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# A robust nonlinear mathematical programming model for design of laterally loaded orthotropic steel plates

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**Abstract.** The main objective of the present paper is to address a formal procedure for orthotropic steel plates design. The theme of the proposed approach is to recast the design procedure into a mathematical programming model. The objective function to be optimized is the total weight of the structure. The total weight is function of its layout parameters and structural element design variables. Mean while the proposed approach takes into consideration the strength and rigidity criteria in addition to other dimensional constraints. A nonlinear programming model is developed which consists of a nonlinear objective function and a set of implicit/explicit nonlinear constraints. A transformation method is adopted for minimization strategy, where the primal model constrained problem is transformed into a sequence of unconstrained minimization models. The search strategy is based on the well-known Fletcher/Powell algorithm. The finite element technique is adopted for discretization and analysis strategies. Mindlin theory is selected to simulate the finite element model and a selective reduced integration scheme is exploited to avoid a shear lock problem.

**Key words:** optimization; orthotropic; steel plates.

## 1. Introduction

In the modern design of steel structures, it is important not only to ensure the safety of the structure, but also to make sure that it is economically optimal. This optimality can be achieved by utilizing efficiently the steel and consequently reducing the dead weight of the structure. Most of the design approaches depend, heavily and basically, on the experience of the practitioner. Therefore, a formal design procedure would be useful. The main objective of the present research work is to address an optimal design procedure, which overcomes the inconsistencies of ad-hoc approaches and minimizes the effect of the subjectivity of practitioners. The kernel of the proposed procedure is the finite element method as an analysis engine.

The orthotropic steel plate as shown in Fig. 1, consists of a top steel plate stiffened longitudinally and transversely by ribs. All members of the structure are welded together. It is quite obvious that the economy criterion of such plates is an inverse function of weight to strength ratio, which means that achieving minimum (optimal) weight and relatively high strength leads to a great economy. The

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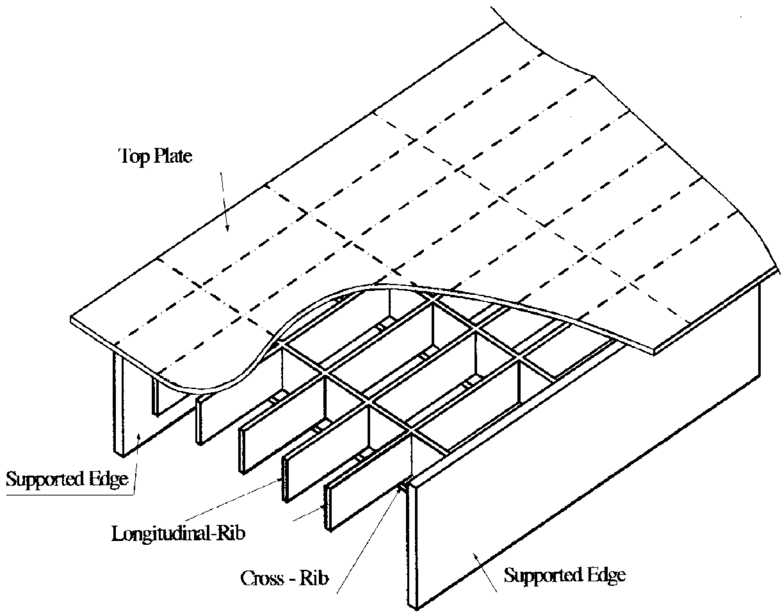


Fig. 1 Orthotropic steel plate

cost of steel structures consists of the costs of material, manufacturing and erection. Obviously, the costs of manufacturing and erection are directly related to the weight of the structure material. Therefore, the least weight satisfying stresses, deflection and environmental conditions will consequently load to the minimum cost. Accordingly, the structure of least weight can be considered as an optimal one.

The structural optimization research over several decades of intensive study has proved to be an effective tool for designers, Atrek (1984). Any structural optimal design model is characterized by the nature of the design variables, state variables, objective function, constraints, structural method used to analyze the current design and the design strategy adopted to evaluate the variations in design variables. In this work the method of structural analysis is kept independent of the numerical method of optimization, the later simply uses the output of the former and provides it with new input data. The numerical methods of structural optimization are mainly classified into two categories, the first one is a direct approach and known by optimality criteria methods. The other approach is indirect and called by transformation methods. The optimality criteria methods need both the objective function and constraints to be explicit functions of the design variables while the transformation methods can deal with any type of functions where the objective and constraints functions are merged together into one augmented function to change the optimization problem to a sequence of unconstrained minimization ones.

