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Analytical model of corrosion-induced cracking of concrete considering the stiffness of reinforcement

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Abstract. The structural deterioration of concrete structures due to reinforcement corrosion is a major worldwide problem. Service life of the age-degraded concrete structures is governed by the protective action provided by the cover concrete against the susceptibility of the reinforcement to the corrosive environment. The corrosion of steel would result in the various corrosion products, which depending on the level of the oxidation may have much greater volume than the original iron that gets consumed by the process of corrosion. This volume expansion would be responsible for exerting the expansive radial pressure at the steel-concrete interface resulting in the development of hoop tensile stresses in the surrounding cover concrete. Once the maximum hoop tensile stress exceeds the tensile strength of the concrete, cracking of cover concrete would take place. The cracking begins at the steel-concrete interface and propagates outwards and eventually resulting in the through cracking of the cover concrete. The cover cracking would indicate the loss of the service life for the corrosion-affected structures. In the present paper, analytical models have been developed considering the residual strength of the cracked concrete and the stiffness provided by the combination of the reinforcement and expansive corrosion products. The problem is modeled as a boundary value problem and the governing equations are expressed in terms of the radial displacement. The analytical solutions are presented considering a simple 2-zone model for the cover concrete viz. cracked or uncracked. A sensitivity analysis has also been carried out to show the influence of the various parameters of the proposed models. The time to cover cracking is found to be function of initial material properties of the cover concrete and reinforcement plus corrosion products combine, type of rust products, rate of corrosion and the residual strength of the cover concrete. The calculated cracking times are correlated against the published experimental and analytical reference data.

Key words: expansive corrosion products; time to cover cracking; steel-concrete interface; porous zone; internal radial pressure; residual strength; tension-softening.

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1. Introduction

Corrosion of reinforcement in concrete has been identified as being one of the most predominant factors responsible for the deterioration of the reinforced concrete structures. The serviceability and the durability of the concrete structures can be seriously affected by the corrosion of reinforcement. Damage to the concrete due to the reinforcement corrosion manifests in the form of expansion, cracking and eventually spalling of the cover. In addition to the loss of cover, the reinforced concrete structure may suffer structural damage due to the loss of bond between reinforcement and concrete and loss of reinforcement cross-sectional area; sometimes to the extent that the structural failure becomes inevitable (Mehta and Monteiro 1997). Therefore, if one has to prevent the premature failure of the reinforced concrete structures then monitoring and control of reinforcement corrosion assume a significant practical importance.

If one has to make certain decisions regarding the inspection, repair, strengthening, replacement and demolition of the age-degraded reinforced concrete structures then it becomes necessary to assess the performance of such structures to withstand the extreme events during their service life. Among other factors, service life of the reinforced concrete structures is governed by the protection of the reinforcement against the contact with the corrosion inducing agents and to be provided by the cover concrete. Similarly, the end of service life of the reinforced concrete structures is generally associated with the loss of protective action to be provided by the cover concrete and this may in turn result in the loss of the bond between corroded reinforcement and reduction of the reinforcement cross-section. The assessment of the effects of the structural deterioration and eventually the determination of the safe residual structural life largely depend on the correct diagnosis of the structural deterioration and the conditions causing it. Therefore, it becomes necessary that analytical models be developed to assess the effect of reinforcement corrosion process in the reinforced concrete structures on the structural performance/deterioration for the reasonable prediction of the safe residual service life of the corroded reinforced concrete structures. The prediction of structural deterioration would in turn be very useful to arrive at the cost-effective strategy in handling of the corrosion affected concrete structures.

In the context of the aforementioned discussion, the prediction of time to cover cracking of reinforced concrete structures may be considered as a useful indicator towards the intensity of the corrosion environment to which the structures are subjected. The quantitative prediction of the cover cracking due to the reinforcement corrosion process is useful in estimating the structural capacity to withstand the possible extreme events during the remaining service life of the structures. There are various analytical models (Cady and Weyers 1984, Bazant 1979b, Andrade *et al*. 1993, Morinaga 1989, Martin-Perez 1998, Liu and Weyers 1998) to predict either the safe residual service life or the time to cover cracking of the corroded reinforced concrete structures. However, due to the complexity of the corrosion process itself in the reinforced concrete structures, there seems to be some difference between the predicted values and the observed data from the field and laboratory, which may be attributed to a number of factors viz. the mathematical model for the rate of corrosion process, the residual strength of the concrete once its tensile strength is exceeded and/or the proper estimation of the material properties of the concrete itself besides the mathematical model to describe the structural response.

Normally, the reinforcement in good quality concrete does not corrode even in presence of sufficient moisture and oxygen due to a thin protective passive oxide film on the surface of the reinforcement in the highly alkaline pore solution of the concrete. For the corrosion process to begin, this passive film must be broken or depassivated. When the alkalinity of the pore water solution of the concrete is decreased to a very low value due to (i) carbonation, or (ii) penetration of sufficient chloride ions to the reinforcement from de-icing salts or seawater or contaminated water used in concrete mix, the protective passive film gets destroyed and the depassivation of the reinforcement takes place. This would result in the oxidation of the iron to form the ferrous ions that can react with the hydroxyl ions and produces ferrous hydroxide. This ferrous hydroxide is further converted to hydrated ferric oxide. This transformation of the metallic iron to the corrosion products is accompanied by an increase in volume, which depending on the level of oxidation, may be up to about 6.5 times the original iron volume (Mehta and Monteiro 1997, Liu and Weyers 1998).

In general the composition of the expansive corrosion products may be expressed as {a.$Fe(OH)_2$ + b.$Fe(OH)_3$ + c.H_2O} (Liu and Weyers 1998), where 'a', 'b' and 'c' are the variables that depend on the alkalinity of the pore water solution of concrete, the oxygen supply and the moisture content. The different corrosion products will have different volume expansions and densities as shown in Fig. 1 and Table 1. The volume increase is believed to be the principal cause of the concrete expansion and ultimately the cover cracking. In the proposed mathematical model, the density 'ρ_r' of the expansive corrosion products has been defined as follows.

$$\rho_r = \frac{\rho_s}{\alpha\alpha_1} \tag{1}$$

Where, 'ρ_s' is the mass density of reinforcing steel, 'α' is the ratio of molecular weight of the iron to the molecular weight of the corrosion products and 'α_1' is the volume expansion ratio for the expansive corrosion products.

The growth of the expansive corrosion products may not follow the linear relationship like Faraday's law (Bazant 1979a), rather it may decrease in proportion to the thickness of the expansive

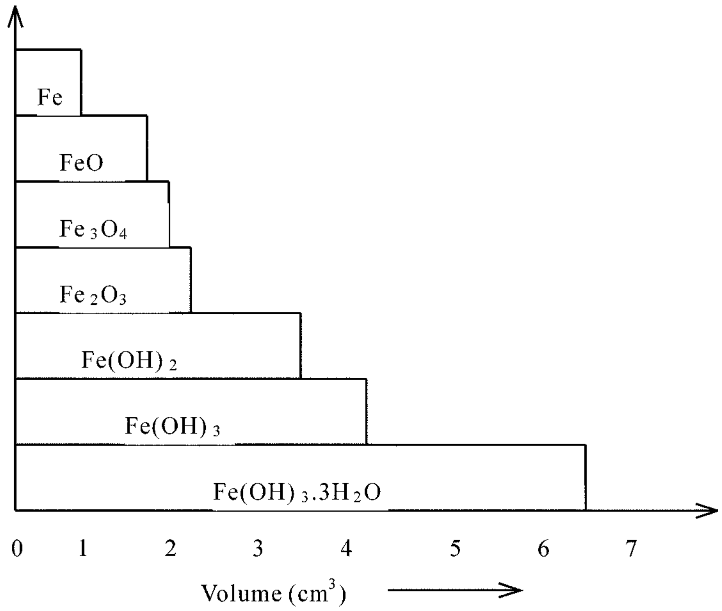


Fig. 1 Relative volume of steel and its corrosion products (Mehta and Monteiro 1997)

Table 1 Correlation between 'α' and 'α_1' (Mehta and Monteiro 1997)

Name of Corrosion Products	'α'	'α_1'
FeO	0.7773	1.81
Fe_3O_4	0.7236	2.06
Fe_2O_3	0.6994	2.19
$Fe(OH)_2$	0.6215	3.75
$Fe(OH)_3$	0.5226	4.20
$Fe(OH)_3.3H_2O$	0.3471	6.40

corrosion products (Liu and Weyers 1998). The governing equation for the rate of production of expansive corrosion products would then be written as follows.

$$\frac{dW_r}{dt} = \frac{k_p}{W_r} \tag{2}$$

Where, 'W_r' is the mass of the expansive corrosion products (mg/mm), 't' is the corrosion time (years) and 'k_p' is the function of the corrosion current.

