Structural Engineering and Mechanics, Vol. 14, No. 6 (2002) 661-677
DOI: http://dx.doi.org/10.12989/sem.2002.14.6.661

The buckling of a cross-ply laminated non-homogeneous orthotropic composite cylindrical thin shell under time dependent external pressure

A.H. Sofiyev†

Department of Civil Engineering, Suleyman Demirel University, Isparta, Turkey

(Received May 28, 2002, Accepted October 4, 2002)

Abstract. The subject of this investigation is to study the buckling of cross-ply laminated orthotropic cylindrical thin shells with variable elasticity moduli and densities in the thickness direction, under external pressure, which is a power function of time. The dynamic stability and compatibility equations are obtained first. These equations are subsequently reduced to a system of time dependent differential equations with variable coefficients by using Galerkin's method. Finally, the critical dynamic and static loads, the corresponding wave numbers, the dynamic factors, critical time and critical impulse are found analytically by applying a modified form of the Ritz type variational method. The dynamic behavior of cross-ply laminated cylindrical shells is investigated with: a) lamina that present variations in the elasticity moduli and densities, b) different numbers and ordering of layers, and c) external pressures which vary with different powers of time. It is concluded that all these factors contribute to appreciable effects on the critical parameters of the problem in question.

Key words: buckling; non-homogeneous; cross-ply laminated; orthotropic shell; external pressure; dynamic critical load; dynamic factor; critical impulse; wave number.

1. Introduction

The improvements of the strength properties of materials used in the production of structural elements in contemporary technology aims at decreasing their sizes and weights. In this way it is essential that computation methods take the actual behavior of materials into consideration. This fact has drawn the attention of researchers to the elasticity problems of objects made of non-homogeneous material in the last decades, see e.g., Lomakin (1976). The non-homogeneity of materials stems from production techniques, surface and thermal polishing processes, effect of radiation, etc. They cause the physical properties of materials to change from point to point as continuous functions of the coordinates. Depending on the production method and the geometry of the structural members, the dependence on the elastic properties can be given by different functions for different coordinates. In actual engineering applications, the variation of the elastic properties of materials remains in a bounded range and small enough, necessitating a restriction on the variation functions. Researchers have given this restriction in different ways.

† Professor

In the referenced works, and in most of available solutions to elastic non-homogoneity, it is assumed that the material is isotropic or orthotropic, the Poisson's ratio is constant, and the elasticity moduli or density is either an exponential or a power function of a spatial variable (Massalas *et al.* 1982, Heyliger and Juliani 1992, Tarn 1994, Mecitoglu 1996, Wang *et al.* 1998, Guiterrez *et al.* 1998, Zhang and Hasebe 1999, Elishakoff 2001, Aksogan and Sofiyev 2000, 2002, Sofiyev and Aksogan 2002).

Laminated structural elements composed of non-homogeneous materials with different elastic properties are frequently used in contemporary engineering applications. The wide use of laminated structural elements is due to the progress in the manufacturing of new composites, leading to materials with the capability of attaining desired strengths and stiffness for specific applications see Reddy (1997). Most materials in nature are of laminated texture. Structural elements made of such materials can be constituted of many thin layers. The theory of such structural elements can be considered as an extension of the classical theory of plates and shells (see Ambartsumian 1964, Vinson and Sierakowski 1986).

In recent years, the vibration, buckling and dynamic stability problems of cross-ply laminated anisotropic shells, using different thin shell theories, has been studied by Jones and Morgan (1975), Soldatos and Tzivanidis (1982), Tong *et al.* (1992, 1993) Argento and Scott (1993), Ng *et al.* (1998, 1999), Greenberg and Stavsky (1980, 1998), Mao *et al.* (1999) and Park *et al*. (2001). In these problems, generally the dynamic stability problem has been solved numerically. This type of solutions necessitate huge amount of computations and yield results, showing peculiar characteristics.

As pointed above, there are many solution methods in order to obtain the static critical loads at different loading and limit conditions, which are consistent with experiments. Research publications on cylindrical shells made of non-homogeneous composite materials under an external pressure, which is a power function of time are very limited in number because of the complexities encountered during manufacture and theoretical analysis; in the case of laminated shells such complexities are further increased. One such problem, not considered till today, is the buckling of laminated orthotropic composite cylindrical shells under the effect of external pressure, which is a power function of time. The solution of a dynamic problem is reduced to the determination of the dynamic factor for certain loading cases. The dynamic factor can be found, using different methods, depending on the manner in which the loading is applied, particularly on the loading speed (Shumik 1970, Ogibalov *et al.* 1975, Sachenkov and Baktieva 1978, Yakushev 1990 and Tazyukov 1991).

The aim of the present research is to study the buckling problem of cross-ply laminated cylindrical thin shells, made of orthotropic composite materials with elasticity moduli and densities varying piecewise continuously in the thickness direction, subjected to external pressure varying as a power function of time, by using the Ritz type variational method.

2. Problem formulation

Consider a circular cylindrical shell as shown in Fig. 1, is assumed to be thin, cross-ply laminated and composed of an N layers of equal thickness δ of non-homogeneous orthotropic composite materials perfectly bonded together. The shell thickness is denoted by $2h$ and the radius to the middle surface by R. The Ox and Oy axes are in the middle plane of the shell in the axial and tangential directions, respectively, and the Oz axis normal to them. The axes of orthotropy in all layers are parallel to Ox and Oy axes. The cross-ply laminates are composed of laminas (plies) with

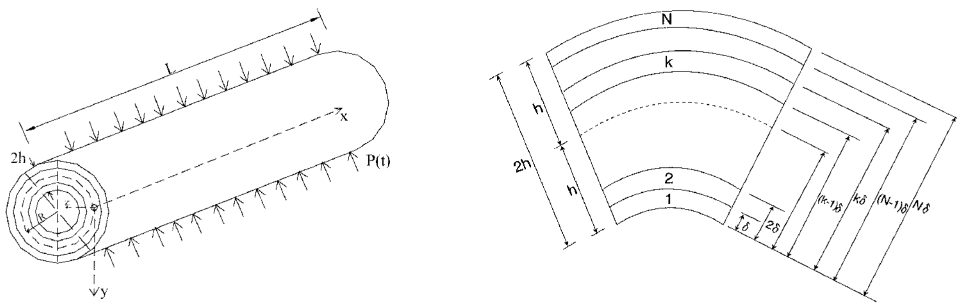

Fig. 1 Geometry and the cross-section of a cylindrical thin shell with *N* layers

their principal material directions (one being the fiber direction) aligned with the axial Ox-axis and the circumferential Oy-axis of the shell. That is, the fibers in one layer are aligned in the axial direction, whereas the fibers in the next layer are aligned in the circumferential direction. Theoretically any sequence of orientations between 0^o (Ox-direction) and 90^o (Oy-direction) can be considered (Jones and Morgan 1975).

The contact condition between any two consecutive layers is one of perfectly rigid bonding, ensuring the satisfaction of the Kirchhoff-Love hypothesis for the whole shell, meaning that there is a single displacement and a single strain expression for the whole shell and that the pressures at the contact surfaces do not need any particular attention. During the deformation there is no slip and no loss of contact between the contact surfaces of the layers. The elasticity moduli and densities of all layers are defined as continuous functions of the thickness coordinate z as:

$$[E_1^{(k+1)}(\bar{z}), E_2^{(k+1)}(\bar{z}), G^{(k+1)}(\bar{z})] = \bar{\varphi}_1^{(k+1)}(\bar{z})[E_{01}^{(k+1)}, E_{02}^{(k+1)}, G_0^{(k+1)}],$$
$$\rho^{(k+1)}(\bar{z}) = \rho_0^{(k+1)}\bar{\varphi}_2^{(k+1)}(\bar{z}), \quad -1 + k\bar{\delta} \le \bar{z} \le -1 + (k+1)\bar{\delta}, \quad \bar{\delta} = \delta/h, \quad \bar{z} = z/h,$$
$$k = 0, 1, 2, \ldots, (N-1) \tag{1}$$

where $E_{01}^{(k+1)}$ and $E_{02}^{(k+1)}$ are the elasticity moduli in the x and y directions for the layer $k+1$, respectively, $G_0^{(k+1)}$ is the shear modulus on the plane of the layer $k+1$, and $\rho_0^{(k+1)}$ is the density of the homogeneous material for the layer $k+1$, $\delta = 2h/N$ is the thickness of the layers. Additionally

$$\bar{\varphi}_i^{(k+1)}(\bar{z}) = 1 + \mu\varphi_i^{(k+1)}(\bar{z}), \quad i = 1, 2 \tag{2}$$

where $\varphi_i^{(k+1)}(\bar{z})$ are continuous functions giving the variations of the elasticity moduli and densities in the layers, satisfying the condition $\left|\varphi_i^{(k+1)}(\bar{z})\right| \le 1$, and μ is a variation coefficient satisfying $0 \le \mu < 1$. The middle surface $z = 0$ is located at a layer interface for even values of N, whereas for odd values of N the middle surface is located at the center of the middle layer.