The optimality criteria methods are used for trusses and frames by Dobbs *et al.* (1978), Fleury (1978 & 1986), Sander (1978), Gellatly (1971), Saka (1991 & 1992) and Hayaliogla (1992). In these methods the authors tried to make the constraints in explicit forms in design variables.

Due to nonlinearity of the objective and constraints functions in addition to the implicit nature of some constraints, the transformation methods are more convenient for such problems, Haug and Arora (1979). The transformation methods are globally convergent which means that they have been proven to converge to a local minimum for any given initial design. Among these methods, there are the sequential unconstrained minimization techniques (SUMTs) in which the problem is reduced to a sequence of unconstrained minimization problems.

The SUMTs-formulation are used for frames by Kanagasundran (1990), cylindrical tanks by Thevendran (1992) and stiffened panels by Patel (1980) and Lund (1974). Systematic search methods are used by Ezeldin (1991) for reinforced concrete beams and by Ostwald (1990) for sandwich cylindrical shells.

Very few papers have been published about the optimization of steel stiffened panels or orthotropic plates. In the present paper the SUMT (interior penalty method) is adopted in the framework of finite element method as a discretizing engine to formulate an optimal weight design model for orthotropic plates. The weight function is subjected to a set of constraints relevant to both strength and design criteria. Three models for the optimization of orthotropic plates are introduced depending on the design variables. A comparison for the three models is discussed for the same example. The optimum weight for an example with different layouts is given by using the third model.

## 2. Discretization strategy

The top plate is discretized into a grid of nonconforming plate bending elements, Hughes (1987); while the stiffeners (ribs) are discretized into isoparametric beam elements. We assume five degrees of freedom at each node of the finite element grid; $\{u_0, v_0, w_0, \theta_{x0}, \theta_{y0}\}$ assuming that the plate is highly rigid in the plane rotation. The displacements $\{U, V, W\}$ at any point can be expressed in terms of the midplane displacements $\{u_0, v_0, w_0\}$ and the transverse rotations $\{\theta_{x0}, \theta_{y0}\}$ according to Mindlin theory, Deb (1988) and Palani (1992) as follows:

$$U(X, Y, Z) = u_0(X, Y) - z\theta_x \tag{1}$$

$$V(X, Y, Z) = v_0(X, Y) - z\theta_y \tag{2}$$

$$W(X, Y, Z) = w_0(X, Y) \tag{3}$$

Where $X$, $Y$, and $Z$ are the reference axes and $z$ is the distance of the point to the midplane of the plate in $Z$-direction. This displacement field is applied for plate and beam elements considering the transverse shear effect. The selective reduced integration scheme, Hughes (1987), is used to overcome the shear locking computational problem.

The degrees of freedom at each node (i) of a stiffener in $X$-direction as shown in Fig. 2 are transformed to be consistent with the plate frame of reference by the following relationship:

$$\begin{bmatrix} u_{si} \\ w_{si} \\ \theta_{xsi} \\ \theta_{ysi} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -e_x & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_{0i} \\ v_{0i} \\ w_{0i} \\ \theta_{x0i} \\ \theta_{y0i} \end{bmatrix} \tag{4}$$

Where $e_x$ is the eccentricity of $X$-stiffener from the mid plane of the top plate. Similarly the

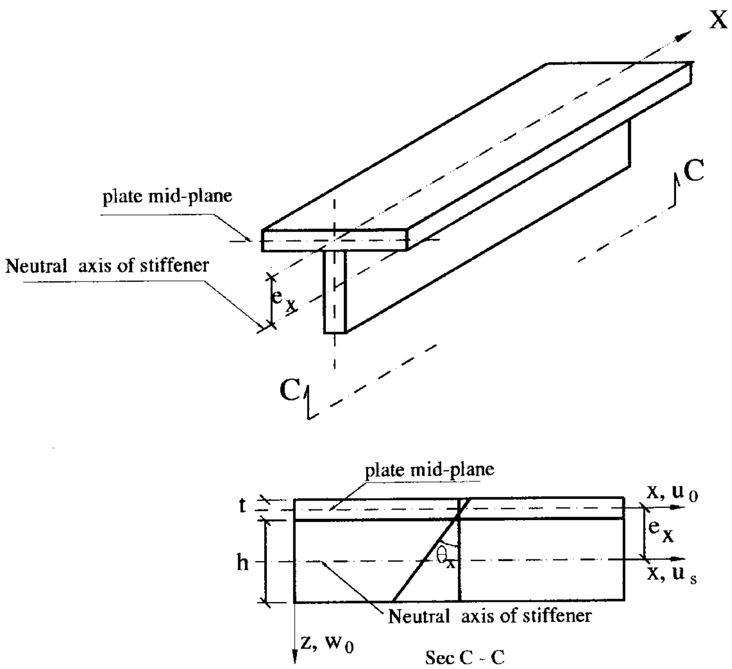


Fig. 2 Plate with an Eccentric *X*-Stiffener

degrees of freedom for each node (i) of a stiffener in *Y*-direction are given by Eq. (5).

