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Predicting the failure modes of monotonically loaded reinforced concrete exterior beam-column joints

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Abstract. This study aims at postulating a simple methodology for predicting the failure modes of monotonically loaded reinforced concrete beam-column joints. All the factors that affect the failure modes of joints are discussed in detail using an experimental database of monotonically loaded exterior beam-column joints. The relative contributions of the strut and truss mechanisms to joint shear strength are determined based on the test results. A simple design equation for the beam longitudinal reinforcement ratio for joints with low, medium and high amount of stirrups is developed. The factors influencing the failure modes of monotonically loaded exterior beam-column joints are investigated in detail. Design charts that predict the failure modes of exterior beam-column connections both with and without stirrups are developed. Experimental data are compared with the design charts. The results show that the simple methodology gives very accurate predictions of the failure modes.

Key words: reinforced concrete; monotonically loaded exterior beam column connection; failure mode.

1. Introduction

It is very important, particularly in the case of reinforced concrete beam-column joints, to estimate, the failure modes. Without a suitable model, designers would not rationally analyze the behavior of the joints and creatively design, where, codes do not provide any solution. The world-wide research on reinforced concrete beam-column joints with few exceptions, has almost been restricted to the cyclically loaded beam-column joints due to beam-column joint failures observed in recent earthquakes (Bakir and Boduroğlu 2002a). Data from experiments on joints subject to cyclic loading, only provides limited information about the behavior of monotonically loaded beam-column joints, because in cyclically loaded beam-column joints, the main problem is the degradation of strength due to extensive cracking of the joint core along each diagonal and the deterioration of the concrete due to the opening and closing of cracks. Thus, the main design goal in cyclically loaded beam-column joints, is to retain sufficient shear stiffness under reversed loading. In monotonically loaded beam-column joints however, the main design requirement is to guarantee that the maximum joint shear strength is not exceeded. Consequently, there is no need to take into account the degradation of either the strength or stiffness in monotonically loaded joints. The design

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methods for cyclically loaded beam-column joints can not be applied to monotonically loaded beam-column joints. The tests under cyclic loading can only furnish a lower bound estimate of the shear strength of the monotonically loaded beam-column joints. Beam-column joints are not designed in countries, such as UK, where seismic effects need not be taken into account, as there are no recommendations for design in either BS8110 or EC2. On the other hand, the US Standards, such as, ACI/ASCE Committee 352 give stipulations both for monotonically loaded exterior beam-column joints and cyclically loaded beam-column joints. Recent tests carried out on monotonically loaded beam-column joints in UK (Scott 1992, Ortiz 1993, Scott & Hamill 1998, Parker & Bullman 1997), reveal that the behavior of monotonically loaded beam-column joints is also very complex and that the existing design recommendations in codes can be unreliable due to shallow presumptions. Monotonically loaded exterior beam-column joints have been the subject of a recent review by Vollum (1998) who has investigated the shear resisting mechanisms of joints. Nevertheless, Vollum has not comprehensively discussed the factors influencing the failure modes of joints. In addition to this, the strut and tie model proposed by Vollum, involves a complex procedure. This paper is complementary to Vollum's study and aims at postulating a simple model that predicts the failure modes of monotonically loaded exterior beam column joints. In addition, a simpler strut and tie model that gives accurate predictions of the failure loads is presented. This study is a part of an ongoing research on beam-column joints and deals only with the monotonically loaded beam-column joints. The studies on the behavior of cyclically loaded beam-column joints will be published separately.

Taylor's model (1974) is the only model in literature for predicting the failure modes of monotonically loaded beam-column joints. Taylor specified an upper limit to the beam longitudinal reinforcement ratio and stated that the equation proposed can be used in design to limit the beam longitudinal reinforcement ratio as shown below:

$$100\rho_b = 100\left(3 + \frac{2d_c}{z_b}\right)\frac{b_c d_c}{b_b d_b}\frac{\beta v_c}{0.87 f_y} \tag{1}$$

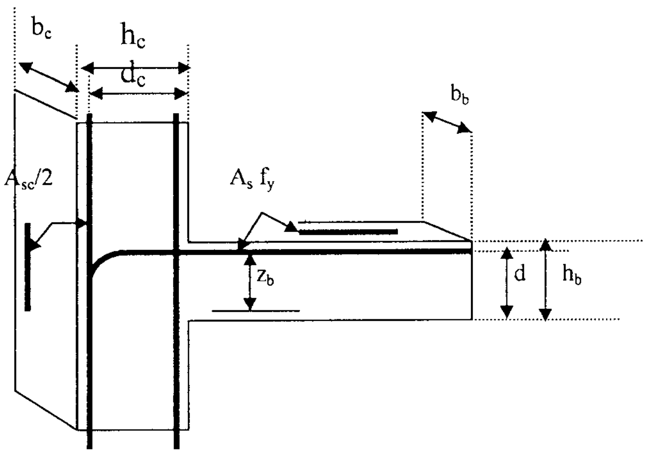


Fig. 1 The notation used in the design formula of Taylor

The notation used in the above formula is shown in Fig. 1. The conclusions of Taylor (1974) were contradicting the experimental evidence at two points:

1. Taylor's conclusions (1974) regarding the influence of transverse reinforcement were erroneous. Taylor believed neither that the transverse reinforcement increased the joint shear strength, nor it changed the failure mode. Taylor placed an extra stirrup to the joints so that the buckling of

Table 1 The geometric data for the experimental database

Investigator	Specimen	Detail	H(mm)	L(mm)	h_c(mm)	d_c(mm)	b_c(mm)	h_b(mm)	d(mm)	b_b(mm)
Ortiz	BCJ 1	L bar	2000	1050	300	275	200	400	367	200
	BCJ 2	L bar	2000	1100	300	275	200	400	367	200
	BCJ 3	L bar	2000	1100	300	275	200	400	367	200
	BCJ 4	L bar	2000	1100	300	275	200	400	367	200
	BCJ 5	L bar	2000	1100	300	275	200	400	367	200
	BCJ 6	L bar	2000	1100	300	275	200	400	367	200
	BCJ 7	L bar	2000	1100	300	275	200	400	367	200
Kordina	RE 2	L bar	3000	1000	200	167	200	400	365	200
	RE 3	L bar	3000	1000	200	167	200	300	265	200
	RE 4	L bar	3000	1000	200	167	200	300	265	200
	RE6	L bar	3000	1000	200	167	200	300	265	200
	RE7	L bar	3000	975	250	217	230	350	315	230
	RE8	U bar	3000	975	250	217	230	350	315	230
	RE9	U bar	3000	975	250	217	230	350	315	230
	RE10	U bar	3000	975	250	217	230	390	355	230
Taylor	P1/41/24	L bar	1290	470	140	110	140	200	170	100
	P2/41/24	L bar	1290	470	140	110	140	200	170	100
	P2/41/24A	L bar	1290	470	140	110	140	200	170	100
	A3/41/24	L bar	1290	470	140	110	140	200	170	100
	D3/41/24	L bar	1290	470	140	110	140	200	170	100
	B3/41/24	L bar	1290	470	140	110	140	200	170	100
	C3/41/24BY	U bar	1290	470	140	110	140	200	170	100
	C3/41/13Y	U bar	1290	470	140	110	140	200	173	100
Scott	C1AL	L bar	1700	750	150	117	150	210	179	110
	C1	L bar	1700	750	150	117	150	210	177	110
	C4	L bar	1700	750	150	117	150	210	177	110
	C4A	L bar	1700	750	150	117	150	210	177	110
	C4AL	L bar	1700	750	150	117	150	210	177	110
	C1A	L bar	1700	750	150	117	150	210	177	110
	C3	U bar	1700	750	150	117	150	210	177	110
	C7	L bar	1700	750	150	117	150	300	267	110
	C3L	U bar	1700	750	150	117	150	210	177	110
	C6	U bar	1700	750	150	117	150	210	177	110
	C6L	U bar	1700	750	150	117	150	210	177	110
	C9	U bar	1700	750	150	117	150	300	267	110

