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Buckling and vibration analysis of stiffened plate subjected to in-plane concentrated load

A. K. L. Srivastava† and P. K. Datta‡

Aerospace Engineering Department, I. I. T. Kharagpur-721302, India

A. H. Sheikh‡†

Department of Ocean Engineering and Naval Architecture I. I. T. Kharagpur-721302, India

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Abstract. The buckling and vibration characteristics of stiffened plates subjected to in-plane concentrated edge loading are studied using finite element method. The problem involves the effects of non-uniform stress distribution over the plate. Buckling loads and vibration frequencies are determined for different plate aspect ratios, boundary edge conditions and load positions. The non-uniform stresses may also be caused due to the supports on the edges. The analysis presented determines the initial stresses all over the region considering the pre-buckling stress state for different kinds of loading and edge conditions. In the structural modeling, the plate and the stiffeners are treated as separate elements where the compatibility between these two types of elements is maintained. The vibration characteristics are discussed and the results are compared with those available in the literature and some interesting new results are obtained.

Key words: finite element method; stiffened plate; buckling and frequency parameter.

1. Introduction

Stiffened plates are structural components consisting of plates reinforced by a system of ribs to enhance their load carrying capacities. These structures are widely used in aircraft, ship, bridge, building, and some other engineering activities. In many circumstances these structures are found to be exposed to in-plane loading. The buckling and vibration characteristics of stiffened plates subjected to uniform and non-uniform in-plane edge loading are of considerable importance to aerospace, naval, mechanical and structural engineers. Aircraft wing skin panels, which are made of thin sheets are usually subjected to non-uniform in-plane stresses caused by concentrated or partial edge loading at the edges, and due to panel stiffener support conditions.

Large number of references in the published literature deal with the buckling and vibration behaviour of rectangular stiffened plates subjected to in-plane uniform initial stresses. In this class

† Research Scholar
‡ Professor
‡† Associate Professor

of problems, the internal stress field remains constant throughout the plate, yielding relatively simpler solution to the problems.

The applied load is seldom uniform and the boundary conditions may be completely arbitrary in practice. The situation in which the loads are applied over only part of the edges has received much less attention. The problem becomes complicated when the loading is non-uniformly distributed over the edges. These studies for most part being concerned with the numerical analysis of the theoretical buckling load and also mostly related to unstiffened plates. Sommerfield (1906) presented a solution for a simply supported plate of infinite length subjected to a pair of concentrated forces acting upon two opposite sides. It was assumed that the in-plane stresses were concentrated only along the line of action of applied forces and hence the results were prone to considerable errors. Timoshenko (1961) arrived at the same result by using the energy method. In Timoshenko's approach also, it was implied that the stresses were concentrated only along the line of action of the forces. Leggett (1937) was able to obtain an approximate non-uniform in-plane stress distribution within the plate subjected to point loading at the two opposite edges. Baker and Pavolic (1982) analyzed the stability of rectangular plates subjected to a pair of patch loading at the center of two opposite edges by using Ritz method. Cases of practical interests arise when concentrated forces acting along the boundaries cause the in-plane stresses. Leissa and Ayoub (1988) have solved the problem of buckling of simply supported rectangular and circular plates subjected to a pair of oppositely directed concentrated compressive and tensile loads anywhere on two opposite edges. Some non-uniform edge loading cases have also been considered by (Dawe 1969, Grimm and Gerdeen 1975, Bassily and Dickinson 1975) in buckling analysis.

A new approach to the plate-buckling problem, which did not require the initial stress field, was set forth by Alfutove and Balabukh (1967). A single term approximate solution of the non-linear equations gave reasonable results for a point loaded square plate. A similar procedure yielding buckling results was presented by Spencer and Surjanhata (1985). Brown (1989) applied yet another approach, called the conjugate load displacement method to the problem of buckling of plates under concentrated edge loading. This method was used earlier by Brown and Yettram (1986) for the elastic stability analysis of square perforated plates. Vibration and buckling calculations for rectangular plates subjected to complicated in-plane stress distribution by using numerical integration in a Raleigh-Ritz analysis are studied by Dickinson and Kalidas (1981). The vibration of simply supported plates subjected to concentrated and partially loads were studied by Deolasi and Datta (1995). Recently Sundersan *et al.* (1998) have studied the influence of partial edge compression on buckling behaviour of angle ply plates for few orientations.

As seen from the literature, the vibration and buckling of isotropic plates subjected to in-plane concentrated edge load has been sparsely treated in the literature and not many results are available on stiffened plates. The authors could not find any work in the literature regarding in-plane concentrated load for stiffened plate. The present paper deals with the problem of vibration and buckling of rectangular stiffened plates subjected to in-plane concentrated load. Finite element formulation is applied for obtaining the non-uniform stress distribution in the plate and also to solve the buckling load and frequency parameters in various modes with different boundary conditions, aspect ratios and various parameters of stiffened plates. The non-uniform loading may also be caused due to the supports on the edges. The analysis presented determines the stresses all over the region. Different cases have been considered and comparison of buckling loads and frequency parameters with previously published results are found to be good. Buckling results show that the stiffened plate is less susceptible to buckling for position of loading near the supported edges and near the position of stiffeners as well.

In the present analysis, the plate is modeled with the nine noded isoparametric quadratic element where the contributions of bending and membrane actions are taken into account. One of the advantages of the element is that it includes the effect of shear deformation and rotary inertia in its formulation. Thus the analysis can be carried out for both thin and thick plates. Moreover it can be applied to a structure having irregular boundaries. The formulation of the stiffener is done in such a manner so that it may lie anywhere within a plate element (Mukhrjee and Mukhopadhyay 1990). In order to maintain compatibility between plate and stiffener, the interpolation functions used for the plate are used for the stiffeners also.

2. Description of the problem

The problem considered here consists of a rectangular plate ($a \times b$) with stiffener of cross-section as shown in Fig. 1. The problem of plate subjected to concentrated load is of practical interest, being the extreme case of non-uniform loading. The stiffened plate is subjected to a pair of concentrated in-plane load at variable location (c/b) on the two opposite edges where c is the distance of the concentrated load position from the lower edge y = 0. The plate area of interest is divided into $M \times N$ finite element mesh, where M is the number of division along x-axis and N is the number of division along y-axis. The mesh division is generally uniform in both the x and y directions. However to accommodate the mesh division of desired load band width or at a desired edge location, the plate is divided into two zones and each zone is meshed separately with a uniform mesh. The loading shown is compressive in nature. The boundaries of the plate are of different boundary conditions along the edges. The length (a) of the stiffened plate considered above is varied keeping its other parameters unchanged. The cross-section of the stiffened plate is shown in Fig. 2. The alignments of the stiffeners concentric and eccentric is shown in Fig. 3.

