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Infilled frames: developments in the evaluation of the stiffening effect of infills

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Abstract. In order to consider the modified seismic response of framed structures in the presence of masonry infills, proper models have to be formulated. Because of the complexity of the problem, a careful definition of a diagonal pin-jointed strut, able to represent the horizontal force-interstorey displacement cyclic law of the actual infill, may be a solution. In this connection the present paper shows a generalized criterion for the determination of the ideal cross-section of the strut mentioned before. The procedure is based on the equivalence between the lateral stiffness of the actual infilled frame scheme during the conventional elastic stage of the response and the lateral stiffness of the same frame stiffened by a strut at the same stage. Unlike the usual empirical approaches available in the literature, the proposed technique involves the axial stiffness of the columns of the frame more than their flexural stiffness. Further, the influence of the bidimensional behaviour of the infill is stressed and, consequently, the dependence of the dimensions of the equivalent pin-jointed strut on the Poisson ratio of the material constituting the infill is also shown. The proposed approach is extended to the case of infills with openings, which is very common in practical applications.

Key words: infilled frames; masonry infill; stiffening effect; simplified model; equivalent strut; identification technique.

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1. Introduction

The last three decades have witnessed a growing interest of the scientific community in the effects of infill walls on the behaviour of frames. It is known that, even though infills are considered non-structural, they radically modify the frame response under lateral loads.

It has been observed that, in a typical situation, an infill panel may stiffen a frame laterally by one order of magnitude and increase its ultimate strength up to four times. These variations are influenced by the system geometry and the mechanical characteristics of the material used for the infill (masonry, reinforced concrete, etc.). Moreover, the frame-infill interaction depends on the height of the infill, which can be partial with respect to that of the frame columns, on the presence of windows or door apertures, on the ratio between the horizontal and vertical loads and the technique used for making the infill.

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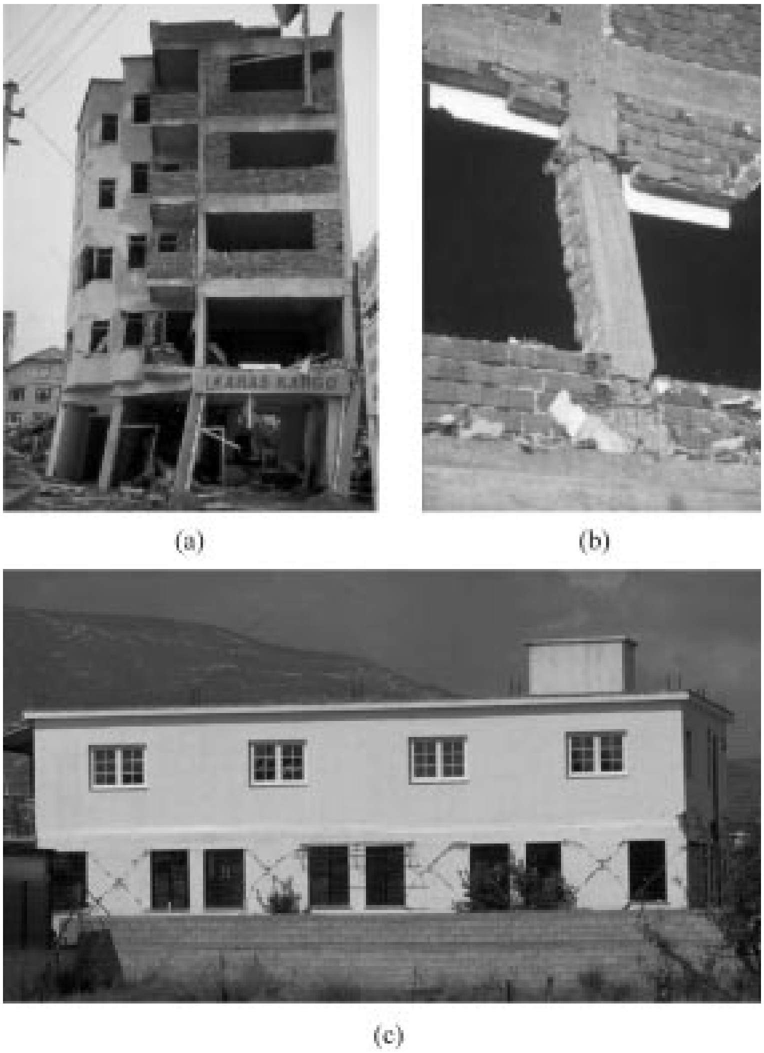

Fig. 1 Some effects of infill on RC frames: (a) soft drift mechanism, (b) column shear collapse for partial height of infill, (c) infill saving RC frames

The interaction between infill and frame may or may not be beneficial to the performance of the structure under seismic loads, as numerous debates and, most of all, experiences in recent earthquakes have demonstrated.

In Fig. 1 some of the positive and negative effects of the infill in framed structures are shown. Fig. 1(a) and Fig. 1(b) show collapse due to a soft drift mechanism and to shear effect on the columns for partial height of the infill, respectively; instead, in Fig. 1(c) damage to the infill without any damage to the structure can be observed.

Many codes give additional design measures for new seismic structures in order to consider the modified behaviour when infill is not taken into account in the calculus model, or suggest introducing infill in the model itself. Further, taking infill into account may be basic in the

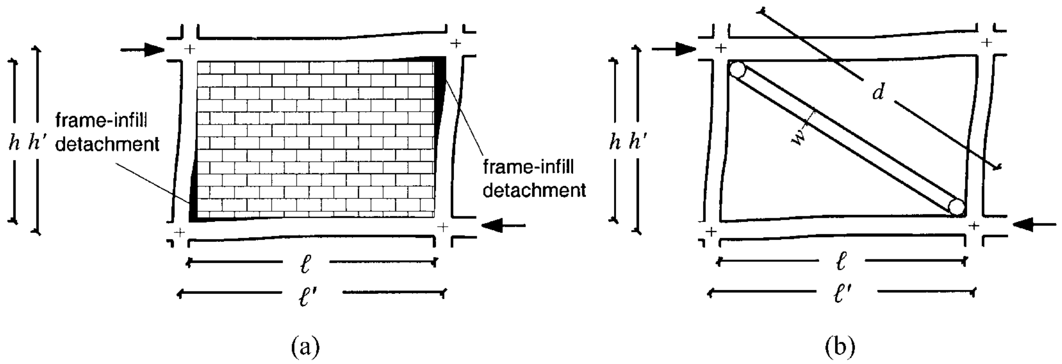

Fig. 2 Infilled mesh: (a) frame-infill joint under horizontal loads on true system, (b) simplified scheme

retrofitting of existing RC structures in order to predict their real seismic vulnerability and to design proper consolidation actions.

