DOI: http://dx.doi.org/10.12989/sem.2002.14.6.733
Structural Engineering and Mechanics, Vol. 14, No. 6 (2002) 733-738

Technical Note

Transverse earthquake-induced forces in continuous bridges

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(Received August 1, 2001, Accepted October 21, 2002)

Abstract. A simplified rational method is developed to evaluate transverse earthquake-induced forces in continuous bridges. This method models the bridge as a beam on elastic foundation, and assumes a sinusoidal curve for both vibration mode shape and deflected shape in the transverse direction. The principle of minimum total potential is used to calculate the displacements and the earthquake-induced forces in the transverse direction. This method is concise and easy to apply, and hence, offers an attractive alternative to a lengthy and time consuming three dimensional modeling of the bridge as given by AASHTO under its Single Mode Spectral Analysis Method.

Key words: earthquake forces; bridge; transverse forces; seismic design; continuous bridge.

1. Introduction

Evaluation of earthquake-induced forces in bridges is given by the American Association of State Highway and Transportation Officials, AASHTO, in Division I-A (AASHTO 1992). The Single Mode Spectral Analysis, SMSA, is the most used method to evaluate earthquake-induced forces in the transverse direction.

Briefly, AASHTO procedures for SMSA method requires the application of a uniform load in the transverse direction to find a corresponding deflected shape using three dimensional space frame analysis. The resulting deflected shape is then used to calculate the period and the earthquake-induced elastic forces in the bridge using some parameters, α, β, and γ which require explicit integration operations (AASHTO 1992) (Buckle, Mayes, and Button 1987). Proportional to this resulting deflected shape which is assumed to be the vibration mode shape, earthquake-induced forces are applied to the bridge accordingly. In order to find the internal elastic forces in the bridge components, mainly, the substructure and the foundations, a three dimensional space frame analysis is needed again.

The SMSA method can be greatly simplified without relying on three dimensional space frame analysis by using the principle of minimum total potential in conjunction with sinusoidal deflected shape (mode of vibration). The sinusoidal mode shape offers the advantage of being readily integrable. Furthermore, the actual vibration modes of beams are sinusoidal (Clough and Penzien 1993).

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2. Problem formulation

The development of the problem may be demonstrated using the example given by AASHTO in its commentary as a reference. The AASHTO example consists of three span continuous deck slab supported directly by two abutments A1 and A2; and two intermediate bents b1 and b2 as shown in Fig. 1. The bridge dimensions are identified as shown in Fig. 1. Each bent consists of three circular columns fixed to the slab at the top and also fixed at the foundation level. As a beam on elastic foundation, the bridge is modeled as a beam, simply supported at A1 & A2; having elastic supports at bent locations with equivalent stiffness at the bent-deck connection as shown in Fig. 2.

With these assumptions, moment of inertia of the bridge deck in the transverse direction is given as $I_d = t.B^3/12$. The equivalent stiffness of the fixed-fixed bent, k_b, is given as:

$$k_b = \sum 12\, EI_c/h^3 \tag{1}$$

AASHTO steps may be followed by applying a uniform load to the bridge in the transverse direction. The resulting deflected shape may now be assumed to be sinusoidal as $v_s = v_o \sin \pi x/L$ as shown in Fig. 2.

The principle of minimum total potential is used to calculate v_o. The total potential, Π, is given as the summation of the strain energy, U, and the potential energy, V (Chen and Lui 1987). The strain energy is the result of the deformation of the deck, U_d, and the deformation of the bents, U_b,