The shell is subjected to a external pressure varying as a power function of time in the form (Yakushev 1990, Aksogan and Sofiyev 2002):

$$N_{11}^0 = 0, \quad N_{22}^0 = -R(P_1 + P_0 t^{\alpha}), \quad N_{12}^0 = 0 \tag{3}$$

where N_{11}^0, N_{22}^0 and N_{12}^0 are the membrane forces for the condition with zero initial moments, P_1 is static external pressure, P_0 is the loading parameter, t is time and $\alpha \ge 1$ is the power expressing the time dependence of the pressure.

According to the shell theory, the stress-strain relations for a thin cross-ply laminated layer are given as follows

$$\begin{pmatrix} \sigma_{11}^{(k+1)} \\ \sigma_{22}^{(k+1)} \\ \sigma_{12}^{(k+1)} \end{pmatrix} = \begin{bmatrix} Q_{11}^{(k+1)} & Q_{12}^{(k+1)} & 0 \\ Q_{12}^{(k+1)} & Q_{22}^{(k+1)} & 0 \\ 0 & 0 & Q_{33}^{(k+1)} \end{bmatrix} \begin{pmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{12} \end{pmatrix} \tag{4}$$

where $\sigma_{ij}^{(k+1)}$, $i, j = 1, 2, 3$ are the stresses in the layers. The quantities $Q_{ij}^{(k+1)}$, $i, j = 1, 2, 3$ for orthotropic lamina are

$$Q_{11}^{(k+1)} = \frac{E_{01}^{(k+1)}\overline{\varphi}_1^{(k+1)}(\overline{z})}{1-\nu_{12}^{(k+1)}\nu_{21}^{(k+1)}}, \quad Q_{22}^{(k+1)} = \frac{E_{02}^{(k+1)}\overline{\varphi}_1^{(k+1)}(\overline{z})}{1-\nu_{12}^{(k+1)}\nu_{21}^{(k+1)}}, \quad Q_{12}^{(k+1)} = \nu_{21}^{(k+1)}Q_{11}^{(k+1)} = \nu_{12}^{(k+1)}Q_{22}^{(k+1)},$$

$$Q_{33}^{(k+1)} = 2G_0^{(k+1)}\overline{\varphi}_1^{(k+1)}(\overline{z}), \quad k = 0, 1, 2, \ldots, (N-1) \tag{5}$$

where $\nu_{12}^{(k+1)}$ and $\nu_{21}^{(k+1)}$ are the Poisson's ratios, assumed to be constant.

By Love's first approximation theory the strain-displacement relations are given by

$$(\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{12}) = (\varepsilon_{11}^0, \varepsilon_{22}^0, \varepsilon_{12}^0) + z(\chi_{11}, \chi_{22}, \chi_{12}) \tag{6}$$

where ε_{11}^0 and ε_{22}^0 are the normal strains in the curvilinear coordinate directions x and y on the middle surface, respectively, whereas ε_{12}^0 is the corresponding shear strain; χ_{11} and χ_{22} are the curvatures of the deformed shell in the directions x and y, respectively, whereas χ_{12} is the twist of the middle surface. The last three entities are given by

$$(\chi_{11}, \chi_{22}, \chi_{12}) = \left(-\frac{\partial^2 W}{\partial x^2}, -\frac{\partial^2 W}{\partial y^2}, -\frac{\partial^2 W}{\partial x \partial y}\right) \tag{7}$$

where W is the small incremental displacement of the middle surface in the normal direction. The well known force and moment resultants are expressed by (Ambartsumian 1964, Leissa 1973, Jones and Morgan 1975, Vinson and Sierakowski 1986)

$$[(N_{11}, N_{22}, N_{12}), (M_{11}, M_{22}, M_{12})] = \sum_{k=0}^{N-1} \int_{-h+k\delta}^{-h+(k+1)\delta} (1, z)(\sigma_{11}^{(k+1)}, \sigma_{22}^{(k+1)}, \sigma_{12}^{(k+1)})dz \tag{8}$$

The relations between the forces N_{11}, N_{22} and N_{12} and the stress function Φ are given by

$$(N_{11}, N_{22}, N_{12}) = \left(\frac{\partial^2 \Phi}{\partial y^2}, \frac{\partial^2 \Phi}{\partial x^2}, -\frac{\partial^2 \Phi}{\partial x \partial y}\right) \tag{9}$$

The dynamic stability and compatibility equations of a cylindrical shell are given respectively by (Agamirov 1990)

$$\frac{\partial^2 M_{11}}{\partial x^2} + 2\frac{\partial^2 M_{12}}{\partial x \partial y} + \frac{\partial^2 M_{22}}{\partial y^2} + \frac{N_{22}}{R} + N_{11}^0 \frac{\partial^2 W}{\partial x^2} + 2N_{12}^0 \frac{\partial^2 W}{\partial x \partial y} + N_{22}^0 \frac{\partial^2 W}{\partial y^2} = \tilde{\rho} \frac{\partial^2 W}{\partial t^2} \tag{10}$$

$$\frac{\partial^2 e_{11}}{\partial y^2} + \frac{\partial^2 e_{22}}{\partial x^2} - 2\frac{\partial^2 e_{12}}{\partial x \partial y} = -\frac{1}{R}\frac{\partial^2 W}{\partial x^2} \tag{11}$$

where

$$\tilde{\rho} = h \sum_{k=0}^{N-1} \rho_0^{(k+1)} \int_{-1+2k/N}^{-1+2(k+1)/N} \bar{\varphi}_2^{(k+1)}(\bar{z}) d\bar{z} \tag{12}$$

Substituting expressions (3-9) in Eqs. (10-11) a system of differential equations for the stress function Φ and the normal displacement of the middle surface W can be obtained

$$c_{12}\frac{\partial^4 \Phi}{\partial x^4} + (c_{11} - 2c_{31} + c_{22})\frac{\partial^4 \Phi}{\partial x^2 \partial y^2} + c_{21}\frac{\partial^4 \Phi}{\partial y^4} - c_{13}\frac{\partial^4 W}{\partial x^4} - (c_{14} + 2c_{32} + c_{23})\frac{\partial^4 W}{\partial x^2 \partial y^2}$$

$$-c_{24}\frac{\partial^4 W}{\partial y^4} + \frac{1}{R}\frac{\partial^2 \Phi}{\partial x^2} - R(P_1 + P_0 t^\alpha)\frac{\partial^2 W}{\partial y^2} = \tilde{\rho}\frac{\partial^2 W}{\partial t^2} \tag{13}$$

$$b_{22}\frac{\partial^4 \Phi}{\partial x^4} + (b_{12} + 2b_{31} + b_{21})\frac{\partial^4 \Phi}{\partial x^2 \partial y^2} + b_{11}\frac{\partial^4 \Phi}{\partial y^4} - b_{23}\frac{\partial^4 W}{\partial x^4}$$

$$-(b_{13} - 2b_{32} + b_{24})\frac{\partial^4 W}{\partial x^2 \partial y^2} - b_{14}\frac{\partial^4 W}{\partial y^4} = -\frac{1}{R}\frac{\partial^2 W}{\partial x^2} \tag{14}$$

where b_{ij}, c_{ij}, $i, j = 1, 2, 3, 4$ are given in the Appendix.