Table 1 Continued

Investigator	Specimen	Detail	H(mm)	L(mm)	h_c(mm)	d_c(mm)	b_c(mm)	h_b(mm)	d(mm)	b_b(mm)
S & Hamil	C4ALNO	L bar	1700	750	150	117	150	210	177	110
	C4ALN1	L bar	1700	750	150	117	150	210	177	110
	C4ALN3	L bar	1700	750	150	117	150	210	177	110
	C4ALN5	L bar	1700	750	150	117	150	210	177	110
	C4ALHO	L bar	1700	750	150	117	150	210	177	110
	C4ALH1	L bar	1700	750	150	117	150	210	177	110
	C4ALH3	L bar	1700	750	150	117	150	210	177	110
	C4ALH5	L bar	1700	750	150	117	150	210	177	110
	C6LNO	U bar	1700	750	150	117	150	210	177	110
	C6LN1	U bar	1700	750	150	117	150	210	177	110
	C6LN3	U bar	1700	750	150	117	150	210	177	110
	C6LN5	U bar	1700	750	150	117	150	210	177	110
	C6LHO	U bar	1700	750	150	117	150	210	177	110
	C6LH1	U bar	1700	750	150	117	150	210	177	110
	C6LH5	U bar	1700	750	150	117	150	210	177	110
	C6LH3	U bar	1700	750	150	117	150	210	177	110
Parker	4a	L bar	2000	850	300	245	300	500	445	250
	4b	L bar	2000	850	300	245	300	500	445	250
	4c	L bar	2000	850	300	245	300	500	445	250
	4d	L bar	2000	850	300	245	300	500	445	250
	4e	L bar	2000	850	300	245	300	500	445	250
	4f	L bar	2000	850	300	245	300	500	445	250
	5a	L bar	2000	850	300	245	300	500	445	250
	5b	L bar	2000	850	300	245	300	500	445	250
	5d	L bar	2000	850	300	245	300	500	445	250
	5e	L bar	2000	850	300	245	300	500	445	250
	5f	L bar	2000	850	300	245	300	500	445	250

column longitudinal reinforcement is prevented. However, the inspection of the experimental database in Table 1 and Eqs. (2) and (5a,b), show that the stirrups increase the joint shear strength of the monotonically loaded joints.

2. Taylor (1974) suggested that the joints with beam longitudinal reinforcement ratios less than the limit proposed in Eq. (1) all exhibited beam failure, whereas the joints with beam longitudinal reinforcement ratios above this limit all exhibited joint shear failure. The above criteria was investigated by Scott (1992). Scott's specimens with beam longitudinal reinforcement ratios above the limit in Eq. (1) all exhibited joint shear failure regardless of the column load or the reinforcement detail. However, the joints with a beam longitudinal reinforcement ratio less than this limit and detailed by either U bars or L bars bent down detail, exhibited beam failure only when a high column load was applied. When subjected to a low column load, these same details gave a joint failure, as did the details of the bent up bars. It was concluded that the beam column-joints subjected to low column load and beam steel percentage of less than 1.2 were not examined by Taylor.

2. The factors influencing the joint shear strength

The authors have investigated both the factors influencing the joint shear strength and the established equations on the basic mechanics of reinforced concrete monotonically loaded exterior beam-column joints in a previous paper (Bakir and Boduroğlu 2000b). The joint mechanics has been also discussed by Paulay (1975) and by Bonacci and Pantazopoulou for interior joints (1992) who have also taken into account the joint deformations. The typical loading system considered in analysis of exterior beam-column joints is shown in Fig. 2. Both of the authors use the average stresses for equilibrium as shown in Fig. 3. The average normal concrete stresses are given in the *x* and *y* directions as:

$$\sigma_x = -\frac{A_s}{b_{eff}h_b}f_s - \frac{A_{sje}}{b_{eff}h_b}f_w \tag{2}$$

$$\sigma_y = -\frac{A_{scol}}{b_{eff}h_c}f_{scol} - \frac{N}{b_{eff}h_c} \tag{3}$$

The tensile stress in the concrete is negligible and therefore $\sigma_1 = 0$, which consequently gives ;

$$\sigma_y = \frac{\tau_{av}^2}{\sigma_x} \tag{4}$$

Eqs. (2), (3) and (4) indicate that the joint shear strength increases by increasing column axial load, column and beam longitudinal reinforcement, and the stirrup ratio. In order to reach a conclusion, the factors influencing the principal tensile strain should be determined because, as shown in Eq. (5), the concrete strength and accordingly the joint shear strength are dependent on the principal tensile strain (Mitchell and Collins 1991).

$$f_{2\max} = \frac{f_c}{(0.8 + 170\varepsilon_1)} \quad \text{[MPa]} \tag{5a}$$

where,

$$\varepsilon_1 = \varepsilon_t + (\varepsilon_t + 0.002)\cot^2\theta \tag{5b}$$

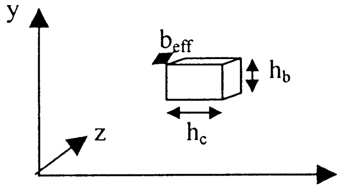


Fig. 2 The joint dimensions

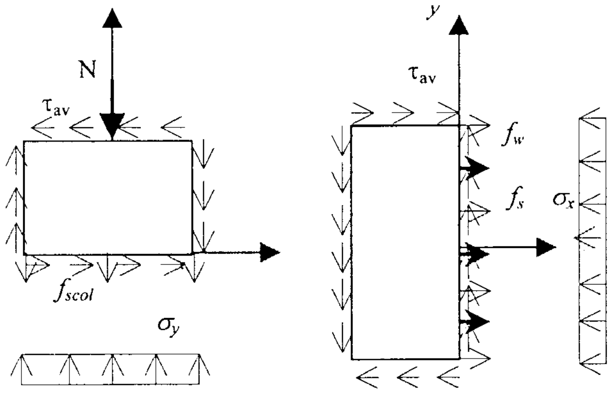


Fig. 3 The stress equilibrium

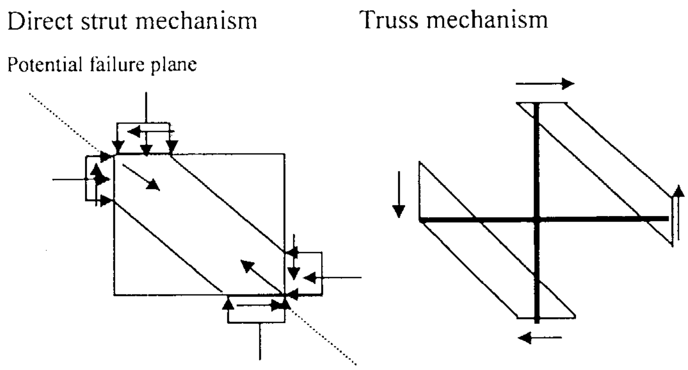


Fig. 4 Idealized behavior of interior beam-column joints according to Paulay

$$f_2 = \left(2\left(\frac{\varepsilon_2}{\varepsilon_{c'}}\right) - \left(\frac{\varepsilon_2}{\varepsilon_{c'}}\right)^2\right) f_{2\max} \tag{5c}$$

The principal tensile strain is determined by Eq. (6) using the established principles of mechanics for joints.

$$\varepsilon_1 = \left(\frac{\varepsilon_x - \varepsilon_y \tan^2\theta}{1 - \tan^2\theta}\right) \tag{6a}$$

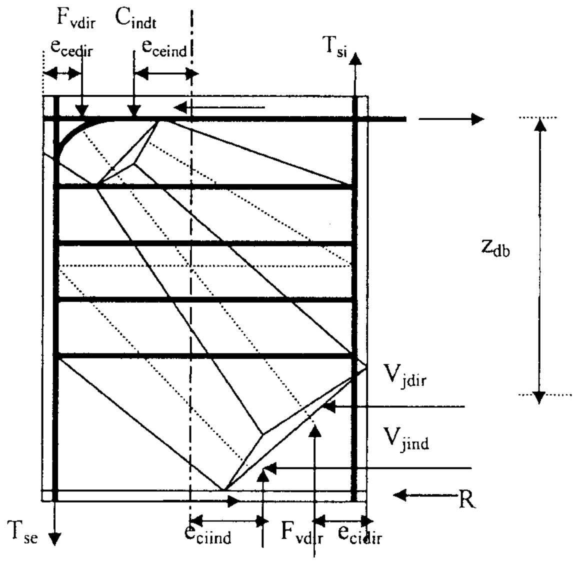


Fig. 5 The strut and tie model of Vollum

where;

$$\varepsilon_x = \frac{f_w}{E_s} = \frac{\tau_{av}\tan\theta}{E_s\left(\frac{A_s\mu}{b_{eff}h_b} + \frac{A_{sje}}{b_{eff}h_b}\right)} \quad \text{and} \quad \varepsilon_y = \frac{f_{scol}}{E_s} = \left(\frac{\tau_{av}}{\tan\theta} - \frac{N}{b_{eff}h_c}\right)\frac{b_{eff}h_c}{A_{scol}E_s} \qquad \text{(6b)\&(6c)}$$

