Structural Engineering and Mechanics, Vol. 15, No. 1 (2003) 39-52
DOI: http://dx.doi.org/10.12989/sem.2003.15.1.039

Optimum design of prestressed concrete beams by a modified grid search method

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(Received December 11, 2000, Revised May 9, 2002, Accepted December 5, 2002)

Abstract. A computer program has been developed for the optimum design of prestressed concrete beams under flexure. Optimum values of prestressing force, tendon configuration, and cross-sectional dimensions are determined subject to constraints on the design variables and stresses. 28 constraints have been used including flexural stresses, cover requirement, the aspect ratios for top and bottom flanges and web part of a beam and ultimate moment. The objective function contains cost of concrete, prestressing force and formwork. Using this function, it is possible to obtain minimum cost design, minimum weight or cross-sectional area of concrete design and minimum prestressing force design. Besides the idealized I-shaped cross-section, which is widely used in literature, a general I-shaped cross-section with eight geometrical design variables are used here. Four examples, one of which is available in the literature and the others are modified form of it, have been solved for minimum cost and minimum cross-sectional area designs and the results are compared. The computer program, which employs modified grid search optimization method, can assist a designer in producing efficient designs rapidly and easily. Considerable savings in computational work are thus made possible.

Key words: prestressed concrete beam design; modified grid search; optimum design; optimization; linear and non-linear programming and cost optimization.

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1. Introduction

Several analytical studies of optimum design of prestressed beams have been reported in the literature (Morris 1978, Taylor 1987, Cohn and MacRae 1983, Jones 1984, Saouma and Murad 1984, Birkeland 1974, Fereig 1994, Wang 1970, Kirsh 1972). In these studies linear programming methods and non-linear programming procedures such as gradient methods have been used as optimization techniques. However, the grid search optimization method, which is simple and effective for programming, has not been used for the optimization of prestressed concrete beams before. In the present study, a modified grid search optimization method (Cagatay 1996) has been applied to the prestressed concrete beam problem, with no restriction on the number and type of constraints on the design variables and stresses.

In this paper, the proposed method is applied to an example problem available in the literature (Taylor and Amirebrahimi 1987) and the results are in good agreement.

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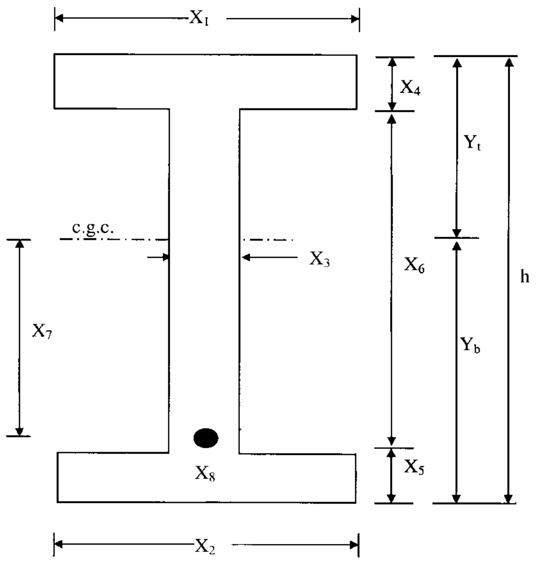

Fig. 1 Cross-section of idealised I-shaped PC beam

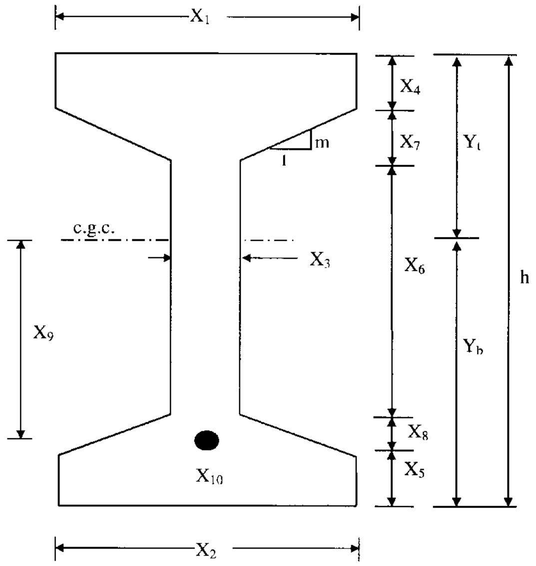

Fig. 2 Cross-section of general I-shaped PC beam

A computer program has been developed employing the modified grid search optimization method, which can assist a designer in producing efficient designs rapidly and easily.