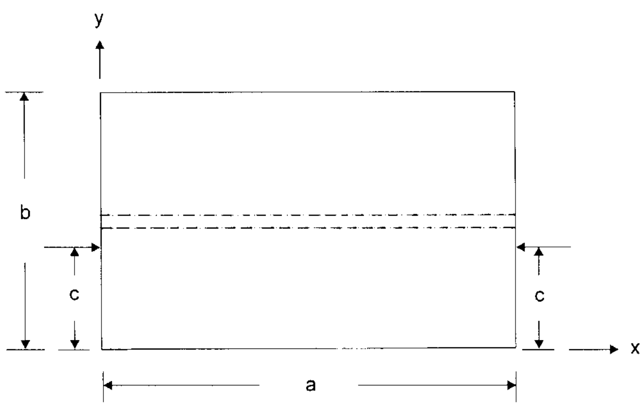

Fig. 1 Concentrated edge edge loading at the two opposite edges of a plate with a central stiffener

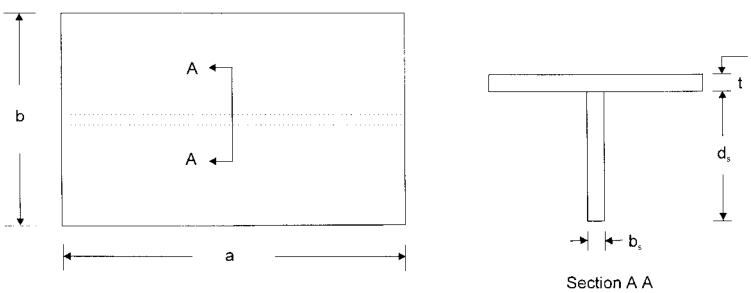

Fig. 2 Stiffened plate cross-section

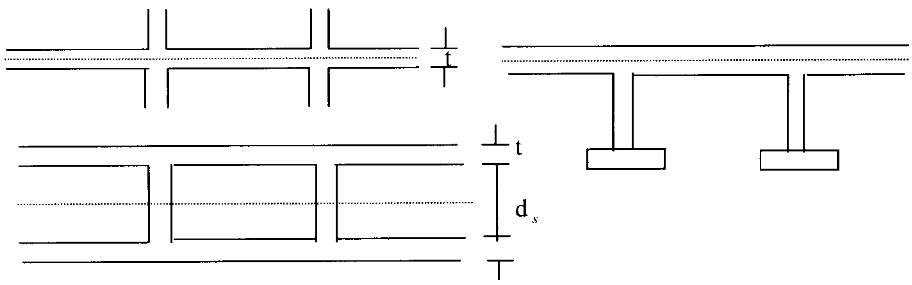

Fig. 3 Concentrically and eccentrically stiffened plates

3. Governing equation

The equation of equilibrium for the stiffened plate subjected to in-plane loads can be written as:

$$[M]\{\ddot{q}\} + [[K_b] - P[K_G]]\{q\} = 0 \tag{1}$$

Eq. (1) can be reduced to the governing equations for buckling and vibration problems as follows:

Buckling problem

$$\{\ddot{q}\} = 0 \tag{2}$$

$$[[K_b] - P[K_G]]\{q\} = 0$$

Free vibration problem:

Assuming that the stiffened plates vibrates harmonically with angular frequency ω, Eq. (1) reduces to:

$$[[K_b] - P_{cr}[K_G] - \omega^2[M]]\{q\} = 0 \tag{3}$$

Both Eqs. (2) and (3) represent the eigenvalue problems. The eigenvalues are the critical buckling loads, P_{cr}, for Eq. (2) or squares of natural frequencies, ω^2, for known load P for Eq. (3). The eigenvectors correspond to the mode shapes of buckling or vibration.

4. Finite element formulation

The formulation is based on Mindlin's plate theory, which will allow for the incorporation of shear deformation. The plate skin and the stiffeners are modeled as separate elements but the compatibility between them is maintained. This is similar to the concept proposed by Mukherjee and Mukhopadhyay (1990). The middle plane of the plate is taken as the reference plane for both the plate and the stiffeners. The effect of in-plane deformations is taken into account in addition to the deformations due to bending, which will help to model the stiffener eccentricity conveniently.

The nine noded isoparametric quadratic element with five degrees of freedom (u, v, w, θ_x, and θ_y) per node is employed in the present analysis. The coordinates at a point within the element are approximated in terms of its nodal co-ordinates as follows

$$x = \sum_{r=1}^{9} N_r x_r \quad \text{and} \quad y = \sum_{r=1}^{9} N_r y_r, \tag{4}$$

In a similar manner, the displacement field variables (u, v, w, θ_x, and θ_y) are also expressed in terms of its nodal variables as

$$u = \sum_{r=1}^{9} N_r u_r, \quad v = \sum_{r=1}^{9} N_r v_r, \quad w = \sum_{r=1}^{9} N_r w_r$$

$$\theta_x = \sum_{r=1}^{9} N_r \theta_{xr} \quad \text{and} \quad \theta_y = \sum_{r=1}^{9} N_r \theta_{yr}. \tag{5}$$

In the above equation, N_r are the shape functions while (x_r and y_r) are the nodal coordinates and (u_r, v_r, w_r, θ_{xr} and θ_{yr}) are the nodal displacements of an element. The shape functions are expressed in terms of non-dimensional parameters ξ and η.