As the beam longitudinal reinforcement ratio and the stirrup ratio increases, the principal tensile strain decreases increasing the concrete strength and the joint shear strength (Eqs. 6a,b). When the beam longitudinal reinforcement ratio and the stirrup ratio increases, the average concrete stress in the x direction increases and this too results in an increase in the joint shear strength (Eq. 2). As the column axial load and the column longitudinal reinforcement ratio increases, the principal tensile strain increases, consequently decreasing the joint shear strength. Thus, the increase in the joint shear strength by the column axial load and the column longitudinal reinforcement in Eq. (3) is offset by the decrease in joint shear strength due to an increase in the principal tensile strain in Eqs. (6a,c). Thus, the column axial load and column bar ratio do not influence the joint shear strength.

3. Development of the design equation

It is widely accepted that the exterior beam-column joints resist the joint shear by two mechanisms as first suggested by Paulay (1975) whose model is shown in Fig. 4. The first of these mechanisms is the strut mechanism which accounts for the concrete contribution to joint shear. The second is the truss mechanism, which accounts for the stirrups' contribution. The horizontal tie in the truss mechanism represents the stirrups that are situated between the top of the beam compressive chord and the beam tensile reinforcement. It should be noted that the diagonal strut mechanism can develop without any bond stress transfer at the beam and column reinforcement within the joint, while the truss mechanism can exist only when a good bond stress transfer is maintained along the beam and column reinforcement. Thus, the increase of the joint shear strength by the stirrups is related to good bond conditions of the beam reinforcement through the joint. The vertical tie in the truss mechanism is used to account for the intermediate column bars. Paulay (1975) suggests that this vertical tie equilibrates the vertical shear in the joint. This, in reality, is not true because it has been proved by Vollum (1998) and Fuji & Morita (1991) that there is a considerable amount of tensile shift in the forces at the columns from that calculated, in the assumption of plane sections remain plane. Thus, intermediate column bars are ineffective in resisting vertical shear in the joint. Vollum has proposed a model, which is similar to that of Fuji and Morita's (1991) as shown in Fig. 5. His model is composed of fan shaped direct and indirect struts. The relative contributions of the strut and truss mechanisms to joint shear strength is yet debatable by many researchers (Paulay 1975, Ortiz 1993, Vollum 1998). The radically different views on this show that the width of the primary strut can best be predicted empirically. Vollum (1998) suggested that the width of the primary strut at failure is 0.4 $h_c/\sin\theta$ using Eq. (5a) suggested by Mitchell and Collins (1991) for the concrete compressive strength. Vollum (1998) used Eq. (9) for determining the angle of inclination of the direct strut where, e_{cedir} and e_{cidir} are determined as shown in Eqs. (7,8).

$$e_{cidir} = 0.5h_c - \frac{(M_{colb} + (T_{si} - T_{se} - P)(0.5h_c - d'))}{F_v} \tag{7}$$

$$e_{cedir} = 0.5h_c - \frac{(M_{colu} - (T_{si} - T_{se})(0.5h_c - d'))}{F_v} \tag{8}$$

$$\tan\theta = \frac{z_{db}}{(h_c - e_{ecedir} - e_{cidir})} \tag{9}$$

Vollum's procedure for determining the angle of inclination is accurate but highly iterative and complex. Paulay (1975) has suggested that when axial force is not applied to the column, the inclination of the strut in Fig. 4 can be accepted as $\tan\theta = h_b/h_c$. The investigation of joint mechanics in the preceding section showed that the joint shear strength is independent of the column axial load. Thus, the angle of inclination of the direct strut mechanism will be independent of the column axial stress and can be accepted as Eq. (10) regardless of the column axial stress, as opposed to Paulay:

$$\tan\psi = \frac{h_b}{h_c} \tag{10}$$

The width of the direct strut is also accepted as $\alpha h_c/\sin\psi$ in the authors' equation. The CEB Code 90 equation given by Eq. (11) is used for the cracked concrete strength due to the practicability it provides and the ultimate joint shear resistance of a joint without stirrups is given by Eq. (12):

$$f_{cr} = 0.6f_c\left(1 - \frac{f_c}{250}\right) \quad \text{[MPa]} \tag{11}$$

$$V_j = wf_{cr}b_{eff}\cos\psi \quad \text{[N]} \tag{12}$$

This formulation is applied to joints without stirrups. The failure loads are predicted most accurately when the equation has a constant of 0.114. The simplified equation is given in Eq. (13) and the vertical shear in the joint is defined as Eq. (14):

$$V_j = 0.114h_c^2(b_c + b_b)\frac{f_c\left(1 - \frac{f_c}{250}\right)}{h_b} \quad \text{[N]} \tag{13}$$

$$F_v = A_{scu}f_{scu} + C_{cu} + A_{stb}f_{stb} \quad \text{[N]} \tag{14}$$

$$N = C_{cu} + A_{scu}f_{scu} - A_{stu}f_{stu} \quad \text{[N]} \tag{15}$$

The horizontal shear in the joint is defined as shown in Eq. (16) and the shear force in the upper column is calculated as Eq. (17). In the present study, the shear force in the upper column is neglected and the joint shear force is calculated as Eq. (18).

$$V_j = A_sf_{sb} - V_{coltop} \quad \text{[N]} \tag{16}$$

$$V_{coltop} = \frac{P(L + 0.5h_c)}{H} \quad [\text{N}] \tag{17}$$

$$V_j = 0.87 A_s f_y \tag{18}$$

If Eqs. (13) and (18) are combined and both sides are divided by the depth and the width of the

Table 2 The material data for the experimental database

Investigator	Specimen	ρ_b	f_c (MPa)	f_y (MPa)	$1000A_{sje}/b_{eff}\,h_c$	Stirrup Ratio	ρ_{blimit}	ρ_c*1000
Ortiz	BCJ 1	0.011	34	720	0	L	0.007	2.19
	BCJ 2	0.011	38	720	1.727	L	0.008	2.19
	BCJ 3	0.011	33	720	0	L	0.007	2.92
	BCJ 4	0.011	34	720	3.369	M	0.009	3.65
	BCJ 5	0.011	38	720	0	L	0.008	3.65
	BCJ 6	0.011	35	720	0	L	0.007	3.65
	BCJ 7	0.011	35	720	7.870	H	0.012	3.65
Kordina	RE 2	0.009	25	420	0	L	0.004	2.41
	RE 3	0.018	40	420	3.910	M	0.014	2.41
	RE 4	0.012	32	420	2.550	L	0.01	2.41
	RE6	0.012	32	463	5.110	M	0.010	2.41
	RE7	0.013	26	448	5.220	M	0.010	1.61
	RE8	0.013	28	464	5.290	M	0.009	1.61
	RE9	0.013	28	454	5.165	M	0.008	1.61
	RE10	0.012	24	459	5.250	M	0.006	1.61
Taylor	P1/41/24	0.024	33	500	3.310	M	0.014	4.1
	P2/41/24	0.024	29	500	3.106	M	0.013	4.1
	P2/41/24A	0.024	47	500	3.427	M	0.019	4.1
	A3/41/24	0.024	27	500	2.997	M	0.012	4.1
	D3/41/24	0.024	53	500	3.360	M	0.021	4.1
	B3/41/24	0.024	22	500	6.765	H	0.012	4.1
	C3/41/24BY	0.024	32	500	3.370	M	0.012	4.1
	C3/41/13Y	0.014	28	500	3.358	M	0.010	4.1
Scott	C1AL	0.011	33	540	2.400	L	0.011	4.29
	C1	0.011	39.9	540	2.890	L	0.013	4.29
	C4	0.021	41	540	2.890	L	0.014	4.29
	C4A	0.021	44	540	2.890	L	0.014	4.29
	C4AL	0.021	36	540	2.907	L	0.012	4.29
	C1A	0.011	48	540	2.890	L	0.015	4.29
	C3	0.011	36	540	2.890	L	0.010	4.29
	C7	0.014	35	540	5.780	H	0.005	4.29
	C3L	0.011	35	540	2.890	L	0.010	4.29
	C6	0.021	40	540	2.890	L	0.011	4.29
	C6L	0.021	46	540	2.890	L	0.012	4.29
	C9	0.014	36	540	5.780	M	0.005	4.29