2. Problem formulation

Most papers on prestressed concrete beam design and optimization deal with idealized I beam section with six dimensions (Morris 1978, Taylor 1987, Cohn and MacRae 1983, Jones 1984) as shown in Fig. 1, due to simplicity. But, here, the method has been formulated for a general I shaped cross-section with eight geometrical design variables denoted by X_1 through X_8 as shown in Fig. 2. The variables X_7 and X_8 are calculated as a function of m, which is the slope of the top and bottom flanges of the cross-section, as shown in Fig. 2, as follows:

$$X_7 = m\frac{X_1 - X_3}{2} \tag{1}$$

$$X_8 = m\frac{X_2 - X_3}{2} \tag{2}$$

If the m value is chosen to be zero, the cross-section of the beam turns into an idealized I-beam, Fig. 1. Also, additional design variables X_9 and X_{10} are considered, which represent the eccentricity and the prestressing force, respectively. The method presented here can also be used to treat T, inverted T and double T sections as special cases of the general I-shaped cross-section as shown in Fig. 3.

The objective function to be minimized is denoted by $f(X)$ and the constraint functions by $g_i(X)$.

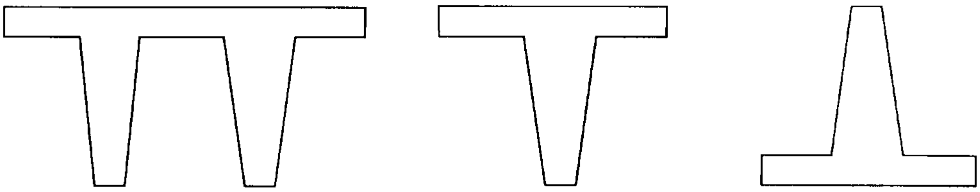

Fig. 3 Prestressed concrete beam cross-sections

The optimization problem can thus be defined as:

$$\text{Minimize} \quad f(X) = LA_cC_c + LW_pA_pC_p + A_fC_f \tag{3}$$

$$\text{Subject to} \quad \begin{matrix} g_i(X) \le 0, \\ X \ge 0 \end{matrix} \quad (i = 1, 2, \ldots, k) \tag{4}$$

where L is the span length; A_c, A_p and A_f are the cross-sectional area, the area of the prestressing steel, and the area of forming, respectively; C_c, C_p and C_f are the unit cost of concrete, the unit cost of prestressing steel, and the unit cost of formwork, respectively; W_p is the unit weight of prestressing steel; and k denotes the number of constraints. The aims of the objective function are to minimize both the cross-sectional area of prestressed concrete and find the minimum prestressing force corresponding to maximum eccentricity. To obtain the minimum prestressing force corresponding to maximum eccentricity, Magnel's graphical method has been used (Magnel 1948).

The following constraints were considered:

a. Flexural stresses

There are two stages of loading to be considered. The first stage is at the transfer of prestressing force to the beam. For computing the flexural stresses at the top and bottom of the beam, following equations can be written:

$$f_{tt} = \frac{P}{A} - \frac{PeY_t}{I} + f_{td} \tag{5}$$

$$f_{tb} = \frac{P}{A} + \frac{PeY_b}{I} - f_{bd} \tag{6}$$

where f_{tt} and f_{tb} are the flexural stresses at the top and bottom of the beam and P and e are the prestressing force and the eccentricity, respectively. Y_t and Y_b denote the distances of the top and bottom fibers of the cross-section from the center of gravity of concrete section (c.g.c.) and I and A denote the gross second moment of area and the gross area of the cross-section, respectively. f_{td} and f_{bd} are the flexural stresses at the top and bottom of the beam due to the dead load.

The second stage is at the service condition when the beam carries live loads in addition to its own weight. The following equations can be written:

$$f_{st} = \frac{P\alpha}{A} - \frac{P\alpha eY_t}{I} + f_{td} + f_{tl} \tag{7}$$

$$f_{sb} = \frac{P\alpha}{A} + \frac{P\alpha e Y_b}{I} - f_{bd} - f_{bl} \tag{8}$$

where f_{st} and f_{sb} are the flexural stresses at the top and bottom of the beam, respectively; α is the loss factor; f_{tl} and f_{bl} are the stresses due to the live load at the top and bottom fibers, respectively.