Historically, the attempts to evaluate the stiffening and the strengthening offered by infills or to define simplified mechanical response laws have been preceded by experimental tests leading to different solution criteria. Holmes (1961) was interested in infilled (steel) frames and, on the basis of the experimental evidence (the detachment of the frame from the infill as shown in Fig. 2a), proposed replacing the panel with an equivalent strut made of the same material, having a width w equal to 1/3 of the infill diagonal length d (Fig. 2b). The lateral strength of the system was obtained by computing the horizontal component of the axial strength of the strut.

Subsequently, Stafford Smith (1966), after an experimental investigation on diagonally and laterally loaded square infilled steel frames, developed the idea of the strut suggested by Holmes, providing an empirical curve for the evaluation of its dimensions. The experimental and analytical investigations showed a certain analogy between the frame-infill contact phenomenon and the behaviour of a beam on an elastic foundation, so that the definition of the nondimensional parameter

$$\lambda h' = h' \sqrt[4]{\frac{E_i t}{4 E_f I_f h}} \tag{1}$$

was proposed in order to characterise the column-infill contact length and, consequently, the stiffness of the system.

In Eq. (1) t and h are the thickness and the height of the infill, respectively; h' is the height of the frame, measured between the centrelines of the beams; E_i is the Young modulus of the infill while E_f and I_f are the Young modulus of the material constituting the frame and the moment of inertia of the cross-sectional area of the frame elements (beams and columns having the same dimensions).

The curve provided by Stafford Smith was based on experimental evidence and on the results of several numerical investigations carried out by means of the finite difference method. It gives the dimensionless parameter w/d for a fixed value of $\lambda h'$.

Referring to infilled frames subjected to vertical and lateral loads, Stafford Smith observed an increase in the horizontal stiffness when a vertical load was applied as a consequence of the increase of the length of contact of the beam on the infill, but no parameters were inserted in order to take this phenomenon into account.

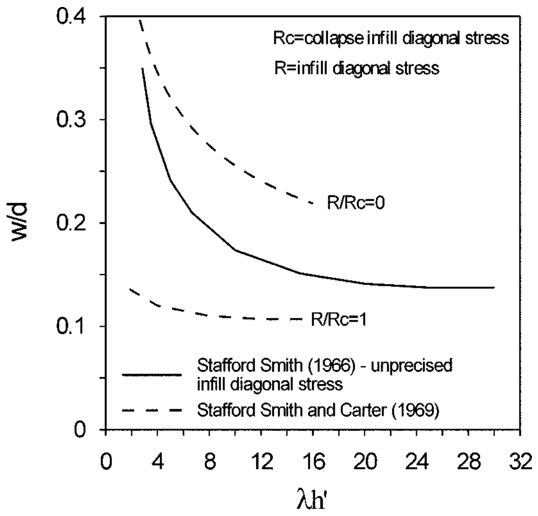

Fig. 3 Available *w/d* against $\lambda h'$ curves for square infilled frames

Later, Stafford Smith and Carter (1969) extended the concepts developed before to the case of rectangular frames, defining the characterising parameter

$$\lambda h' = h' \sqrt[4]{\frac{E_i t \sin(2\theta)}{4 E_f I_c h}} \tag{2}$$

θ being the slope of the infill diagonal, obtained by the expression $\theta = arctg(h/\ell)$, and I_c the moment of inertia of the columns.

Different curves w/d-$\lambda h'$ were defined with variation in the value of the slope θ. Further, the influence of the infill stress state along the diagonal direction was considered in the evaluation of *w*, due to the different secant stiffness observed with variation in the lateral load. So a set of curves was derived, for different stress levels and fixed ratio ℓ/h. Comparing these curves with the one provided by Stafford Smith (1966), it is not clear how the former are related to the latter. Note that no analytical form of the curves mentioned before is provided so every comparison has to be performed graphically, as is done in Fig. 3 for the case of square infilled frames.

Klingner and Bertero (1978), basing their work on the conclusions of Mainstone (1974), proposed calculating the width of the strut equivalent to the infill for frames having proportions 2.4 (length) against 1 (height) by means of the following expression:

$$\frac{w}{d} = 0.175(\lambda h')^{0.4} \tag{3}$$

This value of *w/d* allows one to calculate the mean lateral stiffness of the infilled frame before the cracking of the infill. In the cases examined by Klingner and Bertero, unlike the more usual ones, the infill was connected to the frame by means of proper reinforcement passing from the infill to the surrounding reinforced concrete frame. Nevertheless, comparing the curve w/d-$\lambda h'$ expressed by Eq. (3) with the curves provided by Stafford Smith and Carter (1969) for an infill having the same aspect ratio, one concludes that in the first case much lower stiffness of the system is obtained with

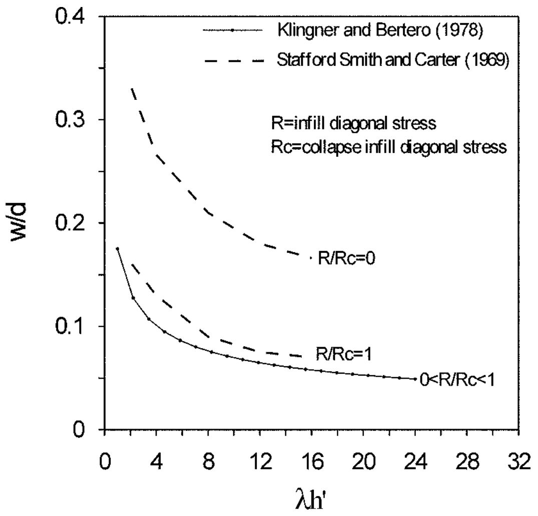

Fig. 4 Available w/d against $\lambda h'$ curves for rectangular infilled frames (ℓ/h = 2.4)

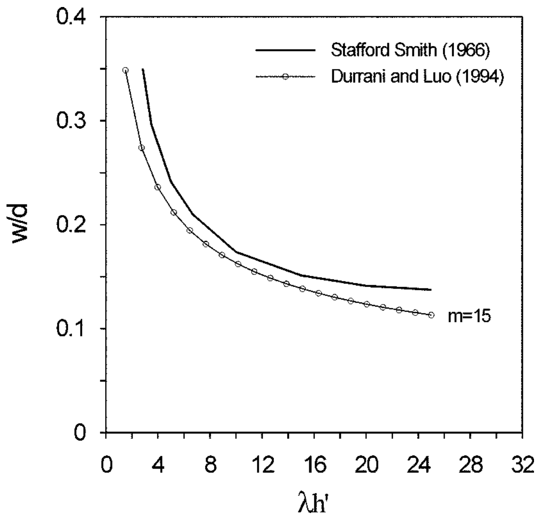

Fig. 5 Comparison of w/d against $\lambda h'$ curves for square infilled frames

respect to that expected in relation to the different frame-infill connection (Fig. 4).