The present paper attempts to formulate a mathematical model for the evaluation of time to cracking of the cover concrete in the corroded reinforced concrete structures. A sensitivity analysis has also been carried out by considering variation of the various parameters of the mathematical model. The proposed analysis evaluates the rate of production of the expansive corrosion products with respect to time and this is used to estimate the expansive radial pressure at the interface of the concrete and reinforcement that would cause tensile stresses in the concrete, which ultimately results in the cracking of cover concrete.

2. Corrosion cracking model

2.1 Problem definition

This problem is a boundary value problem, wherein the internal circular boundary at the interface of the reinforcement and the concrete is displaced to accommodate the expansive corrosion products resulting in the evolution of the expansive radial pressure at the boundary. It is assumed that, there is a porous zone around the interface of the concrete and steel (Liu and Weyers 1998) as shown in Fig. 2(a), wherein the reinforcing bar of initial diameter 'D_i' is embedded in the concrete with a clear cover to the reinforcement being 'C' and the thickness of the porous zone around the interface of reinforcement and concrete being 'd_o'. Fig. 2(b) represents the free expansion of the corrosion products at the surface of the reinforcement depending on the level of oxidation, wherein the combined diameter of the reinforcement plus the corrosion products is denoted by 'D_2' and the reduction in the initial diameter of the reinforcement due to corrosion is denoted by '$2d_1$. Initially, the porous zone will gradually be filled with the corrosion products and at this stage, the surrounding concrete is not subjected to any pressure. Once this porous zone around the interface of concrete and steel gets completely filled with the corrosion products, the free expansion of the corrosion products beyond the porous zone is restrained by the surrounding concrete and at this stage further production of expansive rust on the surface of steel would result in the surrounding

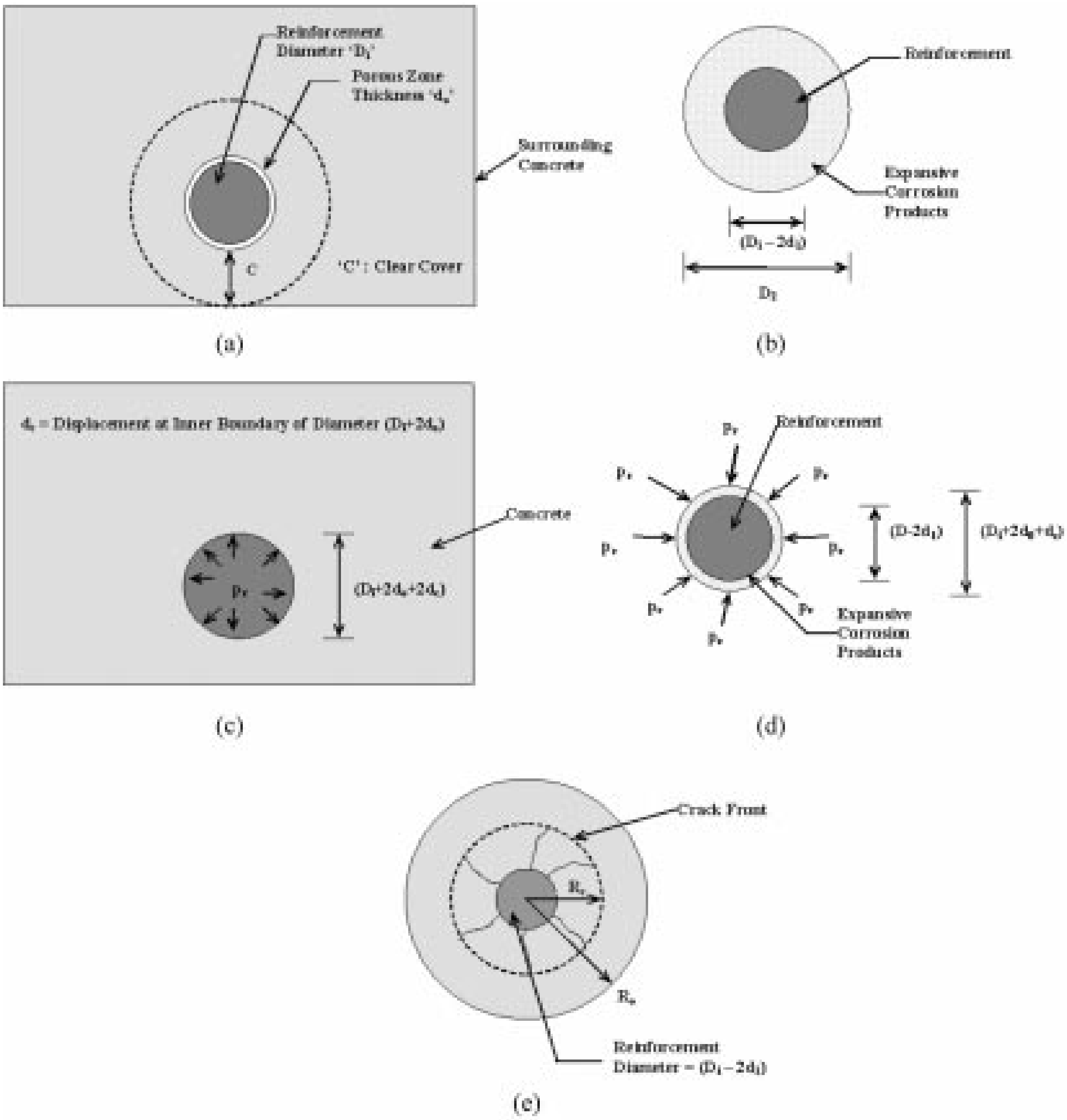


Fig. 2(a) Concrete block and reinforcement in unrestrained condition (a thick walled cylinder model), (b) Reinforcement and expansive corrosion products in unrestrained condition, (c) Concrete block subjected to internal radial pressure in restrained condition, (d) Reinforcement and expansive corrosion products subjected to external radial pressure in restrained condition, (e) Propagation of radial splitting cracks through cover concrete

concrete subjected to a radial pressure 'p_r' and getting displaced by an amount 'd_c' i.e. the thickness of the expansive corrosion products deposited around the reinforcement at the internal boundary as shown in Fig. 2(c). The reinforcement plus corrosion products combine would be subjected to an equivalent external pressure of 'p_r' under equilibrium conditions as shown in Fig. 2(d). The radial pressure increases with the increase in the corrosion products and cracking occurs, when the maximum hoop stress exceeds the tensile strength of the concrete. The cracking begins at the steel-concrete interface and propagates outwards.

In the formulation the concrete is assumed to be a homogeneous, isotropic and linear-elastic material and the inner radius of the thick concrete cylinder being $R_i = (D_i + 2d_0)/2$, while the outer radius being $R_o = (R_i + C)$. Initially, the concrete is assumed to be intact and subsequently with the application of the uniform internal pressure 'p_r', the radial splitting cracks will propagate in all the directions to the same distance 'R_c' as shown in Fig. 2(e). 'R_c' is defined as a radius of the crack front at which the tensile capacity of the concrete is reached. With further progress of the corrosion process, there would be increase in the uniform internal pressure 'p_r' over the surrounding concrete and this would result in the further propagation of the crack front or further increase in the radius of the crack front 'R_c'. The concrete cover is assumed to be fully cracked, once 'R_c' becomes equal to 'R_o'.