3. Solution of the differential equations

Assuming the cylindrical shell to have simply supports at the ends, the solution of equation set (13-14) in general is sought in the following form (Volmir 1967):

$$W = \sum_m \sum_n \xi_{mn}(t) \sin\frac{m_1 x}{R} \sin\frac{ny}{R}, \quad \Phi = \sum_m \sum_n \zeta_{mn}(t) \sin\frac{m_1 x}{R} \sin\frac{ny}{R} \tag{15}$$

where $m_1 = m\pi R/L$, m is the half wave length in the direction of the Ox axis, n is the wave number in the direction of the Oy axis, $\xi_{mn}(t)$ and $\zeta_{mn}(t)$ are the time dependent amplitudes. Substituting expressions (15) in the equation set (13-14) and applying Galerkin's method in the ranges $0 \le x \le L$ and $0 \le y \le 2\pi R$, in obtained equation set, the terms that are included by (15) expression, are solved besides so one of this terms are selected and eliminating $\zeta_{mn}(t)$, a differential equation is obtained:

$$\frac{d^2 \xi_{mn}(\tau)}{d\tau^2} + (\lambda_1 - \lambda_0 \tau^\alpha)\xi_{mn}(\tau) = 0 \tag{16}$$

where the following definitions apply:

$$\lambda_0 = \frac{P_0 t_{cr}^{\alpha+2}}{\tilde{\rho}} \frac{n^2}{R} \tag{17}$$

$$\lambda_1 = \frac{t_{cr}^2}{\tilde{\rho} R^4} \Big\{ [c_{13} m_1^4 + (c_{14} + 2c_{32} + c_{23}) m_1^2 n^2 + c_{24} n^4] + \frac{[m_1^2 R - c_{12} m_1^4 - (c_{11} - 2c_{31} + c_{22}) m_1^2 n^2 - c_{21} n^4]}{b_{22} m_1^4 + (b_{12} + 2b_{31} + b_{21}) m_1^2 n^2 + b_{11} n^4}$$
$$\times [b_{23} m_1^4 + (b_{13} - 2b_{32} + b_{24}) m_1^2 n^2 + b_{14} n^4 + m_1^2 R] - P_1 n^2 R^3 \Big\} \tag{18}$$

An approximating function will be chosen as a first approximation as

$$\xi_{mn}(\tau) = A_{mn} \xi(\tau) = A_{mn} e^{50\tau} \tau [52/51 - \tau] \tag{19}$$

satisfying the initial conditions

$$\xi(0) = 0, \quad \text{and} \quad \frac{\partial \xi(1)}{\partial \tau} \equiv \xi'(1) = 0 \tag{20}$$

where A_{mn} is found from the condition of transition to the static condition (Sachenkov and Baktieva 1978).

Multiplying (16) by $\xi'(\tau)$ and integrating it with respect to τ, from 0 to τ and from 0 to 1, in that order, the Ritz type variational method yields the following characteristic equation for finding the critical load (see appendix of the paper Sofiyev and Aksogan 2002):

$$P_0 t_{cr}^{\alpha} = \frac{\tilde{\rho} R}{B_2(\alpha) t_{cr}^2 n^2} [B_0(\alpha) \lambda_1 + B_1(\alpha)] \tag{21}$$

in which the new constants are defined as follows:

$$B_0(\alpha) = \int_0^1 [\xi(\tau)]^2 d\tau, \quad B_1(\alpha) = \int_0^1 [\xi'(\tau)]^2 d\tau, \quad B_2(\alpha) = 2\int_0^1 \int_0^{\tau} \eta^{\alpha} \xi'(\eta) \xi(\eta) d\eta d\tau \tag{22}$$

In cylindrical shells of medium length, the wave number n satisfies the inequality $n^4 \gg m_1^4$, then Eq. (21) becomes (Aksogan and Sofiyev 2000):

$$P_0 t_{cr}^{\alpha} = \frac{B_0(\alpha)}{B_2(\alpha)} \left[\left(\frac{c_{24} b_{11} - c_{21} b_{14}}{b_{11} R^3} \right) n^2 + \frac{1}{b_{11} R} \frac{m_1^4}{n^6} - P_1 \right] + \frac{B_1(\alpha) \tilde{\rho} R}{B_2(\alpha) t_{cr}^2 n^2} \tag{23}$$

If expression (23) is minimized with respect to the parameter n_1^2, after some mathematical operations the following equation is found for finding the minimum critical load:

$$P_0 t_{cr}^{\alpha} = 2\frac{B_0(\alpha)}{B_2(\alpha)} \left[\left(\frac{c_{24} b_{11} - c_{21} b_{14}}{b_{11} R^3} \right) n^2 - \frac{1}{R b_{11}} \frac{m_1^4}{n^6} - \frac{P_1}{2} \right] \tag{24}$$

Eliminating t_{cr} from Eqs. (23) and (24) one gets

$$(1 - 3\Omega)^{2\alpha/(1+\alpha)} (1 - \Omega - 0.5 \overline{P}_1 \Omega^{1/4})^{4/(1+\alpha)} = \Lambda^{2\alpha/1+\alpha} \Omega \tag{25}$$

where the following definitions apply:

$$\Omega = \frac{R^2}{(c_{24}b_{11} - c_{21}b_{14})}\frac{m_1^4}{n^8}, \qquad \overline{P}_1 = \frac{P_1 R^{5/2} b_{11}}{(c_{24}b_{11} - c_{21}b_{14})^{3/4} m_1} \tag{26ab}$$

$$\Lambda = \frac{B_1(\alpha) P_0^{2/\alpha} R^{(5+3\alpha)/\alpha} b_{11}^{(2+\alpha)/\alpha} (B_2(\alpha))^{2/\alpha} \tilde{\rho}}{2^{2/\alpha} (B_0(\alpha))^{(2+\alpha)/\alpha} m_1^{2(1+\alpha)/\alpha} (c_{24}b_{11} - c_{21}b_{14})^{(3+\alpha)/(2\alpha)}} \tag{27}$$

For $P_1 = 0$ and very large P_0 values Λ » 1. The solution of (25) yields

$$\Omega = \Lambda^{-2\alpha/(1+\alpha)} \tag{28}$$

Considering expression (28) in the expression (26a), the following expression is found for the wave number corresponding to the dynamic critical load:

$$n_d = \frac{B_1(\alpha) P_0^{2/\alpha} R^{(9+7\alpha)/\alpha} b_{11}^{(2+\alpha)/\alpha} m_1^{(8+8\alpha)/\alpha} \tilde{\rho}}{2^{2/\alpha} (c_{24}b_{11} - c_{21}b_{14})^{(7+5\alpha)/2\alpha} [B_0(\alpha)]^{(2+\alpha)/\alpha} [B_2(\alpha)]^{2/\alpha}} \tag{29}$$

Substituting expression (29) in Eq. (24), the dynamic critical load is found as follows:

$$P_{cr}^d = P_0 t_{cr}^{\alpha} = \left[\frac{4B_0(\alpha) B_1(\alpha) P_0^{2/\alpha} (c_{24}b_{11} - c_{21}b_{14}) \tilde{\rho}}{B_2^2(\alpha) R^2 b_{11}}\right]^{\alpha/(2+2\alpha)} \tag{30}$$

In the static case $(t_{cr} \to \infty, S_0 \to 0)$, the following equation is found for the wave number corresponding to the static critical load:

$$n_{st}^2 = \left(\frac{3R^2 m_1^4}{c_{24}b_{11} - c_{21}b_{14}}\right)^{1/4} \tag{31}$$