Durrani and Luo (1994), on the basis of the experimental work of Mainstone (1974), proposed the following analytical relation for the evaluation of the width of the strut:

$$\frac{w}{d} = 0.32\sin^{1.5}(2\theta)\left(\frac{E_i t h'^4}{m E_c I_c h}\right)^{0.1} \tag{4}$$

where

$$m = 6\left(1 + \frac{6}{\pi} arctg \frac{I_b h'}{I_c \ell'}\right) \tag{5}$$

and I_b is the moment of inertia of the beam cross-section. Fig. 5 shows the results provided by Eq. (4) for square infilled frames with I_b/I_c = 1 and the ones given in Stafford Smith (1966) commented on above. It is evident that there is good agreement.

With reference to a further approach relating the initial lateral stiffness of the equivalent strut to the collapse condition of the system, in Saneinejad and Hobbs (1995) the dimensions of the strut are assumed to be constant with variation in the stress level, while the initial value of the Young modulus is assumed to be twice the secant modulus derived from the maximum resistance condition. This criterion does not allow one to compare the cross-sectional dimensions of the strut with those derived by Stafford Smith and Carter (1969) and by Mainstone (1971, 1974) varying with the level of the diagonal stress. Nevertheless, if the comparison is made in terms of initial lateral stiffness, very different values of w/d are obtained.

This partial review of the experimental and analytical investigations shows that the results obtained by different researchers are strongly influenced by the types of infill and test, and this conclusion is confirmed by examining and comparing results of other researches (e.g. Bertero and Brokken 1983, Valiasis and Stylianidis 1993, Panagiotakos and Fardis 1996, Mehrabi and Benson Shing 1997, Madan *et al.* 1997) not commented on in detail for brevity's sake. On the whole, it is possible to derive qualitative rather than quantitative considerations, without the possibility of generalising the very different empirical expressions proposed for the evaluation of the lateral stiffness for practical applications. Analogous comments can be made with reference to the expressions which have been proposed for the evaluation of the lateral collapse load and for laws modelling the hysteretic behaviour.

Referring to the possibility of obtaining generalised tools, the work presented in this paper is an attempt to make a contribution to the definition of a general procedure for modelling the behaviour of infilled frames to be adapted to any particular situation. In this first stage the elastic behaviour of the system is studied in order to obtain the dimensions of the equivalent strut and define the first branch of a more complex law for the prediction of the response under cyclic loads (ultimate load, softening branch, hysteretic characteristics, etc).

In contrast with the available methods for the determination of the width of the strut and the definition of the lateral stiffness of the system, it is shown here that the cross-section of the strut also depends on the axial stiffness of the elements constituting the frame, especially the columns.

The analysis implies the resolution of the frame-infill system by a so-called micromodel approach, performed by adopting for the infill a discretization in agreement with the Boundary Element Method. This method allows an easy and reliable resolution of the contact problem in the regions in which frame and infill transmit compressive stress to each other. The shear stress in the same regions is assumed to be governed by the Coulomb friction law, this assumption being different from the constant distribution of normal and shear stresses considered by Saneinejad and Hobbs (1995). Moreover, since the infill is considered to be a plate in plane stress state characterised by an elastic modulus and Poisson ratio, the width of the equivalent strut is also recognised to depend on the latter parameter.

By using the same procedure as proposed for the evaluation of the strut equivalent to a full infill, the case of infill with a window opening is also analysed, relating the reduction in the lateral stiffness to the dimensions of the window itself.

2. Identification of equivalent pin-jointed strut

The identification of the section of the equivalent pin-jointed strut can be made by imposing the condition that the initial stiffness of the actual system in Fig. 2(a) be equal to the initial stiffness of the equivalent braced frame in Fig. 2(b); further parameters able to describe the nonlinear behaviour of the panel will be defined in a subsequent study concerning adequate characteristics to be given to the strut.

The response of the scheme in Fig. 2(a) can be obtained by using a micromodel approach in which every structural element is modelled maintaining its geometrical and mechanical features. Assuming the columns to have the same dimensions and orientations in plane, in order to attain the anticipated aim, a micromodel that formally reproduces the scheme shown in Fig. 6(a) is used here, while the simplified model corresponding to the equivalent braced frame is reduced to the scheme in Fig. 6(b). Note that columns of both schemes are constrained at the base. Hence these schemes do not exactly represent a generic mesh of a framed structure (as shown in Fig. 2) because the lower beam is assumed to be rigid. Nevertheless, this assumption is in agreement with the conclusions of many experimental tests, showing that the flexural stiffness of the beams does not influence the lateral stiffness of the infilled mesh (Mainstone 1971, 1974, Stafford Smith and Carter 1969).

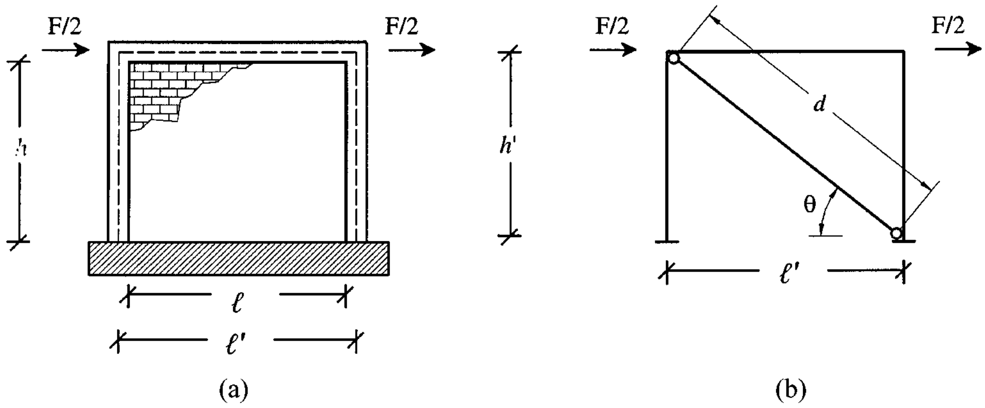

Fig. 6 Structural schemes: (a) infilled mesh, (b) braced frame with equivalent strut

If the problem is first solved by means of a micromodelling approach and subsequently by means of the simplified scheme in Fig. 6(b) (macromodelling approach) then, by imposing the equivalence of the stiffnesses obtained from the two models, the dimension of the strut can be evaluated. Therefore, denoting as $\overline{D}_i$ and D_i the stiffnesses of the two different schemes, the condition of equivalence can be written as

$$D_i = \overline{D}_i \tag{6}$$

When this equivalence is imposed, assuming the Young modulus and the thickness of the strut to be the same as for the infill, the width w of the strut can be determined, it being the only unknown quantity.