4.1 Formulation of plate element for eccentrically stiffened plate

Displacement field of plate $\{f\}$ is expressed as,

$$\{f\} = \begin{Bmatrix} U \\ V \\ W \end{Bmatrix} = \begin{Bmatrix} u - z\theta_x \\ v - z\theta_y \\ w \end{Bmatrix} \tag{6}$$

Strain displacement relation can be written by

$$\{\varepsilon\} = \Sigma[B_p]_r\{\delta\}_r = [B_p]\{\delta\} \tag{7}$$

where $$[B_P] = [[B_P]_1 \;\; [B_P]_2 \;\; \dots \;\; [B_P]_r \;\; \dots \;\; [B_P]_9]$$

$$[B_P]_r = \begin{bmatrix} \frac{\partial N_r}{\partial x} & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial N_r}{\partial y} & 0 & 0 & 0 \\ \frac{\partial N_r}{\partial y} & \frac{\partial N_r}{\partial x} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\partial N_r}{\partial x} & 0 \\ 0 & 0 & 0 & 0 & -\frac{\partial N_r}{\partial y} \\ 0 & 0 & 0 & -\frac{\partial N_r}{\partial y} & -\frac{\partial N_r}{\partial x} \\ 0 & 0 & \frac{\partial N_r}{\partial x} & -N_r & 0 \\ 0 & 0 & \frac{\partial N_r}{\partial y} & 0 & -N_r \end{bmatrix}$$

Stress Strain relationship:

$$\{\sigma_p\} = [D_p]\{\varepsilon_p\} \qquad (8)$$

$$\{\sigma_p\}^T = \{\sigma_x \ \ \sigma_y \ \ \tau_{xy} \ \ \tau_{zx} \ \ \tau_{zy}\}$$

Where

$$[D_p] = \begin{bmatrix} \frac{Et}{1-\nu^2} & \frac{\nu Et}{1-\nu^2} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{\nu Et}{1-\nu^2} & \frac{Et}{1-\nu^2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{Et}{2(1+\nu)} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{Et^3}{12(1-\nu^2)} & \frac{\nu Et^3}{12(1-\nu^2)} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\nu Et^3}{12(1-\nu^2)} & \frac{Et^3}{12(1-\nu^2)} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{Et^3}{24(1+\nu)} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{Et}{2.4(1+\nu)} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{Et}{2.4(1+\nu)} \end{bmatrix}$$

Element stiffness matrix is expressed as

$$[K_p] = \int_{-1}^{+1}\int_{-1}^{+1}[B_p]^T[D_p][B_p]|J_p|\,d\xi d\eta \tag{9}$$

and $$|J_p| = |J|.$$

Consistent mass matrix:

The element mass matrix can be expressed in isoparametric coordinate as

$$[M_p] = \int_{-1}^{+1}\int_{-1}^{+1}[N]^T[m_p][N]|J_p|\,d\xi d\eta \tag{10}$$

where

$$[m_p] = \begin{bmatrix} \rho t & 0 & 0 & 0 & 0 \\ 0 & \rho t & 0 & 0 & 0 \\ 0 & 0 & \rho t & 0 & 0 \\ 0 & 0 & 0 & \dfrac{\rho t^3}{12} & 0 \\ 0 & 0 & 0 & 0 & \dfrac{\rho t^3}{12} \end{bmatrix}$$

Geometric stiffness matrix $[K_G]$:

For buckling analysis the bending strains developed due to the action of in-plane loads are considered. Thus the stiffness matrix is modified by another matrix $[K_G]$ and an eigenvalue is encountered.

Strain at middle plane of the plate

$$\{\varepsilon\} = \{\varepsilon_p\}_E + \{\varepsilon_p\}_G \tag{11}$$

$$\{\varepsilon\} = \begin{Bmatrix} \dfrac{\partial U}{\partial x} \\ \dfrac{\partial V}{\partial x} \\ \dfrac{\partial U}{\partial y} + \dfrac{\partial V}{\partial y} \\ \dfrac{\partial W}{\partial x} \\ \dfrac{\partial W}{\partial y} \end{Bmatrix} + \begin{Bmatrix} \dfrac{1}{2}\left(\dfrac{\partial W}{\partial x}\right)^2 + \dfrac{1}{2}\left(\dfrac{\partial U}{\partial x}\right)^2 + \dfrac{1}{2}\left(\dfrac{\partial V}{\partial x}\right)^2 \\ \dfrac{1}{2}\left(\dfrac{\partial W}{\partial y}\right)^2 + \dfrac{1}{2}\left(\dfrac{\partial U}{\partial y}\right)^2 + \dfrac{1}{2}\left(\dfrac{\partial V}{\partial y}\right)^2 \\ \left(\dfrac{\partial W}{\partial x}\right)\left(\dfrac{\partial W}{\partial y}\right) + \left(\dfrac{\partial U}{\partial x}\right)\left(\dfrac{\partial U}{\partial y}\right) + \left(\dfrac{\partial V}{\partial x}\right)\left(\dfrac{\partial V}{\partial y}\right) \\ 0 \\ 0 \end{Bmatrix} \tag{12}$$

Considering a concentric stiffener case, the displacement field can be written as

$$\{\varepsilon\} = \begin{Bmatrix} U \\ V \\ W \end{Bmatrix} = \begin{Bmatrix} -z\theta_X \\ -z\theta_y \\ w \end{Bmatrix} \tag{13}$$

$$\{\overline{\varepsilon}_p\}_G = \sum_{r=1}^{9} [B_{GP}]_r \{q\}_r \tag{14}$$

$$[B_{GP}] = [[B_{GP}]_1 \ [B_{GP}]_2 \ \ldots \ [B_{GP}]_r \ \ldots \ [B_{GP}]_9]$$

Where

$$[B_{G_P}]_r = \begin{bmatrix} 0 & 0 & \frac{\partial N_r}{\partial x} & 0 & 0 \\ 0 & 0 & \frac{\partial N_r}{\partial y} & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial N_r}{\partial x} & 0 \\ 0 & 0 & 0 & 0 & \frac{\partial N_r}{\partial y} \\ 0 & 0 & 0 & 0 & \frac{\partial N_r}{\partial x} \\ 0 & 0 & 0 & \frac{\partial N_r}{\partial y} & 0 \end{bmatrix},$$

When expressed in isoparametric coordinates the geometric stiffness matrix can be expressed as

$$[K_{G_p}] = \int_{-1}^{+1}\int_{-1}^{+1} [B_{G_p}]^T [\sigma_p][B_{G_p}] |J_p| d\xi d\eta \tag{15}$$

4.2 Stiffener element formulation

Similar to the plate element, the elastic stiffness matrix $[K_S]$, geometric stiffness matrix $[K_{GS}]$ and mass matrix $[M_S]$ of a stiffener element placed anywhere within a plate element and oriented in the direction of x may be derived as expressed as