Substituting Eq. (31) into (24) and replacing $P_0 t_{cr}^{\alpha} B_2(\alpha)/B_0(\alpha)$ by P_{cr}^{st}, the static critical load is found to be

$$P_{cr}^{st} = \frac{4m_1 (c_{24}b_{11} - c_{21}b_{14})^{3/4}}{3^{3/4} b_{11} R^{5/2}} \tag{32}$$

and from $K_d = P_{cr}^d / P_{cr}^{st}$, the dynamic factor is given by

$$K_d = \left[\frac{3^{(3+3\alpha)/(2\alpha)} B_0(\alpha) B_1(\alpha) P_0^{2/\alpha} R^{(5+3\alpha)/\alpha} b_{11}^{(2+\alpha)/\alpha} \tilde{\rho}}{4^{(2+\alpha)/\alpha} (c_{24}b_{11} - c_{21}b_{14})^{(3+\alpha)/(2\alpha)} [B_2(\alpha)]^2 m_1^{(2+2\alpha)/\alpha}}\right]^{\alpha/(2+2\alpha)} \tag{33}$$

The critical time is obtained as

$$t_{cr} = \left[\frac{4B_0(\alpha) B_1(\alpha) (c_{24}b_{11} - c_{21}b_{14}) \tilde{\rho}}{[B_2(\alpha)]^2 R^2 b_{11} P_0^2}\right]^{1/(2+2\alpha)} \tag{34}$$

The corresponding critical impulse is obtained as

$$I_{cr} = \frac{P_0 t_{cr}^{1+\alpha}}{\alpha + 1} = \left[\frac{4B_1(\alpha) B_0(\alpha) \tilde{\rho} (c_{24}b_{11} - c_{21}b_{14})}{[B_2(\alpha)]^2 (1+\alpha)^2 b_{11} R^2}\right]^{1/2} \tag{35}$$

It is indicated in Eq. (35), in the large values of the loading parameter, the values of the critical impulse is independent from the length of the shell.

When, $\mu = 0$, $N = 1$ the appropriate formulas for a single layer cylindrical shell made of a homogeneous isotropic material are found as a special case (Sachenkov Baktieva 1978):

$$P_{cr}^{d} = 3.696\frac{h_1}{R}\left[\frac{RP_0}{V}\frac{E_0}{(1-\nu^2)^{1/2}}\right]^{1/2} \tag{36}$$

$$P_{cr}^{st} = 0.886\frac{E_0}{(1-\nu^2)^{3/4}}\frac{R}{L}\left(\frac{h_1}{R}\right)^{5/2} \tag{37}$$

$$K_d = 4.2087\left(\frac{1-\nu^2}{E_0}\frac{P_0 R}{V}\frac{RL^2}{h_1^3}\right)^{1/2} \tag{38}$$

$$t_{cr} = 3.696\left[\frac{h_1^2}{RVP_0}\frac{E_0}{(1-\nu^2)^{1/2}}\right]^{1/2} \tag{39}$$

$$I_{cr} = 6.8302\frac{h_1^2}{RV}\frac{E_0}{(1-\nu^2)^{1/2}} \tag{40}$$

where $\rho_0 = E_0/V^2$, ρ_0 is density of the homogeneous isotropic material, V is speed of sound propagation in a isotropic material, E_0 is the modulus of elasticity of the homogeneous isotropic material, $h_1 = 2h$ is thickness of the one layered cylindrical shell.

4. Numerical computations and results

For the numerical computations cross-ply laminated truncated conical shells up to 10 layers are considered. The numerical computations were carried out for glass/epoxy and graphite/epoxy composites with the following material properties (Jones and Morgan 1975, Vinson and Sierakowski 1986, Reddy 1997, Ng and Lam 1998, Mao 1999),

$$E_{01}^{(k+1)} = 5.38\times10^4\text{MPa},\quad E_{02}^{(k+1)} = 1.793\times10^4\text{MPa},\quad \nu_{12}^{(k+1)} = 0.25,\quad \nu_{21}^{(k+1)} = 0.0833,$$

$$\rho_0^{(k+1)} = 2.004\times10^3\,\text{kg/m}^3,$$

$$E_{01}^{(k+1)} = 1.724\times10^5\,\text{MPa},\quad E_{02}^{(k+1)} = 7.79\times10^3\,\text{MPa},$$

$$\nu_{12}^{(k+1)} = 0.35,\quad \nu_{21}^{(k+1)} = 0.016,\quad \rho_0^{(k+1)} = 1.53\times10^3\,\text{kg/m}^3,$$

shell and loading parameters (Agamirov 1990):

$$R/(2h) = 112.5,\quad L/R = 2.2,\quad P_0 = 100\div650\,\text{MPa/s}^{\alpha}$$

For the homogeneous case and an even number of layers the values of the critical parameters are independent of the stacking sequence (cases 90°/0°/...) and (0°/90°/...) of cross-ply laminas (Table 1, 2).

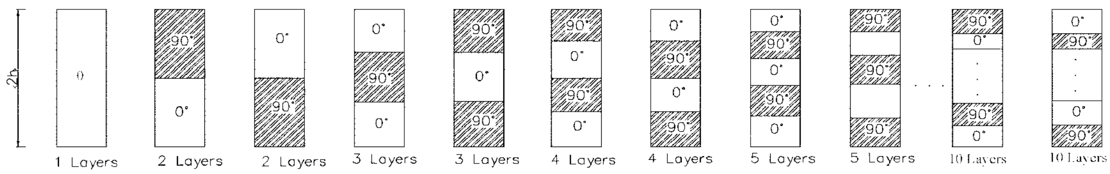

Fig. 2 Number and ordering of layers

In Table 1 are presented the variation of the dynamic critical load, dynamic factor and critical impulse of cross-ply laminated orthotropic composite cylindrical thin shells with the number and ordering of layers, when elasticity moduli and densities functions are given as $\varphi_i^{(k+1)}(\bar{z}) = \pm\bar{z}\,(i = 1, 2)$. Examining Table 1, we can see from the given variation function $\varphi_i^{(k+1)}(\bar{z}) = \pm\bar{z}$

Table 1 The variation of the dynamic critical load, dynamic factor and critical impulse with linear functions of elasticity moduli and densities, number and ordering of layers ($P = P_0t$, $P_0 = 100$ MPa/s, $R = 0.09$ m, $2h = 0.0008$ m, $L = 0.2$ m, $k = 0, 1, 2, 3, 4$)

Number and positioning of layers		$\varphi_i^{(k+1)}(\bar{z}) = \bar{z}$						$\varphi_i^{(k+1)}(\bar{z}) = -\bar{z}$		
		P_{cr}^d (MPa)		K_d		$I_{cr}\times 10$ (MPa × s)		P_{cr}^d (MPa)	K_d	$I_{cr}\times 10$ (MPa · s)
		$\mu = 0$	$\mu = 0.9$, $i = 1$	$\mu = 0$	$\mu = 0.9$, $i = 1$	$\mu = 0$	$\mu = 0.9$, $i = 1$	$\mu = 0.9$, $i = 1$	$\mu = 0.9$, $i = 1$	$\mu = 0.9$, $i = 1$
Glass/epoxy										
1	0°	1.3640	1.2608	19.851	23.234	0.0930	0.0795	1.2608	23.3337	0.0795
1	90°	1.7953	1.6594	15.083	17.653	0.1611	0.1377	1.6594	17.6529	0.1377
2	(0°/90°)	1.5401	1.4257	17.210	21.407	0.1186	0.1016	1.4581	18.2539	0.1063
2	(90°/0°)	1.5401	1.4581	17.210	18.254	0.1186	0.1063	1.4257	21.4089	0.1016
3	(0°/90°/0°)	1.3886	1.3238	20.371	22.414	0.0964	0.0876	1.3238	22.4143	0.0876
3	90°/0°/90°	1.7841	1.6089	13.425	16.507	0.1591	0.1294	1.6089	16.5070	0.1294
4	(0°/90°/...)	1.6028	1.5381	15.890	17.779	0.1285	0.1183	1.4284	19.4824	0.1020
4	(90°/0°/...)	1.6028	1.4284	15.890	19.482	0.1285	0.1020	1.5381	17.7786	0.1183
5	(0°/90°/...)	1.4880	1.4018	18.003	20.286	0.1107	0.0982	1.4018	20.2859	0.0982
5	(90°/0°/...)	1.7295	1.5722	14.012	16.957	0.1496	0.1236	1.5722	16.9566	0.1236
Graphite/epoxy										
1	0°	1.0312	0.9532	21.204	24.817	0.0532	0.0454	0.9532	24.8170	0.0454
1	90°	2.2366	2.0674	9.7760	11.442	0.2501	0.2137	2.0674	11.4419	0.2137
2	(0°/90°)	1.4872	1.5115	11.975	13.236	0.1106	0.1142	1.3555	13.2269	0.0919
2	(90°/0°)	1.4872	1.3535	11.975	13.227	0.1106	0.0919	1.5115	13.2361	0.1142
3	0°/90°/0°	1.1915	1.1733	17.457	18.004	0.0710	0.0688	1.1733	18.0043	0.0688
3	90°/0°/90°	2.2166	1.9653	5.9036	7.5100	0.2457	0.1931	1.9653	7.5100	0.1931
4	(0°/90°/...)	1.8225	1.8231	7.9750	8.4409	0.1661	0.1662	1.5497	10.5252	0.1201
4	(90°/0°/...)	1.8225	1.5497	7.9750	10.525	0.1661	0.1201	1.8231	8.4409	0.1662
5	(0°/90°/...)	1.5716	1.5072	10.284	11.181	0.1235	0.1136	1.5072	11.1811	0.1136
5	(90°/0°/...)	2.1162	1.9001	6.2208	7.7164	0.2239	0.1805	1.9001	7.7164	0.1805