Table 2 Continued

Investigator	Specimen	ρ_b	f_c (MPa)	f_y (MPa)	$1000A_{sje}/b_{eff}\,h_c$	Stirrup Ratio	ρ_{blimit}	ρ_c*1000
S & Hamil	C4ALNO	0.021	42	522	0	L	0.014	4.29
	C4ALN1	0.021	46	522	2.910	L	0.015	4.29
	C4ALN3	0.021	42	522	5.799	H	0.021	4.29
	C4ALN5	0.021	50	522	9.490	H	0.024	4.29
	C4ALHO	0.021	104	522	0	L	0.025	4.29
	C4ALH1	0.021	95.2	522	2.890	L	0.024	4.29
	C4ALH3	0.021	105.6	522	5.800	H	0.037	4.29
	C4ALH5	0.021	98.4	522	9	H	0.036	4.29
	C6LNO	0.021	51	522	0	L	0.014	4.29
	C6LN1	0.021	51	522	2.536	L	0.014	4.29
	C6LN3	0.021	49	522	5.760	H	0.02	4.29
	C6LN5	0.021	37	522	8.700	H	0.016	4.29
	C6LHO	0.021	101	522	0	L	0.021	4.29
	C6LH1	0.021	102	522	2.899	L	0.021	4.29
	C6LH5	0.021	100	522	9	H	0.036	4.29
	C6LH3	0.021	97	522	5.760	H	0.030	4.29
Parker	4a	0.009	39	570	0	L	0.008	1.09
	4b	0.009	39	570	0	L	0.008	1.09
	4c	0.009	37	570	0	L	0.008	1.09
	4d	0.009	39	570	0	L	0.008	4.37
	4e	0.009	40	570	0	L	0.008	4.37
	4f	0.009	38	570	0	L	0.008	4.37
	5a	0.009	42	485	5.450	M	0.011	2.67
	5b	0.009	43	485	5.464	M	0.011	2.67
	5d	0.014	43	515	8.197	H	0.017	2.67
	5e	0.014	45	515	8.240	H	0.017	2.67
	5f	0.014	43	515	8.197	H	0.017	2.67

Notes: L; low amount of stirrups, M; medium amount of stirrups, H; high amount of stirrups

beam, Eq. (19) is obtained. To stay on the conservative side, Eq. (19) is increased by 15% and Eq. (20) is obtained.

$$\rho_{b\ \mathrm{lim}} = 0.13h_c^2(b_c + b_b)\frac{f_c\left(1 - \frac{f_c}{250}\right)}{h_b b_b d f_y} \tag{19}$$

$$\rho_{b\ \mathrm{lim}} = 0.15h_c^2(b_c + b_b)\frac{f_c\left(1 - \frac{f_c}{250}\right)}{h_b b_b d f_y} \tag{20}$$

In order to determine the joint shear strength, the reported failure load P is used to calculate the moment in the beam:

$$M_b = P(L + 0.5d') \tag{21}$$

An initial value is assumed for the strain of concrete. The compression reinforcement of the beam is neglected. The force in the tensile reinforcement in the beam is calculated so as to maintain

Table 3 The failure modes

Investigator	Specimen	Detail	N(kN)	$N/b_c d_c f_c$	P(kN)	Failure mode	Predicted Failure mode
Ortiz	BCJ 1	L bar	0	0	118	JS	JS
	BCJ 2	L bar	0	0	125	JS	JS
	BCJ 3	L bar	0	0	118	JS	JS
	BCJ 4	L bar	0	0	130	JS	JS
	BCJ 5	L bar	300	0.14	115	JS	JS
	BCJ 6	L bar	300	0.16	115	JS	JS
	BCJ 7	L bar	300	0.16	170	B	B
Kordina	RE 2	L bar	240	0.29	67	JS	JS
	RE 3	L bar	400	0.3	80	JS	JS
	RE 4	L bar	51	0.05	51	JS	JS
	RE6	L bar	213	0.2	66	JS	JS
	RE7	L bar	650	0.5	117	JS	JS
	RE8	U bar	525	0.37	105	JS	JS
	RE9	U bar	770	0.55	110	JS	JS
	RE10	U bar	551	0.46	100	JS	JS
Taylor	P1/41/24	L bar	240	0.47	35	JS	JS
	P2/41/24	L bar	240	0.54	35	JS	JS
	P2/41/24A	L bar	240	0.33	47	JS	JS
	A3/41/24	L bar	240	0.58	35	JS	JS
	D3/41/24	L bar	60	0.07	50	JS	JS
	B3/41/24	L bar	240	0.70	30	JS	JS
	C3/41/24BY	U bar	240	0.48	29	JS	JS
	C3/41/13Y	U bar	240	0.55	27	JS	JS
Scott	C1AL	L bar	50	0.086	22	JS	JS
	C1	L bar	275	0.39	26.2	B	B
	C4	L bar	275	0.38	30	JS	JS
	C4A	L bar	275	0.35	32	JS	JS
	C4AL	L bar	50	0.079	28	JS	JS
	C1A	L bar	275	0.33	26.8	B	B
	C3	U bar	275	0.43	25.9	B	B
	C7	L bar	275	0.45	32	JS	JS
	C3L	U bar	50	0.08	22	JS	JS
	C6	U bar	275	0.39	22	JS	JS
	C6L	U bar	50	0.06	26	JS	JS
	C9	U bar	275	0.43	28	JS	JS

Table 3 Continued

Investigator	Specimen	Detail	N (kN)	$N/b_c d_c f_c$	P (kN)	Failure mode	Predicted Failure mode
S & Hamil	C4ALNO	L bar	50	0.067	27	P	P
	C4ALN1	L bar	50	0.062	34	JS	JS
	C4ALN3	L bar	50	0.068	35	JS	JS
	C4ALN5	L bar	50	0.057	40	JS	JS
	C4ALHO	L bar	100	0.055	43	P	P
	C4ALH1	L bar	100	0.059	43.4	B	B
	C4ALH3	L bar	100	0.054	46.1	B	B
	C4ALH5	L bar	100	0.058	48.7	B	B
	C6LNO	U bar	50	0.056	24	JS	JS
	C6LN1	U bar	100	0.112	25	JS	JS
	C6LN3	U bar	50	0.058	29	JS	JS
	C6LN5	U bar	50	0.077	34	JS	JS
	C6LHO	U bar	100	0.056	36	JS	JS
	C6LH1	U bar	100	0.056	37	JS	JS
	C6LH5	U bar	100	0.057	51.4	B	B
	C6LH3	U bar	100	0.059	41	JS	JS
Parker	4a	L bar	0	0	118	C	C
	4b	L bar	300	0.104	138	JS	JS
	4c	L bar	600	0.22	170	JS	JS
	4d	L bar	0	0	150	JS	JS
	4e	L bar	300	0.102	160	JS	JS
	4f	L bar	600	0.215	183	JS	JS
	5a	L bar	0	0	213	C	C
	5b	L bar	300	0.094	236	JS	JS
	5d	L bar	0	0	226	C	C
	5e	L bar	300	0.090	295	C	C
	5f	L bar	600	0.19	322	JS	JS

Notes: B; beam failure, P; connection zone reinforcement pull out, JS; joint shear failure, C; column failure