Element stiffness matrix for stiffener

$$[K_S] = \int_{-1}^{+1} [B_S]^T [D_S][B_S] |J_S| d\xi, \tag{16}$$

where
$$[B_S]_r = \begin{bmatrix} \frac{\partial N_r}{\partial x} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\partial N_r}{\partial x} & 0 \\ 0 & 0 & 0 & 0 & \frac{\partial N_r}{\partial x} \\ 0 & 0 & \frac{\partial N_r}{\partial x} & -N_r & 0 \end{bmatrix}$$

and
$$[\overline{D_S}] = \begin{bmatrix} EA_S & EF_S & 0 & 0 \\ EF_S & ES_S & 0 & 0 \\ 0 & 0 & GT_S & 0 \\ 0 & 0 & 0 & GA_S/1.2 \end{bmatrix}$$

The element mass matrix can be expressed in isoparametric coordinate as

$$[M_S] = \int_{-1}^{+1} [N]^T [m_S][N]|J| d\xi \tag{17}$$

where
$$[m_S] = \rho \begin{bmatrix} A_S & 0 & 0 & 0 & 0 \\ 0 & A_S & 0 & 0 & 0 \\ 0 & 0 & A_S & 0 & 0 \\ 0 & 0 & 0 & F_S & 0 \\ 0 & 0 & 0 & 0 & P_S \end{bmatrix}$$

The geometric stiffness of the stiffener element can be expressed as

$$[K_{GS}] = \int_{-1}^{+1} [B_{GS}]^T [\sigma_S][B_{GS}]|J_S| d\xi \tag{18}$$

where A_S is the area, F_S is the first moment of area about reference plane, S_S is the second moment of area about reference plane, T_S is the torsion constant and P_S is the polar moment of area of the stiffener cross-section.

$|J_S|$ is the jacobian of the stiffener, which is half of its actual length within an element.

The element matrices for the plate and stiffener are generated separately and then added up to form overall matrices.

5. Computer program

A computer program is developed to perform all the necessary computations. The geometric stiffness matrix is essentially a function of the in-plane stress distribution in the element due to applied edge loading. Since the stress field is non-uniform, for a given edge loading and boundary conditions, the static equation, i.e., $[K]\{\delta\} = \{F\}$ is solved to get these stresses. The geometric stiffness matrix is now constructed with the known in-plane stresses. The computer program developed accepts two sets of boundary conditions, the first for the static analysis and second for the buckling analysis. In the present case, a three-point integration scheme is adopted for the evaluation of all the matrices except the portion of the stiffness matrix related to shear strain components.

The overall elastic stiffness matrix, geometric stiffness matrix and mass matrix are generated from the assembly of those element matrices and stored in a single array where the variable bandwidth profile storage scheme is used. The solution of eigenvalues is performed by the simultaneous iteration technique proposed by Corr and Jennings (1976).

6. Result and discussion

The loads and frequencies are converted into non-dimensional form according to the following definitions.

$$\text{Non-dimensional load, } \lambda = \frac{Pb}{D} \tag{19}$$

$$\text{Non-dimensional frequency, } \bar{\omega} = \omega b^2 \sqrt{\rho h / D} \tag{20}$$

The stiffener parameter terms δ and γ are defined as:

$\delta = \dfrac{A_S}{bt}$ = Ratio of cross-sectional area of the stiffener to the plate, where A_S is the area of the stiffener.

$\gamma = \dfrac{EI_S}{bD}$ = Ratio of bending stiffness rigidity of stiffener to the plate, where I_S is the moment of inertia of the stiffener cross-section about reference axis.

6.1 Convergence and validation studies with previous results

It is desirable to determine the critical (or lowest) value of load, which causes buckling before studying the effects of concentrated forces upon the buckling and vibration of a stiffened plate. It is necessary to keep the applied forces below their critical values in the subsequent vibration analysis. In a finite element analysis, it is desired to have the convergence studies to estimate the order of mesh size to be necessary for the numerical solution. The convergence studies have been carried out for frequency of vibration and buckling loads for a plate subjected to concentrated load for various plate geometries and load positions and the results are shown in Table 1. As the convergence study shows that a mesh size of 10 × 10 is sufficient enough to get a reasonable order of accuracy. The analysis in the subsequent problems is carried out with this mesh size.

In the buckling and vibration analysis, it is necessary to mention the mode shapes. A nomenclature of the mode shapes is adopted wherein, m denotes the number of approximate half waves in x-direction, and n denotes the number of approximate half waves in y-direction. Table 2

Table 1 Convergence of non-dimensional frequency (ω) for a simply supported square unstiffened plate subjected to in-plane concentrated loads

c/b	Mesh divison	Non-dimensional frequency parameter			
	P/P_{cr}	–1	–0.5	0	0.5
0.25	4 × 4	26.52	23.74	19.75	14.23
	6 × 6	26.43	23.51	19.73	14.29
	8 × 8	26.54	23.56	19.73	14.37
	10 × 10	26.37	23.47	19.73	14.41
	12 × 12	26.39	23.54	19.73	14.37
0.5	4 × 4	27.54	24.01	19.75	14.07
	6 × 6	27.59	24.02	19.73	14.06
	8 × 8	27.64	24.04	19.73	14.05
	10 × 10	27.67	24.05	19.73	14.05
	12 × 12	27.69	24.06	19.73	14.04

Table 2 Comparison of non-dimensional frequency parameter with P/P_{cr} for simply supported square unstiffened plate subjected to in-plane concentrated load

Non-dimensional natural frequencies (ω)					
c/b	Mode shape	P/P_{cr}			
		–1	–0.5	0	0.5
0.25	Mode 11	26.37 (26.48) (26.41)	23.47 (23.54) (23.50)	19.73 (19.739) (19.729)	14.41 (14.339) (14.36)
	Mode 12	59.31 (59.43) (59.09)	55.28 (55.39) (55.12)	49.31 (49.34) (49.148)	39.86 (39.62) (39.515)
	Mode 21	53.00 (53.14)	51.15 (51.23)	49.31 (49.34)	47.40 (47.41)
	Mode 22	90.53 (91.12)	84.65 (84.89)	78.86 (78.95)	73.51 (73.42)
0.5	Mode 11	27.67 (27.75) (27.732)	24.05 (24.09) (24.07)	19.73 (19.73) (19.729)	14.05 (14.027) (14.022)
	Mode 12	63.73 (63.82)	57.12 (57.18)	49.31 (49.34)	39.58 (39.61)
	Mode 21	49.47 (49.36)	49.40 (49.36)	49.31 (49.34)	49.21 (49.31)
	Mode 22	83.36 (83.12)	81.15 (81.07)	78.86 (78.95)	76.47 (76.76)

Note: The values within the first brackets are from Leissa and Ayoub (1988) and second bracket from Deolasi and Datta (1995).