The stresses computed by Eqs. (5-8) should not exceed the allowable stresses, which are specified in design codes. Rearranging Eqs. (5-8), and considering the allowable stresses, the following inequalities are obtained:

$$\left(-\frac{1}{A} + \frac{eY_t}{I}\right)\frac{1}{(f_{IT} + f_{td})} \le \frac{1}{P} \tag{9}$$

$$\left(\frac{1}{A} + \frac{eY_b}{I}\right)\frac{1}{(f_{IC} + f_{bd})} \le \frac{1}{P} \tag{10}$$

$$\left(\frac{1}{A} - \frac{eY_t}{I}\right)\left(\frac{\alpha}{f_{FC} - f_{td} - f_{tl}}\right) \le \frac{1}{P} \tag{11}$$

$$\left(\frac{1}{A} + \frac{eY_b}{I}\right)\frac{\alpha}{(-f_{FT} + f_{bd} + f_{bl})} \ge \frac{1}{P} \tag{12}$$

where f_{IT} and f_{IC} are the allowable tensile and compressive stresses in the concrete at transfer condition and f_{FC} and f_{FT} are the allowable compressive and tensile stresses in the concrete at service condition, respectively.

b. Prestressing force and eccentricity

For the selection of prestressing force and eccentricity, Magnel's graphical technique (Magnel 1948) has been used and the following inequalities are obtained:

$$E3 \ge E1 \tag{13}$$

$$E4 \ge E2 \tag{14}$$

in which

$$E1 = \frac{1}{(f_{IT} + f_{td})} \tag{15}$$

$$E2 = \frac{1}{(f_{IC} + f_{bd})} \tag{16}$$

$$E3 = \frac{\alpha}{(-f_{FC} + f_{td} + f_{tl})} \tag{17}$$

$$E4 = \frac{\alpha}{(-f_{FT} + f_{bd} + f_{bl})} \tag{18}$$

If Eqs. (13-14) are satisfied, then the cross-sectional area is adequate under the given loading condition. These two constraints are active constraints. It is assumed that for maximum efficiency, the eccentricity takes its largest allowable value at mid-span (with minimum cover). Therefore,

$$e \le Y_b - e_{cover} \tag{19}$$

in which e_{cover} is the required concrete cover.

c. Cross-sectional dimensions

Aspect ratios of the web and flanges cannot exceed a prescribed limiting value without any conditions concerning slenderness. This limit is presently taken as 8. The value is adjustable at the discretion of the designer. Hence, for any component part of the cross-section,

$$\lambda_t \ge \frac{X_1}{X_4} \tag{20}$$

$$\lambda_b \ge \frac{X_2}{X_5} \tag{21}$$

$$\lambda_w \ge \frac{X_6 + X_7 + X_8}{X_3} \tag{22}$$

where λ_t, λ_b, λ_w are the aspect ratios of top and bottom flanges, and web, respectively.

Further constraints have also been introduced for the cross-sectional dimensions as follows:

$$(X_i)_{\min} \le X_i \le (X_i)_{\max} \quad (i = 1, 2, \ldots, n) \tag{23}$$

where n is the number of variables.

d. Ultimate moment

Ultimate moment design is based on the solution of the equations of equilibrium, using the equivalent rectangular stress block shown in Fig. 4. An equivalent rectangular stress block is used

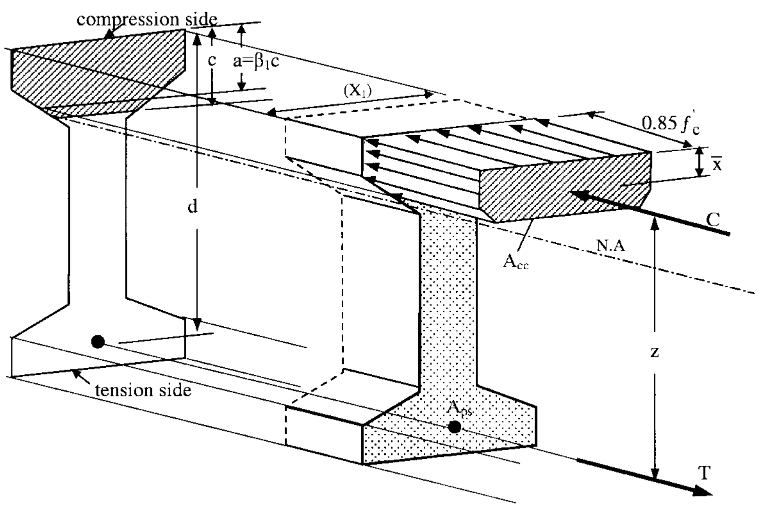

Fig. 4 Assumed stress distribution in the cross-section of the beam at ultimate limit state

with ease and without loss of accuracy to calculate the compressive force and hence the flexural moment strength of the section. The depth of the rectangular stress block is related to the depth of the neutral axis c by the following equation:

$$a = \beta_1 c \tag{24}$$

where β_1 is the dimensionless coefficient and is a function of the concrete strength which is 0.85 for $f_c' = 20\text{-}27$ MPa, 0.80 for $f_c' = 34$ MPa, 0.75 for $f_c' = 41$ MPa, and 0.7 for $f_c' = 48$ MPa.