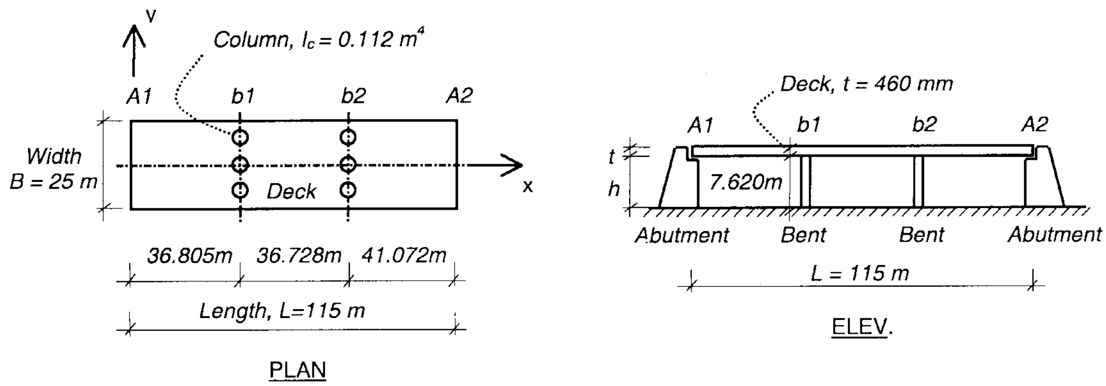

Fig. 1 Bridge layout and dimensions

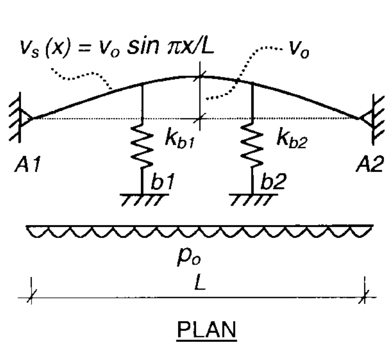

Fig. 2 Statical system of the bridge under transverse uniform load, p_o

whereas the potential energy is given as the negative of the work done by the external loads, W_{po}. These quantities may be evaluated as

$$U_d = \int \frac{1}{2} EI_d (v'')^2 dx = EI_d/2 \int (v_o)^2 (\pi/L)^4 \sin^2(\pi x/L) dx$$
$$= \frac{1}{4} EI_d (\pi^4/L^3)(v_o)^2 \tag{2}$$

$$U_b = \sum \frac{1}{2} k_b (v_b)^2 = \sum \frac{1}{2} k_b (v_o)^2 \sin^2(\pi x_b/L) \tag{3}$$

$$w_{po} = \int p_o v_s dx = p_o \int v_o \sin(\pi x/L) dx = (2p_o L/\pi) v_o \tag{4}$$

Adding Eq. (2), Eq. (3), and Eq. (4), we get the total potential energy, Π, as

$$\Pi = U + V = U_d + U_b - W_{po}$$

v_o can be found by taking the first derivative of Π with respect to v_o, i.e., $\partial \Pi / \partial v_o = 0$.

Once v_o is found, and noting that in most cases, bridges have uniform distribution of mass (weight) along their longitudinal axes, i.e., $w(x) = w_o$, the parameters α, β, and γ as given by AASHTO can be calculated as

$$\alpha = \int v_s(x) dx = \int v_o \sin(\pi x/L) dx = 2Lv_o/\pi$$

$$\beta = \int w(x) v_s(x) dx = w_o v_o \int \sin(\pi x/L) dx = 2L w_o v_o/\pi$$

$$\gamma = \int w(x) v_s(x)^2 dx = w_o (v_o)^2 \int \sin^2(\pi x/L) dx = w_o (v_o)^2 L/2$$

Substituting the above values in the period, T, and elastic force, p_e, expressions as given by AASHTO, we get

$$T = 2\pi \sqrt{\frac{\gamma}{p_o g \alpha}} = \sqrt{\frac{\pi^3 w_o v_o}{p_o g}} \tag{5}$$

$$p_e = (\beta C_s/\gamma) w(x) v_o \sin(\pi x/L)$$
$$= (4/\pi) C_s w_o \sin(\pi x/L) = p_{eo} \sin(\pi x/L) \tag{6}$$

where C_s is a normalized acceleration response spectrum. C_s is given by the following expression as recommended by NEHRP (FEMA 1994)

$$C_s = 1.2\, AS/T^{2/3} \leq 2.5A \tag{7}$$

where A and S are the seismic zone factor and soil profile parameter respectively.

The force in the columns due to the earthquake loading p_e can be also found by similar simplified procedures making use of the sinusoidal deflected shape again. By applying the earthquake load, $p_e = p_{eo} \sin(\pi x/L)$, to the bridge as shown in Fig. 3, the resulting deflected shape, v_e, due to p_e is also assumed to be sinusoidal, i.e., $v_e = v_{eo} \sin(\pi x/L)$. Similar to the calculations of v_o, principle of minimum total potential energy is used to find v_{eo}

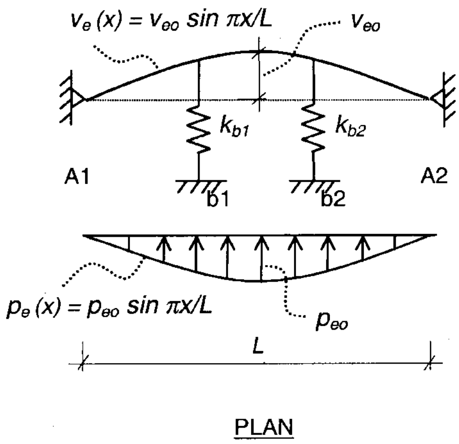

Fig. 3 Bridge deflection under earthquake-induced forces, p_e

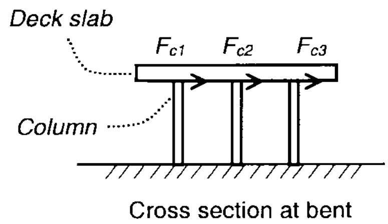

Fig. 4 Earthquake-induced forces in columns

$$U_d = \frac{1}{4} EI_d (\pi^4/L^3)\,(v_{eo})^2 \tag{8}$$

$$U_b = \sum \frac{1}{2}\, k_b (v_{eo})^2 \sin^2(\pi x_b/L) \tag{9}$$

The work done by p_e is calculated as

$$W_{pe} = \int p_e v_e dx = \int p_{eo} \sin(\pi x/L) v_{eo} \sin(\pi x/L) dx$$
$$= p_{eo} v_{eo}\, L/2 \tag{10}$$