Table 3 Variation of buckling load parameter with position of load for simply supported square unstiffened plate

	Buckling load parameter (λ)	
c/b	Deolasi (1996)	Present
0	40.83	40.84
0.1	44.94	45.00
0.2	38.67	38.71
0.3	31.38	31.32
0.4	27.51	27.43
0.5	25.52	26.62

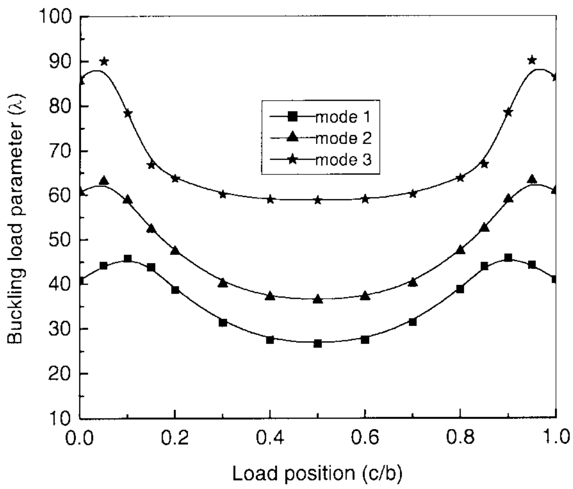

Fig. 4 Variation of buckling load parameter (λ) with position of load for simply supported square unstiffenned plate subjected to concentrated load

gives the comparison of vibration results for unstiffened plate for c/b = 0.25, 0.5 for different load ratios for various modes with those obtained by (Leissa and Ayoub 1988, Deolasi and Datta 1995).

Now the validation studies are done for buckling of unstiffened plate subjected to a pair of concentrated loads at different positions and the results are compared as shown with Deolasi and Datta (1996) in Table 3. It represents values of non-dimensional critical buckling load ($\lambda = P_{cr}b/D$) for the aspect ratios a/b = 1. The buckling load parameters are obtained in three modes and the values so obtained are plotted as c/b vs load parameter in Fig. 4. It is found that the lowest buckling load will occur for centrally applied concentrated loads, i.e., when c/b = 0.5 for all modes. Buckling load parameters variation at different load positions is also plotted in Figs. 5, 6 for simply supported and clamped edge conditions respectively for different aspect ratios.

It is observed that buckling load parameter of a plate simply supported along all the edges increase as the loads are nearer to the support. It is also observed that for plates with small aspect ratios, the boundary condition on the loaded edge has the significant effect on the load required to cause elastic stability.

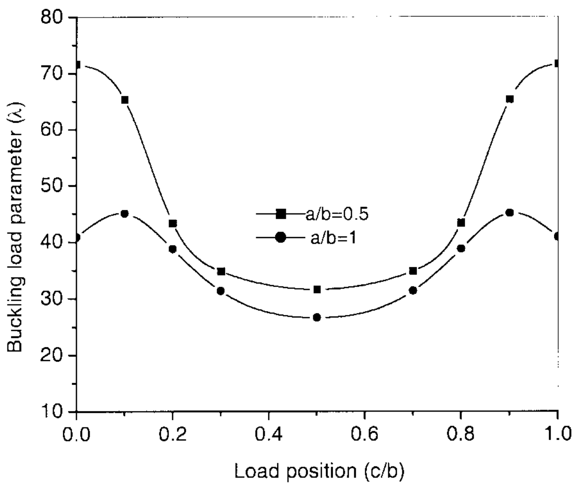

Fig. 5 Variation of buckling load parameter (λ) with position of load for simply supported unstiffenned plate subjected to concentrated load

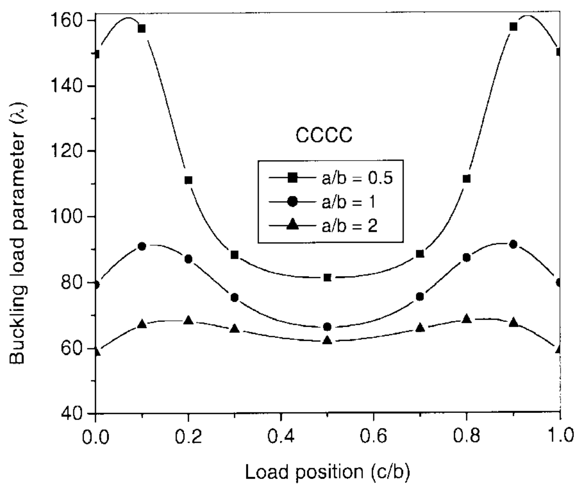

Fig. 6 Variation of buckling load parameter (λ) with position of load for clamped unstiffenned plate subjected to concentrated load

6.2 Buckling studies for stiffened plate under concentrated edge loading

To study the behaviour of stiffened plates under concentrated in-plane edge load, stiffened plates for various boundary conditions, aspect ratios, stiffener parameters are considered. The cross-section of the stiffened plates is shown in Fig. 2 and the data are as follows: a = 600 mm, b = 600 mm, t = 6 mm, b_S = 12.7 and d_S = 22.2 mm.

6.2.1 Effect of position of load with aspect ratio

As a first case, the effects of position of concentrated load and aspect ratios for buckling are studied by taking different values of c/b and results obtained are shown in Fig. 7 for different aspect ratios a/b = 0.5, 1, 2 considering in-plane displacements. In order to study the effect of aspect ratio (a/b) of a stiffened plate on its buckling behaviour, the length (a) of the stiffened plate considered above is varied keeping its other parameters unchanged. It can be observed from Fig. 7 that the variation of buckling load with the position of the load on the edges is more pronounced for the stiffened plates of the smaller aspect ratios. For the stiffened plates of higher aspect ratios, the buckling loads are not as much affected with the position of the load. This is due to the fact that for long stiffened plates subjected to concentrated load, a major part of the plate area is subjected to fairly uniform stress distribution comparatively. It can be observed that buckling behaviour of the stiffened plates with the aspect ratio is similar to that of a simply supported unstiffened plate under similar loading (1996).