(i = 1, 2) that the variation of the material densities has no effect on the critical parameters.

When elasticity moduli and densities functions of the glass/epoxy (graphite/epoxy) composites are given as $\varphi_i^{(k+1)}(\bar{z}) = \bar{z}$ the maximum effect on the critical load and critical impulse was observed to be in the four layered shell, in which the ordering of the layers was (90°/0°/90°/0°), being 10.88% and 20.62% (14.97% and 27.69%) respectively, on the dynamic factor was observed to be in the three layered shell, in which the ordering of the layers was (90°/0°/90°), being 22.96% (27.21%).

When elasticity moduli and densities functions of the glass/epoxy (graphite/epoxy) composites are given as $\varphi_i^{(k+1)}(\bar{z}) = -\bar{z}$ the maximum effect on the critical load and critical impulse was observed to be in the four layered shell, in which the ordering of the layers was (0°/90°/0°/90°), being 10.88% and 20.62% (14.97% and 27.69%) respectively, on the dynamic factor was observed to be in the three layered shell, in which the ordering of the layers was (0°/90°/0°), being 24.40% (27.70%). The values in the parenthesis are for the shells, which are made of graphite/epoxy (Table 1).

In Table 2 are presented the variation of the dynamic critical load and dynamic factor of cross-ply laminated orthotropic composite cylindrical thin shells with the number and ordering of layers, when elasticity moduli and densities functions are given as $\varphi_i^{(k+1)}(\bar{z}) = \pm\bar{z}^2$.

When the elasticity moduli and densities functions of the glass/epoxy (graphite/epoxy) composites were varied (the variation function is given as $\varphi_i^{(k+1)}(\bar{z}) = \bar{z}^2,\ i = 1, 2$), the maximum effect on the dynamic critical load, was observed to be in the five layered shell, in which the ordering of the layers was (90°/0°/…), being 20.27% (20.55%). When the densities were kept constant and the elasticity moduli were varied (the variation function is given as $\varphi_i^{(k+1)}(\bar{z}) = \bar{z}^2,\ i = 1$), the maximum effect on the dynamic factor was observed to be in the five layered shell, in which the ordering of the layers was (90°/0°/…), being 24.89% (28.36%) (Table 2).

When the elasticity moduli and densities functions of the glass/epoxy (graphite/epoxy) composites were varied (the variation function is given as $\varphi_i^{(k+1)}(\bar{z}) = -\bar{z}^2,\ i = 1, 2$), the maximum effect on the dynamic critical load, was observed to be in the five layered shell, in which the ordering of the layers was (90°/0°/…), being 26.92% (28.36%). When the densities were kept constant and the elasticity moduli were varied (the variation function is given as $\varphi_i^{(k+1)}(\bar{z}) = -\bar{z}^2,\ i = 1$), the maximum effect on the dynamic factor was observed to be in the five layered shell, in which the ordering of the layers was (90°/0°/…), being 68% (70.93%) (Table 2).

In the foregoing, the elasticity moduli and densities of the materials of the layers vary linear and parabolic functions in the thickness direction, both together and each at a time, as separate cases. It is observed that, both in the homogeneous and in the non-homogeneous cases, the number and ordering of the layers affect the values of the critical parameters appreciably. The effect of the non-homogeneity on the critical parameters also changes with the number and ordering of layers. It is also observed that, compared to homogeneous case, the foregoing effect of the variation of the material properties in the thickness direction is more pronounced for the linear variation than that for the parabolic one. The variation effect of elasticity moduli and densities on the values of critical parameters of the shell made of graphite/epoxy materials is bigger than the shell, which is made of glass/epoxy composites (Table 1, 2).

Fig. 3 shows the values of the dynamic critical load and dynamic factor for cross-ply laminated orthotropic composite cylindrical thin shells made of glass/epoxy composites, versus the number and ordering of layers, when elasticity moduli and densities functions are given as $\varphi_i^{(k+1)}(\bar{z}) = e^{-0.1|\bar{z}|}(i = 1, 2)$. The maximum effect on dynamic critical load occurs at five layered (0°/90°/...) ordered shells, the maximum effect on the dynamic factor occurs at five layered (0°/90°/...) ordered shells. In homogeneous and non-homogeneous cases, at the (0°/90°/...) ordered and (90°/

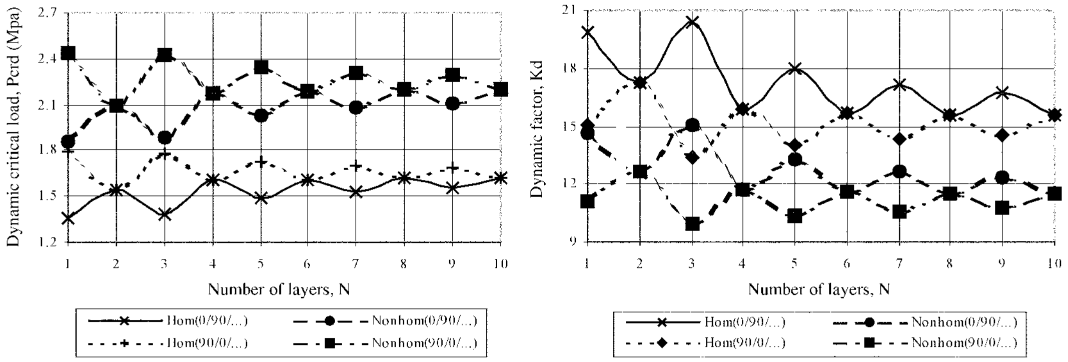

Fig. 3 The variation of the dynamic critical load and dynamic factor with exponential functions of elasticity moduli and densities, number and ordering of layers ($P = P_0 t$, $P_0 = 100$ MPa/s, $R = 0.09$ m, $2h = 0.0008$ m, $L = 0.2$ m, $\mu = 0.9$, $k = 0, 1, 2, ..., 9$)

Table 2 The variation of the dynamic critical load and dynamic factor with parabolic functions of elasticity moduli and densities, number and ordering of layers ($P = P_0 t$, $P_0 = 100$ MPa/s, $R = 0.09$ m, $2h = 0.0008$ m, $L = 0.2$ m, $\mu = 0.9$, $k = 0, 1, 2, 3, 4$)