It can be observed that the results which will be shown later are obtained by considering the panel made of homogenous and isotropic material to be affected by the Young modulus value derived from compression diagonal tests or correlated to that derived from a compression load acting orthogonally to the bed joint direction by using an adequate reduction coefficient (Jones 1975). This assumption, which could be removed, simplifies the micromodelling procedure, while maintaining the same level of precision as the approaches described above.

3. Lateral stiffness of equivalent braced frame

The lateral stiffness of the scheme in Fig. 6(b), equivalent to the scheme in Fig. 6(a), can be evaluated with good approximation by imposing the condition that the horizontal forces to be applied to the schemes in Fig. 7(b) and Fig. 7(c) produce unitary displacement of the point P in the middle span of the beam. It can easily be found that the following value D_d of lateral stiffness is obtained for the scheme in Fig. 7(b):

$$D_d = \frac{k_d \cos^2\theta}{1 + \frac{k_d}{k_c}\mathrm{sen}^2\theta + \frac{1}{4}\frac{k_d}{k_b}\cos^2\theta} \tag{7}$$

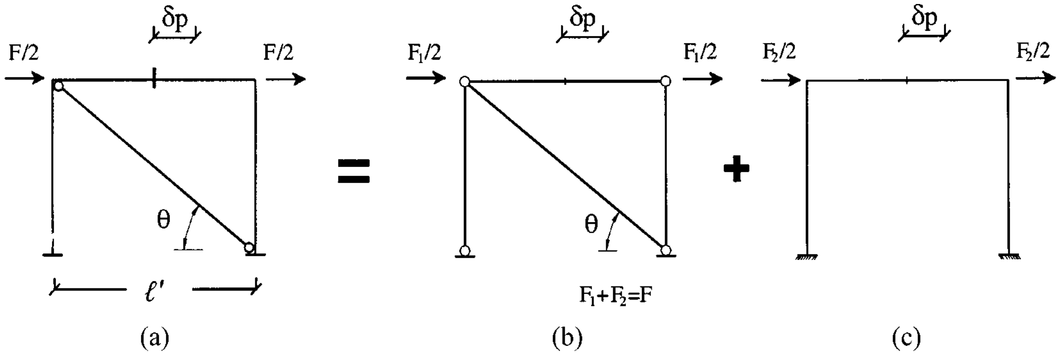

Fig. 7 Decomposition of lateral stiffness of macromodel

where the following equivalencies hold:

$$k_d = \frac{E_d t w}{d}; \quad k_c = \frac{E_f A_c}{h'}; \quad k_b = \frac{E_f A_b}{\ell'} \tag{8}$$

In Eq. (8) k_d, k_c and k_b are the axial stiffnesses of the diagonal strut, of the columns and of the beam, respectively; E_d, E_f are the Young modulus of the infill along the diagonal direction and the Young modulus of the frame respectively; t is the thickness of the infill; w is the previously defined width of the equivalent strut; A_c and A_b are the cross-sectional areas of the columns and the beam; θ defines the diagonal direction as specified before; finally, h' and ℓ' are the height and the length of the frame in agreement with Fig. 6.

The lateral stiffness D_f of the frame in Fig. 7(c) can be simply evaluated using the expression

$$D_f = 24\frac{E_f I_c}{h'^3}\left(1 - 1.5\left(3\frac{I_b}{I_c}\frac{h'}{\ell'} + 2\right)^{-1}\right) \tag{9}$$

where I_c and I_b are the moments of inertia of the columns and the beam sections respectively. Hence the global stiffness of the simplified scheme constituting the braced frame in Fig. 7(a) can be assumed to be:

$$D_i = D_f + D_d \tag{10}$$

4. Lateral stiffness of infilled frame

4.1 Modelling of the frame-infill system

The modelling of the infilled frame for the evaluation of the lateral stiffness is based on a different discretization of the infill and of the frame. For the former the boundary element method (B.E.M.) is used while for the latter the finite element method is applied. The typical discretizations adopted are shown in Fig. 8. The bases of the columns (nodes B and C) are fully constrained on the rigid lower beam (the validity of this assumption has been better clarified in the previous sections) and

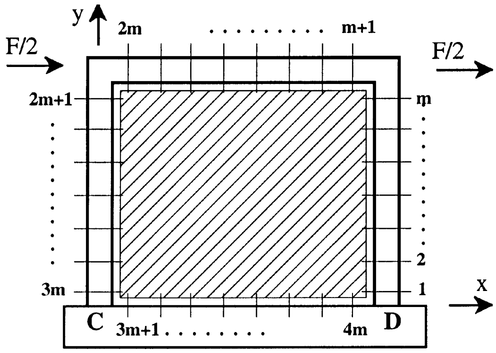

Fig. 8 Micromodel approach: discretization of infilled frame

their deformations are defined as a function of the three degrees of freedom of each of the $3m$ nodes belonging to the frame, m being the number of nodes that defines the beam and each of the two columns.

The boundary of the infill is divided into $4m$ elements having the same dimensions. By a discretization technique commonly used in the B.E.M., the corners are removed in the model, considering two distinct nodes very close to the actual edges, one being in contact with the beam and the other with the column. This strategy allows one to distinguish shear and normal stresses for these regions also, maintaining a good level of approximation in the results.

The infill is assumed to be a plate in plane stress state, made of homogeneous and isotropic material, having elastic modulus E_d and Poisson ratio ν. The path of displacements and stresses along the horizontal and vertical directions is assumed to be linear from the generic node to the subsequent one. In agreement with this assumption, every frame element is modelled as a Timoshenko beam so that, considering two consecutive nodes where infill and frame are in contact, the compatibility of the displacements is ensured. In agreement with the discretization adopted for the infill, the nodal points m, $m + 1$ and $2m$, $2m + 1$ are connected by rigid elements. The horizontal forces are concentrated at the nodes $m + 1$ and $2m$ marked in Fig. 8. The stresses transferred from the infill to the frame (and vice versa) can be decomposed into a normal component and a tangential component. Normal stresses are admitted only in the case of compression; in this condition mutual tangential sliding of corresponding nodes belonging to the two sub-structures occurs, and the mutual tangential stress can be calculated by means of the Coulomb friction law. Therefore, denoting as τ the shear stress and σ the normal compressive stress, the following equation holds:

$$|\tau| = \mu\sigma \tag{11}$$

μ being the friction coefficient. On the other hand, when the assumption that corresponding nodes are affected by the same normal displacement would lead to mutual normal tensile stress, complete detachment between these nodes is considered. Obviously, the solving procedure proves to be iterative. The next section shows the criterion by which each step can be carried out, with reference also to the sign to be given to the stress τ in Eq. (11).