It is observed that a stiffened plate with one central stiffener loaded centrally by concentrated loads (c/b = 0.5) buckles with the maximum load. This is because of stiffener placed at the centre. This reason can be attributed by considering stiffened plate with two equispaced and three-equispaced stiffener shown in Fig. 8. The dimension taken for this case are as follows: b =100 mm, t =1 mm, δ = 0.1 and γ =10, a/b = 2. It is quite obvious from Fig. 8 that stiffened plate gets maximum buckling load when the load position coincide with the location of the stiffeners. It is

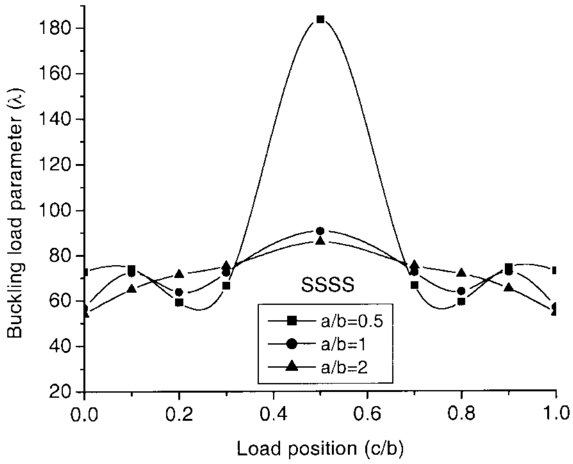

Fig 7 Variation of buckling load parameter (λ) with position of load for simply supported stiffened plate subjected to concentrated load

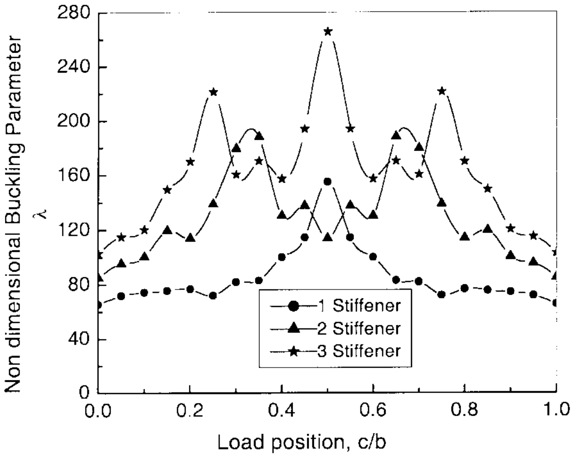

Fig 8 Variation of buckling load parameter (λ) with position of load for stiffened plate subjected to concentrated load. b =100 mm, t =1 mm, δ = 0.1 and γ=10, a/b = 2

observed that this behaviour of stiffened plate is distinctly different to that of corresponding unstiffened plate (1996).

An unstiffened plate loaded centrally by concentrated loads (c/b = 0.5) buckles with the lowest load. For unstiffened plate, it is observed from the Table 3 that the buckling load is higher for the location of loads near the ends of the edges. The buckling load decreases rapidly as the load is moved toward the centre of two opposite edges and becomes minimum for a concentrated loading at the centre. The lower values of the buckling load for loading near the centre of the edges is because of the fact that a substantial compressive zone under such loading lies near the centre of the plate where the edge restraints are not much effective.

6.2.2 Effect of position of load with boundary condition

As a second case, the effect of boundary condition with different position of the concentrated load is studied for buckling and the results obtained are shown in Fig. 9, considering in-plane displacements.

It can be observed from Fig. 9 that the variation of buckling load parameter for boundary conditions (SSSS, CCCC, CSCS) with the position of loads are similar as described above for Fig. 7. The buckling strength of clamped boundary edges are more than SSSS and CSCS edge condition. It is observed that for small value of c/b, the buckling load is usually higher and it decreases gradually upto some values of c/b depending upon degree of edge restraint. This may be due to the fact that for loading near the corners, the edge restraints (x = 0) increase the capacity to withstand the higher loads and as c/b increases, the effect of edge restraints decreases resulting in lower buckling loads. Again as the position of concentrated load is nearer to the location of stiffener, the effect of stiffener predominates. As a consequence, in the case of the stiffened plate problem considered here, the value of buckling load parameter goes on increasing near the location of stiffener and becomes maximum at stiffener position.

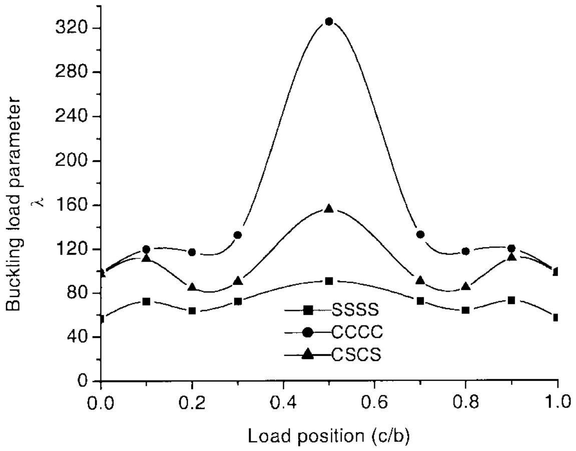

Fig 9 Variation of buckling load parameter (λ) with position of load for square stiffened plate subjected to concentrated load

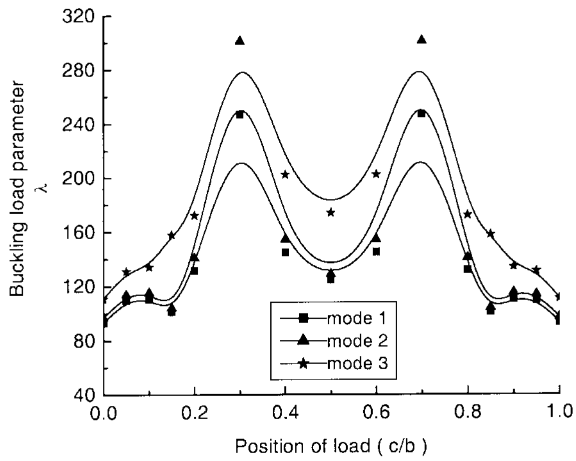

Fig. 10 Variation of buckling load parameter vs position of load (*c/b*) for simply supported square stiffened plate with two equispaced concentric stiffeners having (δ = 0.1 and γ = 10)

The buckling load parameter for square stiffened plate with one central stiffener (δ = 0.1 and γ = 10) is also studied for various boundary conditions (SSSS, CCCC, CCSS, CSCS) at different load position i.e. *c*/*b* from 0 to 1 with an interval of 0.1 and also for full uniform load. The maximum buckling load will occur for centrally applied concentrated load, i.e. when *c*/*b* = 0.5 where stiffener is placed for all boundary conditions.