Analytical models have been developed with various assumptions about the strength of the cracked concrete : (i) it retains its original tensile strength, (ii) it does not have any residual strength, (iii) it possesses some residual strength which degrades with increasing strain. The assumption (iii) seems to be the most realistic one. However, analyses with the first two assumptions have been carried out to obtain some boundary values. In this process of formulation, smeared cracking approach is adopted and therefore the formulation is presented in terms of the average stresses and strains.

2.2 Basic assumptions

The following assumptions are made in the proposed model while formulating the corrosion-cracking model to determine the time for the concrete cover cracking due to the reinforcement corrosion : (i) although the real problem is practically a three-dimensional one, but for the sake of simplicity and computational efficiency a two-dimensional approach is proposed for the model, (ii) the buildup of the corrosion products over the reinforcement is spatially uniform, although in real practical problem there may be instances, where the corrosion process is actually more pronounced on one side of the reinforcement only, (iii) the rate and initiation of corrosion are known in advance, (iv) due to the complex nature of the problem, the proposed model is restricted to the stresses resulting from the expansion of corrosion products only, (v) the basic material properties of the cover concrete are time-independent, however, the corrosion process and the material properties for concrete themselves are time-dependent phenomena, (vi) mechanical properties viz. modulus of elasticity and Poisson's ratio of the corrosion products have been assumed to be same as that of reinforcement, (vii) no amount of the corrosion products shall be accommodated within the open radial cracks during the progress of the crack front.

2.3 Governing equilibrium equations

The boundary value problem is represented by a thick walled hollow concrete cylinder having inner radius R_i and outer radius R_o under radially symmetric conditions and this problem is modeled and solved with reference to Fig. 2 as a plane stress problem. The governing stress equilibrium equation is as follows (Timoshenko and Goodier 1970).

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \tag{3}$$

The governing strain displacement equations are as follows.

$$\varepsilon_r = \frac{du}{dr}; \quad \varepsilon_\theta = \frac{u}{r} \tag{4}$$

Where, u = radial displacement at any radius 'r' and the other symbols have their usual meanings. The stresses are given as follows.

$$\sigma_r = \frac{E_c}{(1-\upsilon_c^2)}(\varepsilon_r + \upsilon_c \varepsilon_\theta); \quad \sigma_\theta = \frac{E_c}{(1-\upsilon_c^2)}(\varepsilon_\theta + \upsilon_c \varepsilon_r) \tag{5}$$

Where, E_c = modulus of elasticity for cover concrete.
υ_c = Poisson's ratio for the cover concrete.

The solution of Eq. (3) using the relations given in Eqs. (4) and (5) will yield the following relations.

$$u = \frac{Ar}{2} + \frac{B}{r} \tag{6}$$

$$\sigma_r = \frac{E_c}{(1-\upsilon_c^2)}\left[\frac{A}{2}(1+\upsilon_c) - \frac{B}{r^2}(1-\upsilon_c)\right] \tag{7a}$$

$$\sigma_\theta = \frac{E_c}{(1-\upsilon_c^2)}\left[\frac{A}{2}(1+\upsilon_c) + \frac{B}{r^2}(1-\upsilon_c)\right] \tag{7b}$$

Where, the constants 'A' and 'B' can be determined by applying the suitable boundary conditions.

2.4 Mathematical formulations for the problem

The formulation given in section 2.4.1 is from the reference (Liu and Weyers 1998), while the formulations given in sections 2.4.2 to 2.4.5 are the present work.

2.4.1 Formulation 1 (FM1) (Liu and Weyers 1998)

In this model, the constants 'A' and 'B' in Eq. (6) are obtained by specifying $u = d_c$ at $r = R_i$ and $\sigma_r = 0$ at $r = R_o$. The radial stress at the inner boundary is given as $\sigma_r = -p_r$. Where, 'p_r' is the radial pressure at the concrete/rust products interface and is expressed as follows.

$$p_r = \frac{2E_{ef}d_c}{(D_i + 2d_o)(m + \upsilon_c)} \tag{8}$$

The notation 'm' is defined as follows.

$$m = \frac{(R_o^2 + R_i^2)}{(R_o^2 - R_i^2)} \tag{9}$$

Where, 'E_{ef}' is the effective modulus of elasticity of the cover concrete given as follows.

$$E_{ef} = \frac{E_c}{1.0 + \theta} \tag{10}$$

Where, θ is the creep coefficient for the cover concrete. Assuming the cover 'C' to be very thin and cover concrete to be fully cracked when the minimum stress required to cause the cracking becomes equal to the tensile strength of the concrete 'f_t', 'p_r' would be defined as follows.

$$p_r = \frac{Cf_t}{R_i} \tag{11}$$

From Eqs. (8) and (11), 'd_c' would be defined as follows.

$$d_c = \frac{Cf_t}{E_{ef}}(m + \upsilon_c) \tag{12}$$

The critical amount of the corrosion products 'W_{crit}' needed to induce cracking of the cover concrete can be estimated as follows.

$$W_{crit} = \rho_r\left(\pi\left[\frac{Cf_t}{E_{ef}}(m + \upsilon_c) + d_o\right]D_i + \frac{W_{st}}{\rho_{st}}\right) \tag{13}$$

After the evaluation of 'W_{crit}' the time to cover cracking is calculated using the diffusion theory. As per the diffusion theory, the rate of production of expansive corrosion products is inversely proportional to the oxide layer thickness because of the fact that, with the increase in the thickness of expansive corrosion products, there shall be an increase in the steel ionic diffusion distance (Liu and Weyers 1998). Integration of the Eq. (2) gives the time to cover cracking 't_c' as follows.

$$t_c = \frac{W_{crit}^2}{2k_p} \tag{14}$$

Where 'W_{crit}' is defined by Eq. (13) and 'k_p' is defined as follows.

$$k_p = \frac{0.091}{\alpha}\pi D_i i_{cor} \tag{15}$$

Where, 'i_{cor}' is the annual mean corrosion rate (μA/cm^2) (Liu and Weyers 1998).

2.4.2 Formulation 2 (FM2)

This formulation considers the effects of the stiffness of the reinforcing bar and the corrosion products considered as one single material besides the structural stiffness of the concrete. All the other considerations related to the production of rust, development of internal radial pressure and the cracking of cover concrete are the same as those in the formulation 'FM1'. Following boundary conditions will be applied for the solution of Eqs. (6) and (7).

For Concrete :

$$u = d_c \quad \text{at} \quad r = R_i \quad \text{and} \quad \sigma_r = 0 \quad \text{at} \quad r = R_o \tag{16a}$$

For the combination of reinforcement and corrosion products :

$$\sigma_r = -p_r \quad \text{at} \quad r = R_i \quad \text{and} \quad u = \{D_2 - (D_i + 2d_o + 2d_c)\}/2 \quad \text{at} \quad r = D_2/2 \tag{16b}$$

In the formulation 'FM1', the radial displacement of concrete at $r = R_i$ is equal to the thickness of the rust layer 'd_c' beyond the porous zone. However in the formulation 'FM2', since the stiffness of concrete as well as the combination of the reinforcement and the corrosion products is concerned, the radial displacement at the inner boundary has to be evaluated from continuity of displacement at $r = R_i$ besides the rust products. This would result in the following relationship for 'd_c'.

$$d_c = \frac{D_2 - 2R_i}{2 + m_1 \dfrac{D_2}{R_i}} \tag{17}$$

Where 'm_1' is obtained as follows.

$$m_1 = \frac{E_{ef}(1.0 - \upsilon_s)}{E_s(m + \upsilon_c)} \tag{18}$$

Where, 'E_s' and 'υ_s' are the effective modulus of elasticity and the Poisson's ratio for the reinforcement plus expansive corrosion products combine.