Number and positioning of layers		$\varphi_i^{(k+1)}(\bar{z}) = \bar{z}^2$						$\varphi_i^{(k+1)}(\bar{z}) = -\bar{z}^2$					
		P_{cr}^d (MPa)			K_d			P_{cr}^d (MPa)			K_d		
		$i = 1, 2$	$i = 1$	$i = 2$	$i = 1, 2$	$i = 1$	$i = 2$	$i = 1, 2$	$i = 1$	$i = 2$	$i = 1, 2$	$i = 1$	$i = 2$
		Glass/epoxy											
1	0º	1.623	1.520	1.457	15.99	14.98	21.20	1.028	1.123	1.248	29.27	32.00	18.16
1	90º	2.135	2.000	1.917	12.15	11.38	16.11	1.352	1.479	1.642	22.24	24.31	13.80
2	0º/90º	1.827	1.711	1.645	13.95	13.06	18.38	1.164	1.273	1.409	25.20	27.55	15.74
2	90º/0º	1.827	1.711	1.645	13.95	13.06	18.38	1.164	1.273	1.409	25.20	27.55	15.74
3	(0º/90º/0º)	1.643	1.539	1.483	16.36	15.32	21.75	1.064	1.164	1.270	29.86	32.64	18.63
3	90º/0º/90º	2.126	1.991	1.905	11.01	10.31	14.34	1.335	1.460	1.632	19.36	21.17	12.28
4	(0º/90º/...)	1.892	1.772	1.712	12.99	12.17	16.97	1.22	1.334	1.466	22.95	25.09	14.53
4	(90º/0º/...)	1.892	1.772	1.712	12.99	12.17	16.97	1.22	1.334	1.466	22.95	25.09	14.53
5	(0º/90º/...)	1.741	1.630	1.589	14.85	13.91	19.22	1.18	1.287	1.361	24.54	26.83	16.48
5	(90º/0º/...)	2.08	1.944	1.847	11.24	10.52	14.96	1.262	1.380	1.582	21.53	23.54	12.82
		Graphite/epoxy											
1	0º	1.227	1.149	1.101	17.09	16.00	22.64	1.737	0.849	0.943	31.26	34.18	19.40
1	90º	2.66	2.492	2.388	7.878	7.378	10.44	3.767	1.842	2.046	14.41	15.76	8.942
2	(0º/90º)	1.731	1.621	1.588	10.08	9.443	12.79	2.566	1.255	1.360	16.83	18.39	10.95
2	(90º/0º)	1.731	1.621	1.588	10.08	9.443	12.79	2.566	1.255	1.360	16.83	18.39	10.95
3	(0º/90º/0º)	1.366	1.279	1.272	14.8	13.86	18.64	2.205	1.078	1.090	22.39	24.47	15.97
3	90⁰/0⁰/90º	2.644	2.476	2.367	4.97	4.655	6.304	3.697	1.808	2.028	8.26	9.030	5.400
4	(0º/90º/...)	2.105	1.972	1.946	6.814	6.382	8.516	3.186	1.557	1.667	10.92	11.94	7.295
4	(90º/0º/...)	2.105	1.972	1.946	6.814	6.382	8.516	3.186	1.558	1.667	10.92	11.94	7.295
5	(0º/90º/...)	1.78	1.667	1.678	8.997	8.426	10.98	2.977	1.456	1.438	12.34	13.56	9.406
5	(90º/0º/...)	2.551	2.389	2.260	5.01	4.692	6.643	3.389	1.657	1.938	9.726	10.63	5.690

Table 3 The variation of the dynamic critical load and dynamic factor with parabolic functions of elasticity moduli and densities, power α and number of layers ($\varphi_i^{(k+1)}(\bar{z}) = -\bar{z}^2$, i = 1, 2, $P = P_0 t^{\alpha}$, P_0 = 200 MPa/s$^{\alpha}$, R = 0.09 m, $2h$ = 0.0008 m, L = 0.2 m)

Glass/epoxy P_{cr}^d (MPa)						
α	(0°)		(0°/90°)		(0°/90°/0°)	
	$\mu = 0$	$\mu = 0.9$	$\mu = 0$	$\mu = 0.9$	$\mu = 0$	$\mu = 0.9$
1.0	1.929	1.453	2.178	1.646	1.964	1.505
2.0	0.420	0.288	0.494	0.340	0.431	0.302
3.0	0.199	0.131	0.240	0.158	0.205	0.138
K_d						
1.0	28.07	41.39	24.34	35.64	28.81	42.22
2.0	6.12	8.21	5.52	7.37	6.32	8.47
3.0	2.91	3.72	2.68	3.41	3.01	3.86

0°/...) ordered shells that have even number of layers and at the (0°/90°/...) ordered shells that have odd number of layers, as the number of layers increases, the values of the dynamic critical load increase, but the values of the dynamic factor decrease whereas at the (90°/0°/...) ordered shells that have odd number of layers, as the number of layers increases, the values of the dynamic critical load decrease, but the values of the dynamic factor increase. However, at the shells that have more than 10 layers, the values of the dynamic critical load and dynamic factor are almost same without considering the order of the layers.

In Table 3 are presented the variation of the dynamic critical load and dynamic factor of cross-ply laminated orthotropic composite cylindrical thin shells with the power α and number of layers, when elasticity moduli and densities functions are given as $\varphi_i^{(k+1)}(\bar{z}) = -\bar{z}^2$. Therefore, as α increases, the values of the dynamic critical load and dynamic factor decrease. Another point to be noted is that as α increases, the effect of the variation of the elasticity moduli on the dynamic critical load increases, whereas, the effect on the dynamic factor decreases, meanwhile the effect of the variation of the elasticity moduli and densities on critical parameters is larger compared with the effect at the one layered (0°) shell.

In Table 4 are presented the variation of the dynamic critical load and dynamic factor of cross-ply laminated orthotropic composite cylindrical thin shells with the ratio h/R and number of layers, when elasticity moduli and densities functions are given as $\varphi_i^{(k+1)}(\bar{z}) = -\bar{z}^2$. With an increase of the ratio h/R, the values of the dynamic critical load increase, however the values of the dynamic factor decrease. The effect of the variations of elasticity moduli and densities to the critical parameters is very necessary and it is independent on the variations of ratio h/R.

Table 5 presents the comparison of the results of the present method, using finite deformation analysis, for the dynamic buckling of an homogeneous orthotropic cylindrical thin shell with one layered having simple supports at the two ends, with those found numerically of Ogibalov *et al.* (1975). The comparisons were carried out for the following material properties and shell parameters:

$$E_{01}/E_{02} = 2, \quad \nu_{12} = 0.12, \quad \rho_0 = 1.84 \times 10^2 \text{ kg} \cdot \text{s}^2/\text{m}^4, \quad 2h/R = 2/143, \quad L/R = 2.6,$$

$$\mu = 0, \quad N = 1, \quad \alpha = 1$$

Table 4 The variation of the dynamic critical load and dynamic factor with parabolic functions of elasticity moduli and densities, number of layers and the ratio $2h/R$ ($\varphi_i^{(k+1)}(\bar{z}) = -\bar{z}^2$, $i = 1, 2$, $P = P_0 t$, $P_0 = 100$ MPa/s, $R = 0.09$ m, $L = 0.2$ m)

	Glass/epoxy P_{cr}^d (MPa)					
α	(0°)		(0°/90°)		(0°/90°/0°)	
$2h/R$	$\mu = 0$	$\mu = 0.9$	$\mu = 0$	$\mu = 0.9$	$\mu = 0$	$\mu = 0.9$
0.0020	0.614	0.462	0.693	0.524	0.625	0.479
0.0025	0.767	0.578	0.866	0.655	0.781	0.559
0.0030	0.921	0.694	1.039	0.786	0.940	0.720
0.0035	1.070	0.809	1.210	0.920	1.090	0.840
0.0040	1.230	0.925	1.390	1.048	1.250	0.960
	K_d					
0.0020	65.76	96.96	57.01	83.48	67.48	98.90
0.0025	47.05	69.38	40.80	59.73	48.29	70.77
0.0030	35.80	52.78	31.03	45.44	36.73	53.83
0.0035	28.41	41.88	24.63	36.06	29.15	42.72
0.0040	23.25	34.28	20.16	29.51	23.86	34.97

Table 5 Comparison of critical parameters with those of Ogibalov *et al.* (1975)

	Ogibalov *et al.* Numerical	Present work
P_0 (MPa/s)	K_d	K_d
250	4.20	4.32
300	4.56	4.73
350	4.94	5.11
400	5.31	5.47

As seen from Table 5, the results of the present work have been compared with the numerical results of Ogibalov *et al.* (1975) for an homogeneous orthotropic elastic cylindrical shell with one layered subject to an external pressure varying linearly with time and a good match has been observed.