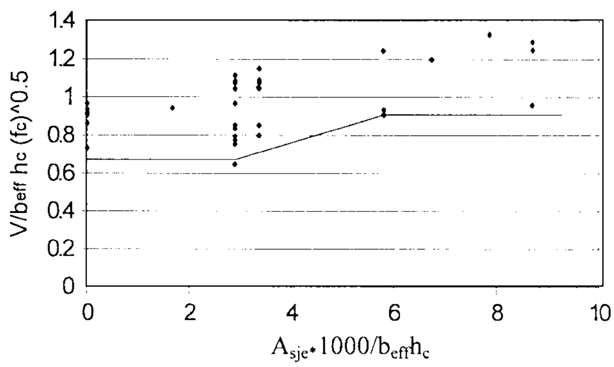


Fig. 6 The influence of the stirrup ratio on the normalized joint shear strength

equilibrium. If the equilibrium is not maintained, the strain value assumed for the concrete at the beginning is increased up until the equilibrium is satisfied in the beam. The normalized joint shear stress is calculated as shown in Eq. (22). As $b_b \le b_c$ in the experimental database in Table 1, b_{eff} is taken as in Eq. (23).

$$v_j = \frac{V_j}{b_{eff}\, h_c \sqrt{f_c}} \tag{22}$$

$$b_{eff} = \frac{(b_c + b_b)}{2} \tag{23}$$

Eqs. (2) and (6a,b) had shown that the joint shear strength is increased by stirrup ratio. All the previously suggested strut and tie models for beam-column joints except the model of Vollum suggest that the stirrups contribute to the joint shear strength as given by Eq. (24).

$$V_j = V_c + A_{sje} f_{yw} \tag{24}$$

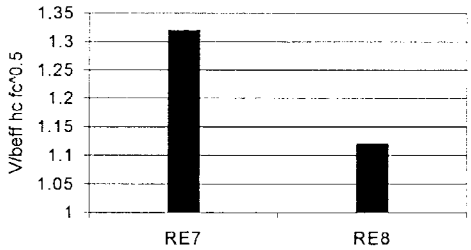
The test data in Table 4 shows that when the joints have a stirrup ratio higher than 0.0055, not all the stirrups yield within the joint. Thus, Eq. (24) can not be used for determining the contribution of the truss mechanism. Fig. 6 shows that the relationship between the stirrup ratio and the joint shear strength is tri-linear. For exterior beam-column joints with stirrup ratios less than 0.003 (low amount of stirrups), the influence of stirrups on the joint shear strength can be neglected. For stirrup ratios between 0.003 and 0.0055 (medium amount of stirrups), there is a substantial increase in the joint shear strength with increasing stirrup ratio. For stirrup ratios higher than 0.0055 (high amount of stirrups), the joints have the highest joint shear strength. A parametric study is carried out to determine the relative contributions of the truss and strut mechanisms in resisting shear in Table 4. Column 2 of Table 4 represents the reported stirrup strains of experiments. Column 4 shows the

Table 4 The parametric study on the relative contributions of truss and strut mechanisms to shear

Specimen	Reported Stirrup Strains	Stirrup Force	Total Stirrup Force	Joint Shear	The Contribution of the Strut Mechanism to Joint Shear	The Expected Contribution of the Truss Mechanism to Joint Shear	The Horizontal Component of the Residual Force in the Strut of the Truss Mechanism
Units	Microstrain	kN	kN	kN	kN	kN	kN
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
BCJ2	y7584	57	57	349.1	330.618	18.482	38.518
BCJ4	1887 y2990 y3003 1139	38 57 57 22	114	368.2	301.397	66.808	47.191
C4ALN3	2100 2880 2340	24 32.5 72	54.0	156.6	110.126	46.525	7.475

stirrup forces calculated from the reported strains. Column 5 shows the joint shear forces calculated by the iterative procedure using the reported failure loads of the specimens. The calculated resistance of the direct strut is reported in Column 6 based on Eq. (13). Column 7 shows the expected contribution of the truss mechanism to the joint shear. It is simply calculated as Column 5 subtracted by Column 6. However, the expected contribution of the truss mechanism in Column 6 is less than the total stirrup force in Column 4. Column 8 shows the horizontal component of the residual force in the strut of the truss mechanism. It is apparent that beam bars can not equilibrate the truss mechanism and some other mechanism should be equilibrating the force in the upper indirect strut in the truss mechanism. In order to maintain equilibrium in the upper node and to equilibrate the force in the indirect strut, another strut coming from the upper column, which is equilibrated, by the upper column ties in the vertical direction and the stirrups of the confinement region of the column in the horizontal direction is necessary. If Table 4 is investigated in detail, it will be apparent that the residual force in Column 8 of Table 4 decreases, as the stirrup ratio increases. This is because, the bond in the beam longitudinal reinforcement increases as the stirrup ratio increases and consequently, the force in the indirect strut is resisted by the beam longitudinal reinforcement rather than the upper column ties. In this study, the influence of stirrups on the joint shear strength for low amount of stirrups will be neglected, as the joint shear strength is not much influenced by low amount of stirrups. Table 4 suggests that the joint shear strength is increased by 18.14% when the joints have medium amount of stirrups. The joint shear strength is increased by 29.7% when the joint is provided by high amount of stirrups. The joint shear mentioned here is the parameter $T_{beam} - V_{coltop}$.

The above percentages are determined by calculating the ratio of Column 7 in Table 4 to Column 5. Failure occurs when the total shear resistance of the strut and truss mechanisms is equal to the horizontal shear in Eq. (16). The proposed simple strut and tie model of authors is shown in Fig. 8. The failure loads of joints in the experimental database are calculated by using the simple model shown in Fig. 8. The average of the ratios of the predicted failure loads of the model to the actual



RE7 has beam bars bent down detail - RE8 has U bars detail.

Fig. 7 The influence of detailing on the normalized joint shear strength

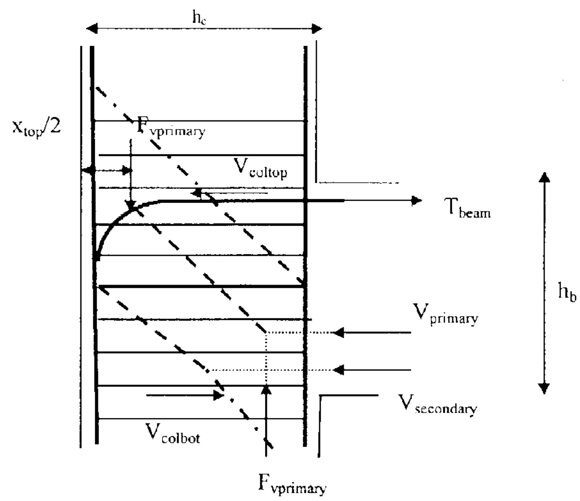


Fig. 8 The proposed simple model

failure loads of the specimens in the experimental database is 0.99. As apparent, the model gives accurate predictions of the failure loads. Eq. (2) shows that the stirrups increase the joint shear strength. Thus, the limit in Eq. (20) should be increased for the joints with medium and high