The solution of Eqs. (8), (11) and (17) would result in the estimation of the combined diameter of the reinforcement plus the freely expanded corrosion products 'D_2'. The mass of the expansive corrosion products per unit length of the reinforcement 'W_r', is obtained as follows.

$$W_r = \frac{\pi}{4}\rho_r \alpha_1 \left[\frac{D_2^2 - D_i^2}{\alpha_1 - 1.0}\right] \tag{19}$$

Where, 'ρ_r' is the mass density of the corrosion products and is defined by Eq. (1).

Once the mass of expansive corrosion products 'W_r' equals the critical value, the time to cover cracking then shall be evaluated by the Eq. (14).

2.4.3 Formulation 3 (FM3)

This model is based fully on thick-walled cylinder approach and considers the propagation of the radial splitting cracks. Fig. 4 shows the schematic representation of the cover concrete into the two zones of cracked and uncracked concrete. The 'zone 1' corresponds to the area of cracked concrete between the reinforcement plus expansive corrosion products combine and the radius of crack front 'R_c', while the 'zone 2' corresponds to the area of uncracked concrete between the radius of crack front 'R_c' and the outer radius 'R_o' of the thick-walled concrete cylinder. It may be mentioned here that, the radius of the crack front 'R_c' is defined by the condition that at 'R_c', the principal hoop stress 'σ_θ' becomes equal to the tensile strength 'f_t' of the cover concrete. The cover concrete is assumed to be fully cracked, when the crack front radius 'R_c' becomes equal to the outer radius of the thick-walled concrete cylinder 'R_o'. It is further assumed that the cover concrete maintains the original strength equal to its tensile capacity in the cracked concrete portion and the modulus of elasticity remains unchanged.

In this formulation, the Eqs. (6) and (7) shall be slightly modified as follows to account for the 'zone 1' and 'zone 2' of the cover concrete.

$$u_1 = \frac{A_1 r}{2} + \frac{B_1}{r}; \quad u_2 = \frac{A_2 r}{2} + \frac{B_2}{r} \tag{20}$$

$$\sigma_{r1} = \frac{E_{c1}}{(1-\upsilon_{c1}^2)}\left[\frac{A_1}{2}(1+\upsilon_{c1}) - \frac{B_1}{r^2}(1-\upsilon_{c1})\right] \tag{21a}$$

$$\sigma_{\theta 1} = \frac{E_{c1}}{(1-\upsilon_{c1}^2)}\left[\frac{A_1}{2}(1+\upsilon_{c1}) + \frac{B_1}{r^2}(1-\upsilon_{c1})\right] \tag{21b}$$

$$\sigma_{r2} = \frac{E_{c2}}{(1-\upsilon_{c2}^2)}\left[\frac{A_2}{2}(1+\upsilon_{c2}) - \frac{B_2}{r^2}(1-\upsilon_{c2})\right] \tag{22a}$$

$$\sigma_{\theta 2} = \frac{E_{c2}}{(1-\upsilon_{c2}^2)}\left[\frac{A_2}{2}(1+\upsilon_{c2}) + \frac{B_2}{r^2}(1-\upsilon_{c2})\right] \tag{22b}$$

Where, the subscripts '1' and '2' to the various symbols will carry the usual meanings of the symbols for 'zone 1' and 'zone 2' concrete respectively. The constants 'A_1', 'B_1', 'A_2' and 'B_2' can be determined by applying the suitable boundary conditions. In this formulation, for the solution of Eqs. (20), (21) and (22), boundary conditions will be defined by Eq. (16).

Using the Eqs. (2), (15) and (19), one can estimate the amount of the corrosion products 'W_r' generated and then the combined diameter of the reinforcement plus the freely expanded corrosion products 'D_2' at any corrosion time 't'. After the evaluation of 'D_2', one can estimate 'd_c' by using Eqs. (17) and (18).

The radius of the crack front 'R_c' at any instant of time is obtained as follows.

$$R_c = \sqrt{\frac{p_r R_i^2 R_o^2}{(f_t R_o^2 - f_t R_i^2 - p_r R_i^2)}} \tag{23}$$

The cover is assumed to be fully cracked, once 'R_c' becomes equal to the outer radius 'R_o' of the thick-walled concrete cylinder.

2.4.4 Formulation 4 (FM4)

This model is similar to the formulation 'FM3' except that like formulation 'FM3', it is assumed that the cracked concrete does not maintain any residual strength, i.e., the inner boundary of the structural concrete is shifted to the radius of the crack front 'R_c'. In this formulation, for the solution of Eqs. (20), (21) and (22), boundary conditions will be defined by Eq. (16). Therefore, if 'R_c' is to be displaced by 'd_c' then the material between 'R_i' and 'R_c' will have to move by an amount 'd_{c1}' for preserving the volume. Here, 'd_{c1}' is defined as follows.

$$d_{c1} = \frac{R_i}{R_c} d_c \tag{24}$$

Following the procedure mentioned for the formulation 'FM3' and making use of Eq. (25) 'd_c' is obtained as follows.

$$d_c = \frac{D_2 - 2R_i}{\left(\frac{D_2 m_1}{R_c} + \frac{2R_c}{R_i}\right)} \tag{25}$$

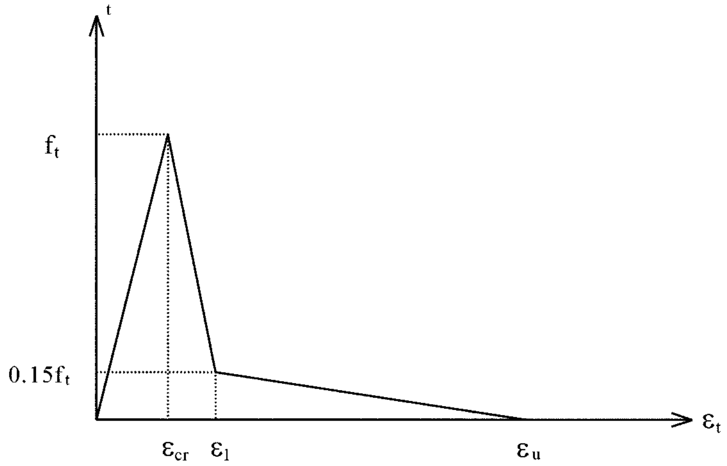


Fig. 3 Idealized stress-strain diagram for cover concrete in uniaxial tension

After the evaluation of 'd_c', the radius of the crack front 'R_c' at any instant of time 't' is defined by Eq. (23).

2.4.5 Formulation 5 (FM5)

This model is based on thick-walled cylinder approach as described for formulations 'FM3' and 'FM4'. It is further assumed that the cracked concrete maintains some residual strength after its tensile capacity is exceeded or in other words the cracking in the cover concrete is modeled as a process of tension-softening, once the principal tensile strain in the hoop direction exceeds the cracking tensile strain value of 'ε_{cr}' as shown in Fig. 3. This behaviour of the concrete has been considered (Pantazopoulou and Papoulia 2001). The present work considers a simple 2-zone model of concrete (cracked or uncracked) and develops an analytical solution instead of a finite difference based solution presented in the reference literature (Pantazopoulou and Papoulia 2001). The modulus of elasticity for the 'zone 1' concrete is defined by the secant slope of the descending branches of the stress-strain curve as shown in Fig. 3 and the modulus of elasticity for the 'zone 2' concrete is defined by its original value.

