth life projection compared well with the pseudo population, the actual RS function depends on the crack growth curves used. In this study, in lieu of actual crack growth curves, the crack growth curves were estimated using constant crack size increment (Eq. 5). This constant incremental crack size, although has been used by the engineers at both the U.S. Navy and Air Force in some of their studies, may compromise the accuracy of the crack growth curve when the curve's gradient is steep. If the crack growth curves were computed numerically to determine the RS functions, one may wish to consider using a variable crack size increment in Eq. (5) to better approximate the crack growth curve.

Before a total life model can be used to predict the service life of an aircraft, one also needs to determine the probability model of the crack initiation time. Unfortunately, the physical model in this area is still very limited at this point.

Acknowledgements

This study was initiated during the first author's visit at the Navy Air Command System, AIR-4.3.3 under the Navy-ASEE Summer Faculty Research Program. Additional support under Navy contract no. N00421-99-M-5859 managed by Dr. Paul Hoffman is gratefully appreciated. The first author is grateful for the valuable discussions with Drs. Paul Hoffman and Margery Hoffman during the Summer Faculty Research Program and subsequent visits.

References

Ang, A.H-S. and Tang, W.H. (1975), *Probability Concepts in Engineering Planning and Design,* **1**, New York, John Wiley & Sons.

Bannantine, J.A., Comer, J.J. and Handrock, J.L. (1990), *Fundamental of Metal Fatigue Analysis*, New York, Prentice Hall.

Benjamin, J.R. and Cornell, C.A. (1970), *Probability, Statistics and Decision for Civil Engineers*, New York, McGraw Hill.

Box, G.E.P. and Wilson, K.B. (1951), "On the experimental attainment of optimum conditions", *Journal of the Royal Statistical Society*, Ser. B, **13**, 1-45.

Chou, K.C. (1998), "Reliability assessment model for aging aircraft" preliminary report to Structures Division, Navy Air Station, Patuxent River, MD, part of Navy-ASEE Summer Faculty Program.

Cox, G.C. (2000), "Probabilistic fatigue crack growth analysis using response surface methodology", Master Thesis, University of Tennessee.

Harr, M.E. (1987), *Reliability Based Design in Civil Engineering*. New York: McGraw Hill.

Myers, R.H., Khuri, A.I. and Carter, W.H. Jr (1989), "Response surface methodology: 1966-1988". Technometrics **31**(2), 137-157.

Rajashekhar, M.R. and Ellingwood, B.R. (1993), "A new look at the response surface approach for reliability analysis", *Structural Safety*, 205-220.

Rosenblueth, E. (1975), "Point estimates for probability moments", *Proc. of National Academy of Science*, U.S.A., **72**(10), 3812-3814.

Rosenblueth, E. (1981), "Two point estimate in probability", *Applied Math. Modelling*, **5**, 329-335.

Wirsching, P.H. (1983), "Statistical summaries of fatigue data for design purposes", NASA Contractor Report 3697, NASA Lewis Research Center.

Yang, J.N., Hsi, W.H. and Manning, S.D. (1985), "Stochastic crack propagation with applications to durability and damage tolerance analyses", Technical Report, Flight Dynamics Lab., AFWAL-TR-85-3062.

Yang, J.N. and Manning, S.D. (1996), "A simple second order approximation for stochastic crack growth analysis", *Engineering Fracture Mechanics*, **53**(5), 677-686.