For uniform isotropic cylindrical shell with one layered the same problem in Shumik (1970) was solved numerically using energetic method, Lagrange equation and Runge-Kutta method, in Tazyukov (1991) was solved analytical using Bessel equations, in Agamirov (1990) numerical and experimental. The comparisons were carried out for the following material properties, shell parameters (Shumik 1970, Agamirov 1990, Tazyukov 1991):

$$E_0 = 7.75 \times 10^4 \text{ MPa}, \quad \nu_0 = 0.3, \quad V = 5 \times 10^3 \text{ m/s}, \quad 2h = 8 \times 10^{-4} \text{ m}, \quad R = 0.09 \text{ m},$$

$$L = 0.2 \text{ m}, \quad \mu = 0, \quad N = 1, \quad \alpha = 1$$

As seen from Table 6, the results of the present work have been compared with the analytical, experimental and numeric results for an homogeneous isotropic elastic cylindrical shell with one

Table 6 Comparison of critical parameters with those of Agamirov (1990), Shumik (1970) and Tazyukov (1991)

P_0 (MPa/s)	Agamirov (1990) numerical		Agamirov (1990) experimental		Shumik (1970) numerical		Tazyukov (1991) theoretical		Present work ($\mu = 0$, $N = 1$)	
	n_d	n_{st}	n_d	n_{st}	n_d	n_{st}	n_d	n_{st}	n_d	n_{st}
200	8	6	7	6	-	-	8	-	7	6
470	9	6	8	6	-	-	9	-	9	6
650	10	6	9	6	-	-	10	-	10	6
	P_{cr}^d (MPa)	K_d	P_{cr}^d (MPa)	K_d	P_{cr}^d (MPa)	K_d	P_{cr}^d (MPa)	K_d	P_{cr}^d (MPa)	K_d
200	0.52	2.180	0.670	2.600	0.5606	2.352	0.550	-	0.561	2.357
470	0.74	3.100	0.890	3.240	0.8594	3.605	0.860	-	0.860	3.612
650	0.88	3.600	1.060	4.000	1.1070	4.239	1.010	-	1.012	4.248

layered that is subjected to an external pressure varying linearly with time and a good match has been observed.

5. Conclusions

The dynamic buckling of a cross-ply laminated non-homogeneous ortohotropic cylindrical shell has been studied employing the Ritz type variational method. At first, the fundamental relations and modified Donnell type dynamic buckling equations have been written for a cylindrical shell subject to an external pressure which is a power function of time. Then, applying Galerkin's method, a time dependent differential equations with variable coefficients has been obtained. Finally, applying the Ritz type variational method, the critical static and dynamic loads, the corresponding wave numbers, dynamic factor and critical impulse have been found analytically. Using the foregoing results, the dependence of the critical parameters on the variation of the elasticity moduli and densities with a linear, parabolic or exponential function in the thickness direction, are studied numerically. Furthermore, the present method has been verified by comparisons of critical parameters with the theoretical and experimental ones given in previous literature for the case of a homogeneous shell subject to a uniform external pressure, which is a linear function of time.

References

Agamirov, V.L. (1990), *Nonlinear Theory of the Shells*, Moscow, Nauka (in Russian).

Aksogan, O. and Sofiyev, A. (2000), "The dynamic stability of a laminated nonhomogeneous orthotropic elastic cylindrical shell under a time dependent external pressure", (In Eds. Becker AA) *Proc. Int. Conf. on Modern Practice in Stress and Vibration Analysis,* Nottingham, UK, 349-360.

Aksogan, O. and Sofiyev, A. (2002), "Dynamic buckling of a cylindrical shell with variable thickness subject to a time dependent external pressure varying as a power function of time ", *J. Sound Vib.*, **254**(4), 693-702.

Ambartsumian, S.A. (1964), *Theory of Anisotropic Shells*, TT F-118, NASA.

Argento, A. and Scott, R.A. (1993), "Dynamic instability of layered anisotropic circular cylindrical shells", Part I: Theoretical development. *J. Sound Vib.*, **162**(2), 311-322.

Elishakoff, I. (2001), “Inverse buckling problem for inhomogeneous columns”, *Int. J. Solids Struct.*, **38**, 457-464.

Greenberg, J.B. and Stavsky, Y. (1980), “Buckling and vibration of orthotropic composite cylindrical shells”, *Acta Mechanica*, **36**, 15-29.

Greenberg, J.B. and Stavsky, Y. (1998), “Vibrations and buckling of composite orthotropic cylindrical shells with non-uniform axial loads”, *Composites Part B-Engineering*, **29**, 695-703.

Gutierrez, R.H., Laura, P.A.A., Bambill, D.V., Jederlinic, V.A. and Hodges, D.H. (1998), “Axisymmetric vibrations of solid circular and annular membranes with continuously varying density”, *J. Sound Vib.*, **212**(4), 611-622.

Heyliger, P.R. and Julani, A. (1992), “The free vibrations of inhomogeneous elastic cylinders and spheres”, *Int. J. Solids Struct.*, **29**, 2689-2708.

Jones, R.M. and Morgan, H.S. (1975), “Buckling and vibration of cross-ply laminated circular cylindrical shells”, *AIAA J.*, **13**(5), 664-671.

Leissa, A.W. (1973), *Vibration of Shells*, NASA SP-288.

Lomakin, V.A. (1976), *The Elasticity Theory of Non-homogeneous Materials*, Moscow, Nauka. (in Russian)

Mao, R.J. and Lu, C.H. (1999), “Buckling analysis of a laminated cylindrical shell under torsion subjected to mixed boundary conditions”, *Int. J. Solids Struct.*, **36**(25), 3821-3835.

Massalas, C., Dalamanagas, D. and Tzivanidis, G. (1981), “Dynamic instability of truncated conical shells with variable modulus of elasticity under periodic compressive forces”, *J. Sound Vib.*, **79**, 519-528.

Mecitoglu, Z. (1996), “Governing equations of a stiffened laminated inhomogeneous conical shell”, *American Institute of Aeronautics and Astronautics Journal*, **34**, 2118-2125.

Ng, T.Y. and Lam, K.Y. (1999), “Dynamic stability analysis of cross-ply laminated cylindrical shells using different thin shell theories”, *Acta Mechanica*, **134**, 147-167.

Ng, T.Y., Lam, K.Y. and Reddy, J.N. (1998), “Dynamic stability of cross-ply laminated composite cylindrical shells”, *Int. J. Mech. Sci.*, **40**, 805-823.

Ogibalov, P.M., Lomakin, V.A. and Kishkin, B.P. (1975), *Mechanics of Polymers*, Moscow State University, Moscow. (in Russian).

Park, H.C., Cho, C.M. and Choi, Y.H. (2001), “Torsional buckling analysis of composite cylinders”, *AIAA J.*, **39**(5), 951-955.

Reddy, J.N. (1997), *Mechanics of Laminated Composite Plates*, Boca Raton, CRC Press.

Sachenkov, A.V. and Baktieva, L.U. (1978), “Approach to the solution of dynamic stability problems of thin shells”, *Research on the Theory of Plates and Shells*, Kazan State University, Kazan, **13**, 137-152 (in Russian).

Shumik, M.A. (1970), “The buckling of a cylindrical shells subjected to a dynamic radial pressure”, VII *Int. Conf. on the Theory of Plates and Shells*, Moskova, 625-628.

Sofiyev, A.H. and Aksogan, O. (2002), “The dynamic stability of a non-homogeneous orthotropic elastic conical shell under a time dependent external pressure”, *Int. J. Struct. Eng. Mech.*, **13**(3), 329-343.