The following assumptions are also made in defining the behaviour of the section at ultimate load:

1. The failure is primarily a flexural failure, with no shear bond, or anchorage failure which might decrease the strength of the section.
2. The steel and concrete are completely bonded.
3. The strain distribution is assumed to be linear.
4. Concrete is weak in tension. Consequently, concrete in the tension zone of the concrete section is neglected.
5. The load considered is the ultimate load obtained as the result of a short static test. Impact, fatigue, or long-time loadings are not considered.

To satisfy the equilibrium of horizontal forces, the compressive force C in the concrete and the tensile force T in the steel should balance each other.

The tensile stress in the prestressing steel is computed by using the empirical equation (ACI 318-95)

$$f_{ps} = f_{pu}\left(1 - \frac{\gamma_p}{\beta_1}\left[\rho_p \frac{f_{pu}}{f_c'}\right]\right) \tag{25}$$

where f_{pu} is the ultimate strength of prestressing steel, ρ_p is the ratio of prestressed reinforcement. The use of Eq. (25) is allowed by the ACI 318 Building Code providing that

$$f_{pe} \geq 0.50 f_{pu} \tag{26}$$

where f_{pe} is effective prestress.

The value of the factor γ_p is based on the criterion that $f_{py} = 0.80\, f_{pu}$ for high-strength prestressing bars, 0.85 for stress-relieved strands and 0.90 for low-relaxation strands. Also,

$$\begin{aligned} \gamma_p &= 0.55 \text{ for } f_{py}/f_{pu} > 0.80 \\ \gamma_p &= 0.40 \text{ for } f_{py}/f_{pu} > 0.85 \\ \gamma_p &= 0.28 \text{ for } f_{py}/f_{pu} > 0.90 \end{aligned} \tag{27}$$

The nominal moment strength of the section is then determined as

$$M_n = A_{ps} f_{ps} z \tag{28}$$

where A_{ps} is the area of prestressing steel and z is the lever arm of the internal couple which is given as

$$z = d - \bar{x} \tag{29}$$

in which $\bar{x}$ is the distance from the extreme compression fiber to the line of action for the resultant compression force (Fig. 4). Design requirement should meet the following condition

Table 1 Constraints

Constraints	No	Statement
$g_1(X) = -f_{tt} - f_{IT} \le 0$	1	Stress at top fibre at transfer
$g_2(X) = f_{tb} - f_{IC} \le 0$	2	Stress at bottom fibre at transfer
$g_3(X) = f_{st} - f_{FC} \le 0$	3	Stress at top fibre at service
$g_4(X) = -f_{sb} - f_{FT} \le 0$	4	Stress at bottom fibre at service
$g_5(X) = E1 - E3 \le 0$	5	Section efficiency at the top fibre
$g_6(X) = E2 - E4 \le 0$	6	Section efficiency at the bottom fibre
$g_7(X) = X_9 - Y_b + e_{cover} \le 0$	7	Eccentricity
$g_{8\text{-}10}(X) = \text{Depth} - \lambda\ \text{Width} \le 0$	8-10	Aspect ratios for the top and bottom flanges
$g_{11}(X) = h - 914.4 \le 0$	11	Overal depth
$g_{12}(X) = M_u - \phi M_n \le 0$	12	Ultimate moment
$g_{12\text{-}27}(X) = X_{i(\min)} \le X_i \le X_{i(\max)}$	13-28	Cross-sectional dimensions

$$\phi M_n \ge M_u \tag{30}$$

where M_u is the applied factored moment at a section; ϕ is the strength reduction factor which is taken to be as 0.9. In view of Eqs. (5-30), the constraint equations are given in Table 1.