4.2 Solving iterative procedure

In the first step of the analysis all the nodes that define the infill (from 1 to $4m$) are assumed to be connected to the corresponding nodes that define the frame. In this condition, for the nodal points from 1 to $3m$ five unknowns have to be calculated: two displacements, one rotation, and the mutual normal and shear stresses. Therefore, considering that for the nodes numbered from $3m + 1$ to $4m$ the displacements and the rotation must be considered to be null, the total number of unknowns is $5x3m + 2xm = 17m$. Further, $3x3m$ equilibrium equations for the nodes of the frame and $2x4m$ boundary integral equations for the infill are available; moreover, the coefficients of the boundary integral equations can be expressed in closed form as a consequence of the degree fixed for the shape functions of the displacements and stresses. Therefore, the first step of the analysis can be performed without any difficulty. Among other things these first results give the lateral stiffness of the system when a reliable frame-infill connection device has to be considered.

At the end of the first step the sign of the normal stress at each node must be checked. If a normal compressive stress is found at the generic node of the columns and of the upper beam, in the next step the unknowns at this node will be tangential displacement of the node considered as belonging to the infill; tangential displacement of the node considered as belonging to the frame; normal displacement of the node considered as belonging to the infill and the frame; rotation of the node considered as belonging to the frame; normal stress. The mutual tangential stress is assumed to be known: its value is calculated by means of Eq. (11), where σ is the normal stress which was found at the previous first stage; its sign remains the same as at that stage. For nodes common to the infill and to the rigid lower beam the following unknowns are assumed: tangential displacement of the node considered as belonging to the infill; normal stress. In this case too the value of the shear stress is evaluated as specified above.

For nodes where normal tensile stress is found, the infill and the frame will be considered disconnected. Hence, if the node is one of those numbered from 1 to $3m$, the unknowns will be the two displacement components and the rotation of the node considered as belonging to the frame, and the two displacement components of the node considered as belonging to the infill. If the node is one of those numbered from $3m + 1$ to $4m$, the unknowns will be the two displacement components of the node considered as belonging to the infill. In each of the two cases the mutual stresses must be considered null.

What has just been said shows that both for mutual compression and mutual tension at the generic node the whole number of unknown quantities to be determined in the next step of procedure remains unvaried.

The following steps are characterised by verification of the normal stress in each node and of the displacement compatibility for the definition of the unknowns. If frame and infill are connected at a node during the previous step (only tangential sliding is allowed) and mutual compression is found again, the unknowns do not vary and the value of the shear stress is updated by using Eq. (11) and assuming its sign in agreement with the relative sliding; if frame and infill are connected in the previous step and normal tensile stress is obtained, in the following step the frame and the infill will be considered disconnected; if frame and infill are not connected at a node in the previous step and displacements producing penetration of the two sub-systems are obtained, in the following step infill and frame will be considered as connected at this node. The criterion by which the solving equations are reordered according to the unknowns to be determined is the same as in Papia (1988), where, however, different degrees of elements and contact laws were adopted.

The numerical analysis shows that, if a proper discretization is adopted, the procedure converges very rapidly, highlighting the part of the infill boundary which remains connected to the frame. Once the convergence is obtained, the lateral stiffness $\overline{D}_i$ of the system can be calculated by means of the ratio between the applied load F and the average of the horizontal displacements obtained for the nodes numbered from $m + 1$ to $2m$.

5. Cross-section of equivalent strut

By substituting the value of D_i obtained from Eq. (10) into Eq. (6), one obtains

$$\overline{D}_i = D_d + D_f \tag{12}$$

Further, by substituting Eq. (7) into Eq. (12) the w/d ratio proves to be expressed by

$$\frac{w}{d} = \frac{\overline{D}_i - D_f}{E_d t \cos^2\theta}\left(1 - \frac{\overline{D}_i - D_f}{k_c}\left(\frac{h'^2}{\ell'^2} + \frac{1}{4}\frac{k_c}{k_b}\right)\right)^{-1} \tag{13}$$

By evaluating the “exact” lateral stiffness of the system $\overline{D}_i$ by the procedure described before, and the bare frame stiffness D_f (Eq. (9)), the value of w/d can be obtained by means of Eq. (13). The bare frame stiffness D_f can be evaluated once the geometric features of the frame elements and the mechanical characteristics of the material are known. If the procedure is repeated many times for different elastic and geometrical features of the infilled frame, a correspondence between the actual features of the generic infilled frame and the characteristics of the equivalent strut can be found.

Eq. (7) and Eq. (9) can be simplified if the upper beam is considered flexurally and axially rigid (this assumption would be in agreement with the effect of the slab), obtaining a simplified version of Eq. (13). In any case it is not acceptable to neglect the axial deformability of the columns in the evaluation of D_d.

Since the procedure is based on columns having the same cross-sections and orientation, when this condition is not verified, average values of moment of inertia and area of the columns have to be assigned in order to obtain a structurally symmetrical ideal scheme like that considered in the proposed approach. In this case the level of approximation in the results can be considered of the same order as that achievable by other models available in the literature, like those discussed above.

Once the investigation mentioned before is concluded, the direct evaluation of the width of the strut, in agreement with the most widespread tendencies in the literature, requires the definition of a parameter λ^* depending on the elastic and the geometric features of the system in such a way that a function $w/d = f(\lambda^*)$ can be defined. In conclusion, the numerical investigation carried out by means of an “exact” model must make it possible to define a direct relation between the infilled frame and the equivalent braced frame, with a strong reduction in the computational effort for practical use in the structural analysis.

6. Definition of parameter λ^*

The definition of a parameter that, concisely and with good reliability, univocally defines the ratio

w/d to be adopted for the simplified model, can be obtained by imposing the condition that the difference $\overline{D}_i - D_f$ on the right side of Eq. (13) be the true lateral stiffness D_p of the infill panel, obtainable from the true load condition on the panel itself.

In order to link D_p to the elastic and geometrical characteristics of the panel, the loading scheme shown in Fig. 9 can be considered. In this scheme the normal and shear stresses transmitted from the frame are assumed to decrease from the corners to the middle point of the panel sides, in agreement with a path experimentally recorded by several researchers and confirmed by the results of the "exact" procedure proposed here, leading to a piece-wise linear stress distribution, in relation to the degree of the stress functions adopted for the boundary elements.

The lateral stiffness of the panel is expressed by the ratio between the horizontal component of the mutual resultant force (R in the figure) and the relative displacement of the opposite corners of the panel (points A and B), projected along the horizontal direction.