Soldatos, K.P. and Tzivanidis, G.J.(1982), “Buckling and vibration of cross-ply laminated circular cylindrical panels”, *J. Applied Mathematics and Physics (ZAMP)*, **33**, 230-240.

Tarn, J.Q. (1994), “An asymptotic theory for dynamic-response of anisotropic inhomogeneous and laminated cylindrical shells”, *J. Mech. Phys. Solids*, **42**, 1633-1650.

Tong, L. and Wang, T.K. (1992), “Simple solutions for buckling of laminated conical shells”, *Int. J. Mech. Sci.*, **34**(2), 93-111.

Tong, L. (1993), “Free vibrations of composite laminated conical shells”, *Int. J. Mech. Sci.*, **35**, 47-61.

Vinson, J.R. and Sierakowski, R.L. (1986), *The Behavior of Structures Composed of Composite Material*, Nijhoft, Dordrecht.

Volmir, A.S. (1967), *Stability of Elastic Systems*, Nauka, Moscow, English Translation: Foreign Technology Division, Air Force Systems Command. Wright-Patterson Air Force Base, Ohio, AD628508.

Wang, B, Han, J. and Du, S. (1998), “Dynamic fracture mechanics analysis for composite material with material nonhomogeneity in thickness direction”, *Acta Mechanica Solid Sinica*, **11**, 84-93.

Yakushev, A.N. (1990), “The stability of orthotropic cylindrical shells under dynamic pressure”, *Research on the Theory of Plates and Shells*, Kazan State University, Kazan, **20**, 215-222 (in Russian).

Zhang, X. and Hasebe, N. (1999), “Elasticity solution for a radially nonhomogeneous hollow circular cylinder”, *J. Appl. Mech.*, ASME, **66**, 598-606.

Appendix

The expressions appearing in Eqs. (13-14), c_{ij} and b_{ij} i, j = 1, 2, 3, 4, are:

$$c_{11} = a_{11}^1 b_{11} + a_{12}^1 b_{21}, \quad c_{12} = a_{11}^1 b_{12} + a_{12}^1 b_{22}, \quad c_{13} = a_{11}^1 b_{13} + a_{12}^1 b_{23} + a_{11}^2,$$

$$c_{14} = a_{11}^1 b_{14} + a_{12}^1 b_{24} + a_{12}^2, \quad c_{21} = a_{21}^1 b_{11} + a_{22}^1 b_{21}, \quad c_{22} = a_{21}^1 b_{12} + a_{22}^1 b_{22},$$

$$c_{23} = a_{21}^1 b_{13} + a_{22}^1 b_{23} + a_{21}^2, \quad c_{24} = a_{21}^1 b_{14} + a_{22}^1 b_{24} + a_{22}^2, \quad c_{31} = a_{33}^1 b_{31}, \quad c_{32} = a_{33}^1 b_{32} + a_{33}^2,$$

$$b_{11} = a_{22}^0 D, \quad b_{12} = -a_{12}^0 D, \quad b_{13} = (a_{12}^0 a_{21}^1 - a_{11}^1 a_{22}^0) D,$$

$$b_{14} = (a_{12}^0 a_{22}^1 - a_{12}^1 a_{22}^0) D,, \quad b_{21} = -a_{21}^0 D, \quad b_{22} = a_{11}^0 D, \quad b_{23} = (a_{21}^0 a_{11}^1 - a_{21}^1 a_{11}^0) D,$$

$$b_{24} = (a_{21}^0 a_{12}^1 - a_{22}^1 a_{11}^0) D,, \quad b_{31} = 1/a_{33}^0, \quad b_{32} = -a_{33}^1/a_{33}^0, \quad D = 1/(a_{11}^0 a_{22}^0 - a_{21}^0 a_{12}^0),$$

Finally, the expressions for the factors a_{ij}^m, i, j = 1, 2, 3, 4, m = 0, 1, 2, are (i, j not from 1 to 4):

$$a_{11}^{k_1} = h^{k_1+1} \sum_{k=0}^{N-1} \frac{E_{01}^{(k+1)} \bar{h}^{(k+1)}}{1 - \nu_{12}^{(k+1)} \nu_{21}^{(k+1)}}, \quad a_{12}^{k_1} = h^{k_1+1} \sum_{k=0}^{N-1} \frac{\nu_{21}^{(k+1)} E_{01}^{(k+1)} \bar{h}^{(k+1)}}{1 - \nu_{12}^{(k+1)} \nu_{21}^{(k+1)}},$$

$$a_{22}^{k_1} = h^{k_1+1} \sum_{k=0}^{N-1} \frac{E_{02}^{(k+1)} \bar{h}^{(k+1)}}{1 - \nu_{12}^{(k+1)} \nu_{21}^{(k+1)}}, \quad a_{21}^{k_1} = h^{k_1+1} \sum_{k=0}^{N-1} \frac{\nu_{12}^{(k+1)} E_{02}^{(k+1)} \bar{h}^{(k+1)}}{1 - \nu_{12}^{(k+1)} \nu_{21}^{(k+1)}},$$

$$a_{33}^{k_1} = 2h^{k_1+1} \sum_{k=0}^{N-1} G_0^{(k+1)} \bar{h}^{(k+1)}, \quad \bar{h}^{(k+1)} = \int_{-1+2k/N}^{-1+2(k+1)/N} \bar{z}^{k_1} \bar{\varphi}_1^{(k+1)}(\bar{z}) d\bar{z}, \quad k_1 = 0, 1, 2$$

Notation

$E_{01}^{(k+1)}, E_{02}^{(k+1)}$	:	Elasticity moduli of the homogeneous orthotropic materials in the layers
E_{01}, E_{02}	:	Elasticity moduli of the orthotropic material in a single layer shell
E_0	:	Elasticity modulus of the isotropic material in a single layer shell
$G_0^{(k+1)}$	:	Shear moduli of the homogeneous materials in the layers
$2h$	:	Thickness of the shell
K_d	:	Dynamic factor
M_{11}, M_{22}, M_{12}	:	Internal moments per unit length of the cross-section of the shell
N_{11}, N_{22}, N_{12}	:	Internal forces per unit length of the cross-section of the shell
$N_{11}^0, N_{22}^0, N_{12}^0$	:	Membrane forces prior to buckling
N	:	Number of layers
n	:	Wave number in the circumferential direction
n_{st}, n_d	:	Wave number corresponding to the static and dynamic critical loads
$Oxyz$	:	Coordinate system on the middle surface of the shell plane
P_{cr}^{st}, P_{cr}^d	:	Static and dynamic critical loads
P_0	:	Loading parameter
R	:	Radii of the cylindrical shell
t	:	Time
t_{cr}	:	Critical time
V	:	Sound propagation in a isotropic materials

W	: Displacement of the middle surface in the inwards normal direction z
α	: Power of time in the external pressure expression
$\chi_{11}, \chi_{22}, \chi_{12}$	: Curvatures of the middle surface
$\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{12}$	: Strains in the curvilinear coordinate directions
$\varepsilon_{11}^0, \varepsilon_{22}^0, \varepsilon_{12}^0$	: Strain components on the middle surface of the cylindrical shell
Φ	: Stress Function
$\varphi_1^{(k+1)}(\bar{z})$	: Variation function of the elasticity moduli in the layers
$\varphi_2^{(k+1)}(\bar{z})$	: Variation function of the density in the layers
μ	: Elasticity moduli and densities variation coefficient
$\nu_{12}^{(k+1)}, \nu_{21}^{(k+1)}$	: Poisson's ratios of the homogeneous orthotropic materials in the layers
ν_{12}, ν_{21}	: Poisson's ratios of the orthotropic materials in a single layer shell
ν_0	: Density of the isotropic material in a single layer shell
$\rho_0^{(k+1)}$	: Density of the homogeneous materials in the layers
ρ_0	: Densities of the homogeneous materials in a single layer shell
$\sigma_{11}, \sigma_{22}, \sigma_{12}$	: Stress components
τ	: Dimensionless time parameter
$\xi_{mn}(t), \zeta_{mn}(t)$	: Time dependent amplitudes