2.1 Costs of materials

The unit costs of the structural materials are not same and depend on many conditions. For example cost of formwork depends on the type of prestressing system, i.e., pretensioning or post-tensioning and the number of times the formwork is used. A formwork for pretensioning system can cost 10.000 \$, but if it is used 100 or 400 times the unit cost of formwork will reduce to 100 \$ or 25 \$, respectively. Also, workmanship and transportation will affect the costs of materials in an unpredictable way. Hence, such factors had to be ignored in the present study. Consequently, as obtained from the local suppliers, the unit costs of concrete, prestressing steel and formwork are assumed to be 145 \$/m^3, 60 \$/kN, and 10 \$/m^2, respectively.

3. Optimization procedure

The algorithm described below was developed by Cagatay (1996) and is a modification of the one given by Walsh (1975). This modified algorithm, which follows, is going to be used in this study.

1. Start with the solution region defined by $(X_i)_{\min} \le X_i \le (X_i)_{\max}$, ($i$ = 1, 2, …, n), in which X_i (i = 1, 2, …, n) are the design variables, $(X_i)_{\min}$ and $(X_i)_{\max}$ are the minimum and maximum values of the corresponding variables, respectively. Take three values for each variable in the region as shown in Fig. 5 with full circles, two of them at the ends and the third at the midpoint. Make a search for the optimum among all possible combinations of computation points, which satisfies the constraints.
2. Reduce the solution region to half the width of the previous one as shown in Fig. 6. Take the range for each variable to be equal to half the range of the previous step, with two additional computation points on the two sides of the previous optimizing computation point while

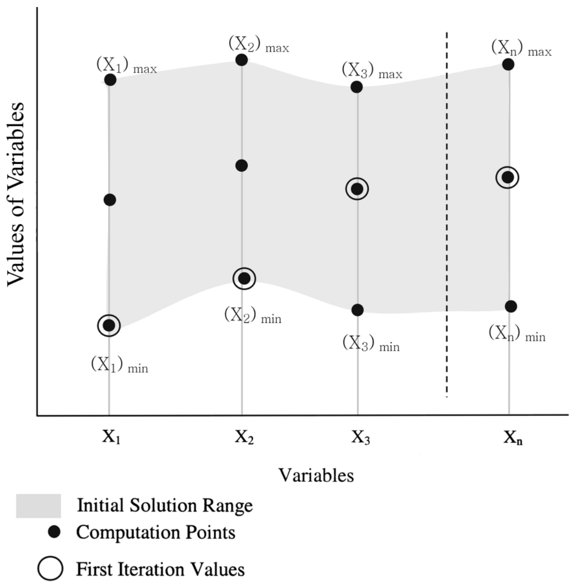

Fig. 5 First iteration results

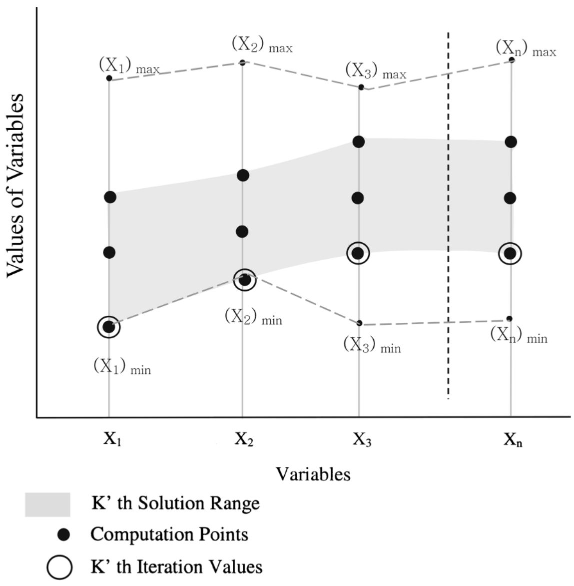

Fig. 6 *K*'th iteration results

making sure to remain within the initial solution range.

3. Repeat step 2 until the variable set is obtained with the desired accuracy. For the accuracy criterion, different options can be used; a fixed percentage accuracy for each variable or a percentage accuracy in the least squares sense, etc. In the present study, fixed percentage accuracy criterion has been employed. That is to say, computation is continued until $\varepsilon_i \leq (\varepsilon_i)_{accuracy}$ $(i = 1, 2, \ldots, n)$, where $\varepsilon_i = \dfrac{(X_i)_{K+1} - (X_i)_K}{(X_i)_K}$, $(i = 1, 2, \ldots, n)$ and $(\varepsilon_i)_{\text{accuracy}}$ are the limits of the respective variables.

4. Numerical examples

Four examples are presented. The first example presents the optimization of a simply supported beam for a least weight design of concrete and is based on an example first discussed by Khachaturian and Gurfinkel (1969), and later by Taylor and Amirebrahimi (1987).