In this formulation, for the solution of Eqs. (20), (21) and (22), boundary conditions will be defined by Eq. (16). In addition to this, at the crack front radius 'R_c', radial stresses due to 'zone 1' and 'zone 2' concretes are equal. Therefore, the solution of Eqs. (20), (21) and (22) would result in the following relationships.

$$A_1 = 2\left[\frac{u_c R_c - d_c R_i}{R_c^2 - R_i^2}\right]; \quad B_1 = R_i R_c\left[\frac{d_c R_c - u_c R_i}{R_c^2 - R_i^2}\right] \tag{26}$$

$$u_c = d_c \frac{E_{ef1}[(1-\upsilon_{c2})R_c^2 + (1+\upsilon_{c2})R_o^2][2R_i R_c]}{E_{ef1}[(1-\upsilon_{c2})R_c^2 + (1+\upsilon_{c2})R_o^2][(1+\upsilon_{c1})R_c^2 + (1-\upsilon_{c1})R_i^2] - E_{ef2}(R_c^2 - R_i^2)(R_c^2 - R_o^2)(1-\upsilon_{c1}^2)} \tag{27}$$

Where, 'u_c' is the radial displacement at the radius of the crack front 'R_c'.

$$A_2 = \frac{2(1-\upsilon_{c2})u_c R_c}{[(1+\upsilon_{c2})R_c^2 + (1-\upsilon_{c2})R_o^2]}; \qquad B_2 = \frac{(1+\upsilon_{c2})u_c R_c R_o^2}{(1+\upsilon_{c2})R_o^2 + (1-\upsilon_{c2})R_c^2} \tag{28}$$

The internal radial pressure at the steel/concrete interface is defined as follows.

$$p_r = \frac{E_{ef1}}{(1-\upsilon_{c1}^2)}\left[\frac{A_1}{2}(1+\upsilon_{c1}) - \frac{B_1}{R_i^2}(1-\upsilon_{c1})\right] \tag{29}$$

The radius of crack front 'R_c' is obtained as follows.

$$R_c = \sqrt{\frac{1}{\left[\frac{f_t(1+\upsilon_{c2})}{E_{ef2}B_2} - \frac{1}{R_o^2}\right]}} \tag{30}$$

For the solution of the problem, first for any value of the radius of crack front 'R_c', the value of 'B_2' shall be evaluated from Eq. (30). After the evaluation of 'B_2', the value of radial displacement at the radius of crack front 'u_c' shall be evaluated from Eq. (28). After getting the value of 'u_c', the value of the thickness of corrosion products 'd_c' deposited around the reinforcement to induce the cracking up to 'R_c' shall be evaluated from Eq. (27). Then, by using Eqs. (26) and (29), the value of 'p_r' at the reinforcement-concrete interface can be estimated. The Eq. (17) would result in the combined diameter of the reinforcement plus the freely expanded corrosion products 'D_2'. The solution of Eqs. (19), (14) and (15) would result in the time required for the crack to reach at the location 'R_c'.

3. Applications and performance of the model

3.1 Numerical analysis

Conditions and results for some of the experimental studies are available in the literature (Liu and Weyers 1998). Numerical analyses were carried out using the proposed models for the input data shown in Table 2. Table 3 presents the brief summary of various considerations in the different formulations adopted for the numerical analyses.

Cracking of cover concrete depends on number of factors viz., the diameter of reinforcement, clear cover to the reinforcement and uniaxial compressive strength of the concrete (Rasheeduzzafar *et al*. 1992, Liu and Weyers 1998, Pantazopoulou and Papoulia 2001). Similarly, it would be worthwhile to mention here that, the lack of understanding of the structural material properties such as the material strengths and their deviations from the anticipated design material strengths may significantly affect the overall structural response. The material strength in the actual structure may be somewhat different than the considered design strength, this being mostly due to the inherent variability of the material properties arising in the manufacture and the errors or deviations arising in the construction. Deviations in the concrete strength may also arise due to variations in the mix proportions, presence of impurities and inadequate compaction or curing. Therefore, the estimation of the suitable material properties viz. modulus of elasticity and tensile strength particularly for the

Table 2 Experimental data taken for analytical evaluation for specimen 1 (Liu and Weyers 1998)

Initial Reinforcement Diameter	= 16 mm
Cover Depth	= 48 mm
Measured Corrosion Rate	= 2.41 μA/cm^2
Uniaxial Compressive Strength of Cover Concrete	= 31.5 MPa
Initial Elastic Modulus for Cover Concrete	= 27000 MPa
Tensile Strength of the Cover Concrete	= 3.3 MPa
Poisson's Ratio for The Cover Concrete	= 0.18
Creep Coefficient of the Cover Concrete	= 2.0

Table 3 Considerations in various formulations

Consideration →	Cracking in Concrete Based on		Number of Components		Strength of Cracked Concrete			Spatial Propagation of Cracks Considered
Formulation ↓	Thin Shell Approach	Thick Shell Approach	One (Only Concrete)	Two (Concrete and Rein-forcement plus Rust Products Combine)	No Strength	Full Original Strength	Appropriate Stress-Strain Relationship (Fig. 3)	
FM1 (Liu & Weyers 1998)	Yes	No	Yes	No	No	No	No	No
FM2 (This Paper)	Yes	No	No	Yes	No	No	No	No
FM3 (This Paper)	No	Yes	No	Yes	No	Yes	No	Yes
FM4 (This Paper)	No	Yes	No	Yes	Yes	No	No	Yes
FM5 (This Paper)	No	Yes	No	Yes	No	No	Yes	Yes

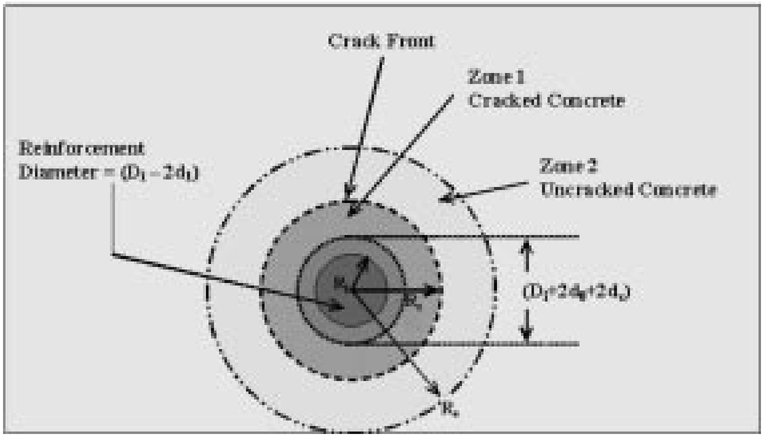


Fig. 4 Schematic representation of the cover concrete into two zones of cracked and uncracked concrete

cover concrete assumes significant importance for the accurate evaluation of the time to cover cracking due to reinforcement corrosion. A sensitivity analysis was carried out by varying the 'E_{ef}', 'f_t', 'E_s', 'i_{cor}'.

Various formulations have been reported in the different international standards (ACI 1985, CEB-FIP 1990, IS 2000) for the evaluation of the initial modulus of elasticity of the concrete and these different formulations would result in the different values of 'E_o'. Similarly, even for the given uniaxial compressive strength, the 'E_o' it is related to the density of the concrete as well. In the proposed formulations, the effective modulus of elasticity 'E_{ef}' for the cover concrete is derived from the initial modulus of elasticity 'E_o' by using the creep coefficient of the concrete. It may be mentioned here that, the creep coefficient of the concrete depends on the constituents of concrete, environmental conditions, stress in concrete, age at loading and duration of loading. Therefore, keeping all these in mind, the value of creep coefficient 'θ' for the cover concrete has been varied over a wide range from 0.0 to 2.0.

The tensile strength of the concrete shows larger variation than the compressive strength as it is significantly influenced by the shape and the surface texture of the aggregates and may be reduced by the environmental effects. Therefore, the tensile strength of the concrete should be taken into account with great caution. Various relations have been reported in the different international standards (ACI 1985, CEB-FIP 1990) for the evaluation of the tensile strength 'f_t' of the concrete, which upon using would result in the different values of 'f_t' even for the same value of the 'f_c' for the concrete. Therefore, the values of 'f_t' have been varied over a wide range from 0.7 MPa to 4.2 MPa.