$$\begin{bmatrix} v_{si} \\ w_{si} \\ \theta_{xsi} \\ \theta_{ysi} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & -e_y \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_{0i} \\ v_{0i} \\ w_{0i} \\ \theta_{x0i} \\ \theta_{y0i} \end{bmatrix} \tag{5}$$

## 3. Optimization strategy

The orthotropic steel plate consists of the following elements: 1) top plate acts as a flange for other members 2) longitudinal ribs welded to the top plate and cross-ribs and 3) cross ribs which are welded to the top plate and the boundary. The weight of the orthotropic plate is a nonlinear objective function in the design variable as will be shown in the different models. This objective function is subjected to linear constraints using bounds on the design variables and implicit constraints on deflection and stresses.

Due to this implicit nature of constraints and nonlinearity of the objective function, the transformation methods are convenient for such type of problems. In these methods, the problem is casted as follows:

$$\text{Minimize } f(x) = \text{weight}$$
$$\text{Subject to } g_i(x) \le 0.0, \quad i = 1, 2, ..., m$$

Where $f(x)$ is the objective function in terms of the design vector $x$; $g_i(x)$ are the constraints and $m$ is the total number of constraints. Both the objective function $f(x)$ and the constraints $g(x)$ are transformed into one augmented function $\phi(x, R)$:

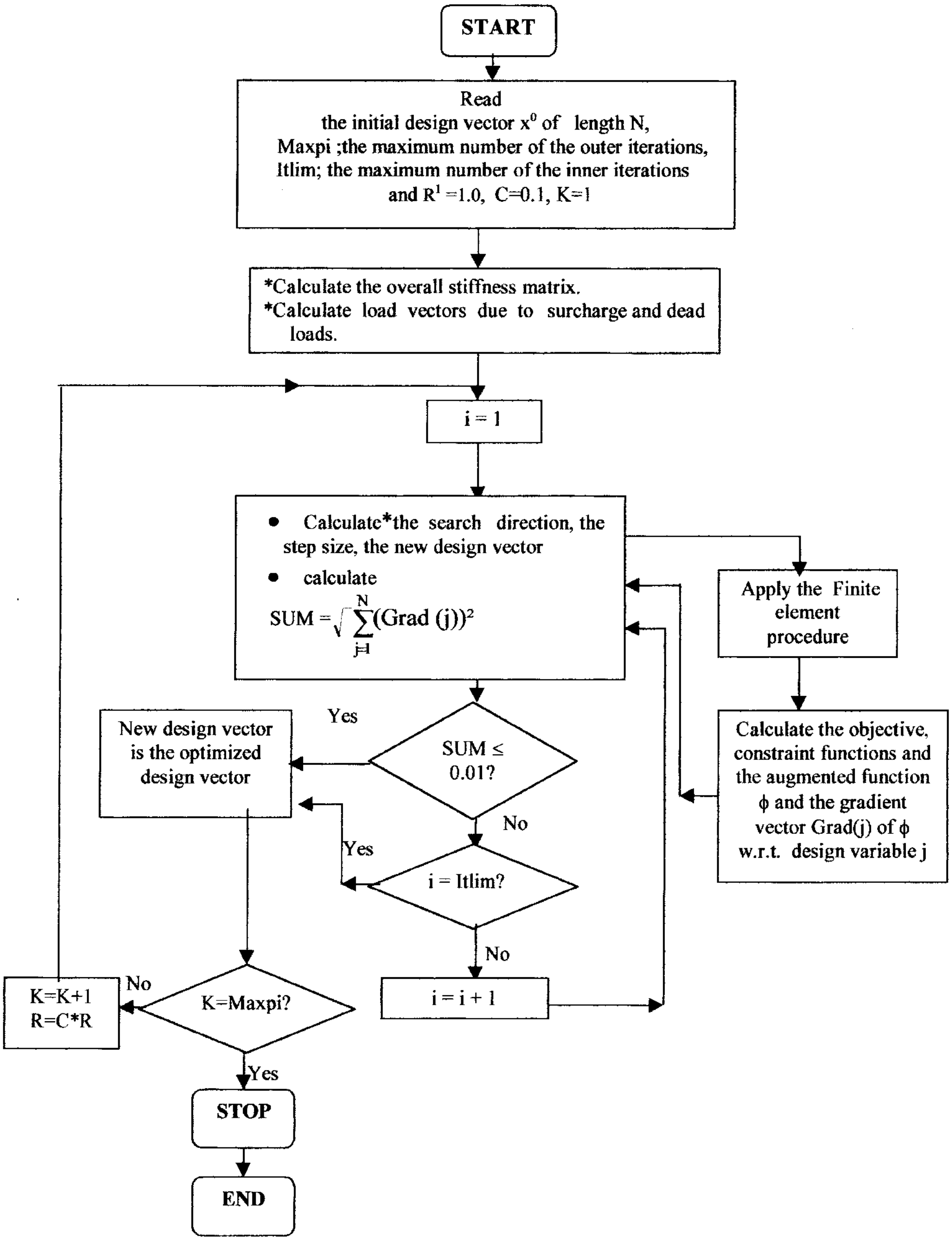


Fig. 3 Algorithm flow chart (*Refer to Fletcher & Powel algorithm in Appendix)

$$\phi(x, R) = f(x) + R\left[\sum_{i=1}^{m} \frac{-1}{g_i(x)}\right] \tag{6}$$

Where the last part of the right hand side represents the penalty function and $R$ is the penalty parameter.

The optimal solution is obtained through a sequence of minimization of a set of unconstrained problems, depending on an initial feasible design vector $x^0$ and the penalty parameter $R$. For iteration $j = 1, 2, \ldots$, the unconstrained problems depicted by Eq. (6) are solved by taking $R^j = R^{(j-1)}/10$ considering $R^0 = 10$. Giving $x^{j-1}$ as a starting design vector, the unconstrained function $\phi^j(x^{(j-1)}, R^j)$ is minimized to obtain $x^j$ by any search strategy such as Fletcher and Powell algorithm, Rao (1984). Appendix contains the details of Fletcher and Powell search algorithm.

A flow chart of the optimization procedure is illustrated in Fig. 3. Three models for the optimization of orthotropic plates are introduced to study the sensitivity of design variables.

## 4. Case studies

Three different models are developed. For each model, the objective function and constraints are described comprehensively.

### 4.1 Model I

In this model the weight of an orthotropic plate shown in Fig. 4 is taken as follows:

$$W_t = \gamma_0[A_0\, B_0\, x(1) + N_s x(2)\, x(3)\, B_0 + N_c\, A_0(x(4)\, x(5) + x(6)\, x(7))]$$

Where $\gamma_0$ is the specific weight of the steel material and $x$ is the design vector, which is considered as follows:

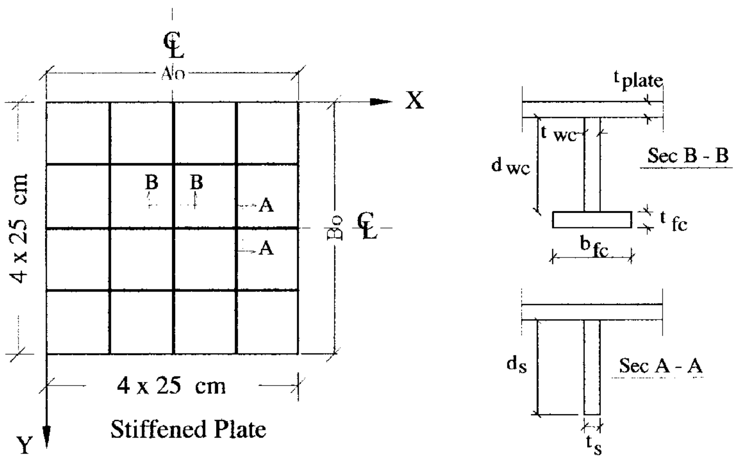


Fig. 4 A simply supported stiffened plate

$x(1)$ : the top plate thickness ($t_{plate}$).
$x(2)$ : the longitudinal rib thickness ($t_s$).
$x(3)$ : the longitudinal rib depth ($d_s$).
$x(4)$ : the cross rib web thickness ($t_{wc}$).
$x(5)$ : the cross rib web depth ($d_{wc}$).
$x(6)$ : the cross rib flange width ($b_{fc}$).
$x(7)$ : the cross rib flange thickness ($t_{fc}$).
$N_s$ : the number of longitudinal ribs.
$N_c$ : the number of cross ribs.
$A_0$, $B_0$ : the length and width of the plate.