In the second example general I-shaped cross-section is used instead of idealized section of Example 1.

Third and fourth examples use the same span and loading as Example 1, but are presented as cost optimization. The third example presents cost optimization of Example 1 and the fourth example presents the cost optimization of the general I-shaped cross-section with eight dimensions for the cross-section of Example 3.

4.1 Example 1

Design a simply supported beam of 16460 mm span subjected to an applied load of 23.34 kN/m,

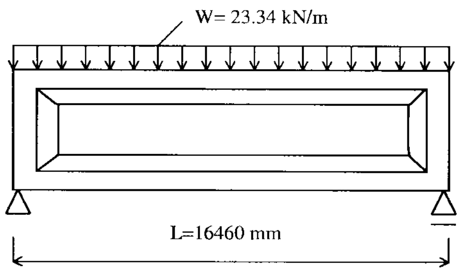

Fig. 7 I Beam elevation

as shown in Fig. 7. The beam is to be pretensioned without non-prestressed reinforcement. Assume the unit weight of concrete is 24 kN/m^3, allowable stresses for compression are at transfer 16.55 MPa, at service 15.51 MPa, those for tension are at transfer –1.31 MPa, at service –2.93 MPa and loss factor is 0.85, f_c' = 34 MPa, f_{pu} = 1862 MPa, and e_{cover} = 50 mm. For this problem, also assume that, because of clearance requirements, the overall depth of the beam, h, cannot exceed 914.4 mm (36 in). Find the minimum cross-section of the beam.

This example was discussed first by Khachaturian and Gurfinkel (1969), and later by Taylor and Amirebrahimi (1987). Khachaturian and Gurfinkel (1969) used the 1963 ACI Building Code. According to the code, allowable stresses are 16.55 MPa and –1.31 MPa in compression and tension, respectively, at transfer stage. Taylor and Amirebrahimi (1987) have taken these values as 13.24 MPa and –0.98 MPa, respectively. Moreover, they did not consider the constraint for the section depth, which should not exceed the value of 914.4 mm.

To obtain the minimum weight design, the cost of prestressing force and the cost of formwork were taken to be zero, and the cost of concrete was taken to be 1 \$/m^3, since it has no significance for this example.

If the program is run with the input data given above, the numerical procedure converges to the optimum solution in 9 iterations (Fig. 8). The solutions are presented in Tables 2-3 and the

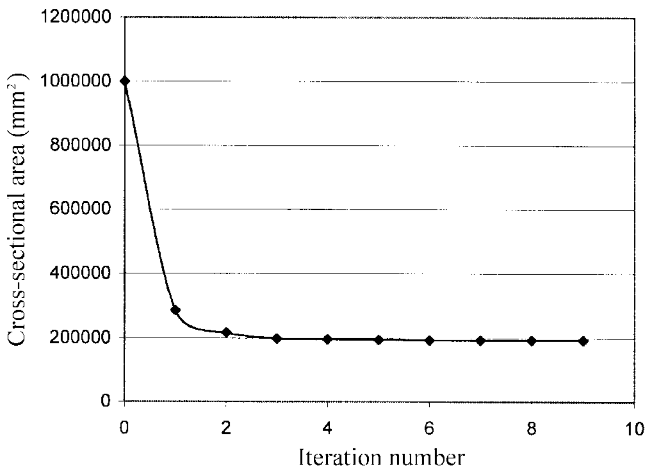

Fig. 8 Cross-sectional area versus iteration number for Example 1

Table 2 Variables, initial values and results for Example 1

Variables	Minimum values (mm)	Maximum values (mm)	Initial values (mm)	Optimum results (mm)
X1	101.6	600.0	350.8	552.3
X2	101.6	600.0	350.8	463.7
X3	101.6	400.0	250.8	101.6
X4	101.6	400.0	250.8	111.5
X5	50.0	150.0	100.0	137.5
X6	400.0	900.0	650.0	664.6

Table 3 Optimum results, the stresses at transfer and service for Example 1

Stage	Location	Stress (MPa)	Allowable stress (MPa)	*P* (kN)	*e* (mm)	Cross-sectional area (mm^2)
Transfer	Top	−1.31	−1.31	1447.0	406.7	192876
	Bottom	16.48	16.55			
Service	Top	15.51	15.51			
	Bottom	−2.93	−2.93			

Table 4 Comparison of the results

	*X*1 (mm)	*X*2 (mm)	*X*3 (mm)	*X*4 (mm)	*X*5 (mm)	*X*6 (mm)	*P* (kN)	*e* (mm)	Cross-sectional area (mm^2)
Khachaturian and Gurfinkel	457.2	439.4	137.2	137.2	152.4	624.8	1650	360.6	219999
Taylor and Amirebrahimi	558.8	457.2	101.6	101.6	177.8	660.4	1673	370.8	205160
Present Study	552.3	463.7	101.6	111.5	137.5	664.6	1447	406.7	192876

comparison of the results with the previous studies is presented in Table 4. Magnel diagram is shown in Fig. 9 and stress diagrams are shown in Fig. 10.