Not much data is reported as far as the material properties of the expansive corrosion products are concerned. Therefore, in the initial calculations the value of 'E_s' is considered as the same value as of reinforcement alone, however, for the sake of parametric study the value of 'E_s' has been varied over a wide range from 105000 MPa to 252000 MPa.

The estimation of the parameter 'k_p' from Eq. (15) depends on 'i_{cor}' and 'α' and therefore, the uncertainties in 'k_p' would greatly affect the accurate evaluation of the time to cracking of cover concrete due to the reinforcement corrosion. In the reference literature (Liu and Weyers 1998) the field measurements and the estimation of the mean corrosion rates have been reported from different test methods. It may be stated here that, there is a marked variation among the reported values of the mean corrosion rates due to different test methods, which may be attributed to the environmental exposure conditions. Therefore, keeping this in mind the annual mean corrosion rate has been varied over a wide range from 0.54 μA/cm^2 to 26.75 μA/cm^2.

While carrying out the sensitivity analysis, only one parameter was varied at a time and the other parameters were kept at their reference value as reported in the reference literature (Liu and Weyers 1998).

3.2 Discussion of results

The results of the numerical analyses for the experimental study as given in Table 2 are presented in Figs. 5 to 14 for the proposed corrosion cracking models.

Fig. 5 demonstrates the variation of time to cover cracking as a function of 'α' for various values of 'E_s'. These results are based on the formulations 'FM1' and 'FM2'. For comparison, the experimentally observed value is also presented. Formulations 'FM1' and 'FM2' are based on thin cylinder approach and they work well for a limited range of dimensions of the specimen. For larger

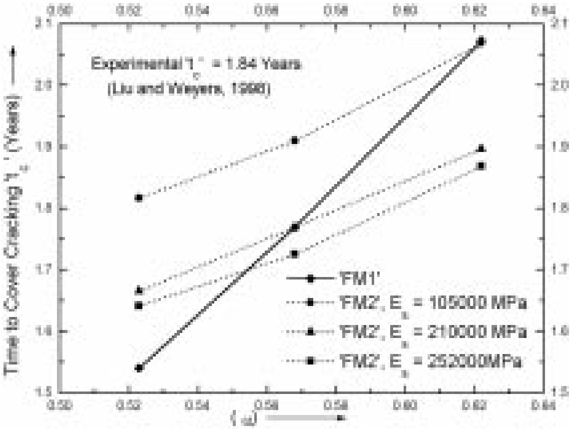


Fig. 5 Time to cover cracking for specimen 1 (Refer Table 2) as per formulation 'FM1' and 'FM2'

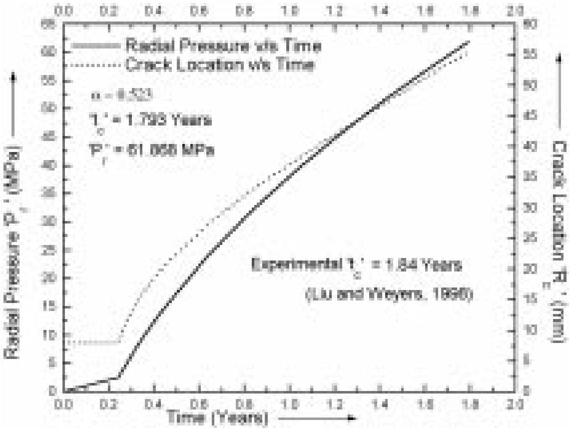


Fig. 6 Radial pressure and crack location with time for specimen 1 (Refer Table 2) as per formulation 'FM3' (Behaviour up to time to cover cracking)

dimensions, a formulation based on thick cylinder approach ('FM3') shall be more appropriate. Typical results with 'FM3' are shown in Fig. 6. It is clear from Figs. 5 and 6 that, the predicted time to cover cracking as per the formulations 'FM2' and 'FM3' are in good agreement with the experimentally observed time to cover cracking.

To facilitate the presentation of results of various formulations in a compact form, following normalized parameters are introduced and the results in Figs. 7 to 13 are presented in normalized forms.

$$T_{c1} = \frac{\textit{Predicted Time to Cover Cracking as per Formulation FM1 Based on Thick Cylinder Approach}(\textit{Years})}{\textit{Predicted Time to Cover Cracking as per Formulation FM3}(\textit{Years})}$$

$$T_{N\theta} = \frac{\textit{Predicted Time to Cover Cracking as per Formulation FM3 or FM4 or FM5 at Any Value of } \theta(\textit{Years})}{\textit{Predicted Time to Cover Cracking as per Formulation FM3 at a } \theta \textit{ Value of 1.0}(\textit{Years})}$$

$$T_{CN} = \frac{\textit{Time from Initiation of Corrosion}(\textit{Years})}{\textit{Predicted Time to Cover Cracking as per Formulation FM3}(\textit{Years})}$$

$$T_{CI} = \frac{\textit{Predicted Time to Cover Cracking as per Formulation FM3 or FM4 or FM5 at Any Value of } i_{cor}(\textit{Years})}{\textit{Predicted Time to Cover Cracking as per Formulation FM3}(\textit{Years})}$$

$$T_{CT} = \frac{\textit{Predicted Time to Cover Cracking as per Formulation FM3 or FM4 or FM5 at Any Value of } f_t(\textit{Years})}{\textit{Predicted Time to Cover Cracking as per Formulation FM3}(\textit{Years})}$$

$$R_{RN} = \frac{\textit{Instantaneous Radial Pressure } P_r \textit{ as per Formulation FM3 or FM4 or FM5}(\textit{MPa})}{\textit{Radial Pressure } P_r \textit{ as per Formulation FM3 at the Time of Cover Cracking}(\textit{MPa})}$$

$$R_{CN} = \frac{\textit{Instantaneous Radius of Crack Front } R_C \textit{ as per Formulation FM3 or FM4 or FM5}(\textit{mm})}{\textit{Outer Radius } R_O \textit{ of the Thick Walled Concrete Cylinder}(\textit{mm})}$$

With these definitions, the range of 'P_{RN}' and 'R_{CN}' shall be from 0.0 to 1.0. Since 'T_{CN}' is normalized with respect to 'FM3', its range shall be from 0.0 to 1.0 for formulation 'FM3'. This

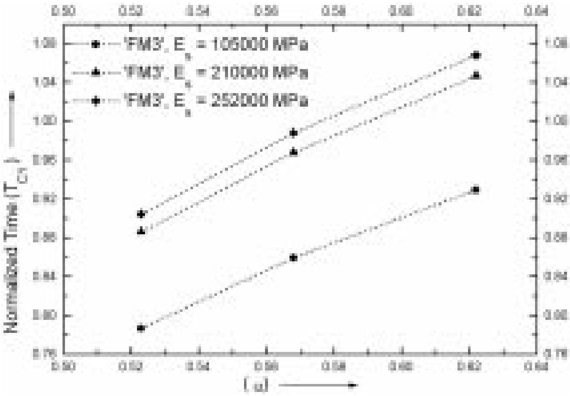


Fig. 7 Normalized time to cover cracking for specimen 1 (Refer Table 2) as per formulation 'FM3'

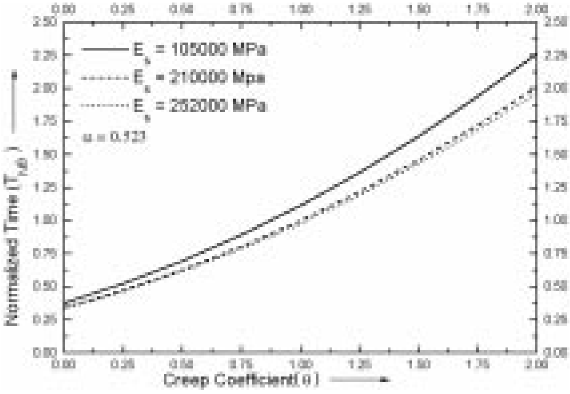


Fig. 8 Influence of creep coefficient on normalized time to cover cracking for specimen 1 (Refer Table 2) as per formulation 'FM3'

method of normalization facilitates an easy comparison of the results taking the results of 'FM3' as the basis of normalization and the base values as per formulation 'FM3' are evaluated for specimen 1 as described in Table 2.