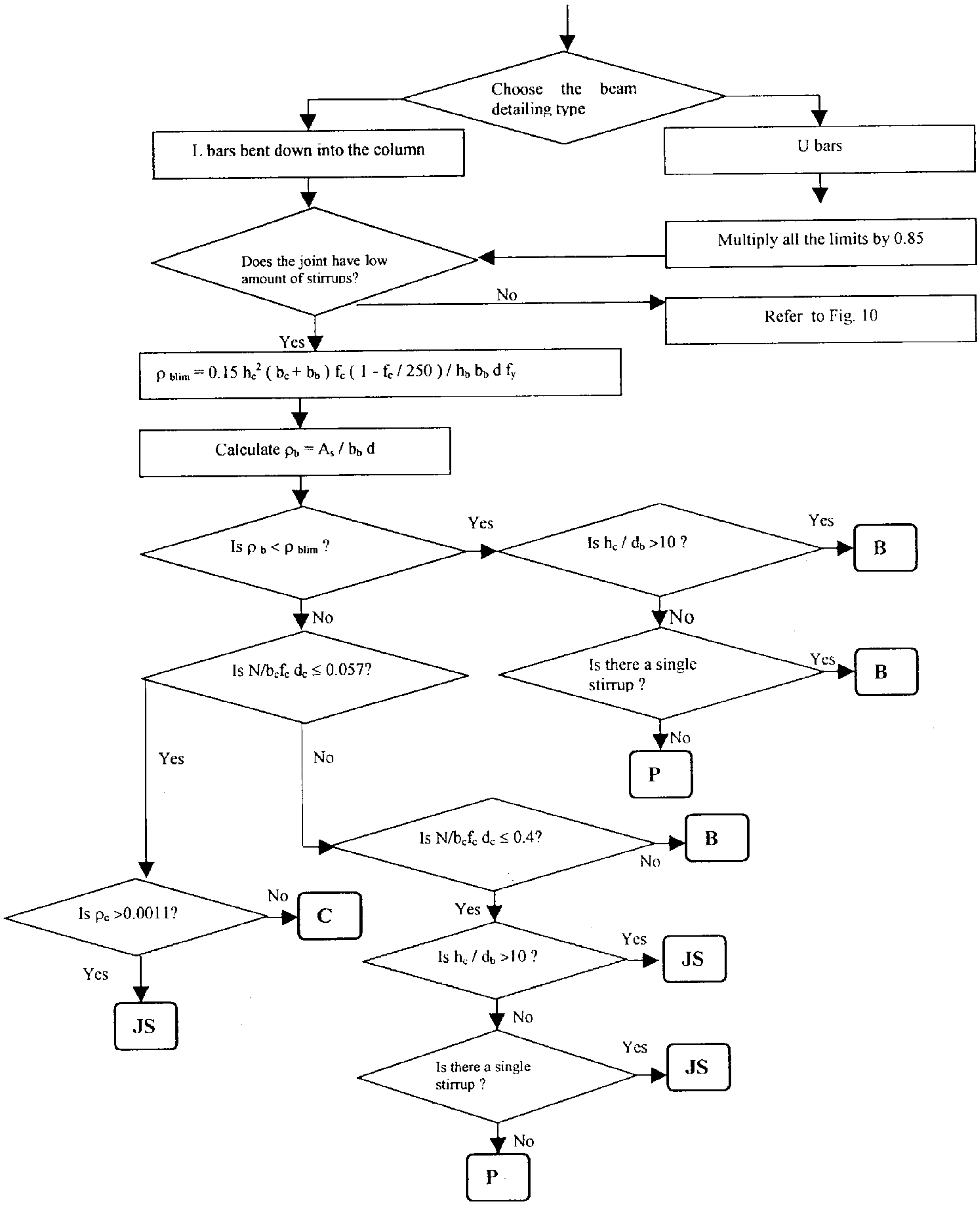
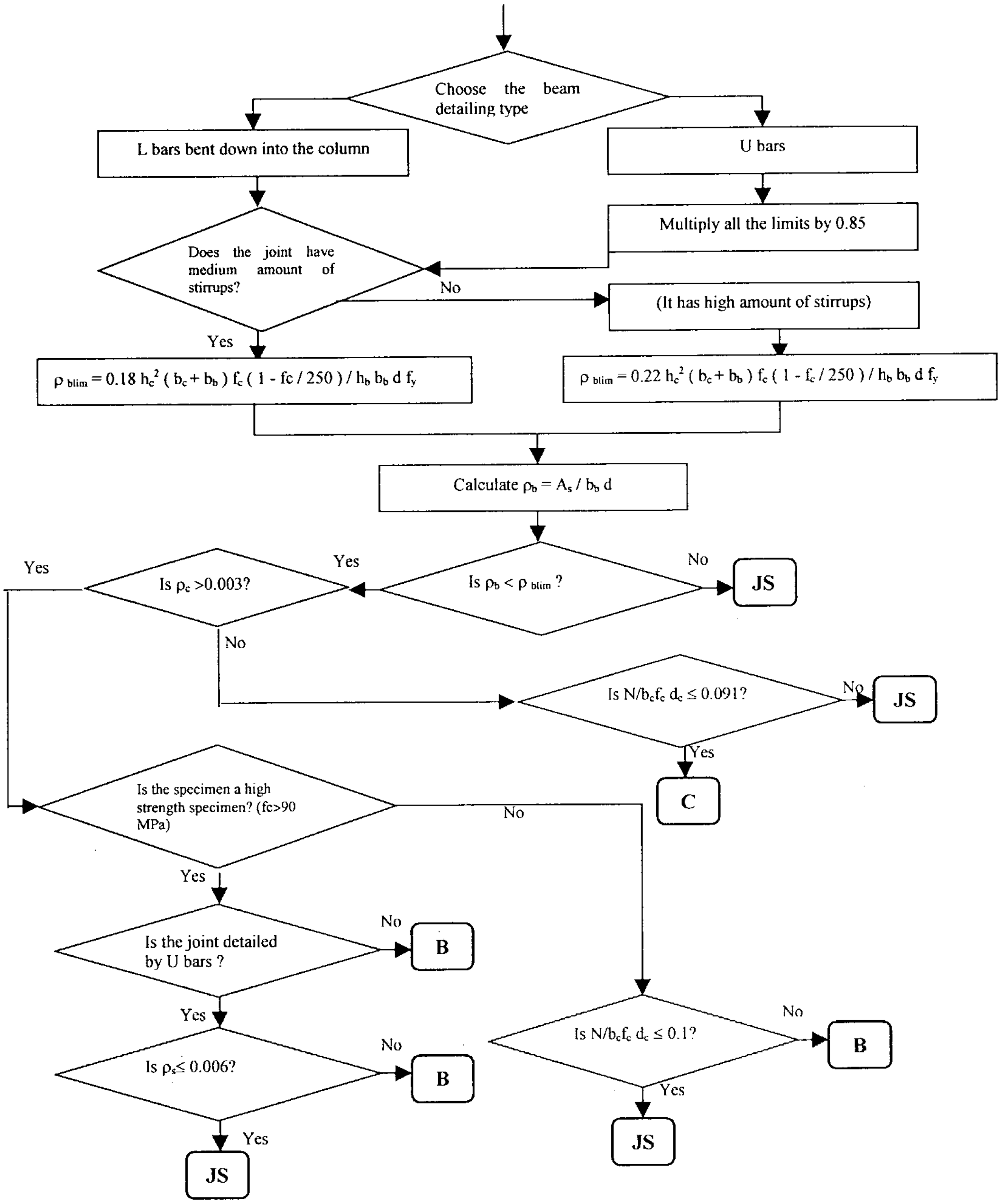


Fig. 9 The methodology for predicting the failure modes of exterior joints with low amount of stirrups or without stirrups



Note:
P represents the connection zone reinforcement pullout
C represents the column failure
B represents the beam failure
JS represents the joint shear failure

Fig. 10 The methodology for predicting the failure modes of exterior joints with medium or high amount of stirrups amount of stirrups as shown.

$$\rho_{b\ \lim} = 0.18h_c^2(b_c + b_b)\frac{f_c\left(1 - \frac{f_c}{250}\right)}{h_b b_b d f_y} \quad \text{for medium amount of stirrups} \tag{25}$$

$$\rho_{b\ \lim} = 0.22h_c^2(b_c + b_b)\frac{f_c\left(1 - \frac{f_c}{250}\right)}{h_b b_b d f_y} \quad \text{for high amount of stirrups} \tag{26}$$

The proposed models for predicting the failure modes are given in Figs. 9 and 10. They are applied on the experimental database given in Table 1. The results show that the model gives good predictions of the failure modes.

4. The factors influencing the failure modes of exterior beam-column joints

Many factors influence the failure modes of exterior joints. A model like Taylor's which takes only a limitation for the beam longitudinal reinforcement ratio will greatly oversimplify the actual behavior of a joint.

4.1 Detail of the beam longitudinal reinforcement

The beam longitudinal reinforcement must necessarily be bent into the joint core for achieving an effective direct strut mechanism as the compressive force in the direct strut is resisted by the bent portion of the beam longitudinal reinforcement. Joints that have beam longitudinal reinforcement bent upward detail, are undesirable in both cyclically and monotonically loaded joints.

The inspection of specimens of RE7 (detailed by L bars bent down detail) and RE8 (detailed by U bars) of Kordina in Fig. 7, showed that the normalized joint shear strength of exterior joints could be increased by 15-20% by the provision of L bars bent down detail reinforcement. Thus Eqs. (20), (25) and (26) should be multiplied by 0.85 when U bar details are used. In joints with high amount of stirrups, inspection of specimens C4ALH3 and C6LH3 of Scott and Hamill (1998) showed that, although both of the specimens had beam longitudinal reinforcement ratio limits lower than that determined by Eq. (26), and column longitudinal reinforcement ratios higher than 0.003, the specimen C4ALH3 that had L bar bent down detail, exhibited beam failure, whereas the specimen C6LH3, that had U bar detail exhibited joint shear failure. The flexural failure of the beam is always preferred to shear failure of the joint. The above considerations show that the behavior of joints are considerably improved if the beam longitudinal reinforcement has L bars bent down detail.