As seen from Table 4, the program gives the minimum cross-sectional area and the minimum prestressing force to be 192876 mm^2, and 1447 kN, respectively. The cross-sectional area is found 14% less than the solution of Khachaturian and Gurfinkel (1969), and 6% less than the solution given by Taylor and Amirebrahimi (1987). The overall depth of the beam is found to be 913.6 mm, which had to be less than 914.4 mm. But, Taylor and Amirebrahimi (1987) obtained this value as 939.8 mm, which exceeded the aforementioned constraint.

In the present study the prestressing force is found to be 14% less than the solution of Khachaturian and Gurfinkel (1969), and 15% less than the solution given by Taylor and Amirebrahimi (1987). The resultant stresses are found to be nearly equal to the allowable stresses. For this reason, feasible region in Magnel diagram shrinks to a point (Fig. 9).

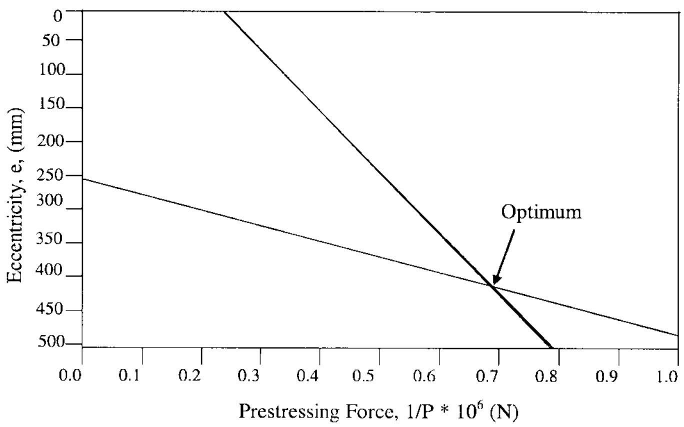

Fig. 9 Magnel diagram for Example 1

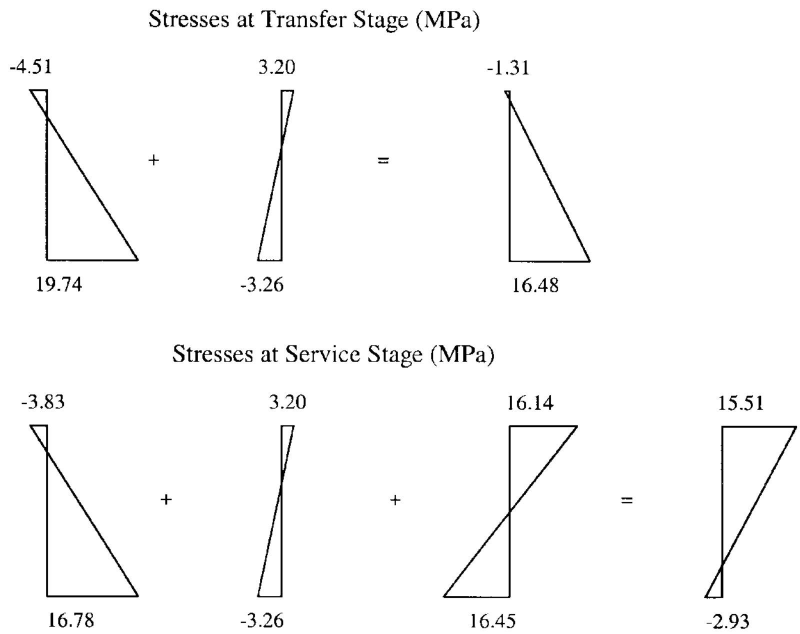

Fig. 10 Stress diagram for Example 1

4.2 Example 2

This example has the same span and loading as Example 1, but the cross-section has a general I shape (Fig. 2).

To obtain optimum solution, *m* is chosen in the range 0 to 1.5. The optimum values are obtained when m = 0.6. The solutions are given in Tables 5 and 6. The cross-sectional area with eight dimensions is found to be 211820 mm^2, which is 10% more than the solution of idealized section considered in Example 1. The prestressing force is 1571 kN which is also 8.5% more than the value obtained for the idealized cross-section.