Fig. 7 presents the normalized predicted time to cover cracking for different values of 'α' in terms of 'T_{C1}' as per the formulation 'FM3'. It is evident that, both the formulations 'FM1' and 'FM3' appear not to show much variation in the prediction of time to cover cracking based on the thick-cylinder approach.

It is seen from Figs. 5, 7, 8 and 12 that, there appears to be a significant variation in the predicted time to cover cracking with the variation in 'E_s'. Lowering the value of 'E_s' would result in decrease of initial stiffness against the expansion of corrosion products and this would eventually result in the delayed cracking of the cover concrete.

Figs. 6, and 11 present the build-up of 'p_r' at the steel-concrete interface and the propagation of crack front from 'R_i' to 'R_o' of the thick concrete cylinder with respect to the continued corrosion process as per the formulation 'FM3' and 'FM5'. These formulations would result in the increase of 'p_r' with respect to the continued corrosion process till the cover concrete is fully cracked. This trend may be attributed to the fact that, in these formulations, it is assumed that the cracked concrete maintains residual strength. This would result in the increase of the initial stiffness against the expansion of the corrosion products and this would eventually result in the increase of 'p_r' at the steel-concrete interface to induce the cover cracking. It is also evident from the same figures that, the formulation 'FM5' would result in a lesser value of 'p_r' at the steel-concrete interface to induce the cover cracking as compared to the formulation 'FM3' and this is because of the lesser value of residual strength for the cover concrete in case of formulation 'FM5' as compared to the formulation 'FM3'.

Fig. 9 presents the build-up of 'p_r' at the steel-concrete interface and the propagation of crack front from 'R_i' to 'R_o' of the thick concrete cylinder with respect to the continued corrosion process as per the formulation 'FM4'. This formulation would result in the decrease of 'p_r' with respect to the continued corrosion process till the cover concrete is fully cracked. This trend may be attributed to the fact that, in this formulation, it is assumed that the cracked concrete does not maintain any

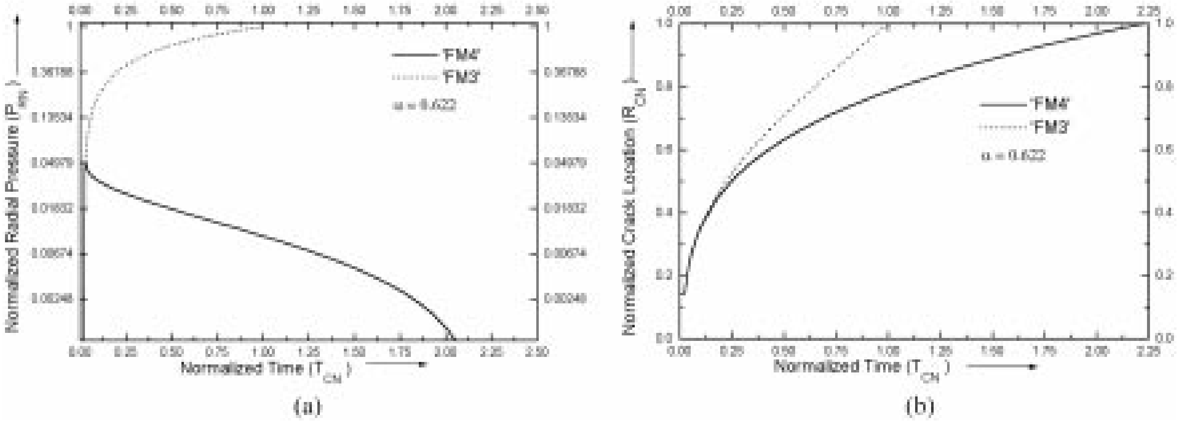


Fig. 9(a) Normalized radial pressure v/s normalized time for specimen 1 (Refer Table 2) for formulations 'FM3' and 'FM4' (Behaviour up to time to cover cracking)

Fig. 9(b) Normalized crack location v/s normalized time for specimen 1 (Refer Table 2) for formulations 'FM3' and 'FM4' (Behaviour up to time to cover cracking)

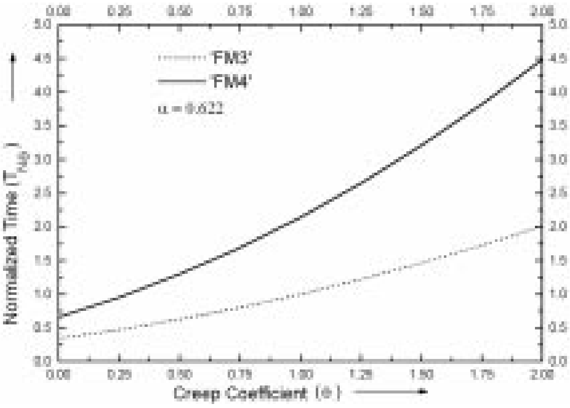


Fig. 10 Influence of creep coefficient on normalized time to cover cracking for specimen 1 (Refer Table 2) as per formulations 'FM3' and 'FM4'

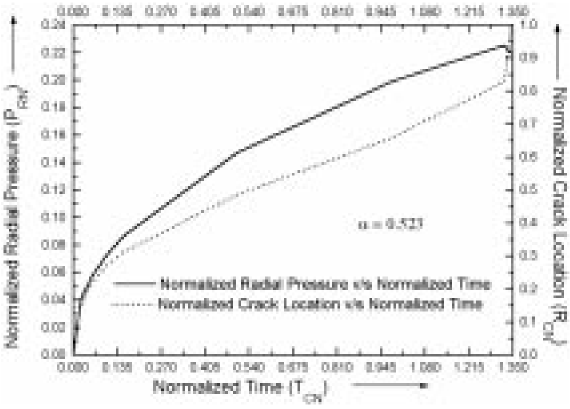


Fig. 11 Normalized radial pressure and normalized crack location v/s normalized time for specimen 1 (Refer Table 2) as per formulation 'FM5' (Behaviour up to time to cover cracking)

residual strength. This would result in the decrease of the initial stiffness against the expansion of the corrosion products and this would eventually result in the decrease of 'p_r' at the steel-concrete interface to induce the cover cracking.

It is seen from the Figs. 8, 10 and 12 that, there appears to be a significant variation in the normalized predicted time to cover cracking with the variation in 'θ' of cover concrete, as this variation would result in the variation of the evaluated effective modulus of elasticity of cover concrete 'E_{ef}'. It is clear from the same figures that, lowering the initial modulus of elasticity for the cover concrete by using higher 'θ' would result in the decrease of initial stiffness against the

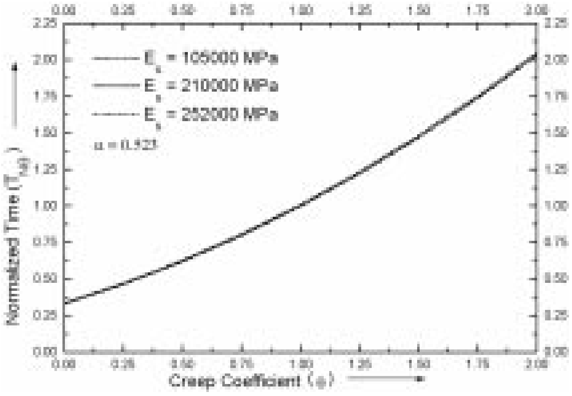
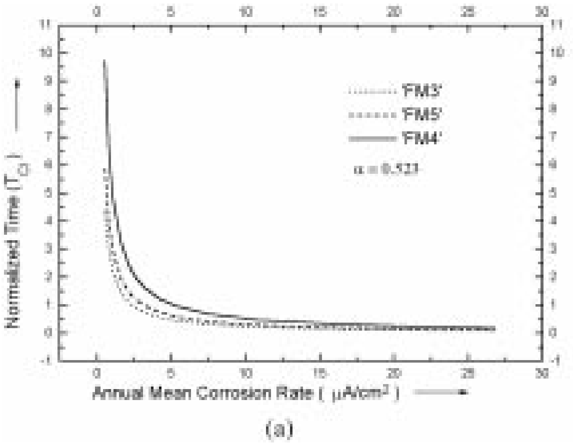