The problem is stated as follows:

$$\text{Minimize } f(x) = W_t$$
$$\text{Subject to } g_i(x) \leq 0.0, \quad i = 1, 2, ..., m$$

Where $g_i(x)$ are the constraints which can be categorized as follows:
a) The design variables constraints:

$$g_j(x) = 1.0 - \frac{x(j)}{x_{j\min}} \leq 0.0, \quad j = 1, 2, \ldots, NDV$$

b) Deflection constraint:

$$g_{NDV+1}(x) = \frac{\delta_{\max}}{\delta_{all}} - 1.0 \leq 0.0$$

c) Stress constraint:

$$g_{NDV+2}(x) = \frac{f_{\max}}{f_{all}} - 1.0 \leq 0.0$$

Where $NDV$ is the number of design variables, $x_{j\min}$ is the minimum allowable dimension of design variable $x(j)$, $\delta_{\max}$ & $\delta_{all}$ are the maximum and allowable deflections and $f_{\max}$ & $f_{all}$ are the

Table 1 Initial and final design values versus the iterations process for a square stiffened plate Model I

| Iterations | Case \ Design variables | $t_{plate}$ (mm) | $t_s$ (mm) | $d_s$ (mm) | $t_{wc}$ (mm) | $d_{wc}$ (mm) | $b_{fc}$ (mm) | $t_{fc}$ (mm) | 1/4 weight (kg) |
|---|---|---|---|---|---|---|---|---|---|
| | Lower bound | 2 | 1 | 20 | 1 | 30 | 20 | 1 | |
| | Initial value | 15 | 2 | 50 | 3 | 120 | 40 | 3 | 32.852 |
| | Initial Gradient | 16.793 | –8.184 | –0.2558 | –12.965 | –1.05 | –1.748 | –18.268 | |
| 4 | Finals | 3.2925 | 4.221 | 49.636 | 2.4299 | 119.79 | 40.383 | 23.016 | 14.881 |
| 8 | | 2.4632 | 2.633 | 49.097 | 1.4779 | 119.36 | 39.452 | 6.283 | 7.796 |
| 12 | | 2.3285 | 2.477 | 49.08 | 1.0955 | 119.35 | 31.399 | 5.8534 | 7.1356 |
| 16 | | 2.2591 | 2.2906 | 49.066 | 1.0237 | 119.35 | 31.348 | 5.804 | 6.8859 |
| 20 | | 2.2441 | 2.2744 | 49.008 | 1.0181 | 119.35 | 31.341 | 5.8007 | 6.8464 |

maximum and allowable stresses.

A square stiffened plate given by Fig. 4 is considered as an example to be optimized. The plate is subjected to uniformly distributed lateral load of 1.0 kg/cm$^2$, modulus of elasticity $2.1 \times 10^6$ kg/cm$^2$ and Poisson's ratio equals 0.3. The allowable deflection and stress equal 0.5 cm and 1400 kg/cm$^2$ respectively. The initial and final values of the design variables are given in Table 1 for different iterations. The $N_s$ and $N_c$ are taken 3 in this example.

### 4.2 Model II

In this model the depth of longitudinal rib and the depth and flange width of cross ribs are excluded from the design variables and taken as constant values equal 5.0, 12.0, and 4.0 respectively. In this case the objective function is considered as follows:

$$W_t = \gamma_0[A_0\, B_0\, x(1) + N_s\, B_0\, x(2)\, d_s + N_c\, A_0\, (x(3)\, d_{wc} + b_{fc}\, x(4))]$$

Where:

$x(1)$ : the top plate thickness, ($t_{plate}$).
$x(2)$ : the longitudinal rib thickness, ($t_s$).
$x(3)$ : the cross rib web thickness, ($t_{wc}$).
$x(4)$ : the cross rib flange thickness, ($t_{fc}$).

The same constraints of Model I are considered except that the thickness of different elements are considered only as design variables.

The same previous example is considered and the initial and final designs are given in Table 2.

### 4.3 Model III

In this model the depth of longitudinal ribs and the depth and flange width of cross ribs are considered as ratios of their thicknesses instead of taken them as constant values as in Model II. In this case the objective function is considered as follows:

$$W_t = \gamma_0[A_0\, B_0\, x(1) + N_s\, B_0\, \alpha_1 x(2)^2 + N_c\, A_0(\alpha_2\, x(3)^2 + \alpha_3\, x(4)^2)]$$

Table 2 Initial and final design values versus the iterations process for a square stiffened plate Model II

| Iterations | Case \ Design variables | $t_{plate}$ (mm) | $t_s$ (mm) | $t_{wc}$ (mm) | $t_{fc}$ (mm) | 1/4 weight (kg) |
|---|---|---|---|---|---|---|
| | Lower bound | 2 | 1 | 1 | 1 | |
| | Initial value | 15 | 2 | 3 | 3 | 32.852 |
| 4 | Finals | 3.2894 | 4.2023 | 2.412 | 22.99 | 14.81 |
| 8 | | 2.4519 | 1.9866 | 1.422 | 11.201 | 9.039 |
| 12 | | 2.3145 | 2.2498 | 1.0813 | 4.698 | 7.0748 |
| 16 | | 2.2806 | 2.2496 | 1.0309 | 4.369 | 6.895 |
| 20 | | 2.2513 | 2.1846 | 1.0184 | 4.3551 | 6.806 |