4.2 The joint stirrups

In cyclically loaded beam-column joints, stirrups are only necessary to improve the ductility, confine the joint core and preserve joint stiffness. The strain in the stirrups as well as the relative contributions of the strut and truss mechanisms to the joint shear strength are dependent on the number of cycles. The greater the number of cycles, the higher the strain in the stirrups will be. In monotonically loaded beam-column joints on the other hand, the joint shear strength will increase as the stirrup ratio is increased. The main function of the truss mechanism is to increase the joint shear strength in monotonically loaded beam-column joints.

In joints without significant amount of stirrups, the inspection of C4ALNO and C4ALN1 of Scott and Hamill (1998) which have h_c/d_b ratios less than 10 and beam longitudinal reinforcement ratios higher than that predicted by Eq. (20), showed that provision of a single stirrup in joints without significant amount of stirrups changes the failure mode from connection zone reinforcement pull out to joint shear failure. In joints without significant amount of stirrups, when the beam longitudinal reinforcement ratio is less than that predicted by Eq. (20), the comparison of specimens C4ALH1 and C4ALHO of Scott and Hamill (1998) showed that C4ALHO, which had no stirrups, exhibited connection zone reinforcement pull out, whereas the specimen C4ALH1, which had a single stirrup, exhibited beam failure. In joints with medium and high amount of stirrups, comparison of the high strength specimen C6LH3 of Scott and Hamill (1998), which had a stirrup ratio less than 0.006, a column longitudinal reinforcement ratio higher than 0.003, a U bar detail and a beam longitudinal reinforcement ratio less than that predicted by Eq. (26); with high strength specimen C6LH5 which was identical to specimen C6LH3 except that it had a stirrup ratio higher than 0.006, showed that C6LH5 exhibited beam failure, whereas C6LH3 exhibited joint shear failure. The analysis of the L bar bent down detail specimen C4ALH3 and C4ALH5 showed that the same minimum limit of 0.006 for stirrup area ratio was not relevant to L bar bent down detail specimens, as both C4ALH3 and C4ALH5 exhibited beam failure mode regardless of the fact that the stirrup ratio of C4ALH5 was increased above a stirrup ratio of 0.006. This shows that the U bar detail specimens with stirrup ratios less than 0.006 are unlikely to exhibit beam failure, although the beam longitudinal reinforcement ratio is less than that predicted by Eq. (26).

4.3 Column axial load

The level of column axial load is anticipated to have a significant effect on the failure modes of joints. The first effect is related with bond. There is reduction of slip with increasing axial load which is due to the confinement of the concrete surrounding the development length zone of the beam longitudinal reinforcement by the column axial force. The greater the axial load in a column, the better the bond environment for the beam bars. Thus, the truss mechanism will be enhanced and the possibility of anchorage failures in the joint will decrease. The second effect is related with the prevention of hinges and column failures in the upper column. Joints exhibit column failures when the inner column longitudinal reinforcement yields. In order to prevent this type of failure, either the column longitudinal reinforcement ratio or the column axial stress should be increased as apparent from Eq. (15). There is also evidence for this from the tests of Parker and Bullman. The comparison of Parker and Bullman (1997) specimen 4a with specimens 4b, 4c, 4e or 4f showed that the former specimen exhibited column failure while the latter four specimens all exhibited joint shear failure. Thus, it can be concluded that the specimens with column axial stress levels less than 0.1 have the possibility to exhibit column failure. The inspection of C6LNO and C6LHO of Scott and Hamill (1998) showed that these specimens exhibited joint shear failure in spite of the fact that they had no stirrups and h_c/d_b ratios less than 10. Thus, the above-mentioned limit of 0.1 for column axial stress should be decreased to as low as 0.057 in order to have consistent results with tests in the flowchart for joints without significant amount of stirrups. The specimens C4 and C3 of Scott (1992) are inspected in order to investigate the influence of the column axial stress higher than 0.4, on the failure mode. The two specimens are identical except the fact that the former has a h_c/d_b ratio less than 10 and the latter higher than 10. The former that exhibited joint shear failure had a column axial stress of 0.38 while the latter that exhibited beam failure had a column axial stress of 0.435. Thus, it is reasonable to assume that in joints without significant amount of stirrups, when the beam longitudinal reinforcement ratio is higher than that determined by Eq. (20), joints can only exhibit beam failure if the column axial stress level is higher than 0.4. In joints with stirrups, the inspection of specimens 5a and 5b of Parker and Bullman (1997) showed that these two specimens were identical except that the first, which exhibited column failure had no column axial stress, while the second, which exhibited joint shear failure had a column axial stress level of 0.095. When the column axial stress level was 0.095, although the column longitudinal reinforcement ratio was less than 0.003, the joints were more likely to exhibit joint shear failure instead of column failure. The comparison of Parker and Bullman (1997) specimens 5e and 5f showed that the above mentioned limit can be decreased to 0.091 as the specimen with a column axial stress level of 0.0907 still exhibited column failure. When the column longitudinal reinforcement ratio is higher than 0.003, beam longitudinal reinforcement ratio is less than that predicted by Eq. (26) and the concrete cylinder strength is less than 90 MPa, the comparison of BCJ7 and C4ALN5 showed that the former which had a column axial stress of 0.057 exhibited beam failure while the latter which had a column longitudinal reinforcement ratio of 0.16 exhibited joint shear failure. It is reasonable to assume that joints with column axial stress levels higher than 0.1 are more likely to exhibit beam failure when the column longitudinal reinforcement ratio is higher than 0.003 and the beam longitudinal reinforcement ratio is less than that predicted by Eq. (26).

4.4 Column longitudinal reinforcement ratio

As mentioned above, joints can exhibit column failure when the inner column bars yield. In order to prevent this type of failure, either the column longitudinal reinforcement ratio or the column axial stress should be increased as apparent from Eq. (15). This is also evident from the comparison of specimens 4a and 4d of Parker and Bullman (1997). The specimen 4a which had a column longitudinal reinforcement ratio of 0.001 exhibited column failure, while 4d which had a column longitudinal reinforcement ratio of 0.004, exhibited joint shear failure. It will be sensible to assume that joints without significant amount of stirrups that have beam longitudinal reinforcement ratios higher than that predicted by Eq. (20) and a column axial stress level lower than 0.06 are likely to exhibit column failure unless the column longitudinal reinforcement ratio is higher than 0.001. Comparison of BCJ7 of Ortiz (1993) and 5e of Parker and Bullman (1997) specimen showed that even though both specimens had beam longitudinal reinforcement ratios less than that predicted by Eqs. (25) and (26), BCJ7 which had a column longitudinal reinforcement ratio of 0.004 exhibited beam failure whereas 5e which had a column longitudinal reinforcement ratio of 0.003 exhibited column failure. Therefore it will be prudent to assume that joints with stirrups that have a beam longitudinal reinforcement ratio less than the proposed limits are still likely to exhibit column failure unless their column longitudinal reinforcement ratios are higher than 0.003.