At *c*/*b* = 0.5, these values are obtained and also for uni-axial full uniform load for the above-mentioned boundary conditions. It can be concluded from the above observation that a square stiffened plate buckles with 140.25, 121, 137, 141.88 percent of the load which could be withstood if the stiffened plate were uniformly loaded all over the two opposite edges.

On the other hand, it is found that the lowest buckling load will occur for centrally applied concentrated loads, i.e. when *c*/*b* = 0.5 for all aspect ratios and boundary conditions for unstiffened plate. A square unstiffened plate buckles with only 65 percent of the load which could be withstood if the plate were uniformly loaded all over the two opposite edges and for a plate with *a*/*b* = 2, this value is only about 48 percent (Leissa 1988).

The variation of buckling load parameter for simply supported square stiffened plate with two-equispaced stiffener having (δ = 0.1 and γ = 10) is also studied in various modes and the results are shown in Fig. 10. It is observed that variation in all modes is almost similar with increased values with modes.

6.3 Vibration studies for stiffened plates under concentrated edge loading

To study the vibration characteristics of stiffened plates under concentrated in-plane edge load, stiffened plates for various boundary conditions, aspect ratios, stiffener parameters are considered. All the boundaries of the stiffened plates are simply supported unless otherwise stated. Vibration frequencies were determined for all edges simply supported and clamped having three aspect ratios

Table 4 Variation of non-dimensional frequency parameter vs P/P_{cr} for square stiffened plate having one stiffener (δ = 0.1 and γ = 10) subjected to concentrated load at c/b = 0.25

Non-dimensional frequency parameter				
Edge condition	SSSS		CCCC	
P/P_{cr}	Mode 1	Mode 2	Mode1	Mode2
–1	44.79	56.93	78.31	88.61
–0.5	43.30	52.88	76.12	84.10
0	40.74	49.14	72.85	79.90
0.2	39.16	47.87	71.04	78.61
0.4	37.11	46.77	68.87	77.39
0.5	35.87	46.26	67.66	76.82
0.6	34.41	45.75	66.34	76.29
0.8	30.26	41.04	60.40	63.26

Table 5 Variation of non-dimensional frequency parameter vs P/P_{cr} for square stiffened plate having one stiffener (δ = 0.1 and γ = 10) subjected to concentrated load at c/b = 0.50

Non-dimensional frequency parameter				
Edge condition	SSSS		CCCC	
P/P_{cr}	Mode 1	Mode 2	Mode1	Mode2
–1	49.66	56.98	71.14	93.85
–0.5	48.93	50.29	72.62	87.74
0	40.75	49.14	72.85	79.99
0.2	36.59	48.76	72.51	76.39
0.4	31.82	48.27	71.64	72.57
0.5	29.12	47.96	70.07	71.43
0.6	26.13	47.59	67.83	70.56
0.8	18.62	45.43	55.34	62.62

(a/b = 0.5, 1, 2) and various locations of point loading.

The variation of non-dimensional frequency parameter with P/P_{cr} for square stiffened plate having one central stiffener (δ = 0.1 and γ = 10) subjected to concentrated load at c/b = 0.25 and 0.5 for simply supported and all edges clamped in various modes is shown in Tables 4, 5 and plotted in Fig. 11. It is observed that the compressive forces decrease the frequencies, and tensile forces increase them. It is also observed that the slope of the curve increases negatively with increasing P/P_{cr} as expected, the values of frequency parameter of clamped edges are more than all edges simply supported cases for all modes and P/P_{cr} values.

The variation of frequency parameter with P/P_{cr} for square stiffened plate having one central stiffener at different position of load is studied for different aspect ratios in Fig. 12. Now the analysis for the same stiffened plate is extended for simply supported and clamped cases and the variation is plotted in Fig. 13.

It is observed that stiffened plates with aspect ratios subjected to concentrated load at different positions have the significant effect on frequency parameters. The variation with position of load is noticeable. The values of vibration frequencies increase with the increase of aspect ratio. It is observed that the vibration frequencies increase as the load position shift away from the supported edge for all aspect ratios as observed from Fig. 12 for all edges simply supported. On the other hand, it is interestingly noted that this variation is somewhat different for all edges clamped as observed by Fig. 13. For clamped case, it is observed that the vibration frequencies decrease as the load position shifts away from the supported edge.

The variation of non-dimensional frequencies with non-dimensional load factor (P/P_{cr}) for unstiffened plate and above dealt stiffened plate with one central stiffener is also carried out for simply supported and clamped edges at load position c/b = 0.5.

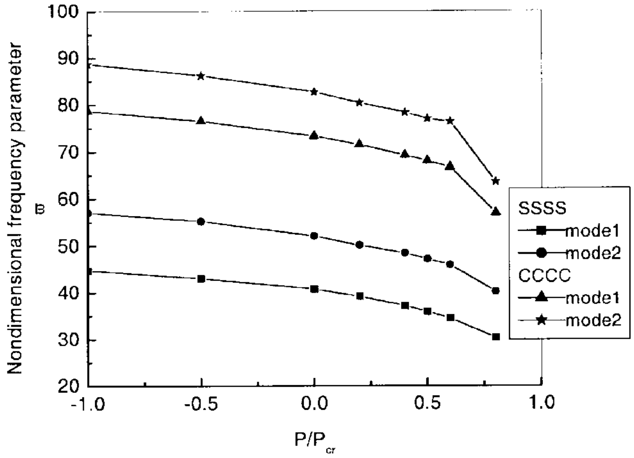

Fig. 11 Variation of frequency parameter with P/P_{cr} for square stiffened plate having one stiffener (δ = 0.1 and γ = 10) subjected to in-plane concentrated load at c/b = 0.25