Table 3 Initial and final design values versus the iterations process for a square stiffened plate Model III alternative 1

| Iterations | Case \ Design variables | $t_{plate}$ (mm) | $t_s$ (mm) | $t_{wc}$ (mm) | $t_{fc}$ (mm) | 1/4 weight (kg) |
|---|---|---|---|---|---|---|
| | Lower bound | 2 | 1 | 1 | 1 | |
| | Initial value | 15 | 2 | 3 | 3 | 37.268 |
| 4 | Finals | 3.1189 | 2.0965 | 1.6646 | 4.7840 | 10.461 |
| 8 | | 2.3701 | 1.6471 | 1.2582 | 3.8373 | 7.2757 |
| 12 | | 2.1535 | 1.5019 | 1.2220 | 3.5618 | 6.5593 |
| 16 | | 2.1223 | 1.4940 | 1.1759 | 3.5876 | 6.4257 |
| 20 | | 2.1145 | 1.4968 | 1.1763 | 3.5885 | 6.4130 |

Table 4 Initial and final design values versus the iterations process for a square stiffened plate Model III alternative 2

| Iterations | Case \ Design variables | $t_{plate}$ (mm) | $t_s$ (mm) | $t_{wc}$ (mm) | $t_{fc}$ (mm) | 1/4 weight (kg) |
|---|---|---|---|---|---|---|
| | Lower bound | 2 | 1 | 1 | 1 | |
| | Initial value | 15 | 2 | 3 | 3 | 35.325 |
| 4 | Finals | 2.8325 | 2.6923 | 1.6612 | 4.2625 | 10.0650 |
| 8 | | 2.3531 | 2.2829 | 1.4395 | 3.3505 | 7.6830 |
| 12 | | 2.1575 | 2.1243 | 1.3576 | 3.3038 | 7.0515 |
| 16 | | 2.1289 | 2.1254 | 1.3207 | 3.2967 | 6.9442 |
| 20 | | 2.1225 | 2.1227 | 1.3138 | 3.2904 | 6.9165 |

Where:

$x$ : the design vector as given in Model II,

$\alpha_1$ : the depth to thickness ratio of longitudinal rib,

$\alpha_2$ : the depth to thickness ratio of the web of cross rib,

$\alpha_3$ : the width to thickness ratio of cross rib flange.

$\alpha_1$, $\alpha_2$, and $\alpha_3$ are taken according to the considered specifications. The same constraints of Models I and II are considered. The same previous example is considered and the initial and final designs for $\alpha_1 = 40$, $\alpha_2 = 120$, and $\alpha_3 = 10$ are given in Table 3 and for $\alpha_1 = 25$, $\alpha_2 = 80$, and $\alpha_3 = 20$ are given in Table 4.

## 5. Discussion of the results of different models

From Table 1, it is noticed that the gradients of the objective function with respect to the thicknesses are high which indicates that it is sensitive to these thicknesses. Also, it is noticed that the depth of ribs and the flange width of cross *T*-rib are slightly changed versus the iterations while the thicknesses are significantly changed. From Table 2, it is noticed that in Model II very slight changes occurred in the final design variables and the weight compared with the results of Model I, but a great reduction in the consumed time is achieved. Therefore the thicknesses of different elements are to be considered as design variables in the design of orthotropic plates.

From Table 3, it is noticed that the optimum weight of Model III for $\alpha_1 = 40$, $\alpha_2 = 120$, and $\alpha_3 = 10$ is about equal the value of Model II given in Table 2 but the values of design variables are changed according to the changes of the depth of longitudinal stiffener and the depth and flange width of cross ribs (floorbeams). The depth $d_s$ in Models I & II equals 5.0 cm while in Model III equals $40 \times 0.14968 = 5.987$ cm. The depth $d_{wc}$ in Models I & II equals 12.0 cm while in Model III equals $120 \times 0.11763 = 14.115$ cm. The flange width $b_{fc}$ in Models I & II equals 4.0 cm while in Model III equals $10 \times 0.35885 = 3.5885$ cm.

When changing the ratios values $\alpha_1$, $\alpha_2$, and $\alpha_3$ to be 25, 80 and 20 respectively, the final optimum weight in this case given in Table 4 is nearly the same as the previous values given in Table 3. The design variables have a remarkable change due to changing in the $\alpha$ ratios. This indicates that these ratios affect on the thickness values to obtain the same optimum weight. Therefore the values of the above $\alpha$ ratios are maintained general and according to the used specifications.