4.5 The ratio of the height of the column to the diameter of the beam bars

In cyclically loaded joints, the bond along the beam reinforcement is lost ultimately, especially after the yielding of the beam longitudinal reinforcement if the strength and size of the reinforcement are not strictly limited. As bond deteriorates, the truss mechanism disappears and the joint shear is resisted mainly by the direct strut mechanism. The tension force which is not transferred to the joint concrete by bond is inevitably resisted by the compressive face of the joint resulting in an increase of the shear stresses in the direct concrete strut mechanism. As the joint core concrete is cracked by the cyclic loading, the concrete compressive strength decreases. Thus, the shear resistance of the direct strut mechanism decreases and the joint eventually exhibits shear failure by the crushing of the concrete due to the compressive stress in the concrete strut. In order to limit slippage of the beam and column bars through the connection, The ACI-ASCE Committee 352 requires that for cyclically loaded joints, the ratio of the column cross-sectional height to the diameter of the beam longitudinal reinforcement should be at least as shown in Eq. (27).

$$\frac{h_c}{d_b} \geq 20 \tag{27}$$

The ACI-ASCE Committee 352 does not give any recommendations on this ratio for monotonically loaded joints. However, the inspection of the experimental database in Tables 1, 2 and 3 has shown that monotonically loaded exterior beam-column joints are also likely to exhibit anchorage failure unless their h_c/d_b ratios are higher than 10 and they are provided by at least a single stirrup as evident from specimens C4ALHO and C4ALH1 of Scott. The comparison of C1A of Scott (1992) that exhibited beam failure (which had a h_c/d_b ratio of 12.5) with C4ALHO of Scott and Hamill (1998) that exhibited anchorage failure (which had a h_c/d_b ratio of 9.38), showed that monotonically loaded joints with beam longitudinal reinforcement ratios less than that predicted by Eq. (20) and without stirrups are still likely to exhibit anchorage failure unless the ratio of h_c/d_b is higher than 10. In joints with significant amount of stirrups, on the other hand, no anchorage failures are observed in the experimental database inspected. Thus, it is recommended that the ratio of the height of the column to the diameter of the beam bars should be as shown in Eq. (28) for monotonically loaded exterior beam-column joints.

$$\frac{h_c}{d_b} \geq 10 \tag{28}$$

5. Conclusions

A model that can predict the failure modes of exterior joints is proposed. The interaction of several parameters, such as, column longitudinal reinforcement ratio, h_c/d_b ratio, the stirrup ratio, the column axial stress, and, the beam longitudinal reinforcement ratio are taken satisfactorily into account. A design equation has been developed for the beam longitudinal reinforcement ratio for joints with and without stirrups. Table 3 shows that the model gives exact predictions of the failure modes. Based on this study, the following recommendations are suggested for design;

1. In cyclically loaded joints, only 90^o hooks are permitted and L bars bent up or U bar detail joints are not permitted. Monotonically loaded joints should also follow the same norm.
2. Column axial load significantly affects the failure mode. The behavior of the beam-column joints will be significantly improved if the joints have high column axial load, high column

longitudinal reinforcement ratio, beam longitudinal reinforcement ratio less than that predicted by Eqs. (25) and (26) and either medium (stirrup area ratio $A_{sje}/b_{eff}h_c$ between 0.003 and 0.0055) or high amount of stirrups (stirrup area ratio higher than 0.0055).

3. In monotonically loaded joints with medium or high amount of stirrups, the joints will never perform adequately unless their beam longitudinal reinforcement ratios are less than that predicted by Eqs. (25) and (26).
4. Stirrups affect the failure modes differently for cyclically loaded or monotonically loaded beam-column joints. Cyclically loaded joints with medium or high amounts of stirrups can exhibit anchorage failure. However, the monotonically loaded joints with medium and high amount of stirrups are unlikely to exhibit anchorage failure.
5. The proposed value of 10 for h_c/d_b is too low for cyclically loaded joints to prevent anchorage failure. However, in monotonically loaded joints without stirrups, this lower limit is adequate to prevent anchorage failures.
6. In joints with medium and high amount of stirrups and with beam longitudinal reinforcement ratios less than that predicted by Eqs. (25) and (26), the joints will exhibit column failure unless the column longitudinal reinforcement ratio and the column axial load are higher than the proposed limits in Fig. 10. Column failures are initiated by the yielding of the inner column reinforcement. As apparent from Eq. (15), yielding of the inner column longitudinal reinforcement can only be prevented if either the column longitudinal reinforcement ratio or the column axial load are increased.
7. The relative contributions of the truss and the strut mechanisms to joint shear are different for joints with low, medium and high amount of stirrups. The joint shear strength is not affected by the provision of low amounts of stirrups (stirrup ratios up to 0.003). In joints with medium amount of stirrups (that have stirrup ratios between 0.003 and 0.0055) the joint shear strength is increased by 18.14%. In joints with high amount of stirrups (joints that have stirrup area ratios higher than 0.0055), the joint shear strength is increased by 29.7% with respect to the joints without stirrups. After the joint stirrup area limit of 0.0055, the increase in joint shear strength is constant and there is no difference in the increase of joint shear strength for joints with stirrup ratios higher than 0.0055.
8. The simple strut and tie model predicts the failure loads as accurate as Vollum model. The average ratio of the predicted failure load of the proposed simple model to the actual failure load of tests is 0.999. The simple strut and tie model shows that another strut coming from the upper column which is resisted by the upper column ties in the horizontal direction and by column longitudinal reinforcement in the vertical direction equilibrates the extra shear that occurs in the secondary strut.
9. The increase of the stirrup ratio above the limit of 0.006 has a more pronounced effect on the failure modes of the U bar detail specimens than the L bars bent down detail specimens. The U bar detail specimens with stirrup ratios less than 0.006 are unlikely to exhibit beam failure although the beam longitudinal reinforcement ratio is less than that predicted by Eq. (20).

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Notation

A_s : cross-sectional area of the beam reinforcement
A_{scol} : cross-sectional area of the total column reinforcement
A_{scu} : area of the compressive reinforcement in the upper column (outer column reinforcement)
A_{stb} : area of the tensile reinforcement in the bottom column
A_{stu} : area of the tensile reinforcement in the upper column (inner column reinforcement)
A_{sje} : area of the stirrups
b_{eff} : average width of the beam and the column
b_c : width of the column cross section
b_b : width of the beam cross section
C_{cu} : compressive force in the upper column rectangular stress block
d_b : diameter of the beam longitudinal reinforcement
d^1 : cover
e_{cedir} : distance from the vertical force that resists the direct strut to the edge of the top column
e_{cidir} : distance from the vertical force that resists the direct strut to the edge of the bottom column
E_s : modulus of elasticity of steel

f_2 : principal compressive stress in concrete
$f_{2\max}$: maximum stress in concrete panels
f_c : mean cylinder strength without any factors of safety in MPa
f_{cr} : cracked concrete strength given by the CEB Model Code 90
f_s : average stress in the beam reinforcement
f_{sb} : stress in the beam longitudinal reinforcement
f_{scol} : average stress in the column reinforcement
f_{scu} : stress in the compressive reinforcement in the upper column
f_{stb} : stress in the tensile reinforcement in the lower column
f_{stu} : stress in the tensile reinforcement in the upper column
f_y : yield strength of the reinforcement
f_{yw} : stirrup yield strength
f_w : average stress in the transverse reinforcement
F_v : vertical shear
$F_{vprimary}$: vertical shear resisted by the primary strut mechanism (same as F_v in the authors' model and F_{vdir} in Vollum model)
h_c, h_b : cross-sectional height of the column and beam respectively
H : total height of the column (distance between horizontal restraints)
L : distance from the point of application of the load to the face of the column
M_{colu}, M_{colb} : upper column moment and lower column moment respectively
N : column axial load
P : applied load
T_{beam} : tensile force in the beam longitudinal reinforcement
T_{si} : tensile force in outer column bars at upper joint boundary
T_{se} : tensile force in inner column bars at lower joint boundary
V_c : joint shear strength of the concrete (without stirrups)
v_c : concrete shear strength of normal beams taken from Table 5 of CP110
V_{colbot} : shear force in the lower column
V_{coltop} : shear force in the top column
$V_{primary}$: horizontal shear force resisted by the primary strut mechanism in the authors' model (V_{jdir} in Vollum model)
$V_{secondary}$: horizontal shear force resisted by the secondary strut mechanism in the authors' model (V_{jind} in Vollum model)
x_{top} : width of the rectangular stress block of the upper column
z_b : distance between the top and bottom beam reinforcement
z_{db} : distance between the horizontal force that resists the strut mechanism and the beam reinforcement
β : moment re-distribution
ψ : angle of inclination of the direct strut
ε_1 : principal tensile strain
ε_2 : compressive strain
ε_c : compressive strain at failure (–0.002)
ε_x : strain in the *x* direction
ε_y : strain in the *y* direction

θ	: angle between the direction of the principle compressive stress and the transverse tensile strain ε_t
σ_x, σ_y	: average normal concrete stresses in the horizontal and vertical directions respectively
μ	: $= f_s / f_w$
ρ_b	: beam steel percentage
τ_{av}	: average shear stress in the joint