4.3 Example 3

This example has also the same span and loading as Example 1, but minimum cost design is carried out in this example. Cost of concrete, cost of prestressing force and cost of formwork are taken as 145 \$/m^3, 60 \$/kN and 10 \$/m^2, respectively, (these values are obtained from local suppliers).

The results are shown in Table 7. The total cost of the beam is found to be \$1182.5 and the area of the cross-section of the beam is found to be 215743 mm^2, the latter of which is 12% greater than the value obtained in Example 1. The prestressing force is found to be 1594 kN, which is 10% greater than the prestressing force obtained in Example 1.

4.4 Example 4

This example is the same as Example 2. But in this example, the aim of the objective function is to find the minimum cost. The results are shown in Table 8. Total cost of the beam is found to be \$1259.5 The area of the cross-section of the beam is found to be 249112 mm^2, which is 18% greater than the value obtained in Example 2. The prestressing force is found to be 1783 kN which is also 13% greater than the value obtained in Example 2.

Table 5 Optimum results, the stresses at transfer and service for Example 2

Stage	Location	Stress (MPa)	Allowable stress (MPa)	P (kN)	e (mm)	Cross-sectional area (mm^2)
Transfer	Top	–1.31	–1.31	1571	382.9	211820
	Bottom	16.19	16.55			
Service	Top	15.48	15.51			
	Bottom	–2.93	–2.93			

Table 6 Optimum results of the variables for Example 2

$X1$ (mm)	$X2$ (mm)	$X3$ (mm)	$X4$ (mm)	$X5$ (mm)	$X6$ (mm)	$X7$ (mm)	$X8$ (mm)	P (kN)	e (mm)	Cross-sectional area (mm^2)
475.4	413.1	101.6	101.6	150.0	444.4	112.1	93.5	1571	382.9	211820

Table 7 Optimum results for Example 3

$X1$ (mm)	$X2$ (mm)	$X3$ (mm)	$X4$ (mm)	$X5$ (mm)	$X6$ (mm)	$X7$ (mm)	$X8$ (mm)	P (kN)	e (mm)	Cross-sectional area (mm^2)
475.4	475.4	138.9	138.9	125.0	448.0	101.0	101.0	1594	383.1	215743

Table 8 Optimum results for Example 4

$X1$ (mm)	$X2$ (mm)	$X3$ (mm)	$X4$ (mm)	$X5$ (mm)	$X6$ (mm)	$X7$ (mm)	$X8$ (mm)	P (kN)	e (mm)	Cross-sectional area (mm^2)
475.4	475.4	176.2	101.6	125.0	470.0	90.0	90.0	1783	349.3	249112

5. Conclusions

The computer program, which is developed and used here, offers a rapid, practical, and interactive method for realizing optimum design of beams of general I, T, double T and inverted T sections. 28 constraints have been used including flexural stresses, cover requirement, the aspect ratios for top and bottom flanges and web part of a beam and ultimate moment. A modified grid search method has been used to minimize the cost, considering costs of concrete, prestressing steel and formwork. To carry out a minimum weight design, what needs to be done is, just, give zero unit costs for the two latter materials and any finite value for that of first.

The program finds the optimum solution in a few iterations. Thus, a considerable saving is obtained in computational work.

A numerical example, which has been introduced first by Khachaturian and Gurfinkel (1969) and later also solved by Taylor and Amirebrahimi (1987), was solved in this study and a 14% smaller cross-sectional area and a 14% smaller prestressing force were obtained.

The constraint about the section depth, h, has not been considered by the latter authors and their results exceeded the maximum section depth of the former authors. Obviously, this constraint has a very important effect on the results.

The idealized I beam section with six dimensions gives 10% smaller value compared with general I-shaped beam section with eight dimensions. In the case of general I section, taking 0.6 for the slope, m, yields the minimum cross-sectional area.

The examples have shown that the cross-sectional area of the prestressed concrete beam of idealized and general I-shaped cross-sections employing minimum cost design are 12% and 18% greater than the corresponding values obtained by using minimum weight design.

The program has also graphical output, which indicates Magnel diagram with feasible region, which helps in determining the prestressing force and the eccentricity values. The stress distribution diagrams corresponding to the optimum design of the member are also presented. Due to space limitation the listing of the program is not given in the paper. PC version and the manual of the program can be obtained free of charge from the authors upon request.

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