The weight of one quarter of the stiffened plate is plotted versus the different iteration in Fig. 5 showing the design history for the four cases. The optimum weight reaches nearly the same value after a few iterations and the curves have the same decreasing trend and there are no unexpected point. The active constraints in all cases are the stress, which equals 1399.9 kg/cm$^2$, the web thickness of cross rib and the thickness of the top plate as given in Tables 1-4.

## 6. Parametric study

To study the effect of the number of longitudinal and cross ribs ($N_s$, $N_c$) on the optimum weight of a stiffened plate (orthotropic plate), the computer program is modified to change these numbers

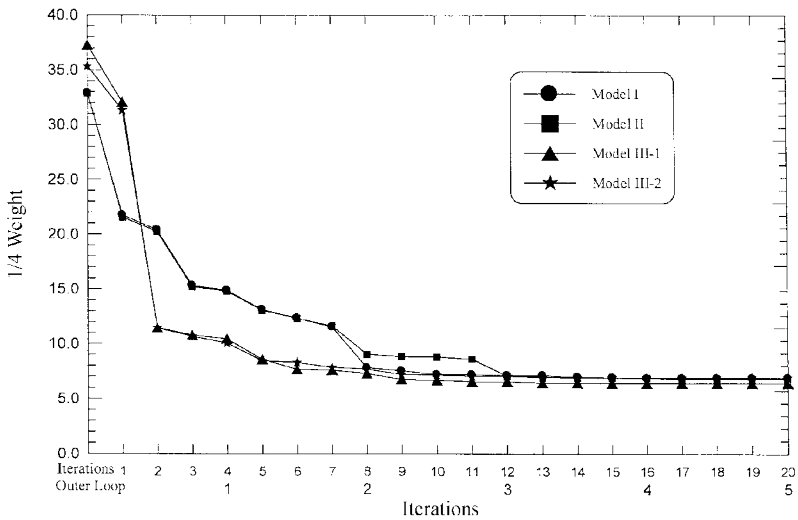


Fig. 5 Weight vs. Iterations (Design history)

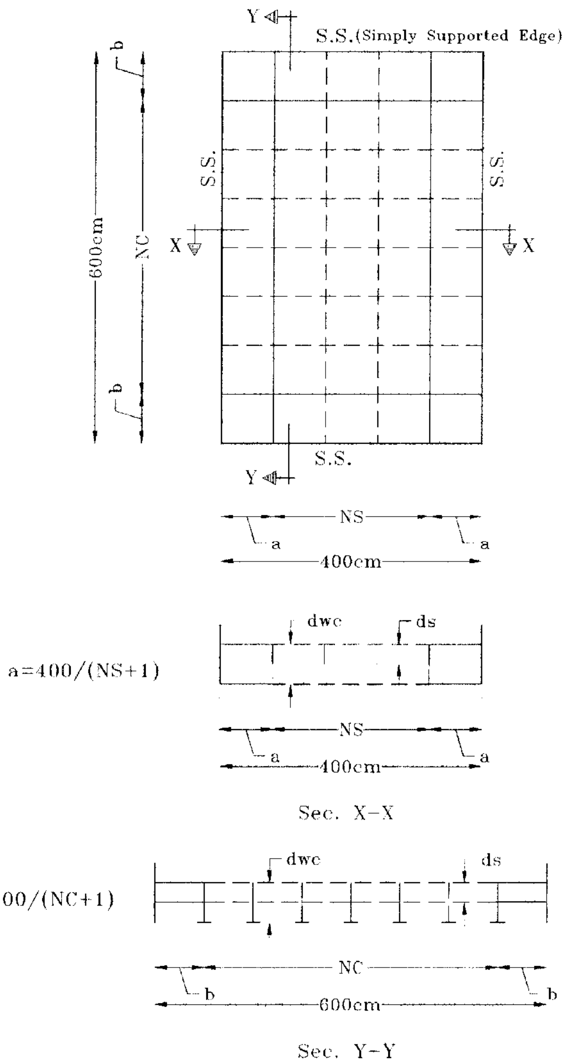


Fig. 6 A simply supported orthotropic plate

automatically by taking the upper and lower numbers for longitudinal and cross-ribs. After solving all combinations the output is the minimum weight for each combination and the optimum of these combinations.