Adding Eq. (8), Eq. (9), and Eq. (10), we get

$$\Pi = U_d + U_b - W_{pe}$$

v_{eo} can be found by taking the first derivative of Π with respect to v_{eo}, i.e., $\partial \Pi / \partial v_{eo} = 0$

Finally, the elastic forces in the in the bent (columns) as shown in Fig. 4 are calculated as

$$F_b = k_b \cdot v_e(x) = k_b \cdot v_{eo} \sin(\pi x/L) \tag{11}$$

$$F_c = k_c \cdot v_e(x) = k_c \cdot v_{eo} \sin(\pi x/L) \tag{12}$$

3. Application of the method

The ease of application and the accuracy of the results may be demonstrated using the dimensions of the same example given by AASHTO as a reference for comparison purposes.

Relevant parameters and properties (taken directly from AASHTO) are:

Seismic parameters:	$A = 0.4$, $S = 1.2$
Modulus of Elasticity:	$E_c = 20700$ MPa

Deck parameters: $I_d = 566\ \text{m}^4$, $L = 115$ m, $B = 25$ m
$w(x)$ = constant = $w_o = 296$ kN /m
Column parameters: $I_c = 0.112\ \text{m}^4$, $h = 7.620$ m

Calculation of initial displacement due to a unit of uniformly distributed force ($p_o = 1$ kip/ft) which is equivalent to 14.59 kN /m will be as follows:

Using Eq. (2): $U_d = 187600\,(v_o)^2$ kN-m
Using Eq. (1): $k_b = 188640$ kN/m
Using Eq. (3): $U_b = 144600\,(v_o)^2$ kN-m
Using Eq. (4): $W_{po} = 1068\ v_o$ kN-m
Accordingly, $\Pi = 187600\,(v_o)^2 + 144600(v_o)^2 - 1068\ v_o$
$\partial\Pi / \partial v_o = 664400 v_o - 1068 = 0$
Resulting in, $v_o = 1.6$ mm vs (1.5 mm in AASHTO example)
Using Eq. (5): $T = 0.310$ sec vs (0.314 in AASHTO)
Using Eq. (7): $C_s = 1.23$
Using Eq. (6): $p_e = 377 \sin(\pi x/L)$ kN/m

Note that p_e is the earthquake-induced forces in the bridge which is proportional to the vibration mode. To find the displacements and the forces in the bridge due to this load, p_e, minimum total potential is utilized again. In this case, the quantities U_d and U_b are readily available by replacing v_o by v_{eo} in the previous v_o calculations, hence

Using Eq. (8): $U_d = 187600\,(v_{eo})^2$ kN-m
Using Eq. (9): $U_b = 144600\,(v_{eo})^2$ kN-m
Using Eq. (10): $W_{pe} = 21678\ v_{eo}$ kN-m
Accordingly, $\Pi = 187600\,(v_{eo})^2 + 144600\,(v_{eo})^2 - 21678\ v_{eo}$
$\partial\Pi / \partial v_{eo} = 664400\ v_{eo} - 21678 = 0$
Resulting in, $v_{eo} = 32.6$ mm vs (31.1 mm in AASHTO example)

The forces in the columns are calculated as function of their stiffness, i.e.,

For bent #1, using Eq. (12): $F_c = 1752$ kN vs (1761 kN in AASHTO example)
For bent #2, using Eq. (12): $F_c = 1879$ kN vs (1886 kN in AASHTO example)

4. Conclusions

Evaluation of earthquake-induced forces in the transverse direction in continuous bridges can be greatly simplified without sacrificing the degree of accuracy offered by AASHTO procedures for the Single Mode Spectral Analysis method. This simplification is attained by assuming that both vibration mode shape and deflected shape in the transverse direction are described by sinusoidal curves. The bridge is modeled as a beam on elastic foundation where the principle of minimum total potential is used to formulate this problem. A great advantage can be taken of the properties of the sinusoidal curves in the integration schemes. Together, these two elements yield final results in simple expressions for period parameters, earthquake-induced forces, and deflection that can be handled with simple hand calculations. It has been demonstrated that the application of this approach is very simple and yet preserves the degree of accuracy given by AASHTO procedures.

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