Considering that the material has been assumed to be elastic, homogeneous and isotropic and, consequently, affected by behaviour only depending on E_d and ν, if the aspect ratio ℓ/h and ν are fixed, the stiffness D_p proves to be independent of R, proportional to the product $E_d t$, and dependent on the coefficients ς_h and ς_ℓ marked in Fig. 9, which govern the direction of R.

Therefore, one can set

$$D_p = \psi E_d t \tag{14}$$

where $\psi = f(\varsigma_h, \varsigma_\ell)$ depends on the unknown extension of the frame-infill contact regions.

On the other hand, setting

$$\lambda^* = \frac{E_d t h'}{E_f A_c}\left(\frac{h'^2}{\ell'^2} + \frac{1}{4}\frac{A_c}{A_b}\frac{\ell'}{h'}\right) \tag{15}$$

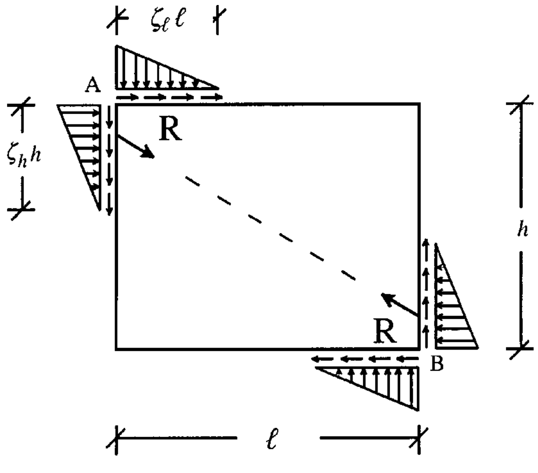

Fig. 9 Distribution of stresses transmitted by frame to infill

and considering Eqs. (6) and (10), Eq. (13) can be written in the form

$$\frac{w}{d} = \frac{1}{\cos^2\theta}\frac{D_p}{E_d t}\frac{1}{1-\frac{D_p}{E_d t}\lambda^*} = \frac{1}{\cos^2\theta}\frac{1}{\frac{E_d t}{D_p}-\lambda^*} \tag{16}$$

which, introducing Eq. (14), becomes

$$\frac{w}{d} = \frac{1}{\cos^2\theta}\frac{1}{\psi^{-1}-\lambda^*} \tag{17}$$

As was stressed previously, the determination of the contact region extension was derived at first by Stafford Smith (1966) by assuming an analogy with the foundation beam on the Winkler soil, but considering experimental tests that do not reproduce the actual lateral load conditions of an infilled frame, because they concerned square infilled frames, made of members of equal sections, and, above all, diagonally loaded. In these conditions, obviously, the flexural rather than the axial stiffness of the frame members is involved; moreover, symmetric stress and strain states occur, unlike the actual state of an infilled frame under lateral loads.

Actually, further tests realizing a more appropriate load condition were carried out by the same author afterwards (see also Stafford Smith and Carter 1969), but they only led him to conclude that the dimensions of the beam cross-section do not influence the value of *w/d* as much as the length of the beam-infill contact zone, which was assumed constant and equal to $\ell/2$, independently of the elastic and geometrical features of the infill and of the beams.

Although this conclusion should have made the proposed analogy insufficiently realistic and reliable, it was assumed to be valid anyway, so that Eq. (2), derived from the subsequent studies mentioned, is obtained from Eq. (1), by characterising the flexural stiffness of the frame only by means of the inertia moment of the sections of the columns, in addition to considering the different geometry of the infill by means of the angle θ.

The comments above concerning the feeble criterion by which the extension of the frame-infill contact regions is determined suggest attempting to correlate this extension (ς_h and ς_ℓ in Fig. 9) to the axial stiffness of the columns rather than to their flexural stiffness, also considering that the former is involved at least as much as the latter in the deformed shape of the structural scheme in Fig. 7(a). Therefore, it is reasonable to foresee that schemes characterised by the same parameter λ^* are affected by the same value of $\psi = f(\varsigma_h, \varsigma_\ell)$, even if this value is unknown.

Under these hypotheses, Eq. (17) shows that, for assigned values of ℓ/h and ν, a curve $w/d = f(\lambda^*)$ can be searched for in a numerical way, by using the "exact" solving method discussed before. The resolution of different schemes featuring the same value of the parameter λ^* (Eq. (15)) will show the validity of the previous assumptions if values of *w/d* which are close enough are found for these schemes.

Finally, it must be observed that if the beam is assumed to be axially rigid, the expression of the parameter λ^* is simplified by assuming $A_b \to \infty$. Nevertheless, by analysing Eq. (15), one can observe that in any case the dimensions of the beam cross-section do not meaningfully influence the width of the equivalent strut, and this confirms what was stressed by the authors cited above.

7. Validity of the role of λ^*

In order to confirm the validity of the approach based on the parameter λ^* defined in the previous section, several infilled frames have been analysed by the "exact" procedure, considering different values of the terms that define the parameter λ^*, but maintaining unvaried an assigned value of this parameter. After very close values of *w/d* have been found using Eq. (13), a new value of λ^* has been assigned, again varying the quantities defining it according to Eq. (15), and so forth.

The analysis has been carried out for square infills (aspect ratio ℓ/h = 1) and rectangular infills with aspect ratio ℓ/h = 1.5, while four values of the Poisson ratio for the infill have been considered: ν = 0, 0.15, 0.30, 0.45.

On the other hand, some data have been maintained constant for every numerical test: thickness of the section of the beam and columns (250 *mm*); Poisson ratio for the material constituting the frame (ν_f = 0.15); shear factor for the section of beam and columns (1.2) (this factor is requested by the Timoshenko theory on beam elements); friction coefficient μ defining the ratio between normal and shear stresses in the contact regions, assumed to be equal to 0.45.

With reference to the latter parameter, it can be observed that the value adopted is the same as was proposed in Saneinejad and Hobbs (1995) and suggested by the ACI Code 530.1-92 (1992) in the case of masonry infill. Nevertheless, some numerical tests have been repeated, also assuming for μ the values 0.30 and 0.60. As a result, it has been recognised that the coefficient μ, in the range of the values considered, does not meaningfully influence the value of $\overline{D}_i$. This occurrence is accounted for by the fact that a different ratio between normal and shear mutual stresses can neither modify the resultant force transmitted from the frame (R in Fig. 9), nor substantially change its direction.

The ranges of variation fixed for the geometrical parameters that concur to the definition of λ^* are expressed in millimetres as follows:

- $300 \le H_b \le 600$;
- $250 \le H_c \le 700$;
- $2250 \le h \le 5500$;
- $125 \le t \le 250$;

H_b and H_c being the height of the cross-sections of the beam and columns respectively. The range of variation fixed for the ratio between the Young modulus of the two materials is $1 \le E_f / E_d \le 10$.