## 6.1 Example

A simply supported orthotropic plate of length 600 cm and width 400 cm as shown in Fig. 6 is considered. The plate is uniformly laterally loaded by 0.1 kg/cm$^2$, modulus of elasticity equals $2.1 \times 10^6$ kg/cm$^2$, Poisson's ratio equals 0.3. The depth to thickness of longitudinal rectangular ribs equals 10, the depth to thickness of cross-ribs web equals 40, and the flange width to its thickness ratio equals 15, within the range of the Egyptian code of practice (1993). The minimum thickness

Table 5 Optimum weight for different $N_c$ and $N_s$

| $N_c$ \ $N_s$ | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|
| 2 | 589.4 | 485.3 | 393.0 | 327.8 | 340.3 |
| 3 | 529.4 | 442.8 | 357.9 | 312.1 | 321.4 |
| 4 | 418.6 | 384.9 | 327.1 | 318.4 | 313.8 |
| 5 | 390.6 | 364.7 | 312.1 | 316.5 | 320.2 |
| 6 | 367.0 | 348.5 | **306.5** | 318.0 | 319.7 |
| 7 | 346.8 | 334.3 | 312.7 | 321.0 | 322.2 |

for all elements is taken equal to 0.40 cm. The number of longitudinal ribs ($N_s$) ranges from 3 to 7 and the number of cross-ribs ($N_c$) ranges from 2 to 7. The results are given in Table 5.

From Table 5, it is clear that the optimum weight occurs at $N_s$ equals 5 and $N_c$ equals 6. Also, it is noticed that for a fixed number of $N_c$ or $N_s$ and increasing the other number, the weight decreases up to a value and then increases, therefore the optimum weight of an orthotropic plate depends on the numbers $N_c$ & $N_s$.

## 7. Conclusions

An optimal model is addressed for the design of laterally loaded orthotropic plates. Transformation method is adopted for minimization procedure in the framework of a finite element model.

Three different cases are worked out to illustrate the versatility of the proposed model. A sensitivity analysis for a set of the design variables is conducted. The effects of the number of longitudinal and cross ribs on the optimal weight of orthotropic plate are studied.

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## Appendix

### *Fletcher-Powell Algorithm*

a) For iterations $j = 1, 2, ...,$ maxpi choose $x^0$ as an initial feasible design vector and $R^1$ equals 1.0,

$$\text{Minimize } \phi(x^{j-1}, R^j) = f(x) + R^j\left[\sum_{i=1}^{m} \frac{-1}{g_i(x)}\right]$$

to obtain $x^j$ considering $R^j = R^{(j-1)}/10$.

b) For $k = 0, 1, 2, \ldots,$ itlim compute the direction $S^k$.

$$S^k = -H^k \nabla\phi^T(x^k, R)$$

$\nabla$ indicates the gradient of the function $\phi$ w.r.t. design variables.

c) By using the cubic interpolation, calculate the step size

$$\alpha = \alpha^k \text{ to minimize } \phi(x^k + \alpha S^k, R).$$

d) Compute the new design vector

$$x^{k+1} = x^k + \alpha^k \cdot S^k$$

e) Calculate the new matrix $H^{k+1}$ as follows:

- Compute $y^k = \nabla\phi^T(x^{k+1}, R) - \nabla\ \phi^T(x^k, R)$
- $\sigma^k = \alpha^k \cdot S^k$

$$\text{matrix } A^k = \frac{\sigma^k \cdot (\sigma^k)^T}{(\sigma^k)^T \cdot y^k}$$

$$\text{matrix } C^k = \frac{H^k \cdot y^k \cdot y^{kT} \cdot H^{kT}}{y^{kT} \cdot H^k \cdot y^k}$$

The new matrix $H^{k+1}$ will take the following formula:

$$H^{k+1} = H^k + A^k + C^k$$

Where $k$ indicates the indirection index.

f) Check the convergence according to the following Euclidean norm criterion of the gradient of augmented function $\phi$:

$$\sqrt{\sum_{i=1}^{N} \left( \frac{\partial \varphi(x, R)}{\partial x_i} \right)^2} \leq EPS$$

Where $N$ is the number of design variables and $EPS$ is the convergence limit. Due to the implicit nature of $\phi(x, R)$, one can use the first two terms of Taylor's expansion as a method to calculate the gradient of the augmented function $\phi$ with respect to design variable $x_i$ as follows:

$$\frac{\partial \varphi(x, R)}{\partial x_i} = \frac{\phi(x_i + \Delta x_i, R) - \phi(x_i, R)}{\Delta x_i}$$

If the convergence criterion is satisfied, the vector $x^{k+1}$ would be considered as feasible design vector for the next iteration $j + 1$ and then go to step ($g$), otherwise return to step (b).

g) As $R^j \to 0$ check the iteration number if it reaches the limit number then terminate the process and take $x^{k+1}$ as the solution, otherwise return to step (a).