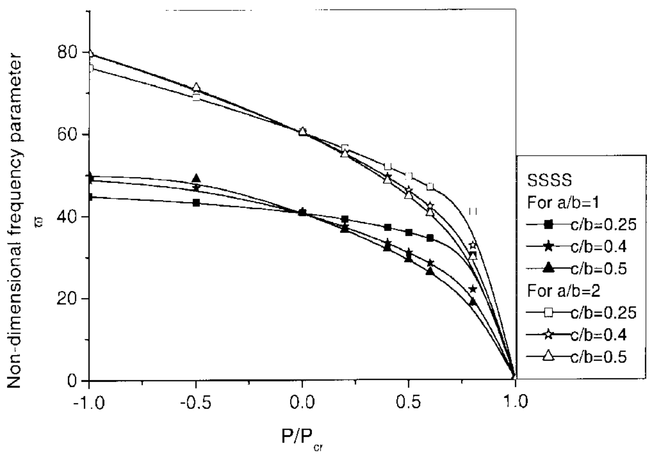

Fig. 12 Variation of frequency parameter with P/P_{cr} for stiffened plate having one stiffener (δ = 0.1 and γ = 10) subjected to concentrated load at different position of load

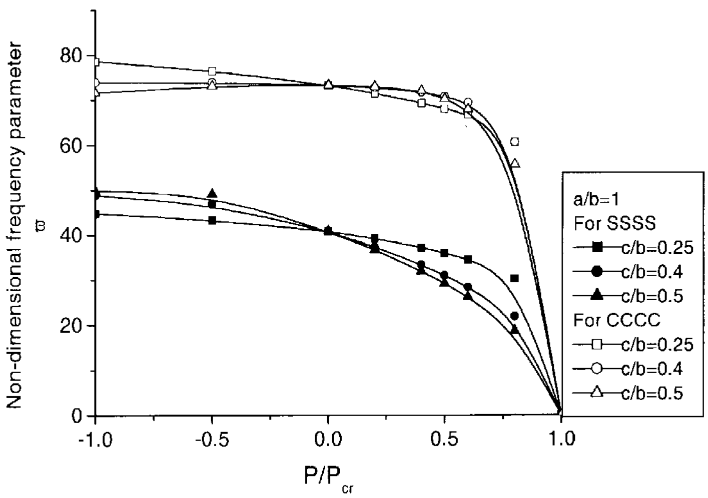

Fig. 13 Variation of frequency parameter with P/P_{cr} for stiffened plate having one stiffener (δ = 0.1 and γ = 10) subjected to in-plane concentrated load at different position of load

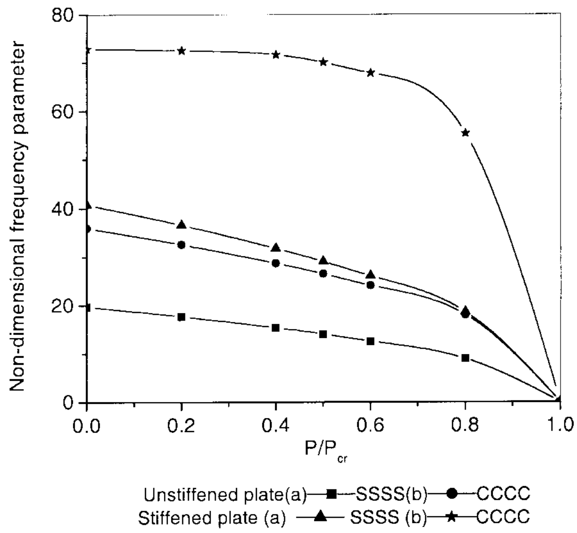

Fig. 14 Variation of non-dimensional frequency parameter with P/P_{cr} for unstiffened plate and stiffened plate having one central stiffener (δ = 0.1 and γ = 10) subjected to concentrated load. c/b = 0.5

The aim is to get the outlook of the comparative study of unstiffened and stiffened plates. The variation is shown Fig. 14. As the applied compressive load increases, the frequencies decrease and finally tend to zero at the respective value of buckling loads.

As expected the values of vibration frequencies for clamped cases are more than simply supported and also the values of frequencies for stiffened plate are more than unstiffened.

7. Conclusions

The results from the study of the compressive buckling and vibration behaviour of a stiffened plate subjected to in-plane concentrated load can be summarized as follows.

The buckling load parameter of a unstiffened plates simply supported along all the edges increases as the loads are nearer to the support. It is also observed that for plates with small aspect ratios, the boundary condition on the loaded edge has the significant effect on the load required to cause elastic instability.

For stiffened plates, compression buckling analysis shows that the variation of buckling load with the position of the load on the edges is more pronounced for the stiffened plates of the smaller aspect ratios. For the stiffened plates of higher aspect ratios, the buckling loads are not as much affected with the position of the load.

The stiffened plate gets maximum buckling load when the load position coincides with the location of the stiffeners.

Natural frequencies decrease as compressive load increases. As expected, the fundamental frequencies become zero at respective values of buckling loads. Also there is significant change in the natural frequencies with position of loading over the edges. The values of frequency parameter of clamped edges are more than all edges simply supported cases for all modes and P/P_{cr} values.

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Notation

a	: Plate dimension in longitudinal direction
b	: Plate dimension in the transverse direction
c	: distance of concentrated load from bottom edge along y direction
t	: Plate thickness
E, G	: Young's and shear moduli for the plate material
υ	: Poisson's ratio
b_s, d_s	: Web thickness and depth of a x-stiffener
ξ, η	: Non-dimensional element coordinate
A_S	: Cross sectional area of the stiffener
I_S	: Moment of inertia of the stiffener cross-section about reference axis
$\{q\}_r$	: Vector of nodal displacement a r^{th} node
$[D_P]$	: Rigidity matrix of plate
$[D_S]$	: Rigidity matrix of stiffener
$[K_e]$	: Elastic stiffness matrix of plate
$[K_S]$	: Elastic stiffness matrix of stiffener
$[M_p]$, $[M_S]$	: Consistent mass matrix of plate, stiffener
$[K_G]$	: Geometric stiffness matrix
$[N]_r$	: Matrix of a shape function of a node r
P_{cr}	: Critical buckling load