As a consequence of the previous assumptions, values of λ^* comprised between 0.35 and 13.30 and between 0.20 and 10.10 have been obtained from Eq. (15) for $\ell/h = 1$ and ℓ/h = 1.5, respectively.

It must be observed that the values of lateral stiffness $\overline{D}_i$ obtained by varying the aforementioned structural parameters prove to be comprised between 90 and 500 kN/mm, denoting the generality of the results obtained.

Once the values of the stiffness $\overline{D}_i$ are known by means of the "exact" analysis, the values of *w/d* are obtained by means of Eq. (13), and for fixed ν and ℓ/h, the dependence of these values only on the parameter λ^* defined by Eq. (15) is also verified.

The results of the numerical analysis are shown in Fig. 10 for infilled frames with square infills, and in Fig. 11 for rectangular infills (ℓ/h = 1.5).

For clarity's sake the results obtained for ν = 0.15 have not been included; however, they are absolutely consistent with those shown in the figures, obtained for ν = 0, ν = 0.30 and ν = 0.45.

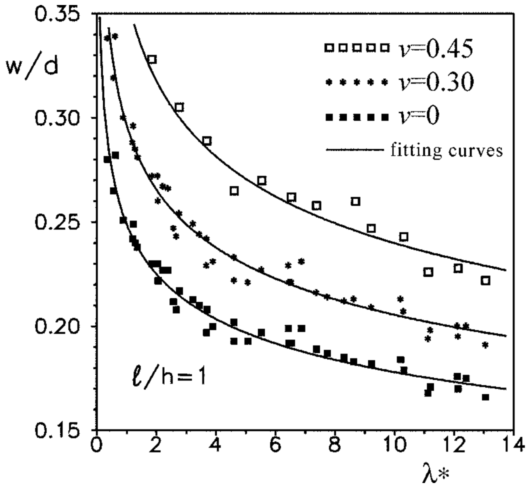

Fig. 10 Numerical values of *w/d* with variation in λ^* and fitting curves for square infills

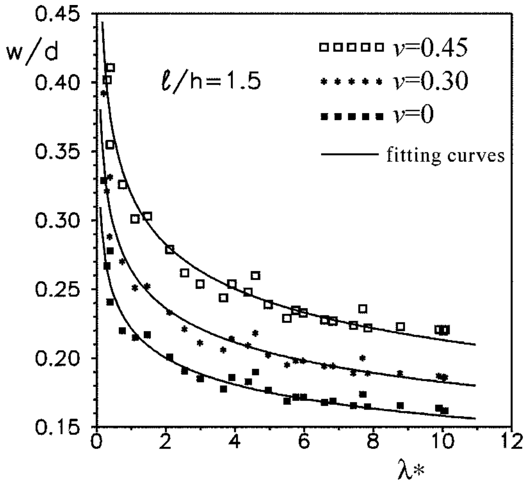

Fig. 11 Numerical values of *w/d* with variation in λ^* and fitting curves for rectangular infills

The values of *w/d* obtained by the numerical investigation can be fitted by the analytical expression:

$$\frac{w}{d} = \frac{c}{z}\frac{1}{(\lambda^*)^\beta} \tag{18}$$

where

$$C = 0.249 - 0.0116\nu + 0.567\nu^2 \tag{19}$$

$$\beta = 0.146 + 0.0073\nu + 0.126\nu^2 \tag{20}$$

$$z = \begin{cases} 1 & \text{if} \quad \ell/h = 1 \\ 1.125 & \text{if} \quad \ell/h = 1.5 \end{cases} \tag{21}$$

For practical applications, Eq. (18) allows the evaluation of the contribution of the infill to the lateral stiffness of the generic mesh of a framed structure without any computational effort.

8. Reliability of proposed model

Considering the different definition of the parameter λ^* with respect to parameters having the same role defined by other authors, a comparison has been made between the ratios *w/d* obtained by the "exact" procedure described above, by Eq. (18) and by the curve provided by Stafford Smith (1966).

Table 1 *w/d* ratios from different approaches

Case	"Exact solution"		Proposed model		Stafford Smith (1966)	
	$\overline{D}_i$ (kN/mm)	w/d	λ^* [Eq. 14]	w/d [Eq. 18]	$\lambda h'$ [Eq. 1]	w/d
1	112	0.242	1.20	0.242	4.00	0.268
2	93	0.238	1.30	0.239	6.20	0.217

Since the substantial difference in the definition of the parameter λ^* (the corresponding parameter is $\lambda h'$ in Eq. (1) or in Eq. (2)) lies in the fact that the axial stiffness of the columns is here considered more basic than their flexural stiffness, the comparison is made considering two infilled frames differing in having columns with the same-cross section but oriented differently, in such a way the axial stiffness does not change while the flexural stiffness changes significantly.

With reference to the symbols defined in this paper, the following data are common to the two examples considered: dimensions of the upper beam cross-section (250*x*600 mm); E_f = 30000 MPa, ν_f = 0.15; $h = \ell$ = 500 cm; t = 250 mm; E_d = 3500 Mpa, ν = 0. The cross-sections of the columns are assumed to be 250*x*600 mm in the first case and 600*x*250 mm in the second one.

The results are summarised in Table 1. It must be observed that for both cases the value of *w/d*, deduced from the "exact" procedure, are calculated by means of Eq. (13) by introducing the value of $\overline{D}_i$ derived from the scheme in Fig. 8 and the value of D_f evaluated by means of Eq. (9). Since this method is affected by the approximation expressed by Eq. (10) and represented in Fig. 7, the reliability of the values of *w/d* has been tested by solving in both cases the braced frame in Fig. 7(a) and by verifying that the values obtained for the lateral stiffness proved to be very close to those ($\overline{D}_i$) inserted in Table 1.

The examples considered show the optimal level of precision obtainable with the model adopted and the reliability of the proposed parameter λ^*.

9. Lateral stiffness of infills with openings

Openings in infill panels can produce a meaningful loss of lateral stiffness, but studies on this specific aspect of the problem are very limited. Some results and references are shown in (Hendry 1998) but definitive conclusions are not available. Results of numerical investigations are presented here, showing that the loss of stiffness due to the opening can be correlated with the ratio between the dimensions of the opening itself and the ones of the infill. Specifically, a reduction factor of the section of the equivalent strut, denoted as r in the following, is obtained for the correction of Eq. (18).

The investigation has been limited to the case of openings having the same aspect ratio of the panel (ℓ/h) and centred with respect to the frame. Under these hypotheses a single parameter can be used for the characterisation of the opening; it has been assumed to be the ratio between one of the dimensions of the opening and the corresponding dimension of the panel.