Fig. 12 Influence of creep coefficient on normalized time to cover cracking for specimen 1 (Refer Table 2) as per formulations 'FM5'



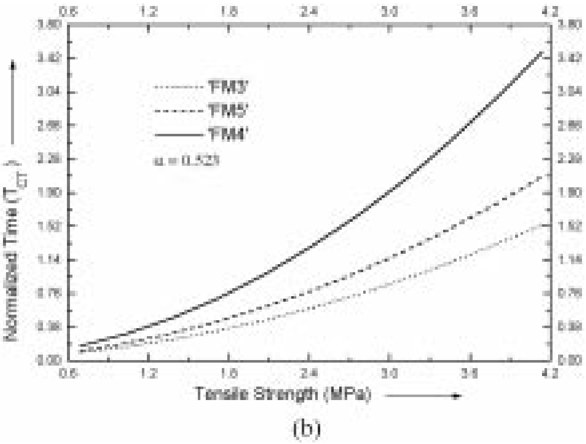


Fig. 13(a) Influence of annual mean corrosion rate on normalized time to cover cracking for speciemen 1 (Refer Table 2) as per formulation 'FM3', 'FM4' and 'FM5'

Fig. 13(b) Influence of tensile strength of cover concrete on normalized time to cover cracking for speciemen 1 (Refer Table 2) as per formulation 'FM3', 'FM4' and 'FM5'

expansion of corrosion products and this would eventually result in the delayed cracking of the cover concrete.

Fig. 13(a) presents the influence of 'i_{cor}' on the normalized predicted time to cover cracking as per the formulations 'FM3', 'FM4' and 'FM5'. Higher 'i_{cor}' would result in the generation of higher amount of the corrosion products for a given period of time and this would further result in the higher build-up of 'p_r' at the steel-concrete interface and hence the lesser predicted time to cover cracking.

Fig. 13(b) presents the influence of 'f_t' of cover concrete on the normalized predicted time to cover cracking as per the formulations 'FM3', 'FM4' and 'FM5'. It is clear that, increase in 'f_t'

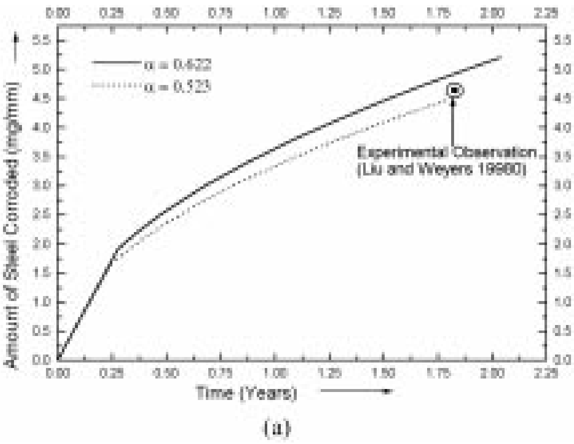


Fig. 14(a) Amount of steel corroded with time for specimen 1 (Refer Table 2) as per formulation 'FM3' (Behaviour up to time to cover cracking)

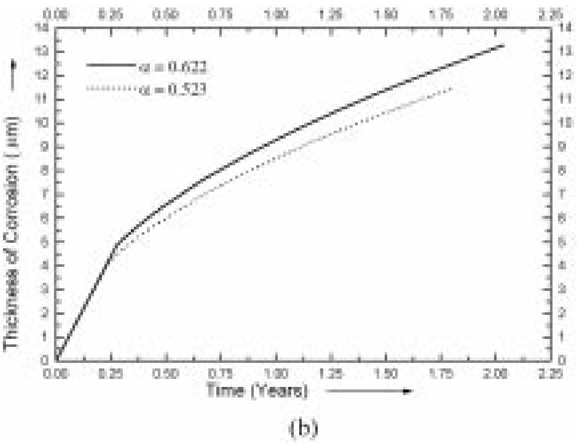


Fig. 14(b) Thickness of corrosion as a function of time for specimen 1 (Refer Table 2) as per formulation 'FM3' (Behaviour up to time to cover cracking)

would result in the increase of cracking strain limit 'ε_{cr}' for cover concrete and this would ultimately result in the delayed cracking of the cover concrete.

As seen from Figs. 5 to 13 that, the predicted time to cover cracking from the formulation 'FM5' is higher than the same from the formulation 'FM3' and lower than that predicted by formulation 'FM4'. This is because the stiffness is minimum for 'FM4' and maximum for 'FM3'.

It is also clear from the Figs. 5 to 13 that, the cover cracking is also dependent on the type of corrosion products. This may be attributed to the fact that, the different corrosion products would have different volume expansion on formation and thus have different mass densities. However, in the present study, the values of the predicted times to cover cracking are reported for two values of 'α' viz. 0.523 and 0.622 and normally these two represent the lower and upper bounds in case of the reinforcement corrosion under chloride environment and therefore, the actual cover cracking time would lie between these two bounds.

Fig. 14 presents some typical results as per formulation 'FM3'. Fig. 14(a) demonstrates the amount of steel corroded per unit length of the bar while, Fig. 14(b) shows the total corrosion loss expressed in terms of thickness, as a function of corrosion time for different values of 'α'. For comparison, the experimentally observed value at the time of cover cracking is also presented in Fig. 14(a). It is evident that, the predicted values are in good agreement with the experimentally observed value.

4. Conclusions

The conclusions from the present study are discussed as follow.

The results of the numerical analyses are presented for the referred experimental data, which show that the cover cracking is mainly dependent on the types of the corrosion products, corrosion rate and the selection of the material properties.

It is evident from the sensitivity analysis that, the most important variables that may affect the predicted time to cover cracking are the initial tangent modulus of elasticity and the tensile strength of the cover concrete and the modulus of elasticity 'E_s' of the reinforcement plus corrosion products combine.

Correlation with the available experimental data shows that the presented models 'FM2', 'FM3', 'FM4' and 'FM5' are mathematically quite simple, capable of reproducing the experimental trends and providing the reasonable estimates of the predicted time to cover cracking.

The process of tension-softening in the cover concrete; once the principal hoop strain exceeds the value of cracking tensile strain as considered in formulation 'FM5', does have a significant effect over the predicted time to cover cracking.

In the proposed formulations, it has been assumed that no amount of the corrosion products shall be accommodated within the open radial cracks; however, in actual practice this may not be true. To quantify this aspect and in particular, the possible convection of the corrosion products away from the reinforcement surface, a rigorous experimentation would be necessary.

The present study may be very useful in identifying the various conditions that may limit the time-dependent structural capacity of the reinforced concrete structures because of the continued corrosion of reinforcement and ultimately cover cracking, which may in turn be helpful to assess the structural performance to withstand the possible extreme events during the remaining service life of the structures.

Formulation 'FM2' is based on thin cylinder approach and an improvement over formulation 'FM1' as it considers the stiffness offered by reinforcement in the evaluation of time to cracking of cover concrete. Formulations 'FM3', 'FM4' and 'FM5' are based on thick cylinder approach and include the stiffness offered by reinforcement. These formulations differ from one another in the manner in which the residual strength of the cracked concrete is modeled. In the present framework of modeling, formulation 'FM5' is the most realistic one.

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