The modelling of the system for the "exact" evaluation of the stiffness $\overline{D}_i$, in agreement with the procedure described in the previous sections, is not modified for the frame and the boundary of the panel; but now the number of unknowns is $17m + 2n$, n being the number of the nodal points along the boundary of the opening. The new $2n$ unknowns, which do not change at each step of the

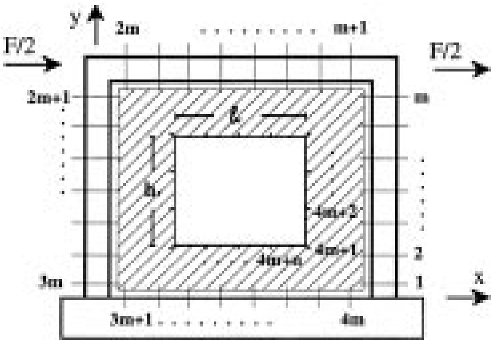

Fig. 12 Micromodel approach: discretization of frame with opened infill

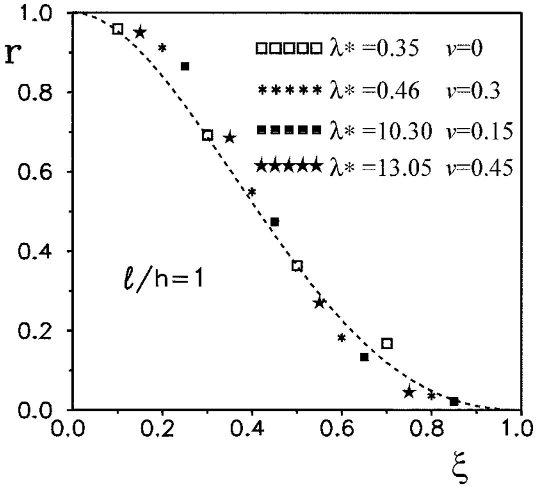

Fig. 13 Reduction in lateral stiffness of infilled frames with centered square openings

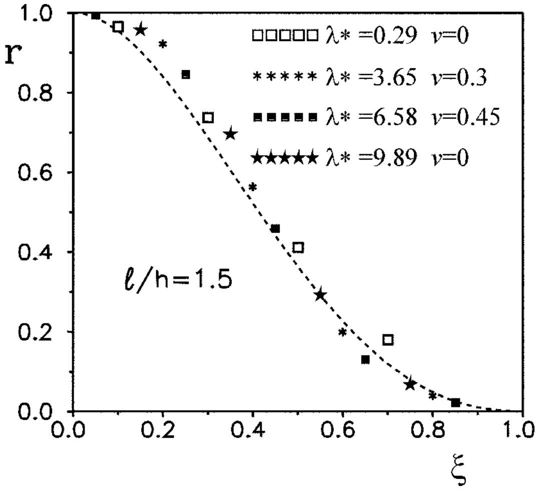

Fig. 14 Reduction in lateral stiffness of infilled frames with centered rectangular openings

procedure, are the components of the displacements of the new nodes along the vertical and horizontal directions; on the other hand $2n$ further integral equations are available at the internal boundary of the panel.

With reference to this new boundary, it must be observed that only one node is required for each corner since only unknown displacements have to be calculated, unlike the external boundary, where stress components can be unknown; therefore, the typical mesh adopted for this analysis is that shown in Fig. 12.

The updating of the first $17m$ unknowns, referring to the external boundary of the panel, has to be performed as explained in Section 4. From the numerical point of view it is worth noting that the greater deformability of the panel with openings allows a higher extension of the contact zones and makes the computational effort to calculate the "exact" stiffness of the system bigger, due to a higher number of iterative steps.

The investigation has been carried out again for infill having geometrical ratios $\ell/h = 1$ and $\ell/h = 1.5$, considering the previous four different values of the Poisson ratio for the infill.

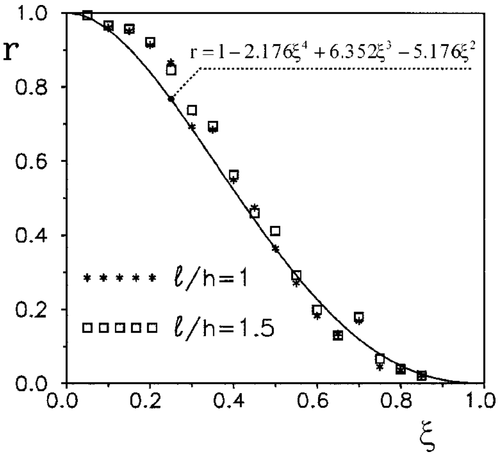

Fig. 15 Comparison of results shown in Fig. 13 and Fig. 14

The results show that the reduction factor r only depends on the parameter ξ (i.e. $r = f(\xi)$) defining the size of the opening: $\xi = h_v/h = \ell_v/\ell$, h_v and ℓ_v being the dimensions of the opening itself. This conclusion is clearly shown in Fig. 13 and Fig. 14, which refer to the cases of square infills and rectangular infills, respectively. It is possible to note that the reduction factor r does not substantially depend on the value of n and on the value of λ^*.

Further, the comparison between the results in Figs. 13 and 14 highlights the fact that the geometry of the infill panel (defined by the ℓ/h ratio) does not influence the lateral stiffness loss either, so a single law $r = f(\xi)$ can be defined, as shown in Fig. 15.

For practical applications ξ usually proves to be in the range 0.2-0.7; it can easily be verified that in this field the polynomial expression of r, marked in Fig. 15, can be replaced by the straight line

$$r = 1.24 - 1.7\xi \tag{22}$$

10. Conclusions

The stiffening effect of infill panels on a generic mesh of a framed structure has been discussed. Then an analytical procedure for the identification of a pin-jointed strut equivalent to the infill has been proposed, and some functions for the practical evaluation of the characteristics of the strut have been provided, summarising the numerical results.

In the paper it is shown that the cross-section size of the strut can be derived as a function of a single parameter depending on the characteristics of the mesh and the infill.

Since this function approximates results of an "exact" solution scheme, it can be used for each kind of infilled mesh, overcoming the limit of many analogous curves given in the literature, obtained by means of empirical approaches and applicable only to specific cases.

The procedure has also been extended to the case of infills with centred openings, showing that the reduction in the lateral stiffness can be related exclusively to the size of the opening itself, at least for the ℓ/h ratios of the panels considered.

Finally, it is worth remarking that the modelling performed here reveals that the calibration of the equivalent strut can be related to the axial stiffness of the columns, unlike what is usually stated in the literature, where only a dependence on their flexural stiffness is underlined.

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